

MAGNETIC FLUX TUBES AND CONVECTION IN THE SUN

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(Received 1963 November 15)*

Summary

Convection in the Sun is apparently cellular and the convective velocities can be estimated. The interaction of cellular convection with a weak magnetic field leads to concentrated ropes of flux in the convective zone with fields of about 5000 gauss and fluxes of 10^{21} maxwells in agreement with observation. These flux tubes tend to float outwards from the centre of the Sun. The magnetic pressure associated with the fields required to halt photospheric convection is comparable with the gas pressure and so fields will generally be diffuse as in bipolar magnetic regions. Occasionally the flux is concentrated to form a sunspot but the cooling is insignificant at more than 1000 km below the surface of the Sun.

1. *Introduction.*—The magnetic field in the solar corona, although it is irregular, appears to be continuous and would be consistent with a uniform field of about 1 gauss in the photosphere (Högbom 1960). However, the Babcocks' measurements (Babcock & Babcock 1955, Babcock 1961) show that magnetic fields in the photosphere are confined to more or less discrete regions. This applies both to the poloidal and the toroidal fields. The latter usually appear as bipolar magnetic regions (BMR) with fluxes of about 5×10^{21} maxwells and average fields of several gauss.

These fields can be described in terms of tubes, or ropes, of magnetic flux emerging from, and subsequently re-entering, the surface of the Sun. The region where they intersect the surface is a centre of activity, associated with the presence of faculae, of flocculi and of chromospheric structure. This structure is related to the direction of the field in the chromosphere (Stepanov 1958) and so the magnetic and the gas pressure above active regions must be comparable.

It seems probable that magnetic fields are confined to discrete ropes of flux throughout the convective zone. Since there is no reason for assuming any motions in the radiative core other than very slow Eddington currents, it is most unlikely that the magnetic fields (which reverse after only 11 years) can penetrate into it. On the other hand, the horizontal scale of BMRs is of the same order as the depth of the convective zone and so the magnetic fields presumably extend throughout this layer.

The aim of this paper is to consider the relationship of the observed magnetic fields to convection in the Sun. Therefore it is first necessary to establish a quantitative description of the convective process. Convective transport and the effects on it of rotation and a magnetic field have been discussed at length elsewhere

* Received in original form 1963 October 17.

(Weiss 1964—to be referred to as I) for the case of a Boussinesq fluid between parallel boundaries. In particular, an attempt was made to express the superadiabatic gradient and the convective velocity in terms of the convective heat flux by balancing the work done by buoyancy forces against the energy passed down to small-scale motions by non-linear interactions and ultimately dissipated by viscosity. In order to apply these results to the Sun it is necessary to assume several approximations. In the photosphere these approximations are of dubious validity; nevertheless, this simple theory appears sufficient to account for the observed dimensions of granules and of penumbral filaments. In the deep convective zone the model is more satisfactory and yields the same results as ordinary mixing-length theory.

It is then suggested that the effect of convection on a weak magnetic field should be to produce flux ropes of the magnitude observed. Moreover, the discussion can be extended to cover sunspots, which are only a special case of BMRs, and to consider the questions: why do sunspots appear and why do they not invariably appear? The approximate treatment also indicates that spots are very shallow phenomena.

Finally, the gross behaviour of flux rings is considered and it is shown that their tendency to float dominates the inward forces except at points close to the axis of the Sun.

2. Convection in the Sun

(i) *Relevant approximations.*—The solar convection zone extends from about $0.85 R_{\odot}$ to the surface and in most of this region the temperature gradient scarcely differs from the adiabatic gradient defined by

$$\frac{d \log \Theta}{d \log P} = \frac{\gamma - 1}{\gamma} = \frac{2}{5} \quad \text{for} \quad \gamma = \frac{5}{3}. \quad (1)$$

In the convective zone this temperature gradient is about $10^{-4} \text{ }^{\circ}\text{K cm}^{-1}$ and the energy carried by radiation amounts to no more than 12 per cent of the total emitted by the Sun. Since the treatment here is only approximate in any case, it will suffice to assume that all the energy transport in the outer parts of the Sun is by convection.

