

AN EXAMINATION OF THE STEADY-STATE MODEL IN THE LIGHT OF SOME RECENT OBSERVATIONS OF RADIO SOURCES

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(Received 1961 January 30)

Summary

The results of some recent observations of radio sources have been used to investigate the "luminosity function" for the sources. Without making any assumptions about their nature, it can be shown from radio observations alone that most of the sources lying in any given range of flux density are extragalactic with an emission at 178 Mc/s which exceeds 10^{24} watts (c/s) $^{-1}$ ster $^{-1}$. If it is assumed either that the physical dimensions or that the optical luminosities of the sources are comparable with those of the Galaxy, then the emission must lie in the range 3×10^{25} – 10^{27} watts (c/s) $^{-1}$ ster $^{-1}$.

From these figures it is possible to derive the expected number-flux density relationship according to different cosmological models and special consideration has been given to the predictions of the steady-state model.

With the new Cambridge interferometer it has become possible to observe sources considerably weaker than those reached in earlier surveys, and hence to make a more accurate determination of the actual number-flux density relationship; the new data has also allowed more detailed corrections for the effects of extended sources and source clustering to be made.

A comparison of these observational results with those predicted by the steady-state model shows a marked discrepancy, the number of sources observed with a flux density in the range 0.5 to 2×10^{-26} watts (c/s) $^{-1}$ m $^{-2}$ being at least 3 ± 0.5 times that predicted by the model.

No attempt has been made to select an alternative model to account for the observations, but the results appear to provide conclusive evidence against the steady-state model.

1. *Introduction.*—Previous observations of radio sources situated more than 20° from the galactic equator have shown there to be an isotropic distribution, with some evidence for non-uniformity in depth (Shakeshaft, Ryle, Baldwin, Elsmore and Thomson 1954; Mills, Slee and Hill 1958; Edge, Scheuer and Shakeshaft 1958; Edge, Shakeshaft, McAdam, Baldwin and Archer 1959). The possibility that this latter result is due to effects of the red-shift was first discussed by Ryle and Scheuer (1955).

A new series of observations has now been made at the Mullard Radio Astronomy Observatory with an instrument having considerably greater resolving power and sensitivity than those used in the previous surveys.

The results obtained are reliable over a considerably greater range of flux density than those of the earlier surveys, and they also allow an examination to be made of selection effects which may have been present previously. In particular analyses have been made of the distribution of the sources in relation to their

surface brightness (Leslie 1961a) to find the effect of partial resolution of extended sources, and also the effects which might be caused by source clustering (Leslie 1961b).

The combination of the results obtained from "whole-sky" surveys made with intermediate resolving power (Elsmore, Ryle and Leslie 1959; Leslie 1961a) with those of a number of limited areas of sky made with the full resolving power of the instrument (Scott, Ryle and Hewish 1961) make it possible to obtain a greatly improved determination of the number-flux density ($N-S$) relationship (Scott and Ryle 1961). This relationship has been extended to still smaller flux densities by applying to the survey of high resolution the statistical method of analysis introduced by Scheuer (1957). This technique has been developed by Hewish (1961) who has shown that if, as in the present observations, the noise level is small, the method can provide information on the $N-S$ relationship even when there is about one source per beam area. A number of authors have shown that a study of individual sources can only give a reliable estimate of source counts for flux densities at which there is not more than one source per 20 beam areas.

In the present observations it has been possible to extend the analysis to flux densities at which there are about 2×10^4 sources per steradian. The results confirm the previous conclusion of an apparent non-uniformity in depth.

In addition to these observations at a frequency of 178 Mc/s, further measurements of the spectral distribution both of radio sources (Whitfield 1957; Harris and Roberts 1960) and of the background emission (Costain 1960) are now available which are relevant to the present investigation.

Before applying the new data to the cosmological problem it is necessary to re-examine the question of the radio luminosity function of the sources. In a previous paper (Ryle 1958) it was shown that, even in the absence of optical identifications, a number of different radio observations could be combined to set restrictions on the ranges of luminosity within which the majority of the observed sources must lie. Although the number of optical identifications has since been increased, it still represents only a small fraction of the sources known; as will be shown in Section 2, however, the optical data are valuable in extending the rigorous, though more conservative, restrictions on the radio luminosity function set by the new radio data. It is now clear that few of the sources are within the Galaxy, and that most of those observed are extra-galactic objects having a radio luminosity approaching that of the Cygnus-A source.

The importance of the red-shift in modifying the number-flux density counts is then evident and consideration is given in Section 3 to the predicted curves for two particular cosmological models: (a) the steady-state model and (b) the Einstein-de Sitter model.

The results of these computations are compared in Section 4 with the observed counts of sources paying particular attention to the observational test of the steady-state model.

