IONIZATION AND EXCITATION EQUILIBRIUM OF Ca II IN THE SOLAR ATMOSPHERE

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ABSTRACT

Ionization and excitation configurations for Ca II are computed for conditions appropriate to the solar chromosphere. The populations of energy levels are found to depart sharply from the LTE values at corresponding values of T_e. The dominant ionization process of Ca II is shown to be via collisions from the 3D level and to exhibit fundamental differences from H, He, and Mg I, as well as some similarities. For electron densities and temperatures such as are expected in most of the solar chromosphere and corona, the ratio Ca II/Ca III is shown to be essentially independent of electron density rather than directly proportional, as predicted by the Saha equation. Computations of opacities and occupation numbers indicate that the Ca II H and K lines in the Fraunhofer spectrum are formed near the top of the hydrogen temperature plateau, where $T_e \approx 15000°$, and that the Ca II emission from spicules arises from regions where $T_e \approx 15000°$ rather than the much higher temperature derived from profiles of hydrogen and helium lines.

I. INTRODUCTION

It is well known that the ionization and excitation of coronal ions and of chromospheric H and He depart markedly from those given by the Saha equation. In this paper we investigate the ionization and excitation properties of Ca II in the solar chromosphere. The problem is of importance for determining the depths in the atmosphere where the Ca II Fraunhofer lines are formed and for interpretation of intensities of Ca II emission lines in the chromosphere and in transient features of solar activity. A specific objective of this investigation is to determine the electron temperature, T_e, for the regions within spicules, where the observed Ca II emission originates. Analysis of profiles of hydrogen, helium, and oxygen lines from spicules indicates a kinetic temperature of about 40000° (Athay 1961). Emission lines of Ca II are also observed in spicules, but with profiles that are inconsistent with those of the atoms mentioned. Furthermore, Zirker (1962) has shown that the intensity of the K-line emission from spicules is incompatible with a temperature higher than about 2×10^4 ° K.

Jefferies and Thomas (1960) and Jefferies (1960) have studied the source functions in the H and K lines of Ca II and have demonstrated that, under certain conditions, marked departures from local thermodynamic equilibrium (LTE) exist in the relative populations of the 4S and 4P levels. Thus we expect that the ionization will also show departures from LTE, although we do not assume that this is necessarily the case.

II. CA II EQUILIBRIUM

In discussing the ionization equilibrium of a complex atom it is necessary, for practical purposes, to use simple atomic models. Experience with H, He, and Mg I has shown that a good approximation is obtained by considering the ground state, the metastable state, if such exists, the upper state of the first resonance line, and the continuum. Thus, for Ca II, we consider the levels 4S, 3D, 4P, and the continuum and label them 1, 2, 3, and κ, respectively. The substates of the 4P and 3D levels are taken into account for computing transition rates and statistical weights but are otherwise ignored.

Using such a model atom, we may solve the equilibrium equations

$$\frac{dn_k}{dt} = \sum_{\substack{j=1 \\ j \neq k}}^{\kappa} (n_j P_{jk}) - n_k P_k = 0 \tag{1}$$

explicitly for the ratios n_j/n_κ. The coefficients P_{jk} are the transition rates from j to k per atom in level j, and P_k is the total transition rate out of level k.

If opacities are small, the radiative transition rates are readily computed from the transition probabilities tabulated by Allen (1955) and Osterbrock (1951) and the observed solar radiation field. For the free-bound continuum transition, the radiative recombination coefficients have been calculated by Burgess and Seaton (1960) and Green (1949). Representative rates are given in Table 1.