Unsöld and Biermann originally used the mixing-length method to find the convective transport but it is only in the photosphere that solar structure is sensitive to the precise nature of the convection. Various models are discussed by de Jager (1959): they assume the existence of rising and sinking columns of gas and apply mixing-length theory to give the heat transport, modified by the radiative conductivity. The theory depends on the arbitrary choice of the mixing-length. In the laboratory this is normally the depth of the convecting layer but in the Sun the density varies drastically within the convection zone so that a smallish disturbance at the bottom of the layer would cause an incredibly rapid motion at the top. It therefore seems that the mixing-length will be less, and probably of the same order as the density scale-height; thus Schwarzschild (1961) recommends “using the mixing-length theory combined with the density scale-height for our purposes until a more accurate theory for convective energy transport has been developed”.

Throughout deeper regions of the convective zone the mean free path of radiation remains close to 1 cm so that the medium is very opaque: it is therefore possible to describe the radiative transport in terms of a radiative conductivity κ so that the energy flux (in cm °K sec⁻¹) is given by

$$F = \kappa \frac{\partial \Theta}{\partial r}. \quad (2)$$

The value of κ in the Sun varies from 10⁹ cm² sec⁻¹ in the convective zone to 10¹⁴ cm² sec⁻¹ in the photosphere. In either case it is very much greater than the molecular conductivity and the latter can therefore be ignored. Moreover, the Prandtl number ($p = \nu/\kappa \sim 10^{-10}$) is likewise very small.

Between the Sun's surface and a depth of 100 000 km the density scale-height, $\rho/(\partial\rho/\partial r)$, increases from 1×10^9 to 5×10^9 cm, so that its own scale-height $[(\partial\rho/\partial r)/(\partial^2\rho/\partial r^2)]$ is about 6×10^9 cm. Under such conditions the Boussinesq approximation will not remain valid.

The only apparent solution to this difficulty is to substitute the density scale-height for the layer depth d wherever it occurs in the equations and to trust that this is not too gross a simplification. Such an approach can be justified by arguments like those advanced by Schwarzschild (1961). Observation shows that solar convection is cellular: suppose that each cell has a depth d ; then, if the cells are packed close together, the continuity equation implies that the convective velocity varies inversely as the density and must decrease downwards. It is very hard to imagine such a circulation and much simpler to suppose that the cell is restricted to a region over which the density does not vary appreciably. In that case, the cell depth will be the scale-height, multiplied by some factor of order unity.

At any given time, then, the convective motions can be pictured in terms of a three-dimensional irregular array of circulating cells, the depth of each being of the order of the local scale-height. Such a set of motions must be quasi-turbulent: individual cells may lose their identity after a time τ comparable with the convective turnover time. So the cellular structure used to describe the motions is only statistical and convection becomes a less regular phenomenon than was supposed in I. Nevertheless, it seems reasonable that the approach used then will still give approximate results, if the scale-height is used instead of the layer depth. Such an assumption is probably adequate provided that $(\partial\rho/\partial r) < \rho(\partial^2\rho/\partial r^2)$: in any case, it seems the best that can be done.

That these assumptions are not hopelessly inaccurate can be seen from observations of the photospheric granulation, notably the beautiful photographs obtained by Project Stratoscope (Bahng & Schwarzschild 1961) and referred to by Schwarzschild (1961). The relevant scale-height is given by him as 150 km at the surface and he is disappointed to find an average granular diameter of 700 km.

It is convenient to introduce a quantity b to describe the width of a cell so that for a cell with semi-diameters a_1 and a_2

$$b^2 \equiv 1 + d^2 \left(\frac{1}{a_1^2} + \frac{1}{a_2^2} \right). \quad (3)$$

Now it was shown in I that if the horizontal and vertical velocities in a cell were equal and the fluid were incompressible, then the continuity equation relates

the half-width a of a cell to the depth d of the layer by

$$b^2 = 1 + \frac{2d^2}{a^2} = \frac{3}{2}$$

and the diameter is therefore four times the depth. At the onset of convection in the classical Rayleigh problem, b^2 does in fact assume this value. The discrepancy between 600 and 700 km is then quite small. Moreover, the convective velocities and temperature differences deduced below agree with those observed; the treatment of sunspot penumbrae by Danielson (1961) likewise concurs with observations*.

