

NEW C¹³ INDICATORS IN STELLAR SPECTRA*

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ABSTRACT

The (2,0) and (3,1) C¹³N¹⁴ bands of the red CN system have been rotationally analyzed in the laboratory. Wavelengths are presented for a total of 437 rotational lines of which 259 pertain to the (2,0) band and 178 lines to the (3,1) band. The wavelengths are compared with the theoretically predicted values of Utsumi. A systematic difference is observed in the residuals, which arises from uncertainties in the molecular constants. If a precision of 0.1 Å is to be achieved, the theoretical calculations must be extremely accurate. Perturbations are indicated for the first few rotational levels in the R_1 -branch.

The laboratory wavelengths have led to a positive identification of numerous individual rotational lines of the isotopic (2,0) and (3,1) bands in the spectra of five carbon stars, which range in type from C1₂ to C9₂. The normal (2,0) band has been scrutinized for regions between C¹²N¹⁴ line groups where only C¹³N¹⁴ lines fall. Twenty-five such regions have been found, which may provide C¹³ indicators, useful for surveys of the C¹³ content in late-type stars on medium-dispersion spectra (50–100 Å/mm).

Relative abundance ratios of C¹² to C¹³ have been derived on the basis of the observed line strengths of two cleanly isolated, single rotational lines, the isotopic $P_2(19)^1$ line at λ 8019.351 and the normal $R_1(40)^n$ line at λ 8038.125. The derived values for the five carbon stars range from well below the equilibrium ratio (3.4:1) of the CNO cycle to well above it (2:1 < a < 11:1). The vibrational temperatures show little correlation with spectral class and are below 2400° K.

The exceptionally high C¹³ content ($a = 2:1$) in such stars as Y CVn and WZ Cas cannot be explained as a product of the equilibrium ratio of the CNO cycle. We suggest that C¹³ in carbon stars is partially synthesized by spallation and processes of neutron irradiation either in the atmospheres of Wolf-Rayet stars or in the carbon stars themselves.

I. INTRODUCTION

In the spectra of carbon stars the (1,0) C₂ Swan sequence shows sometimes two or even three band series due to the C¹²C¹², C¹²C¹³, and C¹³C¹³ molecules. For many years, especially because of McKellar's epoch-making studies, these bands have been used as C¹³ indicators (McKellar 1948). Relative abundances of C¹² to C¹³ can be derived from comparative band strengths. McKellar also established the well-known division of carbon stars into C¹³-rich and C¹³-poor objects.

When the (1,0) C₂ Swan band at λ 4737 is used, such determinations, especially in the case of the late R stars and all N stars, are beset with saturation problems. The head of this band ($\Delta\lambda = 6$ Å) comprises some sixty rotational lines which rapidly bring about saturation of the head intensity for moderate abundances. The blue-violet continuum depression, probably due to polyatomic molecules like C₃ and SiC₂ presents another difficulty. This depression is especially disturbing in studying the N stars, where the spectra of the latest members are difficult to record with normal exposures shortward of H β at λ 4861.

Such factors led us to investigate C¹² to C¹³ abundance determinations from the (0,2) C₂ Swan band at λ 6191 and the (4,0), (2,0) red CN bands at λ 6192 and λ 7873, respectively (Wyller 1960). In these spectral regions the continuum placement is less questionable and the open-band structure of the red CN bands diminishes the crowding of lines and the resulting saturations. Furthermore, we are using an optically more accessible region of the red carbon stars.

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This study, however, only considered the integrated band structures. Ideally, in the case of the red CN bands, one should work with clearly separated, individual rotational lines. They should afford a truer measure of the relative abundances of C¹² to C¹³. The localization of such features has so far been hampered by the lack of observational data on wavelengths for the isotopic rotational lines.¹

Important work has recently been carried out by Phillips (1964) and Climenhaga (1964) on theoretically predicted, isotopic features in the (4,0) red CN band for the region around λ 6206. Utsumi (1963) has completed an extensive study of the isotopic (2,0) red CN band in the spectrum of Y CVn. His study, too, is based on theoretically predicted wavelengths for the isotopic features.

Jenkins, Root, and Mulliken (1932) showed evidence for rotational perturbations in some of the higher vibrational bands of the red CN system, which indicates the need for an analysis of the isotopic (2,0) band in the laboratory. High-precision wavelengths (perhaps to within 0.01 Å) are also needed for a search in high-dispersion stellar spectra; such precision cannot yet be provided by theory. The present paper gives the results of some laboratory and stellar studies of this band. The (2,0) red CN band is chosen over the (4,0) band as its structure is more open and shows greater freedom from atomic-line interferences. The band also lies in a spectral region which is closer to the radiative intensity maximum for these cool, late-type stars.

II. LABORATORY OBSERVATIONS

A device originally constructed by Smythe and Townes (1939) at the California Institute of Technology was intended to produce the isotopic C₂ Swan bands and the red CN bands. This equipment, however, could not be found, so other modes for the excitation of these bands had to be considered. Hollow cathode discharge tubes and Geissler tubes were suggested and rejected because they show a relatively low luminous efficiency, and their construction necessitates a careful production technique in order to avoid contamination when electrodes are inserted into the tubes.

The design of an electrodeless discharge tube with excitation by a microwave generator was ultimately adopted. This technique was inspired by Meggers and Westfall (1950), who strongly advocate the use of such tubes because of the simplicity of construction, longer tube life, uncontaminated spectra, and sharper lines as compared with open-arc spectra. Such a tube was therefore constructed by Fred Wild in the glass-blowing section of the physics department at the California Institute of Technology. Following the method of Jenkins and Wooldridge (1938), the first tube was filled with normal methane (i.e., with only the C¹² isotope), nitrogen, and argon at pressures of $\frac{1}{2}$, $\frac{1}{2}$, and 15 mm Hg, respectively. The tube was pre-excited with a leak detector of the Tesla coil type and then kept active with a Raytheon microwave generator, which nominally puts out 100 watts at a frequency of 2450 Mc/s. The actual operating power was considerably lower than this because of an inefficient antenna director. After a burning-in period of $\frac{1}{2}$ hour, the tube gave off a stable, brilliant light. Even during an exposure time of 1 hour, no cooling of the tube was needed. The rotational temperature as derived from the maxima of rotational-line intensities in the three branches of the (2,0) red CN band was about 500° K.

When the tube was tested in Dr. R. B. King's laboratory with the 21-foot spectrograph, it yielded a well-exposed spectrum of the (1,0) C₂ Swan band within 5 minutes at a dispersion of 1.1 Å/mm. This luminous efficiency is high compared with that of the Smythe and Townes device, which needed 2½ hours to record the same spectral feature at a dispersion of 3.7 Å/mm. These figures indicate a gain in luminous efficiency by a factor of 100, only part of which is a result of viewing the tube end-on when recording the spectrum. A similar gain is registered over the device used by Jenkins and Wooldridge

¹ In the following discussions, "normal" refers to the C¹²N¹⁴ molecule and "isotopic" to the C¹³N¹⁴ molecule.

(1938). It is, of course, essential to have a highly luminous tube for the recording of high-dispersion, infrared CN spectra in a region where the photographic plates are slower and where the bands may be weaker because of the microwave excitation technique.

With the luminosity problem solved, new tubes were constructed by Wild out of quartz glass. The tubes were 12 cm long with an inner bore of 5 mm and a bulbous protrusion for viewing end-on. They were filled with C^{13} -enriched methane ($C^{12}:C^{13} \approx 1:1$), nitrogen, and argon at pressures identical to those in the earlier tubes.

One of the advantages in using electrodeless discharge tubes is the portability of the equipment. The microwave generator plugs into any conventional, grounded outlet and can be carried by one person. Because of the infrared work at hand, the Mount Wilson coude equipment was used. Several of the gratings are blazed in the first order for the infrared region (λ 12000), the equipment is in good alignment and there are excellent facilities for the hypersensitization and development of the plates.

Accordingly, the discharge tube was aligned in the optical path of the comparison spectrum for the 100-inch coude spectrograph with the Fe-Ne hollow cathode tube temporarily removed. A field lens was inserted to enhance the brilliance of the light source in the infrared. A Wratten K-29 filter was used to avoid overlapping orders. The (2,0) and (3,1) $C^{12}N^{14}$ and $C^{13}N^{14}$ bands were then recorded on a IV-N emulsion (water sensitized for uniformity) at a dispersion of 1.4 Å/mm with grating 133 B in the second order. The exposure time was 1 hour.

The usefulness of the electrodeless discharge tubes in stellar-band spectroscopy cannot be overemphasized. If one uses the tube as a comparison spectrum instead of the regular Fe arc, one has a useful tool for a rapid survey on medium-dispersion spectra of the C^{13} content in a variety of stars that display the red CN bands. The upper comparison spectrum can be produced with light from the tube which has a negligible C^{13} content, and the lower comparison spectrum with the isotope-enriched tube. The advantage of this arrangement is that both the comparison and the stellar spectra are produced with identical dispersions and thus provide identical blending of the rotational line features. This was not the aim of the present investigation but is suggested for future applications.

Through the kindness of Professor J. G. Phillips of the Berkeley Astronomical Department of the University of California, the plates were measured on a semiautomatic plate-measuring machine. The measurements were recorded directly on punched cards together with data on blends and relative line intensities. All the data were then processed in a computer to give both the wavelengths and the wavenumbers using Edlén's formula for the vacuum correction.