TABLE 1

TRANSITION RATES, $n_e = 10^{11}$

RATE	$T_e \times 10^{-4}$					
	1	1 5	2	3	4	5
$C_{12}\times 10^{-3}$	4	6	7	8	8	8
$C_{13}\times 10^{-3}$	3	9	14	21	24	26
$C_{1\kappa}$	0 0005	0.03	0 5	7	20	40
$C_{23}\times 10^{-4}$	1	1 4	1 7	1 8	2	2
$C_{21}\times 10^{-3}$	5	4	3.5	2.7	2.4	2.2
$C_{2\kappa}$	0 03	1 4	10	70	190	330
$C_{31}\times 10^{-4}$	4	3 3	2 8	2 3	2 0	1 8
$C_{32}\times 10^{-4}$	9	7	6.5	5.5	4.5	4
$C_{3\kappa}$	.05	2.8	13	80	190	330
$A_{\kappa 1}$	0 0003	0 0003	0 0002	0 0002	0 0001	0 0001
$A_{\kappa 2}$	04	0 03	0 02	0.01	0.01	0.01
$A_{\kappa 3}$	02	0 01	0 01	0 008	0 006	0 005

$A_{12}=0\ 05$ $A_{13}=2\ 2\times 10^4$ $A_{1\kappa}=1\times 10^{-4}$
$A_{21}=1\ 3$ $A_{23}=5\times 10^4$ $A_{2\kappa}=9\times 10^{-2}$
$A_{31}=1\ 5\times 10^8$ $A_{32}=1\ 3\times 10^7$ $A_{3\kappa}=7\times 10^{-2}$

The only fundamental difficulty involved in computing transition rates is for photoionizations from the 3D and 4P levels. Both of these involve the radiation intensity in Lyman-α of hydrogen, which is unknown in the lower regions of the chromosphere. We have computed the photoionization rates by means of Hinteregger's (1961) observed photon fluxes. At $T_e \approx 10^4$ and $n_e \approx 10^{11}$, photoionization from the 3D and 4P levels are of a magnitude comparable to that of collisional ionizations. For lower values of T_e the photoionization terms will dominate and become a critical part of the problem. Thus the unknown Lyman-α radiation field presents a basic difficulty in the low chromosphere, which we shall not consider in this paper.

For the stronger lines, opacities may be large, and it is not sufficient, in this case either, to use the observed radiation field. Ideally, we should solve the radiative-transfer equation simultaneously with the equilibrium equations. This approach is impractical, however, until we have more physical insight into the problem. Thus we shall introduce the radiation field into the equilibrium equations through the net radiative bracket ρ_{kj} (cf. Thomas and Athay 1961), which can be evaluated from solutions to the transfer equation. The solutions to the equilibrium equations can be carried out with ρ_{kj} as parameters, which effectively carries the solutions to the transfer equations as parameters. In radiative detailed balance, $\rho_{kj} = 0$, and for zero radiation field $\rho_{kj} = 1$. Under certain conditions, ρ_{kj} may fall outside these limits, but we do not expect this to be the case here.

Collisional transition rates for the bound levels are computed from cross-sections given by van Regemorter (1960, 1961), and for ionization from the 4P and 3D levels from the equation for the cross-section

$$\sigma_j(E_i) = \frac{2E_H}{aE_{0j}\sqrt{3}}\int_0^{E'} \bar{g}\,\frac{a_j(E')}{(E_{0j}+E')}\,dE' \tag{2}$$

given by Seaton (1961). Here E_H and E_{0j} are the ionization energies of hydrogen and level j, respectively, a is the fine-structure constant, a_j is the photo-recombination coefficient, and $\bar{g}$ is a correction factor. For the 4S level, a_j is small, and equation (2) probably underestimates σ_j because of the neglect of quadrupole terms. We therefore adopt, following a suggestion by Burgess (private communication), the experimental hydrogenic cross-section σ_{1S} corrected by a factor $\bar{g}/3$. The factor $\frac{1}{3}$ arises from an average for the contribution of quadrupole terms in the ionization cross-section and, for Ca II, $\bar{g}$ is taken as 0.26 (van Regemorter 1960). Representative collision rates are given in Table 1, along with the radiative rates.

