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ON THE CIRCULATION THEORY OF SPIRAL STRUCTURE

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Abstract: In continuation of earlier work on the dynamics of galaxies and on the possibility of a quasi-stationary spiral structure a more detailed and somewhat revised circulation theory of spiral structure is described.

The morphological age of spiral galaxies as estimated e. g. by V. C. Reddish [1] and M. S. Roberts [2] from considerations of the evolutionary process connected with star formation from gaseous matter range between 10^9 and 10^{10} years. In consequence it is natural to assume that the typical spiral structure is not an ephemeral phenomenon in the systems but has a certain steadiness in time. Under the influence of the differential rotation in approximately circular orbits a considerable deterioration of a spiral structure should be expected already in time intervals of the order of 10^8 years. The interstellar gas of the spiral arms, which will be partly ionized, may be appreciately affected in its motion by magnetic fields. Whether in our own Galaxy magnetic fields exist which are of character and strength to give a stabilizing effect on the spiral structure of the system is as yet unknown. Even admitting that magnetic forces may come into play in the problem, it seems therefore worth while to investigate how far gravitational forces alone can explain a spiral structure of a fair degree of permanence.

In the approach to the problem of the development of spiral structure in a galaxy by numerical computations P. O. Lindblad [3, 4] starts from a system of annular formations, and shows how small deviations in shape and density of a bisymmetrical nature can cause a development of spiral structure. In a typical case, after a series of transitory stages, the system shows trailing arms which during a time of about $2 \cdot 10^8$ years will be fairly preserved in shape, though successively somewhat tighter wound.

At a certain stage, however, in this development the spiral structure is broken up in such a way that, passing through certain stages of intermediate structures, the system will again develop a typical «trailing» arm structure with rather massive arms. The spiral structure may in this way be a quasi-periodical phenomenon, and the effective life-time of spiral structure in a system can be very effectively prolonged. In a recent paper [5] the present author has tried to show that under certain circumstances general continuous circulation of matter in a system may occur which instead of a quasi-periodic spiral structure gives a quasi-steady formation of the spiral type.

The primary fact on which this circulation theory of spiral structure is based is the way in which a spiral arm will attract and accumulate matter from the surrounding regions. We assume here, as was made by P. O. Lindblad, that the spiral arms are of small mass compared with the total mass of the system, and that there is a regular field of force of rotational symmetry caused mainly by the sub-systems of population II, strongly concentrated towards the centre. We assume at first that in this field the matter of the spiral arms follows nearly circular motions about the centre.

A material point P moving in a circular orbit at the distance R_0 from the centre may be overtaken in its rotation by the parts of a spiral arm S of $R < R_0$. In the disturbed motion of P it will coincide with the spiral arm at a point on the arm with $R = R_s$, where $R_s < R_0$. The motion of P at this point will coincide nearly with the circular motion at $R = R_s$, that is with the motion of the arm. This may be shown in the following way. Let R, θ, z be cylindrical coordinates in a non-rotating coordinate system where R, θ are polar coordinates in the invariable plane about the centre of gravity. The corresponding linear velocity components may be Π, Θ, Z. If $\varphi(R, \theta, z, t)$ is the gravitational potential function, twice the energy of a particle of unit mass I_1 and twice the area velocity in the fundamental plane I_2 are

$$I_1 = \Pi^2 + \Theta^2 + Z^2 - 2\varphi \; ; I_2 = R\Theta .$$

Following the particle in its motion we have

$$\frac{DI_1}{Dt} = -2\frac{\partial\varphi}{\partial t}, \quad \frac{DI_2}{Dt} = \frac{\partial\varphi}{\partial\theta} .$$

If φ is constant in a coordinate system of uniform angular speed of rotation ω, we have

$$\frac{\partial\varphi}{\partial t} = -\omega\frac{\partial\varphi}{\partial\theta},$$

and we have, following a particle in its motion

$$\frac{D}{Dt}(I_1 - 2\,\omega\, I_2) = 0$$

i.e.