Clean, unblended rotational $C^{12}N^{14}$ lines which are interspersed with the isotopic spectrum were used as reference lines. The wavelengths of these normal lines are measured, according to Phillips, with a precision of at least 0.01 Å in the *Atlas of the Red CN System* (Phillips and Davis 1963), which is sufficiently accurate for stellar wavelength determinations. Fifteen normal CN lines were used for $\lambda\lambda$ 7900–8065 and thirteen lines for the region $\lambda\lambda$ 8060–8225. The lines were all selected for their sharp profiles and were evenly distributed throughout the regions. The determinations were anchored on the lines: $R_1(5)$ at λ 7896.395 and $Q_1(34)$ at λ 8062.558 in the first wavelength interval and $P_2(28)$, $R_2(43)$ at λ 8060.246, and (3,1) $P_2(24)$ at λ 8222.224 in the second interval. A least-squares solution for measuring comparison lines was fitted to the *Atlas* wavelengths. This yielded the plate constants from which the wavelengths of the isotopic CN lines were calculated by the computer.

When the derived wavelengths for 134 rotational lines of the (2,0) normal band were compared with those listed in Phillips' *Atlas*, the mean absolute deviation was 0.010 Å. Phillips (*loc. cit.*, p. 13) found that the average deviation between measurements on two plates for 256 lines was 0.013 Å. Thus the wavelengths for the $C^{13}N^{14}$ lines given in Table 1 should be accurate to 0.02 wavenumber units or 0.01 Å.

The following method was found convenient in analyzing the rotational structure of

TABLE 1A--Continued

N	$R_2(N)$		$Q_2(N)$		$P_2(N)$	
	λ_{air}	σ_{vac}	λ_{air}	σ_{vac}	λ_{air}	σ_{vac}
0	7918.900	12624.54				
1	18.046	25.91	7921.131	12620.99		
2	17.392	26.95	22.580	18.68	7925.656	12613.78
3	16.965	27.63	24.221	16.07	29.392	07.84
4	16.746	27.98	26.084	13.10	33.305	01.62
5	16.746	27.98	28.152	09.81	37.460	12595.02
6	16.965	27.63	30.426	06.20	41.790	88.16
7	17.392	26.95	32.922	02.23	46.383	80.88
8	18.046	25.91	35.627	12597.93	51.158	73.33
9	18.900	24.54	38.585	93.24	56.185	65.38
10	20.019	22.76	41.710	88.28	61.424	57.11
11	21.318	20.69	45.098	82.92	66.863	48.54
12	22.906	18.16	48.693	77.23	72.577	39.55
13	24.656	15.38	52.555	71.12	78.511	30.22
14	26.677	12.16	56.630	64.68	84.695	20.52
15	28.944	08.55	60.963	57.84	91.111	10.46
16	31.457	04.56	65.550	50.61	97.801	00.00
17	34.198	00.20	70.336	43.07	8004.781	12489.10
18	37.173	12595.48	75.401	35.11	11.937	77.95
19	40.418	90.33	80.743	26.72	19.351	66.41
20	43.895	84.82	86.283	18.03	27.048	54.46
21	47.654	78.87	92.114	08.89	34.986	42.15
22	51.630	72.58	98.205	12499.37	43.232	29.40
23	55.921	65.80	8004.554	89.45	51.731	16.28
24	60.441	58.66	11.163	79.15	60.507	02.76
25	65.223	51.12	18.058	68.42	69.549	12388.86
26	70.275	43.17	25.187	57.34	78.885	74.54
27	75.567	34.85	32.614	45.83	88.485	59.86
28	81.147	26.08	40.299	33.93	98.382	44.75
29	86.930	17.01	48.271	21.61	8108.549	29.27
30	93.100	07.35	56.504	08.92	19.043	13.34
31	99.421	12497.47	65.027	12395.81	29.766	12297.10
32	8006.060	87.11	73.815	82.31	40.817	80.40
33	13.061	76.20	82.929	68.35	52.181	63.28
34	20.224	65.05	92.300	54.03	63.786	45.85
35	27.767	53.34	8101.961	39.30	75.719	27.98
36	35.523	41.32	11.878	24.21		
37	43.562	28.89	22.150	08.63		
38	51.894	16.02	32.689	12292.68		
39	60.507	02.76	43.522	76.32		
40	69.451	12389.01	54.701	59.50		
41	78.615	74.96	66.075	42.42		
42	88.095	60.45	77.822	24.83		
43	97.885	45.51	89.853	06.88		
44	8107.962	30.17	8202.232	12188.45		
45	18.464	14.22	14.904	69.65		
46	29.014	12298.23				
47	39.967	81.69				
48	51.206	64.75				
49	62.864	47.24				
50	74.744	29.44				

$P_{12}(N)$							
N			N			N	
4	7969.017	12545.15	9	7995.173	12504.11	14	8029.258
5	73.593	37.95	10	8001.368	12494.43	15	37.055
6	78.511	30.22	11	07.883	84.26	16	45.127
7	83.737	22.02	12	14.692	73.66		
8	89.298	13.30	13	21.846	62.53		

TABLE 1B
WAVELENGTHS AND VACUUM WAVE NUMBERS OF THE RED (3,1) C¹³N¹⁴ BAND

R ₁ (N)			Q ₁ (N)			P ₁ (N)		
N	λ_{air}	σ_{vac}	λ_{air}	σ_{vac}		λ_{air}	σ_{vac}	
0	8144.284	12275.18						
1	41.543	79.31	8146.648	12271.61		8149.550	12267.25	
2	39.171	82.89	46.318	72.11		51.397	64.46	
3	37.107	86.00	46.318	72.11		53.481	61.33	
4	35.378	88.61	46.648	71.61		55.854	57.76	
5	33.982	90.72	47.288	70.65		58.567	53.69	
6	32.908	92.35	48.260	69.19		61.609	49.12	
7	32.119	93.54	49.545	67.25		65.003	44.03	
8	31.738	94.11	51.186	64.78		68.686	38.51	
9	31.614	94.30	53.129	61.86		72.719	32.47	
10	31.788	94.04	55.404	58.44		77.073	25.95	
11	32.316	93.24	57.929	54.64		81.755	18.96	
12	33.137	92.00	60.866	50.23		86.759	11.49	
13	34.262	90.30	64.078	45.41		92.093	03.54	
14	35.712	88.11	67.580	40.16		97.719	12195.16	
15	37.426	85.52	71.406	34.43		8203.695	86.28	
16	39.452	82.46	75.532	28.26		09.970	76.97	
17	41.803	78.92	79.971	21.62		16.555	67.21	
18	44.413	74.98	84.700	14.56				
19	47.323	70.60	89.747	07.03				
20	50.557	65.73	95.094	12199.07				
21	54.086	60.42	8200.739	90.67				
22	57.979	54.57	06.689	81.83				
23	62.042	48.47	12.940	72.56				
24	66.416	41.91	19.491	62.86				
25	71.118	34.86						
26	76.118	27.38						
27	81.403	19.48						
28	87.002	11.13						
29	92.890	02.35						
30	99.074	12193.15						
31	8205.563	83.51						
32	12.345	73.44						
33	19.491	62.86						

S _{R₂₁} (N)								
N			N			N		
0	8107.400	12331.02	5	8094.611	12350.50	10	8087.517	12361.34
1	04.370	35.63	6	92.725	53.38	11	86.816	62.41
2	01.605	39.84	7	91.068	55.91	12	86.351	63.12
3	8099.055	43.73	8	89.670	58.05	13	86.173	63.39
4	96.726	47.28	9	88.485	59.86	14	86.173	63.39
						15	86.465	62.94

TABLE 1B---Continued

N	$R_2(N)$		$Q_2(N)$		$P_2(N)$	
	λ_{air}	σ_{vac}	λ_{air}	σ_{vac}	λ_{air}	σ_{vac}
0	8110.615	12326.13				
1	09.861	27.28	8112.978	12322.54		
2	09.200	28.28	14.457	20.30	8117.691	12315.39
3	08.698	29.05	16.195	17.66	21.496	09.62
4	08.502	29.34	18.142	14.70	25.644	03.34
5	08.502	29.34	20.332	11.38	29.977	12296.78
6	08.698	29.05	22.691	07.81	34.262	90.30
7	09.200	28.28	25.301	03.85	39.280	82.72
8	09.861	27.28	28.140	12299.56	44.284	75.18
9	10.804	25.84	31.213	94.91	49.545	67.25
10	11.953	24.10	34.518	89.91	54.981	59.07
11	13.303	22.05	38.055	84.57	60.694	50.49
12	14.951	19.55	41.828	78.88	66.643	41.57
13	16.925	16.55	45.885	72.76	72.852	32.27
14	19.080	13.28	50.164	66.32	79.319	22.60
15	21.443	09.70	54.701	59.50	86.008	12.61
16	24.082	05.70	59.492	52.30	93.008	02.18
17	26.971	01.33	64.541	44.72	8200.260	12191.38
18	30.153	12296.51	69.847	36.77	07.765	80.24
19	33.563	91.36	75.432	28.41	15.548	68.70
20	37.265	85.76	81.274	19.68		
21	41.220	79.80	87.383	10.56		
22	45.440	73.43	93.769	01.04		
23	49.949	66.64	8200.423	12191.14		
24	54.701	59.50	07.368	80.83		
25	59.730	51.94	14.586	70.12		
26	65.003	44.03	22.087	59.02		
27	70.645	35.57				
28	76.503	26.81				
29	82.652	17.62				
30	89.086	08.02				
31	95.803	12198.01				
32	8202.810	87.59				
33	10.102	76.77				
34	17.693	65.52				
$P_{12}(N)$						
N			N			
4	8163.059	12246.94	9	8196.733	12196.63	
5	67.796	39.84	10	8203.610	86.41	
6	78.366	24.02	11	10.695	75.99	
7	84.157	15.37	12	18.091	64.93	
8	90.279	06.24				

the isotopic band. Once the wavelengths of the isotopic heads were established, the spacing of successive rotational lines was calculated from the relation $\Delta\lambda_i = \rho^2\Delta\lambda_n$, $\rho = \sqrt{(\mu/\mu_i)}$. The reduced molecular mass is μ . The wavelength differences, $\Delta\lambda_n$, were taken from Phillips' *Atlas* (1963). The method proved to be a surprisingly accurate means of rapidly locating various members of the branches associated with a given rotational quantum number.