Once values are adopted for the transition rates, the evaluation of n_j/n_κ is straightforward. Several alternative, equivalent expressions may be written for the solutions n_j/n_κ, depending on the particular expansion of the determinants of the coefficients and on the way various rates are combined. We adopt the following forms:

$$\frac{n_1}{n_\kappa} = \frac{P_\kappa\,[p_{\kappa 1}(1 - p_{23}p_{32}) + p_{\kappa 2}(p_{21} + p_{23}p_{31}) + p_{\kappa 3}(p_{31} + p_{32}p_{21})]}{P_1\,[p_{1\kappa}(1 - p_{23}p_{32}) + p_{12}(p_{2\kappa} + p_{23}p_{3\kappa}) + p_{13}(p_{3\kappa} + p_{32}p_{2\kappa})]}, \tag{3}$$

$$\frac{n_1}{n_2} = \frac{P_2\,[p_{21}(1 - p_{3\kappa}p_{\kappa 3}) + p_{23}(p_{31} + p_{3\kappa}p_{\kappa 1}) + p_{2\kappa}(p_{\kappa 1} + p_{\kappa 3}p_{31})]}{P_1\,[p_{12}(1 - p_{3\kappa}p_{\kappa 3}) + p_{13}(p_{32} + p_{3\kappa}p_{\kappa 2}) + p_{1\kappa}(p_{\kappa 2} + p_{\kappa 3}p_{32})]}, \tag{4}$$

and

$$\frac{n_1}{n_3} = \frac{P_3\,[p_{31}(1 - p_{2\kappa}p_{\kappa 2}) + p_{32}(p_{21} + p_{2\kappa}p_{\kappa 1}) + p_{3\kappa}(p_{\kappa 1} + p_{\kappa 2}p_{21})]}{P_1\,[p_{13}(1 - p_{2\kappa}p_{\kappa 2}) + p_{12}(p_{23} + p_{2\kappa}p_{\kappa 3}) + p_{1\kappa}(p_{\kappa 3} + p_{\kappa 2}p_{23})]}, \tag{5}$$

where $p_{kj} = P_{kj}/P_k$. It is helpful in subsequent discussion to reduce equations (3), (4), and (5) to simpler forms by restricting the range of n_e and T_e to within limits appropriate to the chromosphere but, at the same time, allowing ρ_{kj} to vary from 0 to 1. For $10^{10} \leq n_e \leq 10^{12}$, $1 \times 10^4 \leq T_e \leq 5 \times 10^4$, and $0 \leq \rho_{kj} \leq 1$, these equations reduce to

$$\frac{n_1}{n_\kappa} = \frac{P_{\kappa 2}(p_{21} + p_{23}p_{31})}{P_1\,[p_{1\kappa}(1 - p_{23}p_{32}) + (p_{12} + p_{13}p_{32})\,p_{2\kappa}]}, \tag{6}$$

$$\frac{n_1}{n_2} = \frac{P_2(p_{21} + p_{23}p_{31})}{P_1(p_{12} + p_{13}p_{32})}, \tag{7}$$

and

$$\frac{n_1}{n_3} = \frac{P_3(p_{31} + p_{32}p_{21})}{P_1(p_{13} + p_{12}p_{23})}. \tag{8}$$

Equations (7) and (8) are accurate to within about 1 per cent or better for the indicated range of n_e, T_e, and ρ_{kj}. Equation (6), however, is accurate to within only about 10 per cent. The term $P_{1\kappa}(1 - p_{23}p_{32})$ in equation (6) is important only when $T_e \geq 40000°$ and $n_e \leq 10^{11}$. The relative importance of the terms containing $p_{1\kappa}$ and $p_{2\kappa}$ depends strongly on ρ_{kj} and T_e, but only weakly on n_e.

a) Ionization and Excitation

In discussing the ionization properties of Ca II in the solar chromosphere it is convenient to consider two situations distinguished from each other by widely different opacities, τ, in the stronger spectral lines. Thus, we define:

Case I: τ_{31} and τ_{32} large and $\tau_{j\kappa}$ small, $j = 1, 2, 3$;

Case II: all τ's small.

In Case I we consider the solutions to the equilibrium equations with $\rho_{31} = \tau_{32} = 0$ and with the photoionization rates determined from the observed solar radiation field. In Case II, we consider the solutions with all radiative-transition rates evaluated from the observed radiation field.