$$I_1 - 2\,\omega\, I_2 = \text{const.}$$

This is an expression of Jacobi's integral for a rotating system, and can be written

$$\Pi^2 + (\Theta - \Theta_0)^2 + Z^2 - 2\left(\varphi + \frac{1}{2}\Theta_0^2\right) = \text{const.}$$

where

$$\Theta_0 = \omega R .$$

We may form the «characteristic diagram» (cf. Galactic Dynamics, Handbuch der Physik, 53, p. 26) with I_2 and I_1 as axes, and the envelope curve E which represents the relation between I_2 and I_1 for circular motions in the plane. If ω_c is the circular angular velocity at the distance R from the centre we have for points along E

$$\left(\frac{dI_1}{dI_2}\right)_E = 2\,\omega_c\,.$$

The disturbance of P in the actual case will correspond approximately to the disturbance by a spiral arm of uniform rotation ω_s, which is taken to be the value of ω_c for $R = R_s$.

We have then by Jacobi's theorem for the displacements ΔI_1 and ΔI_2 of the particle P during the motion

$$\frac{\Delta I_1}{\Delta I_2} = 2\,\omega_s\,,$$

and

$$\frac{\Delta I_1}{\Delta I_2} \text{ for } P \text{ equal to } \left(\frac{dI_1}{dI_2}\right)_E \text{ for } R = R_s\,.$$

The angular circular velocity for the original position of P at $R = R_0$ may be ω_0, and as the angular velocity is supposed to decease with increasing R,

$$\omega_s > \omega_0\,.$$

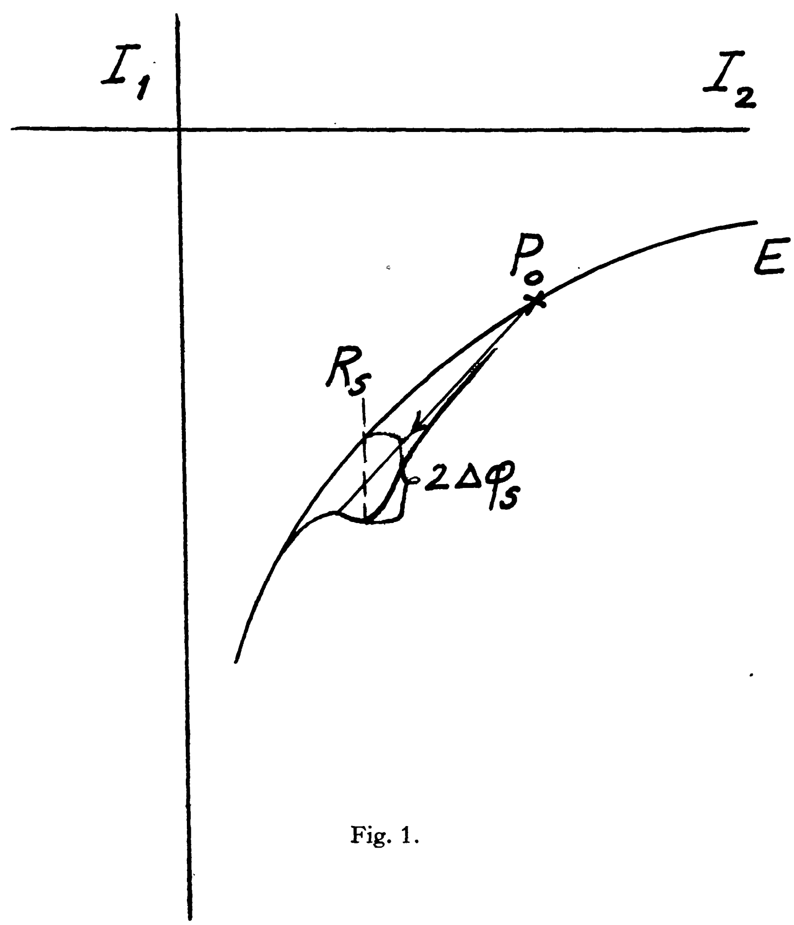

Fig. 1.