The resulting wavenumbers and wavelengths are listed in Table 1. Some 259 rotational lines pertaining to the (2,0) C¹³N¹⁴ band and 178 lines pertaining to the (3,1) isotopic band have been measured, a total of 437 rotational line positions. Only 29 wavelengths are missing which would make the analysis complete up to the (4,2) normal band at λ 8270.84. Centering of the (2,0) and (3,1) bands on the IV-N plate leads to an arbitrary cutoff longward of λ 8223.

In comparing observed wavelengths with the isotopic wavelengths predicted by the rule $\Delta\lambda_i = \rho^2\Delta\lambda_n$, one observes a clear-cut deviation in the R_1 -head for N -values of 1, 2, 3, 4, 5, 6. This may indicate that rotational perturbations affect levels forming the head. The isotopic R_1 -head of the (2,0) band is observed at λ 7938.69, while Utsumi predicted its position at λ 7937.67. Shajn and Gaze (1950) list the "observed" head from stellar spectra at λ 7937.2, which is 0.1 Å from their predicted value. The same large discrepancies are found in the Q_1 -head position which is observed at λ 7952.93 in the laboratory. Utsumi predicts the head at λ 7951.78, while Shajn and Gaze list an "observed" head at λ 7956.2 (see Table 2).

In this connection one should note the importance of accurate calculations of the rotational isotopic wavelength shifts. The differential shifts are considerable, as exemplified by the line trio $R_2(36)$, $R_1(35)$, and $P_{12}(15)$ of the (2,0) band. The C¹²N¹⁴ lines coalesce to one feature located at λ 7995.640. The C¹³N¹⁴ lines, owing to differential rotational isotope shifts, are separated by over 0.7 Å at observed wavelengths of $R_2(36)$: λ 8035.523; $R_1(35)$: λ 8036.232; and $P_{12}(15)$: λ 8037.055, respectively.

It may be tempting to advocate the general use of theoretically predicted line positions because of the ease in computing by modern electronic computers. In order to assess the accuracy of such methods, it is instructive to compare the presently observed wavelengths with those predicted theoretically by Utsumi for the (2,0) C¹³N¹⁴ band. In Figures 1 and 2 we see the residuals, $\lambda(W - U)$, plotted as ordinates versus the rotational quantum number N .

One clearly distinguishes a systematic trend in the residuals for the various branches. In the R_2 -, Q_2 -, and P_2 -branches the agreement is fairly satisfactory, in some cases remarkably good. A systematic trend begins to be discernible for the higher N -values (above 30) in the R_2 -branch, and here the residuals remain approximately at 0.40 Å. In the R_1 -, Q_1 -, and P_1 -branches the agreement is considerably less satisfactory even at low rotational quantum numbers. In fact the residuals are largest for the lowest N -values (>1 Å) and then diminish toward the higher N -values. This points to a systematic error, which may be related to Utsumi's correction of 0.9 cm⁻¹ to the value for the band origin given by Herzberg and Phillips (1948). The removal of this correction in Utsumi's predicted values leads to improved agreement in the R_1 -, Q_1 -, P_1 -branches, but it upsets the relatively good agreement in the R_2 -, Q_2 -, P_2 -branches. (Dr. Utsumi has later informed me that his calculations were based only on the first approximation for the line shifts as quoted in his paper.)

It is worth noting that Phillips (private communication) has found it necessary in the theoretical predictions of the C¹³N¹⁴ line positions of the (4,0) red CN band to adjust various molecular constants. In particular, the value of the coupling constant A is modified in order to minimize the residuals between the predicted values and those observed in stellar spectra. This adjustment process, he feels, can lead to an average precision within 0.1 Å of the predictions. It does appear, though, that the technique adopted in this paper for the production of isotopic bands, whether of C₂, CN, TiO, ZrO, or the

TABLE 2
COMPARISON WITH PREVIOUS WORK

Branch-head	$(2,0)C^{13}N^{14}$			$(2,0)C^{12}N^{14}$
	Shajn and Gaze Stellar Observ.	Utsumi Theory	Wyller Laboratory	Phillips Laboratory
R_2	7915.7	7917.04	7916.75	7872.67
R_1	7937.2	7937.67	7938.69	7894.07
Q_1	7956.2	7951.78	7952.93	7908.67
Branch-head	$(3,1)C^{13}N^{14}$			$(3,1)C^{12}N^{14}$
	Shajn and Gaze Stellar Observ.	S. and G. Theory	Wyller Laboratory	Phillips Laboratory
R_2	8108.2	8107.9	8108.50	8067.07
R_1	8129.3	8130.5	8131.61	8089.70
Q_1	8148.5	8145.5	8146.32	8104.70

TABLE 3
PLATE MATERIAL

Star	m_v	Sp. Class		Coudé Plate No.	Disp. A/mm	Date	Emuls.	Expos. Min.
		Shane	Keenan Morgan					
HD 156074	7.0	R1	C1 ₂	Cel7094	20.4	May 2, '64*	IV-N	410
Y CVn	5.0	N3	C5 ₄	Cel2715	5.9	June 15, '59*	IV-N	240
19 Psc	5.3	N0	C6 ₂	Cel2720	5.9	June 16, '59*	IV-N	240
DS Peg	6.4	N1	C6 ₃	Cel7282	20.4	Aug. 31, '64†	I-N	302
WZ Cas	8	N1 _p	C9 ₁	Pc6143	13.4	Sept. 22, '61‡	I-N	170

Observers: *A.A. Wyller; ‡A. J. Deutsch; †I. J. Danziger

metallic hydrides, is a preferable and rapid way to obtain incontrovertible data on the wavelengths of isotopic rotational lines.

III. STELLAR OBSERVATIONS

The I-N emulsion has so far been almost exclusively used to record stellar spectra in the region $\lambda\lambda$ 7600–8800. It is not generally recognized that the IV-N emulsion, when properly sensitized, is about as fast as the IN emulsion. In addition, one has the advantage of finer grain and increased photographic resolving power. As early as 1943 Sanford (1950) used this emulsion to record the spectra of carbon stars. The present author employed IV-N plates in 1959 to record some high-dispersion spectra (5.9 Å/mm) of two carbon stars of $m_v = 5$ with exposure times no longer than 4 hours under condi-

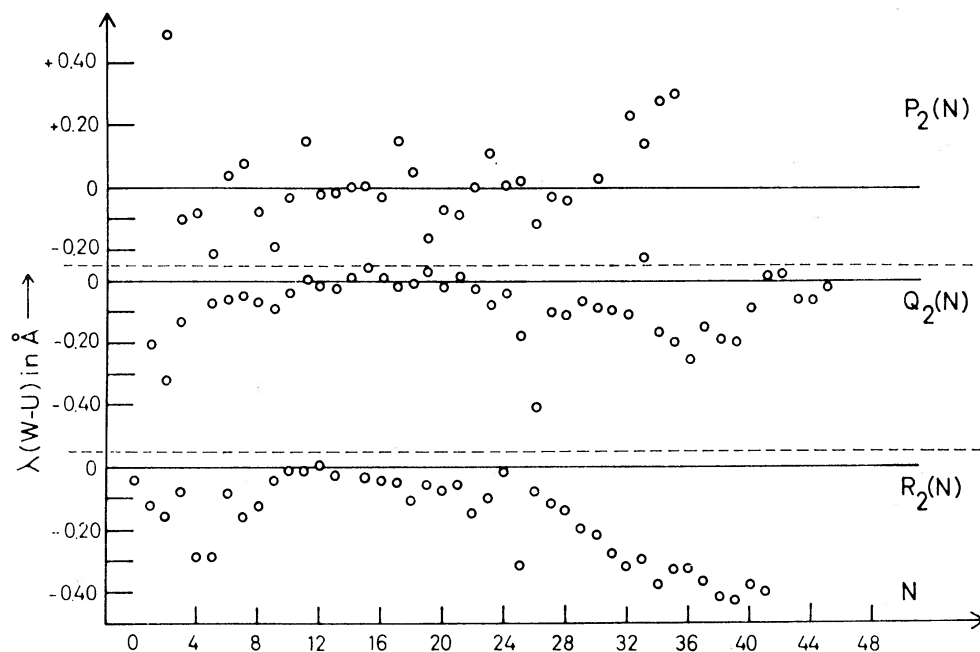


FIG. 1.—Wavelength residuals, $\lambda(W, \text{lab.}) - \lambda(U, \text{theory})$, for the R_2 -, Q_2 -, P_2 -branches of the red (2,0) C¹³N¹⁴ band as a function of the rotational quantum number, N .

tions of average seeing. Recently Spinrad, Münch, and Kaplan (1963) used the IV-N emulsion to detect water vapor in the spectrum of Mars. The fine grain was crucial to the success of their observations. This emulsion is highly recommended for future studies of late-type spectra in the near-infrared region.