At a height of 500 km in the chromosphere, the total hydrogen density is of the order of 10^{14} cm^{-3} (cf. Thomas and Athay 1961). If we take the relative abundance of calcium to hydrogen as 2×10^{-6} and a scale height of 10^8 cm, the maximum possible number of Ca II absorbing atoms in a radial column above 500 km is 2×10^{16} cm^{-2}. The absorption coefficient at the head of the free-bound continuum arising from the 3D level of Ca II is 1.7×10^{-17} cm^2, which gives a maximum opacity of 0.4. Similarly, the continua arising from the 4S and 4P levels would have maximum opacities of 2×10^{-3} and 0.1, respectively. The lines, on the other hand, have much larger absorption coefficients, and the opacities at line centers may well exceed unity by rather larger factors at 500 km. Hence the two cases we are considering represent the extremes of the conditions expected in the chromosphere.

(i) *Case I.*—Under conditions of detailed balance in the transitions 1–3 and 2–3, Case I, equation (6) reduces to

$$\frac{n_1}{n_\kappa} \approx \frac{A_\kappa (C_{21} + C_{23}C_{31}/C_3)}{(A_{2\kappa} + C_{2\kappa})(C_{12} + C_{32}C_{13}/C_3)} \equiv \frac{A_\kappa C_{21}}{(A_{2\kappa} + C_{2\kappa})C_{12}}, \tag{9}$$

where the A's are radiative rates and the C's are collisional rates. The second equality arises from fixed relationships among the C's that require $C_{23}C_{31}/C_{32}C_{13} \equiv C_{21}/C_{12}$. From equations (7) and (8) we obtain

$$\frac{n_1}{n_2} \approx \frac{C_{21} + C_{23}C_{31}/C_3}{C_{12} + C_{32}C_{13}/C_3} \equiv \frac{C_{21}}{C_{12}} \tag{10}$$

and

$$\frac{n_1}{n_3} \approx \frac{C_{31} + C_{21}C_{32}/C_2}{C_{13} + C_{12}C_{23}/C_2} \equiv \frac{C_{31}}{C_{13}}. \tag{11}$$

In Case I, therefore, the relative populations of levels 1, 2, and 3 follow a Boltzmann distribution at the local value of T_e. Stated in terms of the b_j factors, which measure departures from LTE, $b_1 = b_2 = b_3$.

We note from equation (9), however, that n_1/n_κ and hence b_1 depart sharply from their LTE value. The rates in Table 1 indicate that for $T_e \geq 1.5 \times 10^4$ and $n_e \geq 10^{11}$ we have $C_{2\kappa} > A_{2\kappa}$. Under these conditions n_1/n_κ is independent of n_e and $b_1 \propto n_e^{-1}$. Furthermore, when $C_{2\kappa} > A_{2\kappa}$, equation (9) gives $n_1/n_\kappa \propto \exp(x_1/kT_e)$, which requires that b_1 be essentially independent of T_e. Since $b_1 = b_2 = b_3$, the same conclusions apply to b_2 and b_3. For $T_e \leq 1 \times 10^4$, $A_{2\kappa} > C_{2\kappa}$, and n_1/n_κ becomes proportional to n_e but essentially independent of T_e. Hence, for low values of T_e, $b_1 \propto \exp(x_2/kT_e)$. This latter condition, however, assumes that the Lyman-α radiation field is independent of the local T_e. So long as $A_{2\kappa} > C_{2\kappa}$, both n_1/n_κ and b_1 are directly proportional to the intensity of Lyman-α radiation. Numerical values of n_1/n_κ and $b_1 n_e$ are given in Table 2. The values given for $T_e = 1 \times 10^4$ apply only for $n_e = 10^{11}$ and for the observed Lyman-α intensity.

Physically, equation (9) expresses the fact that ionization from the ground state occurs primarily by collisional excitation to level 2, either directly or via level 3, followed by ionization from level 2. Recombinations occur mainly to level 2, followed by collisional de-excitation to the ground state.