2. *The derivation of the radio luminosity.*—Considerable progress has been made in the identification of radio sources with optical objects (Dewhurst 1958, 1961; Bolton 1960), although the number of identified sources is still only a small fraction of the total. It is, however, now clear that most sources are extremely faint optical objects (Minkowski 1960; Long and Marks 1961) and the observations are consistent with a radio luminosity function in which most of the sources observed in a given range of flux density produce a radio emission which

is at least 5 per cent that of Cygnus-A. This interpretation is supported by theoretical arguments concerning the mechanism and supply of energy for the emission, the gravitational potential of the source, and the minimum optical emission expected (Tunmer 1960).

As will be seen later, the luminosity function derived from optical work is entirely consistent with the limits derived from the radio data, but since it must be based on a small fraction of the sources the distribution might be seriously in error if a class of radio source exists having a low intrinsic optical luminosity.

The observation of additional sources of small angular size in the area of sky covered by the "spur" of galactic emission at $l^{\text{II}} = 30^\circ$ was first reported by Edge *et al.* (1959) and this result has been confirmed by recent unpublished observations at Cambridge. The presence at high galactic latitudes of radio sources which are presumably in the Galaxy and which are apparently unrelated to optical objects must raise the question of whether there are other optically invisible galactic sources.

A further difficulty is illustrated by the problem of accounting for the source Hercules-A, which recent observations have shown to be a double source with an angular separation of about $2'$ arc between the components (Boishot 1960; Williams, Dewhirst and Leslie 1961); the sources may represent a similar situation to the Cygnus-A object, although this explanation would require a physical separation of the two emitting regions about 200 kpc to either side of the optical galaxy. If the intensities of the two components had differed by more than 3:1 it would have been difficult to recognize the source as double and the apparent radio position would have been displaced from the optical galaxy by several times the probable error. In such circumstances a very low limit would have been wrongly attributed to the optical luminosity of the related object.

Similar difficulties also apply to the derivation of the distance and hence the radio luminosity from measurements of the angular diameter of a source. The complex nature and large extent of Hercules-A (and to a lesser degree Cygnus-A) suggest that the distances of similar but more remote objects might be seriously underestimated. On the other hand a source of small physical size but having a large volume emissivity might be interpreted as a more powerful and distant source if it were assumed to be of galactic dimensions.

For these reasons it is important to re-examine the limits which may be set to the radio luminosity function from radio observations alone. Having established definite but perhaps less stringent limits in this way, the evidence provided by optical observations and from theories of the radio emission may allow closer restrictions to be set.

(a) *The evidence from radio observations*

In an analysis based on the results of earlier surveys it was shown (Ryle 1958) that limits could be set to the radio luminosity (P) from a correlation of the following information:

- (i) The number of sources observed in a given range of flux density.
- (ii) The average volume emissivity of different regions of space required to account for the observed background emission.
- (iii) The observed isotropic distribution of sources situated at galactic latitudes $> 20^\circ$.

Using the earlier observations at a frequency of 81.5 Mc/s, it was shown that most of the sources must belong either to:

(i) a class of galactic object having $P \simeq 10^{14}$ watt (c/s) $^{-1}$ ster $^{-1}$ (at 81.5 Mc/s) and an average space density throughout the galactic halo of $\rho \simeq 10^{-7}$ pc $^{-3}$; or

(ii) a class of powerful extragalactic emitters, having $P > 10^{23}$ watts (c/s) $^{-1}$ ster $^{-1}$ and $\rho < 10^{-19}$ pc $^{-3}$.

This analysis will now be extended in the light of the new observations which are available. Since most of these observations were made at a frequency of 178 Mc/s it will be convenient to give the numerical values derived for source luminosity, volume emissivity etc. appropriate to this frequency; these quantities will differ by a factor of about two from the corresponding earlier figures. It will also be convenient, when considering extragalactic sources, to adopt the more recent figure of 100 km sec $^{-1}$ Mpc $^{-1}$ for Hubble's constant (Sandage 1958) instead of the value 180 km sec $^{-1}$ Mpc $^{-1}$, previously used.

As before, a uniform but random distribution will be assumed for the region containing the more intense sources, with a dispersion of the luminosity of the form:

$$n(P) \propto \exp [- (\log_{10} P/P_0)^2 / 2]$$

where $n(P)$ represents the number of sources of luminosity P which occur in a sample producing flux densities within a given range. It may be rewritten to give the true space density $\rho(P)$ of sources of different P :

$$\rho(P) \propto P^{-3/2} \exp [- (\log_{10} P/P_0)^2 / 2].$$

By analogy with an undispersed luminosity function the constant of proportionality in the first relation may be written in the form $\rho_0 P_0^{3/2}$; ρ_0 is then seen to represent a weighted mean density of sources. In later considerations it will be convenient to work in terms of ρ_0 and P_0 .

This dispersion covers a range of P similar to that used in the earlier analysis but its form is more probable. A smaller dispersion seems unlikely in view of the far larger values evident in optical stars and galaxies; the adopted dispersion is comparable with that found amongst identified radio sources (Long and Marks 1961). If the actual dispersion were greater than that adopted it would impose more stringent restrictions than those derived below.