Full details of the general sensitization procedure as recommended by Mr. W. Miller, photographer at the Mount Wilson and Palomar Observatories, have been given elsewhere (Wyller 1964). We shall only mention that the IV-N emulsion will take a higher degree of ammoniation than the I-N emulsion (8–10 per cent of 28 per cent NH₃ for 8–10 minutes as against the usual 4 minutes in a 4 per cent solution).

The plate material used in the present investigation is summarized in Table 3.

a) High-Dispersion Features

The high-dispersion spectra of 19 Psc and Y CVn enabled us to make a very detailed search for individual C¹³N¹⁴ rotational lines. Several identifications have been established and are the first positive identifications of individual C¹³N¹⁴ rotational lines in stellar spectra. Until now, all identifications of C¹³ features have used vibrational band heads

and integrated band structures, but in Figures 3 and 4 one can see individual rotational lines to which a specific branch and rotational quantum number can be given.

From the integrated, isotopic, C_2 band intensities in the red spectral region, one surmises that 19 Psc is relatively C^{13} -poor while Y CVn is relatively rich in C^{13} (Wyller 1960). This is shown by line intensities of the $C^{13}N^{14}$ features compared with those of adjacent $C^{12}N^{14}$. The line $P_1(19)^i$ is considerably stronger relative to the adjacent $C^{12}N^{14}$ lines in the spectrum of Y CVn than in 19 Psc; the line $P_2(19)^i$ is enhanced in Y CVn as compared with the corresponding feature in 19 Psc. The group of lines $R_2(36)^i$, $Q_1(26)^i$, and $R_1(35)^i$ at λ 8035.523, λ 8036.031, and λ 8036.232, depresses the intensity profile in Y CVn while remaining weak in 19 Psc.

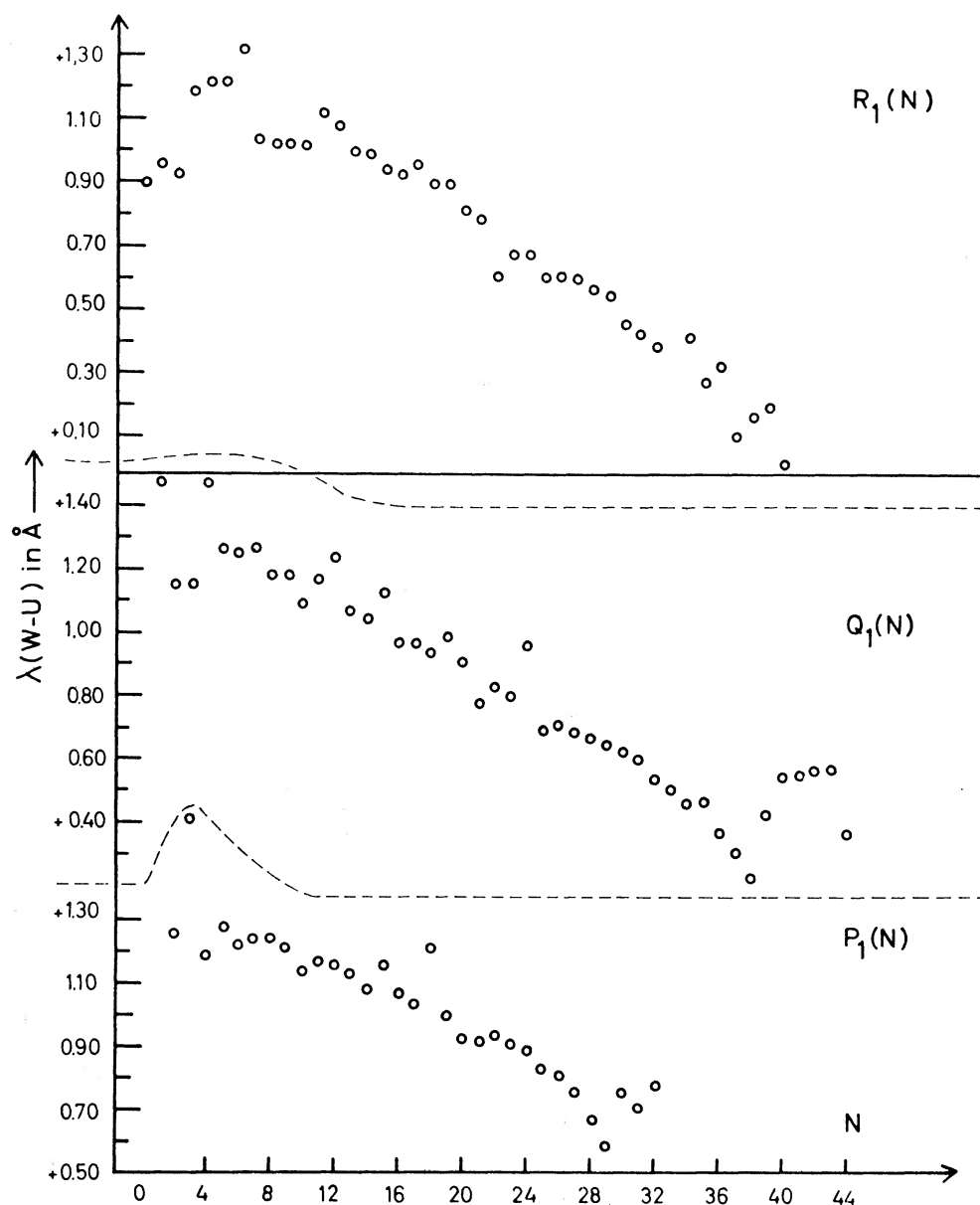


FIG. 2.—Same as in Fig. 1 for the R_1 -, Q_1 -, and P_1 -branches of the $(2,0)$ $C^{13}N^{14}$ band

b) *Medium-Dispersion Features*

The utilization of the present list of wavelengths of isotopic rotational lines for extensive stellar C^{13} surveys depends largely on the possibility of finding isolated $C^{13}N^{14}$ lines, which are discernible even on medium-dispersion plates. A list of C^{13} indicators would clearly be useful for surveys attempting to relate C^{13} content to particular groups of carbon stars, such as the high-velocity C-stars and Bouigue's (1954) two subgroups. Figures 5 and 6 show some such possible C^{13} indicators. The dispersion used for the spectra of WZ Cas and DS Peg are, respectively, 13.4 and 20.4 Å/mm.

The star WZ Cas is found to be extremely rich in C^{13} while DS Peg is a star of intermediate C^{13} content which very likely displays even less C^{13} than 19 Psc. The figures show that at a dispersion of 20 Å/mm, isotopic lines like $Q_1(24)^i$ at λ 8022.668 and $P_1(19)^i$ at λ 8033.707 are clearly distinguishable, revealing differences in C^{13} abundances from star to star.

It was thus thought worthwhile to scrutinize the (2,0) isotopic band from the satellite head region λ 7900 to the (3,1) normal head at λ 8067.08 for regions where only $C^{13}N^{14}$

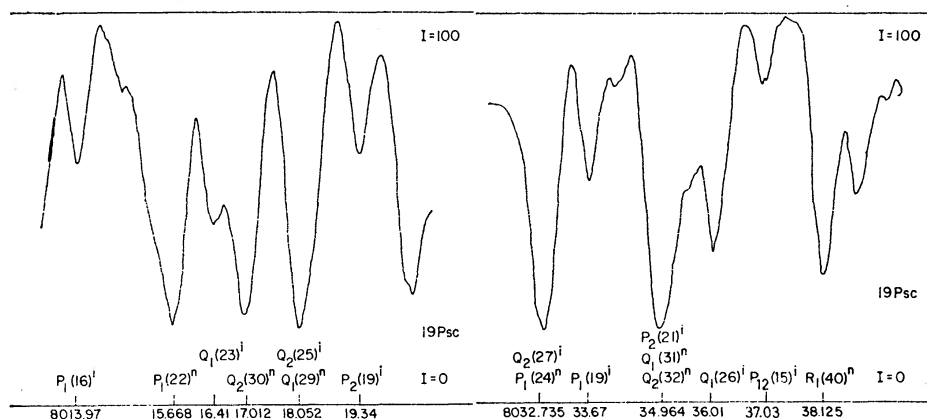


FIG. 3.—Selected isotopic ($C^{13}N^{14}$) and normal ($C^{12}N^{14}$) rotational lines in the region $\lambda\lambda$ 8014–40 of the spectrum of 19 Piscium (5.9 Å/mm).