As $2\,\omega_0 = \left(\frac{dI_1}{dI_2}\right)_E$ for $R = R_0$ is smaller than $2\,\omega_s = \left(\frac{dI_1}{dI_2}\right)_E$ for $R = R_s$, the new representative point for P in the diagram will lie below the curve E. We may represent the spiral arm about $R = R_s$ on E with a «potential pocket» (Fig. 1) along E extending $2\Delta\varphi$ below E where $\Delta\varphi(R)$ is the potential due to the arm about $R = R_s$ in the general direction of P. Let the value of $\Delta\varphi$ for $R = R_s$ be $\Delta\varphi_s$. At $R = R_s$ we have by supposition for the centre of the arm section, if $\Theta_s = \omega_s R_s$,

$$\Pi^2 + (\Theta - \Theta_s)^2 + Z^2 = 0$$

The intersection between the line

$$\Delta I_1 = 2\,\omega_s\,\Delta I_2\,,$$

where ΔI_1 and ΔI_2 are reckoned from the original position of P in the diagram, and the line $I_2 = R_s\,\Theta_s$ occurs well below E. This gives for the square of the peculiar velocity relative to the centre of the arm section, which has the velocity $\Pi = 0$, $\Theta = \Theta_s$, $Z = 0$,

$$\Pi^2 + (\Theta - \Theta_s)^2 + Z^2 < 2\,\Delta\varphi_s\,,$$

which means that the relative velocity is smaller than the velocity of escape from the arm. The particle will therefore be effectively captured by the arm. In the plane the motion of P in a coordinate system following ω_s will be a small epicyclic motion.

A corresponding effect will take place if the particle P at $R = R_0$ moves with higher angular speed than the parts of a spiral arm S with $R > R_0$. Also in this case the outward motion of P will lead to capture by the arm S.

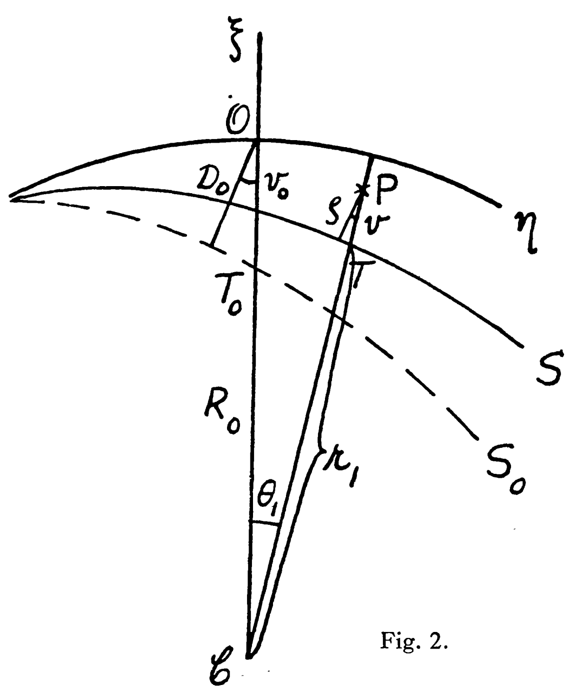

Fig. 2.

Figure 2 illustrates the position of P in a coordinate system ξ , η in which the origin O follows the circular motion at $R = R_0$. P coincides with O for $t = 0$. The positions of the spiral arm for $t = 0$ and t are indicated by S_0 and S . The distance from O to S_0 is indicated by D_0 . The distance from P to S is denoted by ϱ . The angles with the radius vector are v_0 and v respectively. The force of attraction of S on P may be written

$$f = \frac{2G\mu}{\varrho} K(\varrho) ,$$

where G is the constant of gravitation, μ the amount of matter per unit length of the arm S , and $K(\varrho)$ a correction factor which includes the effect of the curvature and of the finite dimensions of the cross section of the arm. We can assume that any cross section of the arm has a core of fairly high density and that the density decreases continuously with the distance from the core. In this way we must have