Of course, the limitations of this model are apparent, particularly in the photosphere, whose properties vary so rapidly with depth. Nevertheless, it is obviously desirable to compare the model with the only region that can be observed. It then appears that this theory is sufficient to explain the observed horizontal scales of photospheric convection.

All in all, the assumption that the scale-height determines the cell-depth seems reasonable provided no precise accuracy is demanded. In the following treatment this will be accepted and the radiative transport will be treated in terms of a simple conductivity: both d and κ vary throughout the layer but it is to be expected that their mean values will determine the convection. As a further simplification we shall consider a uniform layer only whose characteristic properties are average values over the relevant region. The parameters used for these models of the convective zone and the photosphere are given in Table I.

TABLE I

Typical values of parameters in the convective zone and photosphere

Quantity	Unit	Convective Zone	Photosphere
Scale height	cm	3×10^9	3×10^7
Pressure	dynes cm ⁻²	4×10^{11}	1×10^5
Temperature	°K	5×10^5	7.5×10^3
Density	g cm ⁻³	7×10^{-3}	2×10^{-7}
Gravity	cm sec ⁻²	4×10^4	3×10^4
κ	cm ² sec ⁻¹	3×10^8	4×10^{12}
η	cm ² sec ⁻¹	10^3	10^7
ν	cm ² sec ⁻¹	10^{-1}	10^3
F	cm°K sec ⁻¹	3×10^4	2×10^9
Temperature gradient	°K cm ⁻¹	1×10^{-4}	4×10^{-4}

(ii) *Convective transport and temperature gradients.*—It is useful to describe the temperature gradients in terms of a modified Rayleigh number that is independent of the viscosity. Let us therefore define

$$R_3 = \frac{g\alpha\beta d^4}{\pi^4 \kappa^2} \quad (4)$$

* It is just possible that the supergranulation studied by Leighton *et al.* (1962) may indicate cells in the convective zone.

where α is the coefficient of thermal expansion ($\alpha = \Theta^{-1}$ for a gas where Θ is the absolute temperature) and β the superadiabatic temperature gradient. Approximate expressions for the convective heat flux

$$F = \frac{E}{c_p \rho} \quad (5)$$

(where E is the energy flux in $\text{ergs cm}^{-2} \text{sec}^{-1}$) were given in terms of the temperature gradient and conductivity in I: if $R_3 \ll 1$, then

$$\beta \sim \left(\frac{4F\Theta^2}{g^2 d^2} \right)^{1/3} \frac{\pi^2 \kappa}{d^2} \quad (6)$$

while the vertical velocity

$$w \sim \left(\frac{gdF}{2\Theta} \right)^{1/3} \quad (7)$$

and is independent of the conductivity κ ; on the other hand, if $R_3 \gg 1$, then

$$\beta \sim 2 \left(\frac{F^2 \Theta}{gd^4} \right)^{1/3} \quad (8)$$

and is independent of κ , while

$$w \sim \left(\frac{gdF}{\Theta} \right)^{1/3} \quad (9)$$

and does not differ much from the value given by (7). But it was pointed out in I that the results for $R_3 \gg 1$ coincide with those derived from mixing-length theory, so the convective velocities should agree always, though the temperature gradients will differ when $R_3 < 1$; in the latter case, the expressions agree with those obtained by Ledoux, Schwarzschild & Spiegel (1961).

When the parameters from Table I are inserted into these equations, it appears that $R_3 < 1$ in the photosphere while $R_3 \gg 1$ in the convective zone proper. The values substituted into (6) and (7) correspond to a depth of 300 km (optical depth $\tau \approx 2$) and it has been assumed that convection carries only 10 per cent of the energy: the resulting gradients and velocity are given in Table II. The agreement with observations is satisfactory, though the approximations are of dubious validity in this region.