(b) *The number-flux density relation*

At distances corresponding to small red-shift the expected number of sources N per unit solid angle which have a flux density greater than S is given by:

$$N = \frac{1}{3} \rho_0 P_0^{3/2} S^{-3/2}.$$

As already indicated the observations are not consistent with a uniform spatial density, but an observational relationship between N and S can be established at a large value of N where the statistical errors are small and yet not near the limit of the observations. The value of $\rho_0 P_0^{3/2}$ derived in this way will be appropriate in the following analysis in which the source counts are related to the observations of the isotropy of the weak sources and to their contribution to the integrated emission.

Using the recent observations a value of $\rho_0 P_0^{3/2} = 1.4 \times 10^{14}$ is found where ρ_0 is expressed in sources pc $^{-3}$ and P_0 in watts (c/s) $^{-1}$ ster $^{-1}$ at a frequency of 178 Mc/s. This result is shown in a plot of $\log P_0$ against $\log \rho_0$ in Fig. 1. Any class of source whose radio luminosity and space density are represented by a point on this line will satisfy the general observations of the number of sources falling in a given range of flux density, although not of course the non-uniformity in the observed relationship.

(c) Isotropy

The observations have shown that, except for low galactic latitudes and the area of the "spur" in the galactic emission at $l^{\text{II}} = 30^\circ$, the sources are distributed isotropically. As has already been demonstrated (Ryle 1958) most of the sources must then either be extragalactic or they must be in the galactic halo. In order to establish closer limits on the spatial density of the sources in the latter case, some new observations have been made (Scott, Ryle and Hewish 1961) which were designed to detect any differences in the distribution of sources in two areas of sky chosen to represent a large and small extent of the galactic halo. It was shown previously that if the smaller extent is r_1 then, in the absence of dispersion in the luminosity, the source distribution will appear different in the two areas if the source density $\rho < 3N/r_1^3$, where N represents the number of sources per steradian which can be observed.

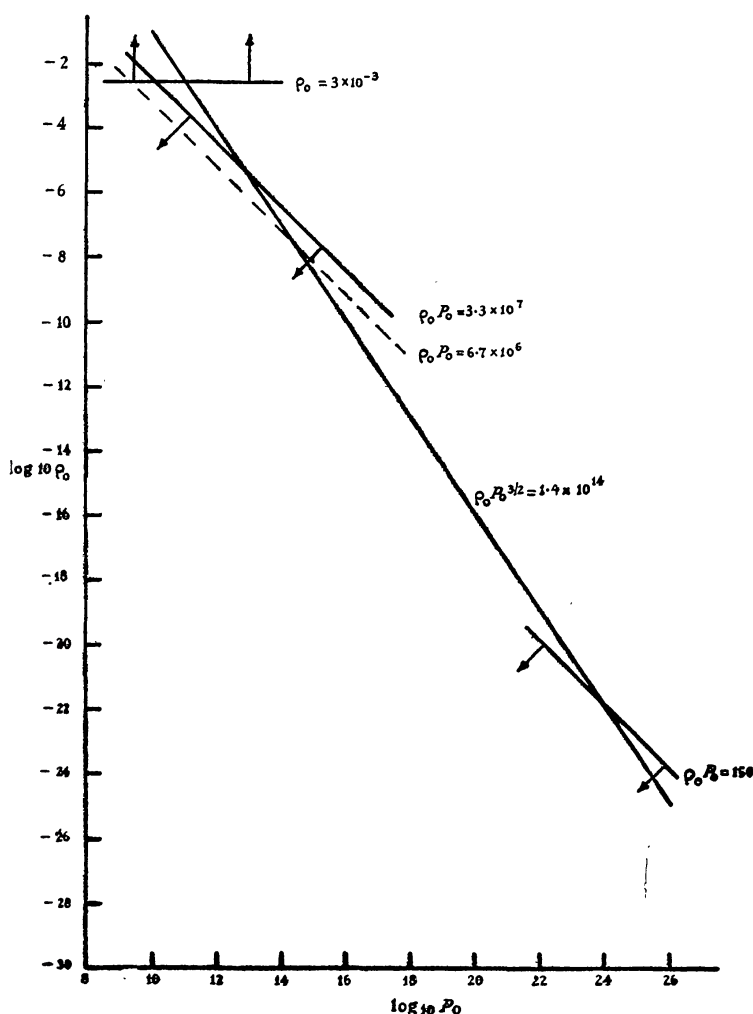


FIG. 1.—Diagram relating the values of the effective radio luminosity P_0 and space density ρ_0 which are compatible with the counts of sources falling in a given range of flux density. The restrictions set by the observed isotropy of the sources, and the maximum contribution to the background emission, are also known.

Observations were made of a strip of sky $02^\circ < \delta < 08^\circ$, and two regions $\alpha = 12^h$ to 16^h and $\alpha = 00^h$ to 04^h were analysed by the statistical method (Scheuer 1957). The positions of the two areas in relation to the emitting region associated with the galactic halo (Baldwin 1955) are shown in Fig. 2.