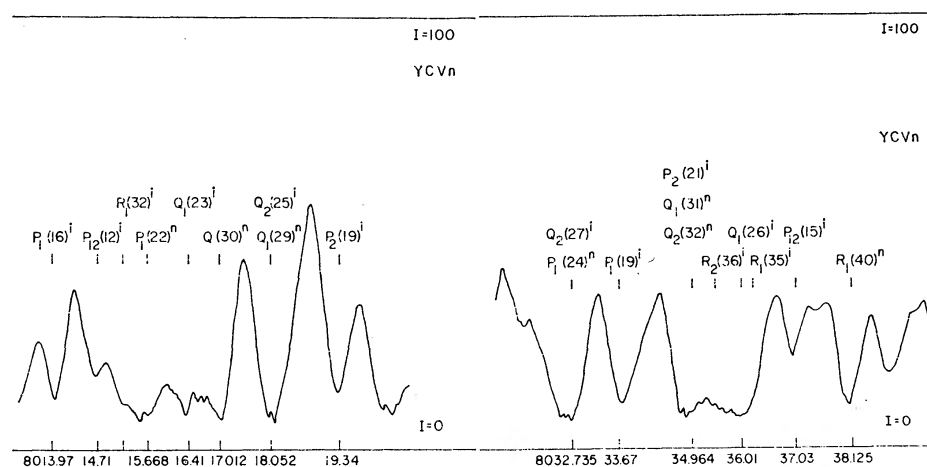


FIG. 4.—Same as in Fig. 3. The star is Y Canes Venaticorum and the dispersion of the spectrum is 5.9 Å/mm.

lines fall singly or in groups well separated on both sides from adjacent $C^{12}N^{14}$ lines. Twenty-five such regions have been found and are listed in Table 4 as " C^{13} Indicators."

The first column contains the identification of a $C^{12}N^{14}$ feature shortward of the C^{13} indicator; the second column gives the separation in angstroms between that feature and the nearest of the $C^{13}N^{14}$ features; the third column lists the identifications of the $C^{13}N^{14}$ features which constitute the C^{13} indicator, and the fourth column, their laboratory wavelengths. The fifth column gives the identified $C^{12}N^{14}$ features longward of the indicator; the sixth column gives the wavelength separation between the nearest $C^{13}N^{14}$ feature in the indicator and these $C^{12}N^{14}$ features; the seventh column lists the wavelength separation of the two $C^{12}N^{14}$ lines between which the C^{13} indicator lies. This latter quantity determines, for the particular dispersion at hand, what C^{13} indicators to choose. In most cases, the separation of the $C^{12}N^{14}$ features is about 2 Å, but in some cases it extends up to 4 Å. Thus, dispersions of 50–100 Å/mm should suffice to detect $C^{13}N^{14}$ in stellar spectra.

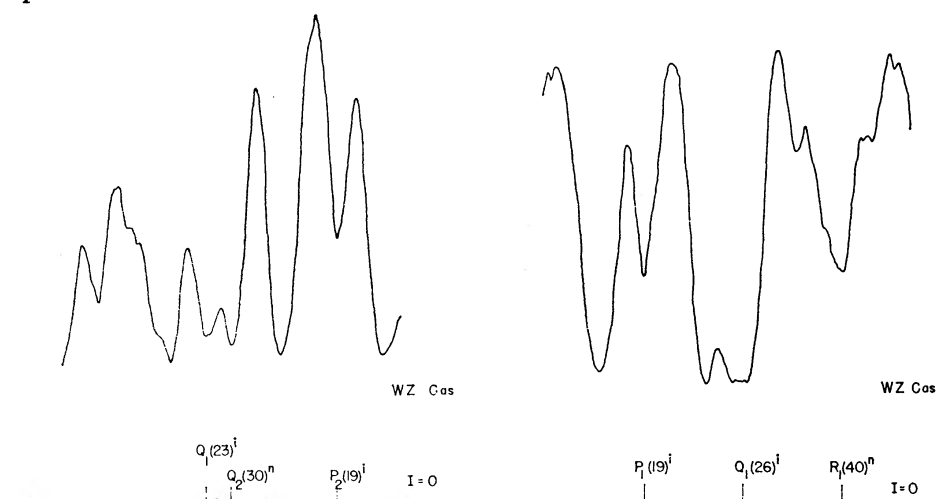


FIG. 5.—Same as in Fig. 3. The star is WZ Cassiopeia and the dispersion of the spectrum is 13.4 Å/mm.

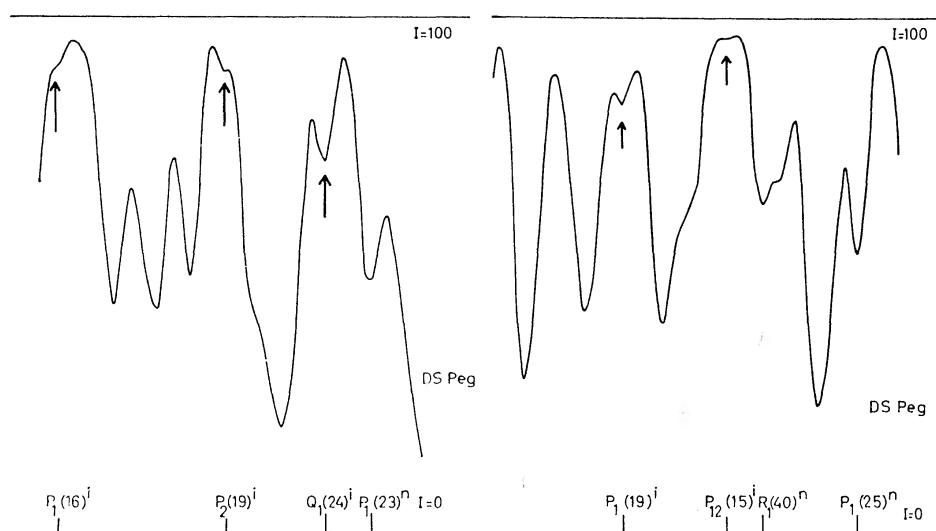


FIG. 6.—Same as in Fig. 3. The star is DS Pegasi and the dispersion of the spectrum is 20.4 Å/mm

TABLE 4
C¹³ INDICATORS

No.	C ¹² N ¹⁴ Shortward		C ¹³ N ¹⁴ Indicator		C ¹² N ¹⁴ Longward		$\Delta C^{12}N^{14}(A)$
	Branch	$\Delta\lambda(A)$	Branch	λ	Branch	$\Delta\lambda(A)$	
1	R ₁ (21)	0.466	R ₂ (4,5) R ₂ (3,6) R ₂ (2,7)	7916.746 16.965 17.392	Q ₁ (10)R ₂ (24)	0.305	1.417
2	P ₁ (5)P ₁₂ (3)	0.543	R ₂ (11)	21.318	R ₂ (25)	1.415	1.958
3	R ₁ (24)	0.480	R ₂ (15) P ₂ (3)	28.944 29.392	Q ₁ (14)	0.377	1.305
4	P ₁ (8)	0.618	R ₂ (16)	31.457	R ₁ (25)Q ₂ (18)	1.690	2.308
5	P ₁₂ (6)	0.613	Q ₂ (8)	35.627	P ₂ (13)	0.751	1.364
6	R ₂ (28)P ₁ (10)	0.128	R ₁ (11) R ₁ (6) R ₁ (12) R ₂ (19)	39.307 40.001 40.078 40.418	P ₁₂ (7)	0.045	1.284
7	P ₁₂ (7)	0.595	R ₁ (5) R ₁ (13) Q ₂ (10) P ₂ (6)	41.058 41.132 41.710 41.790	Q ₁ (17)	0.242	1.569
8	Q ₁ (18)	0.924	R ₂ (21) R ₁ (1,17) Q ₂ (12)	47.654 48.226 48.693	P ₁ (12)R ₁ (28)	0.108	2.071
9	P ₁₂ (9)	0.295	Q ₂ (13) Q ₁ (2,3) Q ₁ (1,4) R ₁ (19) Q ₁ (5)	52.555 52.933 53.248 53.479 53.843	P ₁ (13)	0.245	1.828
10	R ₁ (30)	0.270	Q ₂ (15) P ₂ (10) Q ₂ (10) P ₁ (4)	60.963 61.424 61.554 62.105	Q ₁ (21)	0.501	1.913
11	P ₂ (17)	0.052	R ₁ (22) Q ₁ (11) P ₁ (5)	63.460 64.022 64.700	R ₂ (32)	0.413	1.705
12	Q ₁ (23)	0.737	Q ₂ (18) R ₂ (27) R ₂ (25) Q ₁ (15)	75.401 75.567 76.024 76.821	Q ₂ (25)	0.397	2.554