TABLE II

Convection in the Sun

			Photosphere	Convective Zone
Depth	h	km	300	6×10^4
Optical depth			2	
Scale height	d	cm	3×10^7	3×10^9
Convected flux	F	$\text{cm}^2 \text{K sec}^{-1}$	2×10^8	3×10^4
Adiabatic gradient		$^\circ \text{K cm}^{-1}$	2×10^{-4}	1×10^{-4}
Rayleigh number	R_3		0.3	6×10^8
Superadiabatic gradient		$^\circ \text{K cm}^{-1}$	2×10^{-4}	1×10^{-9}
Convective speed		cm sec^{-1}	4×10^5	2×10^4
Temperature excess or deficit of a convecting element		$^\circ \text{K}$	1.5×10^3	2

In the deep convective zone, the approximate theory may be applied with more confidence. It will be shown elsewhere that rotation does not significantly alter the pattern of convection. Then the relevant quantities are as given in Table II and agree with those derived from simple mixing-length considerations. The cell dimensions are of order 3×10^9 cm.

3. *Flux ropes and convection.*—Now that the nature of the convection has been established, it can be shown that interaction with a magnetic field should produce flux tubes with the observed field strengths, dimensions and frequency of occurrence.

Convection sets in as overstability for an inviscid, conducting fluid in the presence of a magnetic field (Chandrasekhar 1961) but Cowling (1957) argued that overstability was unimportant. Danielson (1961) showed that the transition from oscillatory to steady instabilities occurred when the kinetic energy became comparable with the magnetic energy density. His treatment is elaborated in I where it is also shown that large-scale turbulent convection should only set in at the same point. It is also suggested that the effect of turbulent convection on a weak magnetic field should be to concentrate the lines of force into narrow regions within which the field would be strong enough almost entirely to suppress convection. This will occur when the magnetic energy density in the region exceeds the kinetic energy of the convection. Such regions would correspond to the discrete ropes of magnetic flux that are observed at the surface of the Sun and it was suggested in Section 1 that magnetic fields are confined to such flux ropes throughout the convective zone.

Now the kinetic energy density in the convective zone is about 1×10^6 ergs cm^{-3} , according to the data in Table II. The mean poloidal field is only around 1 gauss; the toroidal field is greater: suppose it averages 10 gauss. Then the magnetic energy density corresponding to a uniform field would only be 4 ergs cm^{-3} and the fields would be completely dominated by convection. Thus the flux will be concentrated into ropes, in which the magnetic energy density is around 10^6 ergs cm^{-3} and therefore prevents convection. The corresponding fields have magnitudes of about 5,000 gauss.

The amount of flux in a rope can be estimated. The surface area of the Sun affords space for several hundred cells in the convective zone; of these, ten to twenty will lie in each polar zone and half of the total within the sunspot zones between $\pm 30^\circ$ latitude. A single flux rope may contain the flux which would have passed through several of these cells (the ropes can bifurcate if need be) which implies nearly the observed density of magnetic regions. The flux through a few cells of area 10^{20} cm^2 with an average field of, say, 5 gauss will then suffice to give a rope with a total flux of 10^{21} maxwells. If its average field is 5×10^3 gauss, then its internal radius will be about 2500 km, a little less than one-tenth of the local scale-height. These flux tubes can run innocuously through the interstices between convection cells. This all fits nicely with the Babcocks' observations of magnetic regions in the photosphere.

If we define a critical field H_0 that will establish equipartition with convection at a given depth, i.e. one that will prevent convection from taking place other than by overstability, then we can tabulate H_0 as a function of depth within the convective zone. This is done in Table III whence it appears that H_0 varies only slowly, between values of 600 gauss in the photosphere and 6 000 gauss at the foot of the