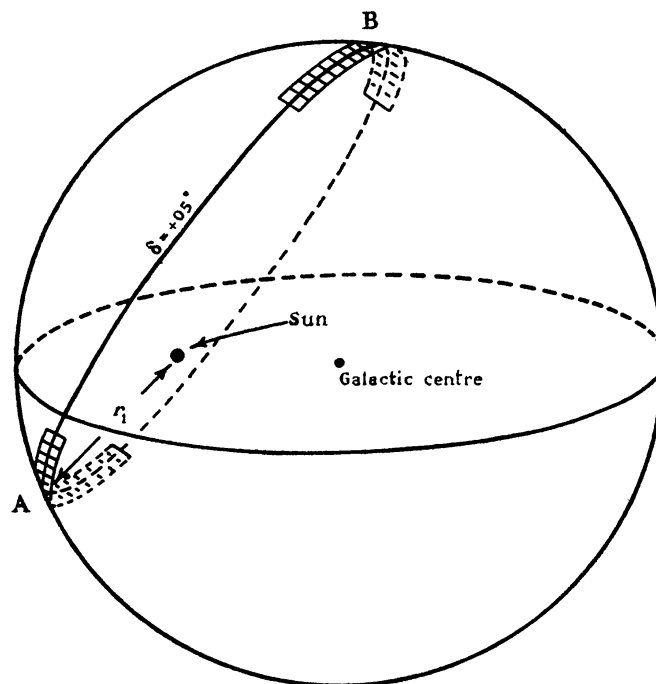


FIG. 2.—The region of emission associated with the galactic halo, showing the two areas examined ($r_1 \simeq 5$ kpc).

Centre of A: $l^\text{II} \simeq 160^\circ$; $b^\text{II} \simeq -55^\circ$.

Centre of B: $l^\text{II} \simeq 355^\circ$; $b^\text{II} \simeq 45^\circ$.

The results of this analysis are shown in Fig. 3 and show no significant differences in the histograms derived for the two areas. Hewish (1961) has computed the theoretical curves for a number of different models of source distribution and in some of these a cut-off of sources below a certain flux-density was adopted to simulate the effect to be expected in the present case. Two of his theoretical curves are shown in Fig. 3; these correspond to similar models, but one of them has a cut-off at 2×10^4 sources/steradian. The differences between the theoretical curves are considerably greater than in the observed curves and it can therefore be concluded that the observations indicate no anisotropy at 2×10^4 sources per steradian.

This result shows that if the sources are mostly in the galactic halo, then with the adopted dispersion of luminosity, the effective space density ρ_0 must be at least 3×10^{-3} pc $^{-3}$.

(d) *The integrated emission from the sources*

Observations of the distribution of the background radiation have allowed the derivation of a model of the galactic emission, and hence the determination of the average emission per unit volume in different parts of the galaxy and also in extra-galactic space (Baldwin 1955). More recently, Costain (1960) has measured

the spectral distribution of some of the components and has thus been able to estimate the volume emissivity at different frequencies, both within the galactic halo and in extra-galactic space.

Without having to know the mechanism of the emission from either of these regions it is clear that the contribution made by the radio sources ($\Sigma \rho P$) cannot exceed, at any frequency, the volume emissivity determined in this way; this analysis then allows an upper limit for $\rho_0 P_0$ to be derived.