$$K \to 0 \text{ for } \varrho \to 0 .$$

We get from Figure 2 with good approximation

$$\varrho = \left[\frac{\dfrac{D_0}{\cos v_0} + \eta \operatorname{tg} v_0}{1 + 2\, At \operatorname{tg} v_0} + \xi \right] \cos v ,$$

where A is the generalized coefficient of differential rotation such that

$$\frac{d\omega}{dR} = - \frac{2A}{R} .$$

We can anticipate that the motion of approach of P to S in the coordinate system ξ , η will include a strong component of the motion parallel to S in the direction of possitive η . The general properties of the function $K(\varrho)$ will then allow to give an approximation of the disturbing force by inserting in f the value of ϱ corresponding to the undisturbed motion of P , i. e. $\xi = 0$, $\eta = 0$, and setting $K = 1$. This gives for f a pure function of the time, viz.

$$f = \frac{2G\mu}{D_0}(1 + 2\, At \operatorname{tg} v_0) .$$

The application to the permanence of the spiral structure in galaxies arises because the outermost part of an arm will be attracted by, and will tend to become assimilated with, the neighbouring parts of the inner structure. In an analoguos way matter close to the nucleus will have a tendency to be assimilated in an outward motion with the inner parts of the spiral structure. A schematic picture of the two regions of circulation is given here in Figure 3. We may suppose that there are two points F in the structure where the angular motion ω_F is equal to the local circular angular velocity. We can introduce a coordinate system which follows ω_F , and on account of the symmetry divide the structure in symmetri-