TABLE III
Magnetic fields inhibiting convection

$r = \frac{R}{R_\odot}$	Depth km	H_0 gauss	Hr_0 cm	$\frac{P_m}{P_0}$	$\Delta\Theta$ °K	H_1 gauss	r_1 cm
0.86	9.8×10^4	5,700	4.8×10^8	6.6×10^{-6}	5.9	2.5×10^6	2.2×10^7
0.88	8.4×10^4	5,500	4.8×10^8	9.2×10^{-6}	6.7	2.0×10^6	2.6×10^7
0.90	7.0×10^4	5,300	5.0×10^8	1.4×10^{-5}	8.5	1.6×10^6	2.8×10^7
0.92	5.6×10^4	5,100	5.0×10^8	2.4×10^{-5}	11	1.2×10^6	3.2×10^7
0.94	4.2×10^4	4,900	5.0×10^8	4.8×10^{-5}	17	7.9×10^5	4.0×10^7
0.96	2.8×10^4	4,700	5.2×10^8	1.3×10^{-4}	30	4.6×10^5	5.2×10^7
0.98	1.4×10^4	3,600	6.0×10^8	4.3×10^{-4}	47	2.0×10^5	8.0×10^7
1-0.007	5.0×10^3	2,500	7.2×10^8	0.0020	70	2.0×10^4	2.6×10^8
1-0.003	2.0×10^3	2,000	8.0×10^8	0.016	320	5.6×10^3	2.8×10^8
1-0.0014	1.0×10^3	1,500	9.2×10^8	0.074	1000	2.0×10^3	8.0×10^8
1-0.0006	4.3×10^2	1,000	11.2×10^8	0.12	1200	9.7×10^2	1.1×10^9
1-0.0004	3.0×10^2	600	14.6×10^8	0.28	1900	4.2×10^2	1.5×10^9

convective zone. These values of H_0 may be taken as estimates of the fields in flux tubes at the relevant depths. For an average tube, the flux is 4×10^{21} maxwells and so it is possible to find the radius r_0 of such a rope at different depths; this is also given in Table III. The cross-section of the tube varies by a factor of ten, while its radius decreases threefold across the convective zone. Over the bulk of this region, between 0.86 and 0.96 $R_\odot$, it suffices to take $H_0 \sim 5,000$ gauss and $r_0 \sim 2.5 \times 10^8$ cm.

4. *Flux tubes and sunspots.*—It is not difficult to see that sunspots can be formed as a special case of magnetic flux ropes, if all the flux is concentrated into a region of the photosphere about 30 000 km in diameter. We shall also attempt to find out why they are relatively infrequent phenomena and to discuss the depth to which they penetrate, as well as some details of their structure.

If a flux tube is in hydrostatic equilibrium with its surroundings, at a pressure P_0 , then the pressure P_1 within the tube will be less in order to satisfy the condition

$$P_1 + P_m = P_0 \quad (10)$$

where the magnetic pressure

$$P_m = \frac{\mu H^2}{8\pi}. \quad (11)$$

The value of P_m corresponding to H_0 can easily be found and thence the ratio P_m/P_0 of the magnetic pressure within a flux rope to the ambient gas pressure outside it. This is given in the fifth column of Table III. Except in the photosphere, the ratio is extremely small, of the order of 10^{-5} ; however, the magnetic pressure associated with the fields required to inhibit convection in the photosphere (i.e. that in sunspots) is about one-quarter of the gas pressure and therefore by no means negligible.

In the deep convective zone there will be no convection of energy outwards along a flux tube and two possibilities arise: either (i) the flux rope is maintained at the ambient temperature by radiative transfer, or (ii) there is a collapse inwards caused by cooling within the tube, until the magnetic pressure becomes comparable with P_0 . Although the first of these states may be unstable and liable to pinch

and to collapse to the second, the latter will only survive until the temperature is raised by radiative transfer. A steady state will presumably be reached with the flux tube slightly cooler than the surrounding material but maintained at a steady temperature. An upper limit to the temperature drop in a flux rope will be given by

$$\frac{\Delta\Theta}{\Theta} = \frac{P_m}{P_0} \quad (12)$$

where Θ is the ambient temperature. This temperature drop is also listed in Table III and is, once again, negligible except in the photosphere.

If we evaluate the field H_1 for which the magnetic pressure exceeds, say, one-eighth of the total pressure, it is only in the photosphere that H_1 is less than H_0 . Thus in this region it is more difficult to establish a magnetic field strong enough to halt convection; for if a flux tube is compressed, the total internal pressure will exceed P_0 before convection is suppressed. Similarly, any flux rope that exists in the photosphere will be metastable. This explains why photospheric fields are generally diffused and weak, whether in the polar regions or in BMRs. Occasionally, random motions may rapidly bring up a "stitch" from below in which no convection is taking place. A sunspot will then be formed but such an occurrence is naturally infrequent. The observed average lifetime of a sunspot is about ten days, of the same order as its thermal decay time of around a week. After this time the anomalous low temperature is liable to be destroyed by radiative transfer and a weak magnetic field will replace the concentrated flux within the sunspot.