TABLE 4--Continued

No.	$C^{12}N^{14}$ Shortward		$C^{13}N^{14}$ Indicators		$C^{12}N^{14}$ Longward		$\Delta C^{12}N^{14}(A)$
	Branch	$\Delta\lambda(A)$	Branch	λ	Branch	$\Delta\lambda(A)$	
13	$Q_1(24)$	1.299	$P_1(10)$ $P_{12}(7)$	7982.434 83.737	$Q_2(26)$	0.878	3.480
14	$Q_1(25)$	1.353	$P_{12}(8)$ $Q_1(18)$ $R_1(28)P_2(15)$ $P_1(12)$ $Q_2(21)$	89.298 89.503 91.111 91.701 92.114	$Q_2(27)P_1(19)$	0.183	4.352
15	$Q_2(27)P_1(19)$	0.803	$R_2(30)$ $Q_1(19)$	93.100 94.308	$P_2(21)$	0.386	2.397
16	$R_1(35)R_2(36)$ $P_{12}(15)$	1.070	$R_1(29)$ $P_1(13)$ $P_2(16)$ $Q_2(22)$	96.710 96.803 97.801 98.205	$P_1(20)$	1.641	4.206
17	$P_{12}(16)$	0.518	$Q_2(23)$ $P_2(17)$ $Q_1(21)$ $R_2(32)$	8004.554 04.781 04.728 06.058	$P_1(21)$	1.524	3.546
18	$R_2(38)$	1.364	$P_1(16)$ $P_{12}(12)$ $R_1(32)$	13.910 14.692 15.166	$P_1(22)$	0.502	3.122
19	$Q_1(29)$	1.299	$P_2(19)$	19.351	$R_1(38)$	0.871	2.170
20	$R_2(39)$	0.369	$P_{12}(13)$ $R_1(33)$ $Q_1(24)$	21.846 21.900 22.668	$P_1(23)$	1.367	2.558
21	$P_1(24)$	0.972	$P_1(19)$	33.707	$Q_1(31)Q_2(32)$	1.257	2.229
22	$Q_1(31)Q_2(32)$	0.559	$R_2(36)$ $Q_1(26)$ $R_1(35)$ $P_{12}(15)$	35.523 36.031 36.232 37.055	$R_1(40)$	1.070	3.161
23	$Q_2(34)$	2.099	$P_1(22)$ $Q_2(30)$	56.188 56.504	$R_1(42)$	0.768	3.183
24	$R_1(42)$	0.969	$Q_1(29)$	58.241	$P_2(28)R_2(43)$	2.005	2.974
25	$Q_2(35)$	0.169	$P_1(23)$ $Q_2(31)$ $Q_1(30)$	64.279 65.027 66.236	$R_2(4,5)$ (3,1)-head	0.838	2.964

The author earlier advocated the use of "continuum windows" for the corroboration of low C¹³ content in some carbon stars (Wyller 1960). At that time, the selection of windows was based on approximate calculations of rotational isotope shifts, which, however, were independent of the uncertainties in the molecular constants. Three of the continuum windows then pointed to, at λ 7988.6, λ 8013.93, and λ 8036.51, coincide with regions that presently seem feasible as C¹³ indicators.

c) The " λ 8192 Window"

Another continuum window at $\lambda\lambda$ 8189–8192 was predicted in 1960 by the author. An isotopic line feature of the (3,1) band was calculated to fall at λ 8189.35. The (3,1) band had not yet been rotationally analyzed, so no rotational assignment could be given to the feature. We now know it to be the $Q_1(19)$ line of the isotopic (3,1) band; the C¹²N¹⁴ component lies at λ 8150.006, while the C¹³N¹⁴ counterpart is observed at λ 8189.747. This feature is easily recognizable in the spectrum of 19 Psc (see Fig. 7). It is also clearly

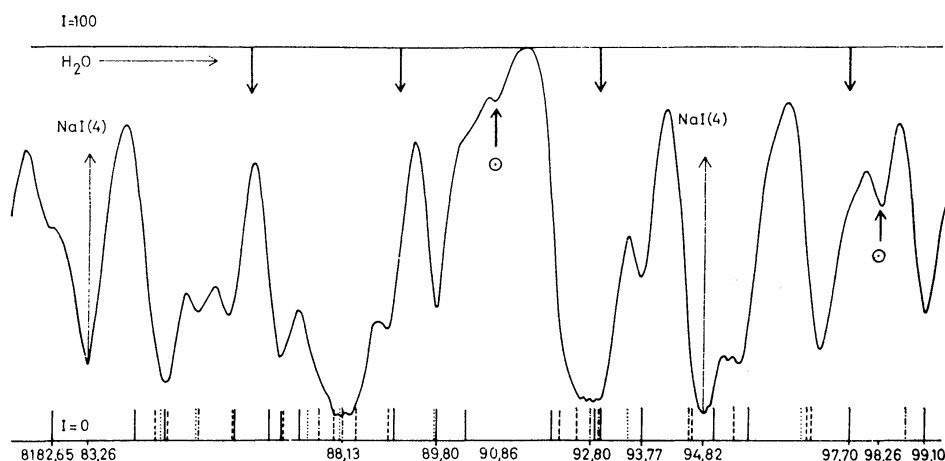


FIG. 7.—The region around the " λ 8192 window" in the spectrum of 19 Piscium (5.9 Å/mm). The wavelength positions of the rotational lines are marked by full lines for the (3,1) C¹³N¹⁴ band, dotted lines for (2,0) C¹³N¹⁴, dashed lines for (3,1) C¹²N¹⁴, and dashed-dotted lines for (2,0) C¹²N¹⁴.

visible in the spectrum of DS Peg at the lower dispersion of 20.4 Å/mm and coincides with a C¹³N¹⁴ line in the (2,0) band, i.e., the line $Q_2(43)$ at λ 8189.853. Together they form an excellent C¹³ indicator for surveys under moderate dispersion.

Fujita and Utsumi (1963) have made a careful study of the region $\lambda\lambda$ 8184–8203 with the aid of theoretically predicted line positions for the isotopic lines of the (2,0) CN band. They concluded that three (2,0) C¹³N¹⁴ features were present at λ 8189.9, λ 8193.6, and λ 8202.3, respectively. As we shall later see, the two first features do not, in the main, represent isotopic (2,0) features but isotopic (3,1) lines.

The contention of Utsumi (1963) that the (3,1) C¹³N¹⁴ band "is so weak that it has little influence on the spectral features" is puzzling. The isotopic (3,1) band should not be weaker than the normal (3,1) band if the isotopic (2,0) band is strongly present. This is seen in the region around the λ 8192 window.

Figure 7 shows the intensity tracing for the spectrum of 19 Psc in $\lambda\lambda$ 8182–8200. The dispersion is slightly higher than in Fujita and Utsumi's tracing for the same star (5.9 Å/mm versus 8 Å/mm), and the graininess of the emulsion is lower (IV-N versus I-N), so the over-all resolution appears slightly better. In Figure 7, wavelength positions are indicated for seven (2,0) normal lines, seven (2,0) isotopic lines, thirteen (3,1) normal lines, and eighteen (3,1) isotopic lines. One can observe that the great absorption troughs at λ 8188, λ 8193, and λ 8195 are mostly cyanogen absorption. The water-vapor lines

were drawn on the basis of observed relative shifts of the O₂ rotational lines and the stellar K I doublet in the atmospheric *A*-band. The shift of the stellar lines relative to the O₂ lines appears to be less than 0.1 Å. The weakness of the water-vapor line at λ 8197.704 indicates that such lines are not major absorbers in this region as compared with the cyanogen lines.

The well-defined profile of Na I(4) appears at λ 8183.26. The other component of the doublet is at λ 8194.82, but the true profile is masked by cyanogen lines.

The strength of the (3,1) isotopic band is clearly seen in the *R*₂(29) line at λ 8182.652. The line profile is not disturbed by any cyanogen lines within 1 Å, although it adjoins the far wing of a Na I(4) line. The previously mentioned clean profile at λ 8189.80 is a genuine C¹³N¹⁴ feature which represents a blend of the isotopic (3,1) *Q*₁(19) line and the (2,0) *Q*₂(43) line. At a temperature of 2350° K for 19 Psc, the (3,1) line should be slightly stronger than the (2,0) line with an intensity ratio of 1.2. The isotopic (3,1) *P*₁₂(8) line also shows as a weak indentation at λ 8190.279. The feature at λ 8193.769 is predominantly the result of the strength of the isotopic (3,1) *Q*₂(22) line, although the isotopic (2,0) *R*₁(51) line may be a minor contributor. In this case the intensity ratio is 2.2 in favor of the (3,1) line at the above temperature.

Thus the first two of Fujita and Utsumi's (2,0) C¹³N¹⁴ indicators owe their intensity as much to the (3,1) as to the (2,0) isotopic band. The third feature listed by them at λ 8202.3 is very likely a bona fide (2,0) C¹³N¹⁴ line. The *Q*₂(44) line of the isotopic (2,0) band appears at λ 8202.232 in the laboratory. It may blend with an isotopic (3,1) *R*₂(32) line at λ 8202.810; it does not appear resolved in the present spectrum of 19 Psc, although it does in Fujita and Utsumi's tracing.

One may also note that the peak continuum intensity at λ 8191.50 is close to or surpasses that at λ 7802, previously taken to represent the infrared continuum in the spectrum of 19 Psc. Weaker absorption features located at λ 8190.86 and λ 8198.26 may represent the unidentified solar lines described in Babcock and Moore's *Atlas of the Solar Spectrum*. These unidentified features point to the need for extensive laboratory analysis of the infrared spectra of the heavier elements of the fifth and sixth periods, far from completely studied at the present moment. I have already pointed this out (Wyller 1964) in a previous study of the spectra of 19 Psc and Y CVn in the IZ region.