For $T_e \geq 1 \times 10^4$ and Case I, $n_2 \geq n_1$. Thus, for computing the degree of ionization of Ca II both levels 1 and 2 must be considered.

TABLE 2

COMPUTED VALUES OF n_j/n_κ AND b_1

n_j/n_κ	CASE	LOG n_e	$T_e \times 10^{-4}$: 1	1 5	2	3	4	5
n_1/n_κ	I	. .	1	0 03	2×10^{-3}	1×10^{-4}	3×10^{-5}	2×10^{-5}
	II	10	2	0 3	0 03	2×10^{-3}	7×10^{-4}	3×10^{-4}
		11	5	0 2	0 02	1×10^{-3}	3×10^{-4}	2×10^{-4}
		12	6	0 1	0 01	6×10^{-4}	2×10^{-4}	1×10^{-4}
n_2/n_κ	II	10	0 07	0 01	2×10^{-3}	1×10^{-4}	3×10^{-5}	2×10^{-5}
		11	0 5	0 03	3×10^{-3}	2×10^{-4}	5×10^{-5}	3×10^{-5}
		12	1 5	0 03	3×10^{-3}	3×10^{-4}	8×10^{-5}	5×10^{-5}
n_3/n_κ	II	10	3×10^{-4}	4×10^{-5}	5×10^{-6}	3×10^{-7}	1×10^{-7}	6×10^{-8}
		11	1×10^{-3}	5×10^{-5}	6×10^{-6}	5×10^{-7}	1×10^{-7}	5×10^{-8}
		12	3×10^{-3}	1×10^{-4}	2×10^{-5}	1×10^{-6}	3×10^{-7}	2×10^{-7}
$n_e b_1 \times 10^{-16}$	I	. .	0 3	1	1	1	1	2
$b_1 \times 10^{-6}$	II	10	0 05	1	2	2	3	6
		11	0 1	1	1	1	2	3
		12	0 1	0 5	0 7	0 7	1	2

(ii) *Case II.*—When the line opacity becomes small, as in Case II, the equilibrium of Ca II assumes a considerably different form. Equation (6), in this case, reduces to

$$\frac{n_1}{n_\kappa} = \frac{A_\kappa (A_{23} + C_{23})}{(A_{2\kappa} + C_{2\kappa})[C_{12} + (C_{13} + A_{13}) A_{32}/A_{31}] + C_{1\kappa}(A_{23} + C_{23})} . \quad (12)$$

Equations (7) and (8) reduce to

$$\frac{n_1}{n_2} = \frac{A_{23} + C_{23}}{C_{12} + (C_{13} + A_{13}) A_{32}/A_{31}} \quad (13)$$

and

$$\frac{n_1}{n_3} = \frac{A_{31} + A_{32}}{C_{12} + C_{13} + A_{13}} . \quad (14)$$

The major changes from Case I are a marked increase in n_1 and a marked decrease in n_3. On the other hand, n_2 differs only slightly from its values in Case I. The term containing $C_{1\kappa}$ in equation (12) is important only for $T_e \geq 4 \times 10^4$ and $n_e \leq 10^{10}$. Hence we note from equation (12) that ionizations and recombinations continue to occur mainly to and from level 2. Equations (12) and (13) both express the additional fact that the equilibrium between levels 1 and 2 occurs mainly by the exchange of electrons through level 3. The equilibrium between levels 1 and 3, however, occurs mainly by direct transitions between these two levels.

Numerical values of n_j/n_k and b_1 for Case II are given in Table 2 for a range of T_e and n_e. By comparison with Case I, at $n_e = 10^{11}$, n_1 has increased by about a factor of 10, n_2 is essentially unchanged, and n_3 is decreased by a factor of 50 at $T_e = 10^4$ and 400 at $T_e = 5 \times 10^4$. The curious behavior of n_1/n_κ with changes in n_e is of special interest. For $T_e \geq 1.5 \times 10^4$, n_1/n_κ decreases slowly as n_e increases. Since most of the electrons are in level 1 in this case, the ratio of Ca II to Ca III will show a similar dependence on n_e. To our knowledge, this is the first time such an effect has been noted.