This discussion can be extended to consider the depth of a sunspot. This is partially a matter of definition: let us define a spot as a region in which convection is largely hindered *and* the gas is at a significantly lower temperature than the ambient material. Now the density will not vary much, so the temperature difference in the umbra of a spot should (see Table III) be about a quarter of the total temperature or 2000 °K. This agrees well enough with observation.

Let us, for a moment, arbitrarily define the perceptible edge of a sunspot as the surface on which

$$P_m = \frac{1}{8}P_0 \quad (13)$$

which corresponds to somewhere near the edge of the umbra. Then the maximum possible radius of a spot with a flux of 4×10^{21} maxwells is r_1 , corresponding to H_1 defined above, and this drops by a factor of ten beneath the photosphere. But the spot will only exist in the region through which $H_0 > H_1$, beneath which convection will be suppressed without a drastic cooling of the gas. So the depth of a spot is only about 0.0007 $R_\odot$ or 500 km, little more than the scale-height at the surface. Of course, this limit depends on the arbitrary choice of equation (13); if a smaller temperature difference had been chosen to define the sunspot, the depth would have been greater. For instance a temperature difference of 1000 °K persists to a depth of 1000 km although it is then only 7 per cent of the total temperature. Nevertheless, the fact remains that sunspots, spectacular as they may appear, can only be very superficial phenomena with a diameter that far exceeds their depth. They are therefore unimportant in considering the fields within the convective zone.

Biermann (1941) first suggested that the darkness of sunspots corresponded to the suppression of convection by the magnetic fields and Danielson (1961) has recently explained penumbral filaments in terms of the convection rolls described

by Chandrasekhar. The umbral structure observed by Bray & Loughhead (1959, 1960) may correspond to overstable disturbances.

Danielson interprets the width of the penumbral filaments as showing that the effective scale-height beneath them is about half that in undisturbed regions. An alternative explanation can be given. We should expect the granulation cells to have $b^2 = 3/2$ and so a diameter equal to four times the local scale-height; but the filaments arise when the Chandrasekhar number

$$Q_3 = \frac{\mu H^2 d^2}{4\pi^3 \rho \kappa^2} \ll 1 \quad (14)$$

and the treatment in I then shows that $b^2 = 2$. Thus their width should only be twice the scale-height since they are one-dimensional. With $d = 150$ km, we then obtain a width of 300 km for the filaments. This agrees well with observation.

Danielson has also related the lifetime of a filament (or a granular element) to the characteristic time τ for exponential growth of an infinitesimal disturbance varying as $e^{t/\tau}$. But the growth-rate is a deduction from the linearized equations, while the lifetime of a cell is controlled by non-linear effects. There is no *a priori* reason why these should in general be related, although an order of magnitude correspondence does exist. In fact, the lifetime might be expected to equal the turnover time for the cell, as has been mentioned in section 2, and this holds for the granulation. Magnetic fields, which absorb a substantial fraction of the kinetic energy in the penumbra, should decrease the convective speeds and hence increase the turnover time for the cell. This may be the reason for the longer lifetime of penumbral filaments.

However, when $Q_3 \ll 1$, the characteristic time τ is equal to the time taken by an Alfvén wave to traverse the cell (see I); at the onset of convection, the Alfvén speed is, in turn, comparable with the convective velocity. Thus τ is roughly equal to the turnover time at the initial onset of instability and therefore to the lifetime of a convective cell. This would not hold for $Q_3 > 1$ or beyond the initial onset of non-oscillatory convection.

5. *Floating flux tubes.*—The flux tubes that give rise to toroidal fields and to sunspots can be thought of (Bullard 1955) as forming girdles around the Sun, resembling Bjerknes' vortex rings. Occasionally, kinks develop and "stitches" are then brought to the surface, giving rise to BMRs or even sunspots. In the chromosphere and corona the lines of force can sometimes be seen as prominence arches.