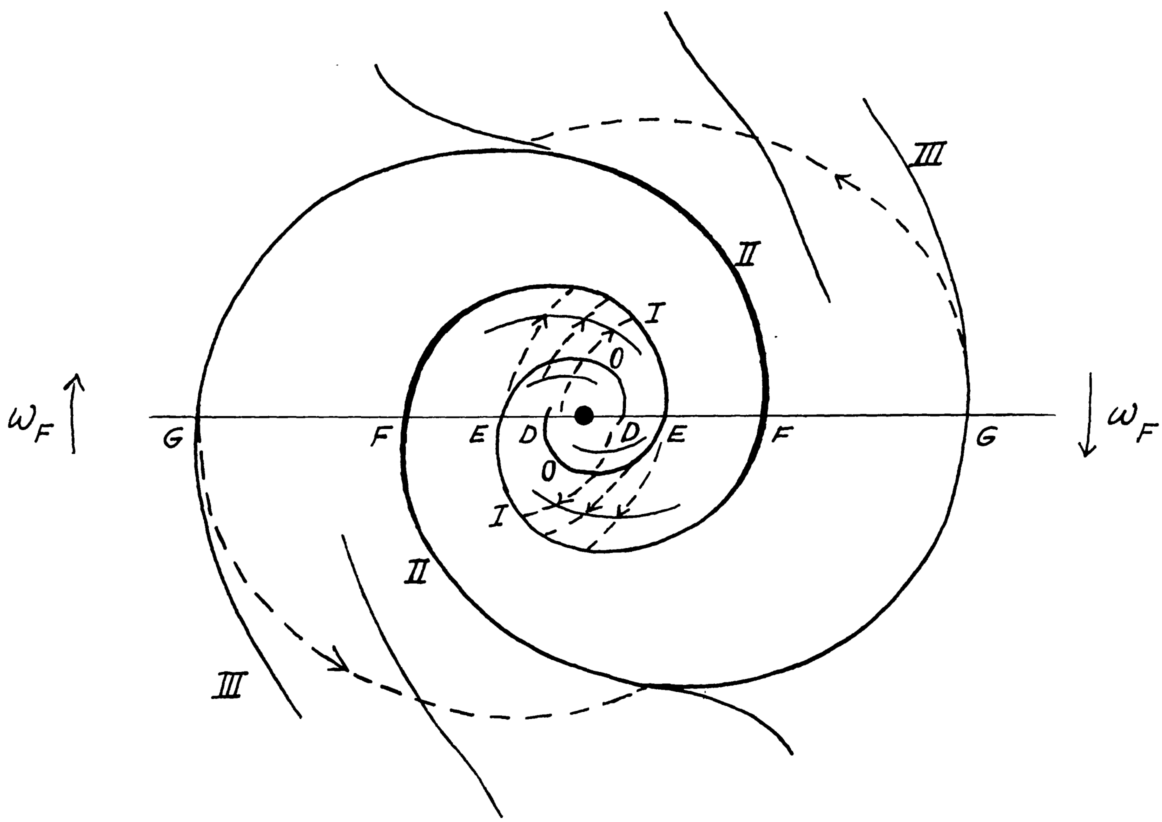

Fig. 3.

cally situated arcs 0 — III , each one covering 180° about the centre. The successive intersections with the diameter through the points F are indicated by the letters D , E , F , G . We make the supposition that, as long as the mass of III is small, the motion of particles in the arcs II is circular motion in the general gravitational field. The gravitational force from the interior spiral structure will not appreciably change the radial direction of the general field, of which it will form an integral part.

We may now consider first the interrelation between the arcs II and III. The arcs II lose matter by differential rotation passing over the points G into the arcs III. The speed of transfer will depend on the quantities μ , A and v_0 . In the natural motion the arc II will also lose angular momentum, because within its fixed interval of 180° in the angle θ the matter is on the average drawn closer to the centre. This loss of momentum will again correspond to a gain of the branch III. A compensation for the loss in mass and angular momentum in II will occur if the mass III is attracted and assimilated in a suitable way with the arc II. The matter passing into III over the points G will have a certain spread in R and, thanks to the attraction from II, will return towards that arc in a shower of orbits, of which one is indicated by the dotted curves in Figure 3. The matter of III is likely to be broken up into branches. Moreover, expanding associations in III will disperse along trajectories marked by the ξ, η coordinates in a coordinate system following the circular motion at $R = R_0$ which is the distance of G from the center. This will give branches with elongation and inclination towards III of the character indicated in Figure 3.

Examples of such branches may be seen for instance in the outer structure of the rich spiral galaxy M 101. In the process of assimilation between II and III the arc II will on the average get an outward motion, which in a steady state should compensate the inward displacement due to differential rotation which has been mentioned above. In order that the return of the matter in III shall actually compensate II for loss of mass and angular momentum, the average orbit in the coordinate system of angular speed ω_F should fulfill certain conditions. We have seen that, as a consequence of the approximate validity of the Jacobi integral of the relative motion, the matter of III will actually be assimilated with II, in which, notwithstanding the slow outward motion, the particles will still move in nearly circular orbits. The loss of angular momentum of III and the gain of II in the process of amalgamation will therefore correspond approximately to changes in angular momentum which are due to transfer between circular orbits of different radius in the general field of force. Whereas the conservation of mass gives a ratio between the values of μ, μ_1 and μ_2, at F and G, respectively, the condition for conservation of angular momentum demands that the average point of contact between II and III should lie at a distance from the centre corresponding to about half way between F and G. As to the value of μ we find that it ought to decrease from F to G along the arc II.

If we choose a certain value of v_0, we can, under the suppositions made, compute a value of μ considered as an average value of the mass per unit length along II. The mass of the spiral structure m_s as a whole can be expressed to the order of magnitude by a circular ring of radius R_0 with the uniform mass μ per unit length, thus $m_s = 2\pi\mu R_0$. Accordingly we measure the relative mass ε by

$$\varepsilon = \frac{2\pi\mu R_0}{M},$$

where M is the entire mass of the system inside R_0.

As to the quantity A a value is accepted in accordance with the relation which is nearly valid up to the «Kepler region» (Cf [3], p. 11)

$$\omega - \frac{1}{2}\varkappa = \text{const},$$

where $\varkappa = 2\sqrt{\omega(\omega - A)}$, and taking $\omega_0 = \varkappa_0$ for $R = R_0$ at the inner limit of the «Kepler region». This gives

$$\omega_0 \geqq A \geqq \frac{3}{4}\omega_0$$

so that we can simply accept $A = \omega_0$.