IV. ISOTOPIC ABUNDANCE RATIOS

The identifications of individual C¹³N¹⁴ lines enable one to determine anew the C¹²/C¹³ abundance ratios in the carbon stars but now in a manner in which the saturation problems are minimized as compared with the C₂ Swan bands. In the present study we have provisionally singled out the *P*₂(19) (2,0) C¹³N¹⁴ line at λ 8019.351 and the *R*₁(40) (2,0) C¹²N¹⁴ line at λ 8038.125 for relative abundance determinations. These lines are free of known atomic line blends, water-vapor lines, or rotational lines from the preceding (7,4) and (8,5) CN bands. As seen in the spectrum of 19 Psc in Figure 4, they show clean and well-defined profiles which are resolved even at dispersions of 20 Å/mm. They may, perhaps, not represent the optimum choices. Another likely band pair is the *Q*₁(24) (2,0) C¹³N¹⁴ and *P*₁(23) (2,0) C¹²N¹⁴ lines at λ 8022.668 and λ 8024.035. This pair may suffer intensity distortions by weak atmospheric lines, by some atomic-line interference, and by weak C¹²N¹⁴ lines owing to the (8,5) CN band. The lines show very clearly in the spectrum of DS Peg (20 Å/mm). In view of these minor uncertainties regarding the true intensities of the second line pair, the relative abundance determinations were made from the first line pair, *P*₂(19)[†] and *R*₁(40)ⁿ.

The forced choice of line pairs makes it impossible to relate normal and isotopic lines pertaining to the same rotational transition, which would have eliminated the need for a separate temperature determination. The line intensities have, therefore, to be calculated from the general formula,

$$I(v', v''; J', J'') = C f_e N_0 i [m_l(J)] R_{v', v''}^2 \exp - [G(v'') + F_l(J'')] \frac{hc}{kT}, \quad (1)$$

where f_e is the electronic transition probability, $i[m_l(J)]$ is a rotational intensity factor, $m = P, Q, \text{ or } R, l = 1 \text{ or } 2$, $\nu_{v',v''}$ is the frequency, $R^2_{v',v''}$ is the vibrational transition probability, N_0 is the molecular abundance, and C is a constant.

When the line-intensity ratios are formed, the factors $C, f_e, R^2_{v',v''}$, and $\exp -[G(v'')hc/kT]$ cancel. Since the molecular abundances are related to the product of the atomic concentrations by the equation of dissociative equilibrium, we finally have, with $a \equiv N(\text{C}^{12})/N(\text{C}^{13})$,

$$\frac{W[R_1(40)^n]}{W[P_2(19)^i]} = \frac{I[R_1(40)^n]}{I[P_2(19)^i]} = 2.4a e^{-(3430/T)}. \quad (2)$$

The intensity factors and the Boltzmann populations have been calculated from established formulae for the CN molecule (Wyller 1960) for a given vibrational temperature. The temperature had to be derived from the relative intensities of the (2,0), (3,1) band pair of the normal CN molecule by the use of pyrometer curves (Wyller 1960, p. 47). Once the equivalent widths, W , of the $P_2(19)^i$ and $R_1(40)^n$ lines have been determined from observations, the abundance ratio, a , of C¹² to C¹³ is readily derived from equation (2).

This simple method takes no account of curve-of-growth effects which may be important when the lines show markedly different intensities, as in the cases of DS Peg and HD 156074. The derived a -values are then lower limits, since the C¹²N¹⁴ lines are invariably the most intense lines. On the other hand, the expected large values for the macroturbulence in these giant stellar atmospheres may raise the flat portion of the curve of growth considerably and thus decrease the effects.

In the case of stars with heavy absorption like WZ Cas, 19 Psc, and Y CVn, the normal and isotopic lines are comparable in intensity. They are, thus, roughly on the same portion of the curve of growth; the differential effects are minimized. Clearly, for a finer precision determination of the a -values, the method of Phillips (1957) should be used, which involves profile computations of the Minnaert type. This may well be possible in the future, as Phillips has informed me that such computing operations are now fully programmed for machine analysis at Berkeley.

Table 5 gives the resulting a -values for five carbon stars. Noteworthy in Table 5 are the generally low vibrational temperatures and the lack of correlation with spectral class. The earliest-type, relatively hot carbon star, HD 156074, C1₂, shows the lowest vibrational temperature ($T_v = 1900^\circ \text{K}$). It is corroborated by an earlier study of the intensity relations of the (4,0), (5,1) red CN band pair and the (1,3), (0,2) C₂ band pair with temperatures of 1890° and 1580°K (Wyller 1960, p. 50). The latest-type carbon star WZ Cas, C9₂, ($T_v = 2400^\circ \text{K}$), does not by any means show the lowest vibrational temperature.

The effective temperatures as determined by the multicolor photometry of Mendoza V and Johnson (1965) have been listed for several of the stars in Table 5. Invariably the molecular band temperatures fall below the effective ones. The striking difference in the case of HD 156074 ($T_e = 4650^\circ \text{K}$, $T_v = 1900^\circ \text{K}$) can best be interpreted as an indication that the band formation takes place well outside the region of atomic-line formation. This is shown by the general lack of sharp rotational-line profiles in the spectra of these stars. They all give evidence of turbulent broadening with half-widths of the order of 0.5–0.7 Å. Phillips (1957) also concludes, from a profile study of the (0,1) C₂ Swan band in a similar star, HD 182040, C1₂, that the molecular excitation temperature lies in the neighborhood of 2000°K , while the effective temperature according to Mendoza V and Johnson is 5520°K .

Two correction factors should be taken into account in obtaining relative abundances from equation (2). One is the saturation effect which implies that $W \propto \sqrt{I}$, i.e., $W \propto \sqrt{(Nf\Gamma)}$; the other is the false strengthening of the normal (3,1) R_2 head due to the blending (2,0) C¹³N¹⁴ lines in this head region. These factors affect the temperature

TABLE 5
RELATIVE ABUNDANCES OF C¹² TO C¹³

Star	Spec. Class	Eq.Widths in Å P ₂ (19) ⁱ R ₁ (40) ⁿ		T _{vib}	T _e (°K)	a ≡ C ¹² /C ¹³	Observer
HD 156074	C1 ₂	0.06	0.12	1900	4650	5.3:1 4.2:1	Wyller 1965 (2,0) Red CN Climenhaga 1960 (1,0)C ₂ Swan
Y CVn	C5 ₄	0.52	0.59	2400		2.4:1 5-10:1 2:1	Wyller 1965 (2,0) Red CN Utsumi 1963 (3,0)C ₂ Phillips Phillips 1964 (4,0) Red CN
19 Psc	C6 ₂	0.14	0.33	2350	2890	4 < a < 10 10:1 8:1	Wyller 1965 (2,0) Red CN Phillips 1964 (4,0) Red CN Climenhaga 1960 (1,0)C ₂ Swan
DS Peg	C6 ₃	0.04	0.27	2560	2840	11 < a < 57 16:1	Wyller 1965 (2,0) Red CN Climenhaga 1960 (1,0) C ₂ Swan
WZ Cas	C9 ₁	0.35	0.40	2400	2560	2:1	Wyller 1965 (2,0) Red CN

determinations in the opposite sense. The saturation effect here leads to underestimating the temperature. The bands do not strengthen as fast as they should if they were on the linear portion of the curve of growth. When the band intensity ratio is larger than 1, the saturation makes the ratio too small and the resulting temperature is too small.

As to the second correction factor, the number of rotational lines and the types of rotational transitions are comparable for the blends of the normal and isotopic (2,0) bands in the (3,1) R_2 head region at $\lambda\lambda$ 8067–8074. The isotopic contribution to the absorption strength of the (3,1) normal R_2 head is then numerically comparable to the blend contributions by the normal (2,0) band, when C¹² and C¹³ are equal in abundance. The actual isotopic intensity contribution to the (3,1) R_2 head is, however, a function of the relative abundance of C¹² to C¹³. In any case, the effect of the blending isotopic (2,0) band leads to an exaggerated high temperature, since the isotopic blend falsely strengthens the (3,1) normal band head, from which the temperature determination is made.

The abundance determinations can thus fall between two limits, one which is derived under the assumption of no saturation and the other with saturation taken into account. In cases like Y CVn and WZ Cas, saturation undoubtedly exists, and only one a -value is given. The star HD 156074 does not show saturated features, and the result for the linear approximation is given. In 19 Psc and DS Peg we have intermediate cases, where one or the other condition may prevail.

The a -value derived for HD 156074 compares favorably with the value derived by Climenhaga (1960) from the suspected existence of the C¹²C¹³ (1,0) band at λ 4744. In the present study, individual C¹³N¹⁴ rotational lines are definitely observed, some of which are pronounced in intensity like the line pair $R_1(35)^i$, $Q_1(26)^i$ at λ 8036.23 and λ 8036.03.

The stars 19 Psc and DS Peg confirm earlier estimates (Wyller 1960; Climenhaga 1960) that they represent a group of carbon stars with C¹³ content intermediate between the CNO cycle equilibrium value (3.4:1) and the terrestrial abundance ratio (90:1). For 19 Psc the saturation value seems more realistic (10:1), considering the line strengths, while for DS Peg the lower limit (11:1) may be somewhat more accurate. Phillips' unpublished profile computations in the (4,0) isotopic red CN band favor the value 10:1 for 19 Psc. In DS Peg the C¹³N¹⁴ lines are weakly but clearly present, so that the terrestrial abundance ratio of 90:1 is out of the question for this star. It is thus most important to obtain the spectrum of HD 182040, since it represents one of the stars for which McKellar (1948) does not find any visible evidence of C¹³ in the C₂ Swan bands.

The stars Y CVn and WZ Cas show an exceptional richness in C¹³, with a -values well below the CNO cycle equilibrium values of 3.4. Fowler has, in a private communication, kindly informed me that, from unpublished analysis, the lowest a -value that the CNO cycle can yield is 2.6, under the special situation of *approach* to the cycle equilibrium (see Caughlan 1965). It therefore becomes an important question to resolve whether or not these C¹³N¹⁴ observations can be interpreted so as to yield a -values at least of the order of 3:1. It is valuable to determine the a -values in some detail for these two stars correcting both for saturation and isotopic blends.