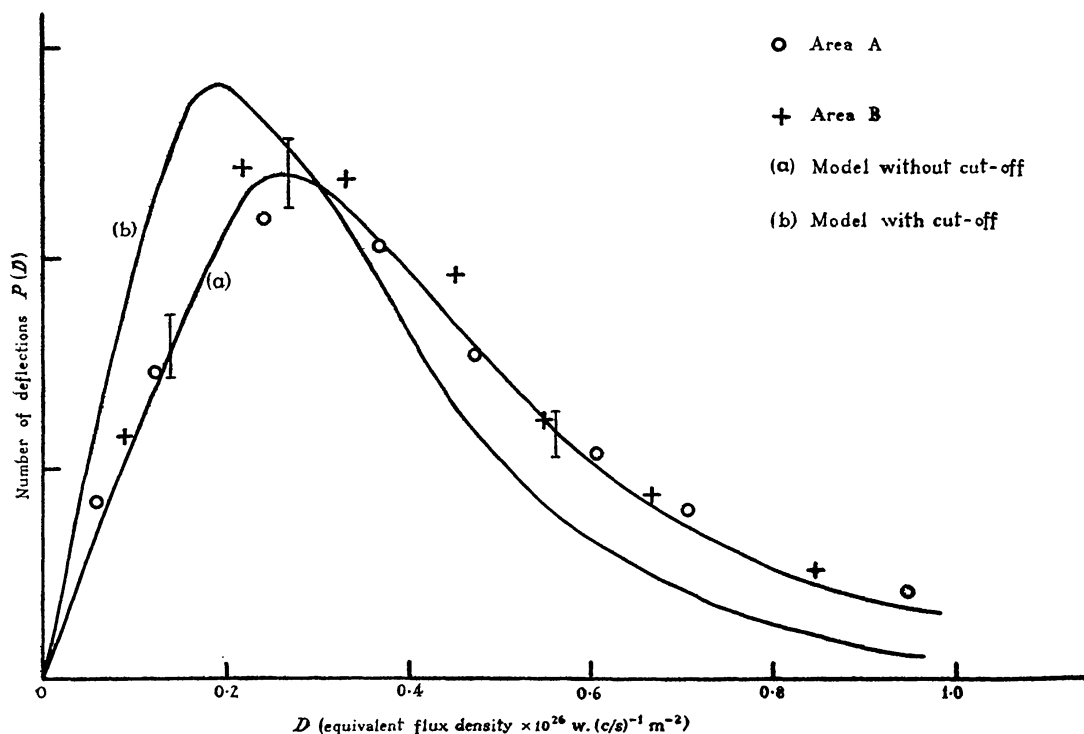


FIG. 3.—Histograms of record deflections for the two areas *A* (anti-centre) and *B* (centre) shown in Fig. 2. The theoretical curves are also shown for two models, one of which has a cut-off at $S = 0.15 \times 10^{-26}$ watts $(c/s)^{-1} m^{-2}$ (2×10^4 sources per steradian).

Except for some of those at low galactic latitudes, most radio sources appear to have an emission which varies as $S \propto f^{-(0.8 \pm 0.2)}$ over the range of frequencies 38–960 Mc/s (Whitfield 1957; Harris and Roberts 1960). The radiation from the galactic halo, on the other hand, has been observed in the range 38–178 Mc/s to have a brightness temperature $T_b \propto f^{-(2.37 \pm 0.04)}$ a relation which corresponds to a volume emissivity proportional to $f^{-(0.37 \pm 0.04)}$. It is therefore clear that at the frequency of the present observations (178 Mc/s) only a fraction $(38/178)^{(0.8-0.37)}$ of the emission from the halo can be due to the sources.

With the adopted dispersion in source luminosity this leads to an upper limit for the volume emissivity of $\rho_0 P_0 < 3.3 \times 10^7$ watts $(c/s)^{-1} pc^{-3}$; this limit is shown in Fig. 1 which shows to it be incompatible with the lower limit of ρ_0 derived from the observed isotropy of the sources.

This analysis may be extended in the light of recent observations of the galactic emission which have now been made over a range of frequencies of 100:1, and which can be fitted with a uniform spectral index of -0.37 (Costain 1960; Chapman 1960; Seeger 1960). Without the adoption of a peculiar distribution of the spectrum of the sources outside the range 38–178 Mc/s, it is difficult to suppose

that they can make a significant contribution to the observed galactic radiation; the combination of the radiation from two types of source having different spectral indices can produce a uniform spectral index only if the spectra of the two components show opposite departures from linearity at approximately the same frequency. With the uncertainties in the present observations it seems unlikely that the sources could contribute more than 10 per cent of the emission from the galactic halo at a frequency of 178 Mc/s. The corresponding upper limit of $\rho_0 P_0 < 6.7 \times 10^6$ watts (c/s) $^{-1}$ ster $^{-1}$ pc $^{-3}$ is shown dotted in Fig. 1.

The discrepancy between the derived limits of $\rho_0 P_0$, the lower limit of ρ_0 and the value of $\rho_0 P_0^{3/2}$ therefore makes it difficult to suppose that more than a few per cent of the sources are situated within the galactic halo.

A similar argument may now be applied to the case in which most of the sources are extra-galactic since their contribution to the total volume emissivity of extra-galactic space must not lead to a total sky brightness greater than that observed. The relation between volume emissivity and sky brightness now depends on the cosmological model adopted (Shakeshaft 1954; Priest 1954).

It is difficult to distinguish observationally between an extra-galactic component of the background emission and one due to the outermost shells of the galactic halo since both give rise to a distribution which is nearly isotropic when viewed from the position of the Sun. Owing to the differences in the spectra of the galactic halo and of the sources it is however, possible to determine the maximum contribution which sources having a given spectral distribution may make to the extragalactic emission.

Costain's observations (1960) can be used in this way to derive an upper limit of 25°K for the maximum sky brightness at 178 Mc/s due to sources having a spectral distribution $S \propto f^{-0.8}$. In the case of the steady-state model and with the adopted dispersion in luminosity this result leads to an upper limit of $\rho_0 P_0 < 150$ watts (c/s) $^{-1}$ ster $^{-1}$ pc $^{-3}$. The intercept of this line with the line $\rho_0 P_0^{3/2} = 1.4 \times 10^{14}$ (Fig. 1) leads to a lower limit of $P_0 > 10^{24}$ watts (c/s) $^{-1}$ ster $^{-1}$.

In most evolutionary models the total sky brightness converges more slowly than for the steady-state model and a lower limit must be set to $\rho_0 P_0$; consequently the lower limit of the effective source luminosity P_0 must be increased for such models.

(e) Discussion

The extension of the previous analysis which has been carried out using the new observational material has effectively eliminated the possibility that a significant fraction of the observed sources are situated within the Galaxy. This possibility is in any case hard to accept in view of the observed non-uniform distribution in depth; if more than a small fraction of the sources were within the galactic halo it would be necessary to postulate a progressive change in luminosity or spatial density of the sources with distance from the Sun, up to several kiloparsec, if both the number-flux density counts and the isotropy were to be explained.