The numerical values of b_1 and $n_e b_1$ in Table 2 are somewhat surprising. At first thought, one might expect values comparable with those of hydrogen, because of the similarity of ionization potentials. In fact, however, the values for Ca II are considerably larger than for hydrogen under conditions of high opacity. For example, in Case I $n_e b_1(\mathrm{H}) \approx 4 \times 10^{12}$ at $T_e = 1 \times 10^4$. In Case II the values of $b_1(\mathrm{H})$ and $b_1(\mathrm{Ca\ II})$ are quite comparable for high values of T_e, but for low values of T_e we have $b_1(\mathrm{Ca\ II}) \gg b_1(\mathrm{H})$.

b) Applications to Spicules and Fraunhofer Spectrum

A complete discussion of the equilibrium conditions in Ca II requires, of course, solutions to the radiative-transfer equations under the specific set of conditions being considered. We are currently carrying out computations along these lines in an attempt to account for the observed intensities of the Ca II lines, as well as those of hydrogen and He I, in chromospheric spicules. Nevertheless, the values in Table 2 give very useful results when applied to spicule observations. They are also useful for estimating the temperature conditions in the chromospheric regions where the centers of the Ca II lines in the Fraunhofer spectrum are formed.

TABLE 3

COMPUTED n_3 AND τ_{jk} FOR $n_e = 10^{11}$

QUANTITY	CASE	T_e					
		1×10^4	1 5	2	3	4	5
τ_{3933}	I	5	0 3	0 02	0 001	0 0003	0 0002
	II	9	2	0 3	0 01	0.003	0 002
τ_{8542}	I	0 5	0 1	0 01	0 0005	0 0002	0 0001
	II	0 2	0 07	0 01	0 0005	0 0002	0 0001
n_3	I	5×10^3	1×10^3	200	20	10	7
	II	8	7	1	0 1	0 02	0 01

Table 3 contains computed opacities in the K line and the λ 8542 line of the infrared triplet arising from the transition 3–2 in our model atom. The opacities are computed by using a scale height of 5×10^7 cm and a Doppler width of the absorption coefficient corresponding to a broadening velocity of 5 km/sec. We have assumed that the total hydrogen density is equal to n_e and that the abundance of Ca relative to hydrogen is 2×10^{-6}.

We note from Table 3 that $\tau_{3933} = 1$ occurs in Case II for $T_e \approx 1.5 \times 10^4$ for $n_e = 10^{11}$ and the assumed scale height and Doppler width. This indicates that the core of the K lines of Ca II is formed somewhat above the top of the hydrogen temperature plateau in the chromosphere (cf. Thomas and Athay 1961, chap. 5) and that higher-temperature regions of the chromosphere are essentially transparent in the K line, in agreement with our former result (Zirker 1962). We note, further, that $\tau_{8542} = 1$ at $T_e \leq 10^4$, which places the region where the core of λ 8542 is formed also in the hydrogen plateau. The total opacity of the chromosphere down to a height of 500 km, where $T_e \approx 6000°$ and $n_H \approx 10^{14}$ (Thomas and Athay 1961, chap. 6), is of the order of 10^4 in the K line and 10^3 in λ 8542, as may be seen by a simple extension of the results in Table 3, using the fact that τ is essentially proportional to n_H at the lower temperatures and assuming that most of the Ca atoms are still singly ionized.

In spicules, there is strong evidence that the K line and λ 8542 are broadened by self-absorption (Athay 1961). The broadening of these lines in excess of that of λ 8498 suggests that τ is of the order of unity in λ 8542 and greater than about 10 in the K line. These serve as boundary conditions on the spicule model. A further useful condition comes from the observed intensity of λ 8498, which indicated that $n_3 \approx 10^2$. The electron density in spicules may be as high as 10^{12} (Thomas and Athay 1961, chap. 7), which would raise the values of τ_{3933} and τ_{8542} in Table 3 by factors of about 5 and 10, respectively. Thus the results in Table 3 require $T_e \approx 15000°$ on this basis. At $n_e = 10^{12}$, the values of n_3 in Table 3 increase by factors of about 10 in Case I and about 30 in Case II. Thus we obtain $n_3 = 10^2$ for $T_e \approx 30000°$ in Case I and $\approx 15000°$ in Case II. It seems, then, that the best value of T_e is near 15000° in the regions within the spicules where the Ca II emission arises and that Case II is the better approximation.