The bodily motions of such a rope can be considered in terms of a highly simplified model: let us restrict our attention to an isolated toroidal flux tube in an axially symmetric and perfectly conducting Sun. In a medium of constant density, pressure equilibrium would be established and the sole force acting would be that caused by the curvature of the lines of force. Thus a ring of flux would collapse into the axis until its dimensions became small enough for ohmic dissipation to be effective, when it would presumably dwindle and disappear. However, in a compressible gas the density ρ_1 within the tube will be less than that without: then the tube will be buoyant and will tend to float outwards. We shall show that this buoyancy dominates the curvature forces in the Sun.

Let us assume that the tube always remains at the temperature of its surroundings. It is easiest to consider a purely azimuthal field H in the flux tube but solar

fields are likely to be twisted and to have a poloidal component G as well. However, we shall now prove that twisting the toroidal field has no effect on the buoyancy forces.

Consider a flux tube symmetrical about the Sun's axis and with a radius of curvature R . Let it be of circular cross-section with an internal radius r_0 such that $r_0 \ll R$. At any point within the tube, take polar co-ordinates (r, θ) centred on its internal, toroidal axis: then the field within the tube has an azimuthal component H and a θ -component G . Let G and H be functions of r only, such that $G(r) = H(r) = 0$ when $r \geq r_0$.

Consider a thin annulus of radius r : the pressure inside it must be greater than that outside because of the curvature of the tangential component of the field and the corresponding density difference might destroy the buoyancy effect. For hydrostatic equilibrium

$$\frac{d}{dr} \left[\frac{\mu(G^2 + H^2)}{8\pi} + P \right] = -\frac{\mu G^2}{4\pi r} \quad (15)$$

whence

$$P_0 - P = \frac{\mu(G^2 + H^2)}{8\pi} + \int_0^r \frac{\mu G^2 dr}{4\pi r} - \int_0^{r_0} \frac{\mu G^2 dr}{4\pi r}. \quad (16)$$

The outward buoyancy force per unit length is

$$F_1 = 2\pi g \int_0^{r_0} (\rho_0 - \rho) r dr = \frac{2\pi g}{\mathcal{R}\Theta} \int_0^{r_0} (P_0 - P) r dr$$

assuming the tube and its surroundings to be at the same temperature Θ , where $\mathcal{R}$ is the Gas Constant per unit mass. Therefore

$$F_1 = \frac{\mu g}{4\mathcal{R}\Theta} \int_0^{r_0} \left[G^2 + H^2 - \int_r^{r_0} \frac{2G^2 dr}{r} \right] r dr \quad (17)$$

which can be integrated by parts to give

$$F_1 = \frac{\mu g}{4\mathcal{R}\Theta} \int_0^{r_0} H^2 r dr \quad (18)$$

which is completely independent of G . This result can be generalized to cover a flux tube of arbitrary cross-section provided that the lines of force lie on axially symmetric surfaces.

The inwards force per unit length is

$$F_2 = \frac{\mu}{4R} \int_0^{r_0} (H^2 - G^2) r dr \quad (19)$$

and is a maximum if G vanishes. In that case, the ratio

$$L(R) \equiv \frac{F_2}{F_1} = \frac{\mathcal{R}\Theta}{gR}. \quad (20)$$

In the equatorial plane of the Sun, $L < 1$ for all $R > 0.17 R_\odot$. Thus any flux ring

outside that radius would float outwards. If we take a ring of latitude λ and distant a from the centre there will be a net force *towards* the axis so long as

$$L(a) > \cos^2 \lambda \quad (21)$$

i.e. a flux ring within this region will experience a net force causing it to collapse inwards to the axis even though it may be floating away from the centre of the Sun.

Acknowledgments.—This work was carried out at the Department of Geodesy and Geophysics, Cambridge and I am grateful to Sir Edward Bullard for his advice and encouragement. The last section was developed in collaboration with Dr D. Lynden-Bell and I owe much to stimulating conversations with him.

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