It may therefore be concluded that most of the sources are extra-galactic with an effective luminosity $P_0 > 10^{24}$ watts (c/s) $^{-1}$ ster $^{-1}$.

It is interesting to compare the limiting luminosity function derived in this way (Fig. 4(a)) with those derived by other arguments.

Measurements of the surface brightness of the sources (Allen, Palmer and Rowson 1960; Leslie 1961 a) allow a luminosity function to be derived if it is assumed that all sources are of galactic dimensions (Fig. 4(b)). An analysis both of sources identified with optical objects, and of the limiting optical magnitudes of unidentified sources, has enabled Long and Marks (1961) to derive a luminosity function on the supposition that all sources have an absolute optical magnitude $M_{pg} = -20.6 \pm 1.0$, a range which includes all the identified sources whose distances have been measured. Two limiting cases of their distribution are shown in Fig. 4(c). In connection with their results, it is interesting to note that sources of intermediate radio luminosity, such as NGC 1275 and M87, cannot represent a large fraction of those observed in a given range of flux-density; some authors have suggested that the presence of such sources would prevent the recognition of any cosmological effect due to more powerful sources.

Minkowski (1960), Bolton (1960) and Mills (1960) have also derived luminosity functions on the supposition that the sources which have been identified with optical objects represent a typical sample; their curves replotted to give the distribution in luminosity of the sources falling in a given range of flux density are shown in Fig. 4(d).

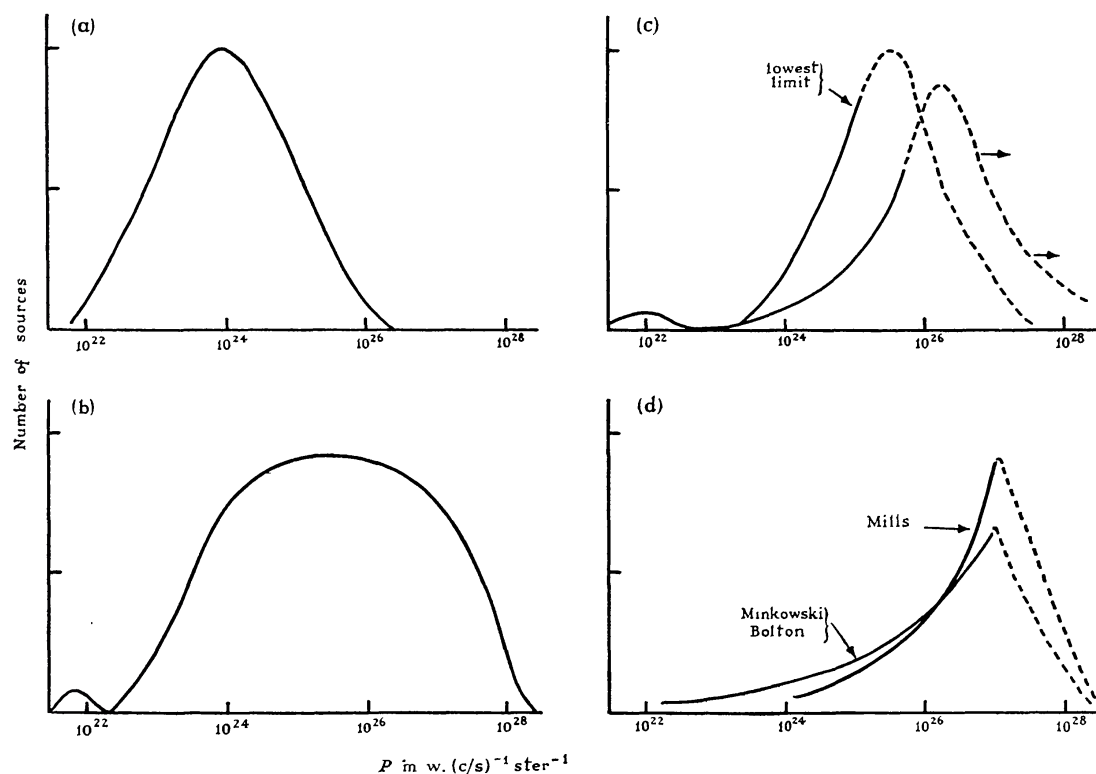


FIG. 4.—The distribution in radio luminosity (P) of the sources falling in a given range of flux density.

(a) Lower limits derived from radio observations without assumptions concerning the nature of the sources.

(b) Model derived from consideration of the distribution of the surface brightness of the sources.

(c) Model derived from limits of $(m_r - m_{pg})$ for identified and unidentified sources (Long and Marks 1961).

(d) Models derived on basis of identified sources (Minkowski 1960; Bolton 1960; Mills 1960).