Profiles of hydrogen, helium, and oxygen lines observed in spicules seem to indicate values of T_e much higher than the estimate of T_e in the regions where the Ca II lines are emitted. It may be possible to reconcile these conflicting temperature estimates by postulating that there are strong temperature gradients within individual spicules and that lines of different ions may originate in different parts of the spicule. A more detailed treatment of the spicule problem is in progress and will be published at a later date.

III. SOURCE FUNCTIONS

Although we shall not make specific application in this paper of the source functions, S_{jk}, for the Ca II lines, it is convenient to discuss them in connection with the equilibrium equations. We adopt the form

$$S_{jk} = \frac{2 h\nu^3 \varpi_j n_k}{c^2 \varpi_k n_j} \tag{15}$$

(Thomas 1957), which neglects stimulated emissions. From equations (7) and (9), we obtain, for the H and K lines,

$$S_{31} = \frac{\int J_\nu \phi_\nu d\nu + \epsilon_{31} B_{31} + \delta_{31}}{1 + \epsilon_{31} + \eta_{31}} = \frac{\epsilon_{31} B_{31} + \delta_{31}}{\rho_{31} + \epsilon_{31} + \eta_{31}}, \tag{16}$$

where J_ν is the radiative intensity averaged over angle and ϕ_ν is the normalized absorption coefficient; B is the Planck function,

$$\epsilon_{31} = \frac{C_{31}}{A_{31}} \approx 2 \times 10^{-4} \quad \text{for} \quad n_e = 10^{11},$$

$$\delta_{31} = \frac{2 h\nu^3}{c^2} \frac{\varpi_1}{\varpi_3} \frac{C_{12} P_{23}}{A_{31} P_2},$$

and

$$\eta_{31} = \frac{C_{21} P_{32}}{A_{31} P_2}.$$

The source function for the 3–2 transition is of the same form as equation (16), where

$$\epsilon_{32}=\frac{C_{32}}{A_{32}}\approx 3\times 10^{-3} \qquad \text{for} \qquad n_e=10^{11},$$

$$\delta_{32}=\frac{2\,h\nu^3\varpi_2 C_{21}P_{13}}{c^2\varpi_3 A_{32}P_1},$$

and

$$\eta_{32}=\frac{C_{12}P_{31}}{A_{32}P_1}.$$

Since $\delta_{31}/\eta_{31}=\epsilon_{31}B/\epsilon_{31}=B_{31}$ and $\delta_{32}/\eta_{32}=\epsilon_{32}B_{32}/\epsilon_{32}=B_{32}$ when ρ_{31} and $\rho_{32}=0$, the source functions reduce to Planck functions. This was implied by the result $b_1=b_2=b_3$ in the preceding discussion of Case I.

In Case II for $T_e\leq 1.5\times 10^4$, which is the region of interest for the Fraunhofer lines, $\delta_{31}\geq\epsilon_{31}B_{31}$ and $\delta_{32}\approx\epsilon_{32}B_{32}$. Also, $\eta_{31}\gg\epsilon_{31}$ and $\eta_{32}\gg\epsilon_{32}$. Hence we note, as has Jefferies (1960), that a two-level atom, neglecting the 3D levels, is not a sufficiently good approximation for Ca II, and that S_{31} is not of the simple form

$$S_{31}=\frac{\int J_\nu\phi_\nu d\nu+\epsilon_{31}B_{31}}{1+\epsilon_{31}}, \tag{17}$$

as used by Jefferies and Thomas (1960). This is particularly true in the atmospheric regions where the centers of the lines are formed, where Case II should be a better approximation than Case I.

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