We can assume further tg $v_0 = \frac{1}{\pi}$, which gives $v_0 = 18°$, in close agreement with the average value accepted by C. G. Danver [6].

A revsion of the analysis in [5] has been made under the suppositions described above, using the formulae for the coordinates ξ, η of P given on p. 7 of [5], where we put $\omega = \varkappa = \omega_0$. At the point of contact defined by $\varrho = 0$ we assume $\xi/D_0 = 0.5$. The resulting value of ε is

$$\varepsilon = 0.09 .$$

This means that the entire mass m_s of the spiral structure should be about 9 percent of the entire mass of the system. As the approximation used for f is certainly an under-estimate of the disturbing force this value of ε is likely to be too large. Moreover, for a smaller value of v_0 the value of ε will decrease.

In the advance of II and III towards a steady configuration, with a continuous circulation of matter, the time spent in the relative orbit from G to the average point of contact between II and III will decrease, which in itself would demand a higher value of ε. However, the outward motion in II which belongs to a steady state will have as consequence an additional ingoing motion of the matter III, after reaching a turning point in the motion determined by the general gravitational field. This circumstance will counteract the demand of a higher value of ε and we have therefore reason to suppose that the value derived above for ε will be approximately valid also in the advanced state.

As to the nuclear regions the differential rotation in the brances 0 and I will tend to make the angle v between the normal of the arm and the radius vector decrease. By the attraction of the branches I matter passing inwards past the points E and D in the differential rotation will tend to be again assimilated by these branches, as indicated schematically in Figure 3. In the coordinate system of angular speed ω_F the motions will occur schematically as indicated by the dotted curves. In a coordinate system following a local circular motion in the nuclear region the motion will occur largely along the arms, converging towards the arm in the direction against the rotation. Local branches of matter and associations disolving in the field will have a tendency to follow these tracks, as indicated by the full-drawn curves between the arms. In the assimilation process with the out-going matter the branches I will be drawn closer to the centre which tends to compensate the decrease of v which would follow from the differential rotation alone.

It may be observed that the two circulation regions and the interchange of matter between the arms means on the whole a comparatively small deviation from a circulation in circular orbits in which the angular speed decreases with increasing R. It is unquestionably a merit for the theory that a quasi-steady state is achieved by introducing only fairly slow outgoing radial motion of the particles in the arcs II and a fairly slow ingoing radial motion in the arcs I.

In the case of our Galaxy it may be observed that the «dualistic» character of the motion in the regions where matter from II and III meet gives a very direct basic explanation to important properties of «star-streaming» in the region about the Sun, superposed on the general velocity ellipsoid in which the ratio of the axes in the galactic plane is determined by the differential rotation. The existence of vast general streams in the outward and inward directions observed especially A — F stars falls in readily with the present ideas. Especially if the angle v_0 is actually smaller in the Galaxy than in the typical spirals like *M* 101, the velocities of the particles in the ares II and III are of an order quite compatible with the observations on star-streaming in our neighbourhood of the Galactic System. A phenomenon which can be interpreted as a confirmation of the theory in the Galaxy is the observed inward streaming of the gas at great distances in the anti-centre region which has been observed by means of the 21–cm line of hydrogen at the Dwingeloo Radio Observatory.

In addition to disturbing forces between the arms there may also appear fairly appreciable disturbing forces along an arm in the case when the mass per unit length μ varies systematically along an arm. If the gradient of μ is positive in the direction of rotation the resulting force will tend to increase the angular momentum of this part of the arm, which will mean the creation of a component of motion outward at right angles to the arms. The tendency of branches of the arcs III to form detached «twigs» converging towards the arc II in the direction of rotation is possibly enhanced by such phenomena which may have a tendency to make the gradient of the «twigs» steeper and more clearly recognizable. The formation of «twigs» in connection with the return of matter in the outermost regions of galaxies has been commented upon extensively in the earlier paper [5] where reference to observed typical systems has been given.

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