All the available evidence therefore indicates that most of the sources observed in a given range of flux density have a luminosity P of 10^{24} – 10^{27} watts $(\text{c/s})^{-1}\text{ster}^{-1}$. Whilst arguments based on measurements of surface brightness and of optical luminosity are both dependent on assumptions concerning the nature of the unidentified sources, the results they provide agree with the lower limits which it has been possible to derive without such assumptions from the radio observations alone.

Even if this more conservative limit is adopted, the region investigated in the survey of individual sources with $S \geq 2 \times 10^{-26}$ watts $(\text{c/s})^{-1}\text{m}^{-2}$ extends to distances of at least 10^9 pc. Any suggestion that the apparent non-uniformity in the distribution of the sources should be attributed to a local irregularity must now be discounted since the volume of space involved represents an appreciable fraction of the observable universe.

In the next section, an investigation is made into the effects of the red-shift on the counts of sources having luminosities of this order.

3. *The predicted number-flux density relationship.*—Several authors have examined the number-flux density ($N-S$) relationship expected for radio sources of given luminosity and have derived curves appropriate to a number of different cosmological models (McVittie 1957; Priest 1958; Davidson 1959).

In most of these models evolutionary effects have been neglected and the density of radio sources has been supposed to vary in the same way as that of galaxies. The problem of accounting for the radio emission from powerful extra-galactic sources of the type now envisaged has however, been examined by a number of authors (Greenstein 1955; Burbidge and Burbidge 1957; Ambartsumian 1958; Burbidge 1958; Shklovsky 1960; Tunmer 1960; Minkowski 1961). The conditions required to account for the emission involve a very considerable supply of energy and some of these authors have concluded that the kinetic energy of the gaseous content of colliding galaxies may represent the only adequate source—a conclusion supported by the nature of a number of the identified sources.

If the radio sources have their origin in galactic collisions then their space density is likely to show more marked changes than that of galaxies, and in the limit for field galaxies would vary as the square of the mean galactic density (Priest 1958; Davidson 1959).

In the present analysis two particular cosmological models have been considered.

- (a) The steady-state model.
- (b) The Einstein-de Sitter model.

In the latter case evolutionary effects and the variation of the total number of sources may both be important.

Curves have been plotted in Fig. 5 showing the variation with S of N/N_0 , the ratio of the number of sources observed having a flux density greater than S , compared with the number which would be observed in a static Euclidean universe. In the Einstein-de Sitter model two limiting cases have been considered in which the density of sources is supposed to vary (i) as the density of galaxies and (ii) as the square of the galactic density.

The relations used in deriving these curves have been given by a number of authors (e.g. Bondi 1952; Davidson 1959). It is convenient to quote the formulae in terms of the red shift $(1+z)$ of the source and the associated distance r which is a function of the model adopted. ρ is the local density of sources and P the luminosity per steradian of the source which is assumed to obey a spectral law $P \propto f^{-x}$.

For all the models considered the flux density per unit bandwidth is

$$P/r^2(1+z)^{1+x}$$

where P has the value appropriate to the frequency of reception, not that of the original emission. In the steady-state model, which has a constant number of sources in a given proper volume, the number of sources in the range r to $(r+dr)$ is $4\pi r^2 dr / (1+z)^3$ with $r = cz/H$ (where H = Hubble's constant, c = velocity of light). In relativistic models the number of galaxies per coordinate volume is constant and for the particular model chosen, the Einstein-de Sitter model, the corresponding relation is $4\pi r^2 dr$ with $r = 2c/H (1 - (1+z)^{-1/2})$. The limiting case of an Einstein-de Sitter universe in which sources are produced by the collisions of galaxies leads to a number of sources which is proportional to the square of the proper density.

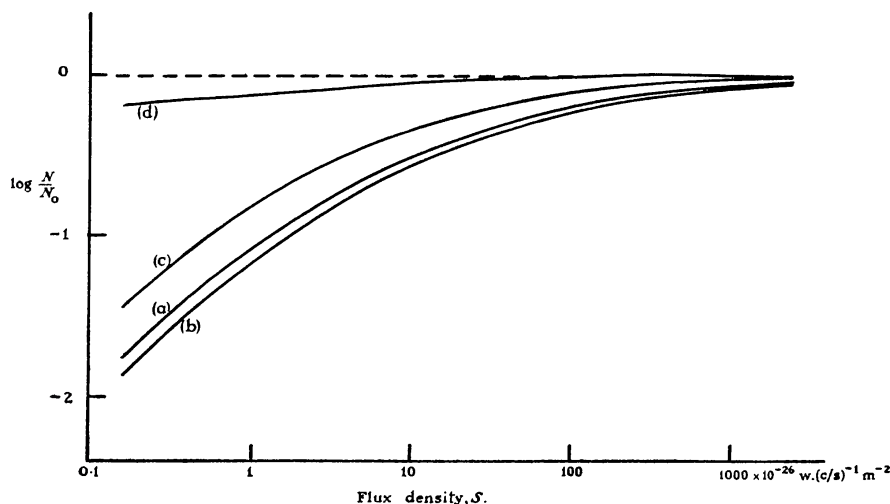


FIG. 5.—Curves showing the variation of the number of sources (N) having a flux density greater than S , compared to the number (N_0) on a static Euclidean model.