A satisfactory treatment of the saturation which governs rotational-line formation in the atmospheres of these stars involves the construction of empirical curves of growth for lines in the (2,0) band. In the absence of such curves we shall adopt the simple damping (i.e., square-root) approximation to represent saturation for selected lines in the spectra of Y CVn and WZ Cas. The molecular damping constant is not known from theory; laboratory observations of molecular collision cross-sections are very scanty for related molecules of interest. However, the normal and isotopic line strengths are comparable in these two stars; their equivalent widths will lie roughly on the same portion of the curve of growth. Thus, the damping constants will cancel out in the abundances.

The resulting a -values should form an upper limit to the a -values for these stars. If the lines are not saturated, the C¹³ content will be even higher. A decision on this point

awaits the construction of empirical curves of growth for the (2,0) CN band, or computations of profiles as was done by Phillips (1957).

The linear approximation for Y CVn yields

$$T_v = 2370^\circ \text{K}, \quad \frac{W[R_1(40)^n]}{W[P_2(19)^i]} = 1.13 = 2.40a e^{-(3430/2370)} \quad (3)$$

or $a = 2.0:1$. The saturation approximation yields

$$T_v = 2660^\circ \text{K}, \quad \frac{W[R_1(40)^n]}{W[P_2(19)^i]} = 1.13 = \sqrt{[2.40a e^{-(3430/2660)}]} \quad (4)$$

which gives $a = 1.9:1$.

Let us accept the saturation for Y CVn. We can then roughly estimate the effect of the isotopic (2,0) blends on the intensity of the normal (3,1) R_2 head in the following way: Theoretical calculations indicate that the isotopic and normal (2,0) blend contributions in the (3,1) head are roughly equal for *equal* abundances of C^{12} and C^{13} . The actual isotopic blend contribution must be 50 per cent of the normal (2,0) blend contribution since we have found $a = 2:1$ for Y CVn. For the derived saturation temperature we find the normal (2,0) blend contribution to the (3,1) head (see Table 6, Wyller 1960). We subtract 50 per cent of this calculated amount from the calculated intensity of the (3,1) head and interpolate in Table 6 of my 1960 paper to a revised temperature from the new (3,1) intensity value. In our particular case the revised temperature becomes 2150°K , and this value is entered into the saturation expression for the equivalent widths in equation (4):

$$\frac{W[R_1(40)^n]}{W[P_2(19)^i]} = 1.13 = \sqrt{[2.40a e^{-(3430/2150)}]} \quad (5)$$

which gives $a = 2.6:1$.

The C^{12} abundance has now increased, so that the estimated isotopic blend contribution to the (3,1) head has to be decreased. The vibrational temperature then increases from 2150° to 2300°K . The process is iterative, and the limiting value for the abundance ratio becomes $a = 2.4:1$.

One might tend to discount such a result for Y CVn because of the extreme condition of saturation. However, Y CVn does not represent an isolated case. Similar corrections for saturation and blends have been applied to the spectrum of WZ Cas, whose spectrum in Figure 5 looks much more like that of 19 Psc, with not excessively strong molecular lines. In this case one is also forced to conclude that within the framework of these approximative considerations it is impossible to raise the a -value significantly above 2.0:1.

The present derivation for Y CVn seems more plausible in view of the fact that Phillips in some unpublished profile computations for this star gives an a -value of 2:1 in order to match the observed (4,0) red CN profile to the computed one. But Utsumi's revised a -value, which is derived from the (3,0) C_2 Phillips band at $\lambda 7714$, is 5–10:1 (Utsumi 1965, unpublished).

Allowing for the above uncertainties, it appears that the range in C^{13} abundances is possibly larger than previously established. Those in excess of the CNO equilibrium value do exist, and the range intermediate between the terrestrial value and the equilibrium value also appears to be populated. It becomes highly desirable to have a large-scale survey of the C^{13} content for all the carbon stars in order to establish the stellar "population spectrum" of C^{12} to C^{13} abundances. The new C^{13} indicators described in this paper should facilitate such a quest.

V. DISCUSSION

Most interpretations of the C¹²/C¹³ abundance ratios in carbon stars are based on the assumption that the C and N constituents in their atmospheres reflect the mixing of CNO cycle equilibrium products from a hydrogen-burning zone up to the stellar surface. In this mixing process any amount of helium-synthesized C¹² can be added from a helium-burning core to give an arbitrarily high a -value above the CNO equilibrium value (e.g., Burbidge 1963). Possible a -values *below* the equilibrium value are not, however, accounted for by this theory.

The results of the present investigation point to the existence of a -values well below the equilibrium ratio of 3.4:1, and the problem arises of how to account for the excessive C¹³. The work of Caughlan (1965) indicates that, during the approach to equilibrium in the CNO cycle, the a -value reaches a minimum of 3:1, revised by Fowler to 2.6:1, which is still not sufficient to account for the extreme results (2:1) found in the present investigation.

Although we are fully aware of the difficulties in the following suggestions, two alternative possibilities will be advanced to explain the exceptionally high C¹³ content in some carbon stars. Both appeal to the spallation and neutron irradiation processes invoked by Fowler, Greenstein, and Hoyle (1962) for the building up of the lighter elements like D, Li, Be, and B during the early history of the solar nebula.

As one alternative, we suggest that the carbon stars of intermediate to late type, C4–C9, (i.e., R5, R8, N) are highly evolved Wolf-Rayet stars where the C¹³ excess has been produced by high-energy particle phenomena in the envelopes. The broad emission lines of carbon and nitrogen in highly ionized stages attest to the strongly disturbed atmospheres of the Wolf-Rayet stars which may be conducive to surface nuclear reactions.

We exclude the early-type carbon stars, C1–C3, which are intrinsically less luminous and show less galactic concentration than do the later types. Observations do not yet show evidence for a -values below the equilibrium ratio in these early stars.

The simplest type of surface nuclear reaction with neutrons would be C¹²(n, γ)C¹³, and C¹²(p, γ)N¹³(β^+, ν)C¹³ with protons. Subsequent captures of neutrons and protons could reduce this yield of C¹³. But disregarding subsequent captures, the conversion of 20 per cent of the existing C¹² into C¹³ suffices to produce an abundance ratio of 2:1, if we start with the CNO equilibrium ratio of, for example, 5:1 in the envelope. In view of the large abundance of C¹², such a massive conversion of C¹² into C¹³ would be accompanied by other secondary reactions which might well affect the abundances of heavier elements. The elements of the fifth and sixth periods do seem enhanced in the spectra of carbon stars (Swings 1958.)

A serious difficulty with this first alternative explanation lies in the binary nature of many Wolf-Rayet stars, a characteristic not shared by the carbon stars. If the Wolf-Rayet star evolves into a red carbon giant with a hot, less-evolved companion, deep eclipses should be observed in some cases. Since none is yet known, we may overcome this difficulty by (1) invoking a possible dissolution of the binary system by mass loss or (2) claiming that the hot companion also evolves into a less luminous red giant.

These difficulties may make more attractive the second alternative which posits that the transformation of C¹² into C¹³ occurs in the envelopes of the carbon stars themselves by high-energy particle bombardment of C¹². In that case one could visualize that the neutrons and protons irradiate a surrounding envelope of graphite particles imbedded in ices of C, N, O, and H compounds. Between the stellar photosphere and the solid-particle envelope there should then exist a gaseous molecular transition zone where the observed band absorption takes place at a temperature relatively independent of the photospheric temperature.

Recently, the neutron irradiation process has been suggested by Greenstein and

Stawikowski (1964) to account for the enhanced C^{13} content in Comet Ikeya ($a = 70:1$). While this value is nearly the terrestrial one, it seems probable that the solar a -value will prove to be considerably higher than the terrestrial (90:1). In carbon stars the conversion of C^{12} to C^{13} will be on a larger scale; the same remarks regarding heavy-element formation apply as in the first alternative.

On the other hand, several carbon stars are known to exhibit unusually high lithium abundances in their atmospheres. According to Wallerstein, Phillips, and Torres-Peimbert (1964) this lithium excess may be produced by spallation processes at the stellar surface. The neutrons required for the above irradiation processes could then originate from the spallation processes themselves in relatively cool stellar environments. If the C^{13} and Li content in carbon stars are the products of spallation processes and neutron irradiation, one may expect a correlation among their abundances in the spectra of carbon stars. In the present study the prototype lithium star, WZ Cas, has been shown to exhibit an exceptionally high abundance of C^{13} . From the study of Wallerstein, Phillips, and Torres-Peimbert (1964) it appears that the other exceptionally C^{13} -rich star, Y CVn, also shows a high lithium content. Furthermore, the high-velocity, C^{13} -poor stars, HD 26 and HD 201626 (Wallerstein and Greenstein 1964) have a low lithium content. It appears highly desirable to combine a C^{13} with a lithium survey in carbon stars.

The present hypothesis for the building up of the excessive abundance of C^{13} in certain carbon stars may well require prohibitively large proton and neutron fluxes to account for conversion of the abundant C^{12} into C^{13} . A decision on this hinges, in turn, on reliable abundance determinations for H, He, C, N, and O in the atmospheres of the carbon stars, of which we presently have a woefully inadequate knowledge.

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