(a) Steady-state ($x = -0.5$).

(b) Steady-state ($x = -1.0$).

(c) Einstein-de Sitter (number of sources proportional to mean galactic density).

(d) Einstein-de Sitter (number of sources proportional to square of galactic density).

The method of presenting the results, in which the numbers of sources are compared with those in a corresponding static Euclidean universe, has the advantage that the predictions of different models may be compared without a knowledge of the absolute local density of sources.

The curves do not depend critically on the value of the spectral index x , as can be seen from Fig. 5, which shows two curves appropriate to the steady-state model with a non-dispersed luminosity $P = 10^{26}$ watts $(c/s)^{-1}$ ster $^{-1}$, and $x = -0.5$ and -1.0 . The curves for the Einstein-de Sitter models in which the number of sources is assumed to be proportional (i) to the density of galaxies and (ii) to the square of the density, are also shown in Fig. 5; a mean spectral index $x = -0.8$ has been adopted and no evolutionary effects have been included.

The purpose of the present paper is not to determine a particular model which accounts for the observations, but to make a critical examination of the predictions of the steady-state model, which allows of no variations except those associated with the spectral index and the source luminosity function. In the next section the steady-state model will be compared with the observations.

4. *Comparison of the observed number-flux density relationship with that predicted on the steady-state model.*—The number-flux density relationship at galactic latitudes $> 20^\circ$ has been established directly for $S \geq 2 \times 10^{-26}$ watts $(\text{c/s})^{-1} \text{ m}^{-2}$ by combining the “whole-sky” observations of Miss Leslie (1961a) with the high resolution observations of 0.6 steradians of sky (Scott and Ryle 1961). By using the statistical method of record analysis on the high resolution observations Hewish (1961) has been able to extend the analysis to cover sources having $5 \times 10^{-28} < S < 2 \times 10^{-26}$ watts $(\text{c/s})^{-1} \text{ m}^{-2}$.

All the observations are combined in Fig. 6 which shows for $S > 2 \times 10^{-26}$ watts $(\text{c/s})^{-1} \text{ m}^{-2}$ the values of N/N_0 derived from individual source counts together with the limits of error due to the statistical uncertainties, possible effects of extended sources, source clustering etc. Three models are shown for $S < 2 \times 10^{-26}$ watts $(\text{c/s})^{-1} \text{ m}^{-2}$, which represent limiting cases of models compatible with the observations.

The marked convergence shown by all three models at small S may be confirmed from a consideration of the contribution which the sources would make to the total background emission of the sky. As already indicated, the contribution to the extra-galactic radiation from sources having a spectral index of -0.8 cannot exceed about 25°K . The total emission from the sources which can be observed individually ($S \geq 2 \times 10^{-26}$ watts $(\text{c/s})^{-1} \text{ m}^{-2}$) corresponds to a sky brightness temperature of about 3°K , and the variation with S of N/N_0 found for $S > 2 \times 10^{-26}$ watts $(\text{c/s})^{-1} \text{ m}^{-2}$ cannot extend in the same way for $S < 2 \times 10^{-27}$ watts $(\text{c/s})^{-1} \text{ m}^{-2}$ or the permissible integrated emission would be exceeded. The three models (i), (ii) and (iii) derived from the statistical analysis of the interferometric records have been integrated for $S > 10^{-27}$ watts $(\text{c/s})^{-1} \text{ m}^{-2}$ and lead to sky brightness temperatures of 7.5 , 8.5 and 11°K , all of which are compatible with the observed sky brightness.

The observed number-flux density relationship may now be compared with that predicted by the steady-state model, using the luminosity functions derived in Section 2. As can be seen from Fig. 5 the curves derived for the steady-state model do not depend critically on the spectral index, and a value of -0.8 has been adopted; a comparison of the intensities of a number of sources at frequencies of 159 and 960 Mc/s (Harris and Roberts 1960) has shown that most sources have over this range a spectral index of 0.8 ± 0.2 and errors in the predicted curve of N/N_0 are therefore likely to be small, at any rate for values of $z < 5$.

Two curves have been computed and are shown in Fig. 6: the first is based on the limiting luminosity derived from radio evidence alone; the second on the assumption that all sources have the same luminosity function as that derived for the identified sources. The point on each curve at which the median of the luminosity function corresponds to $z = 0.5$ is shown.

Both theoretical curves have been drawn to pass through the experimentally determined point at $S = 30 \times 10^{-26}$ watts $(\text{c/s})^{-1} \text{ m}^{-2}$. This is the largest value of S for which the statistical uncertainties are small enough to provide a useful value of N .

A comparison of the predicted and observed curves shows a marked discrepancy, even when the smallest permissible source luminosity is adopted; the observed number of sources in the range $0.5 < S < 2 \times 10^{-26}$ watts (c/s) $^{-1}$ m $^{-2}$ is 3 ± 0.5 times that predicted by the steady-state model. If a luminosity function similar to that of the identified sources is assumed, the discrepancy is 11 ± 2 .

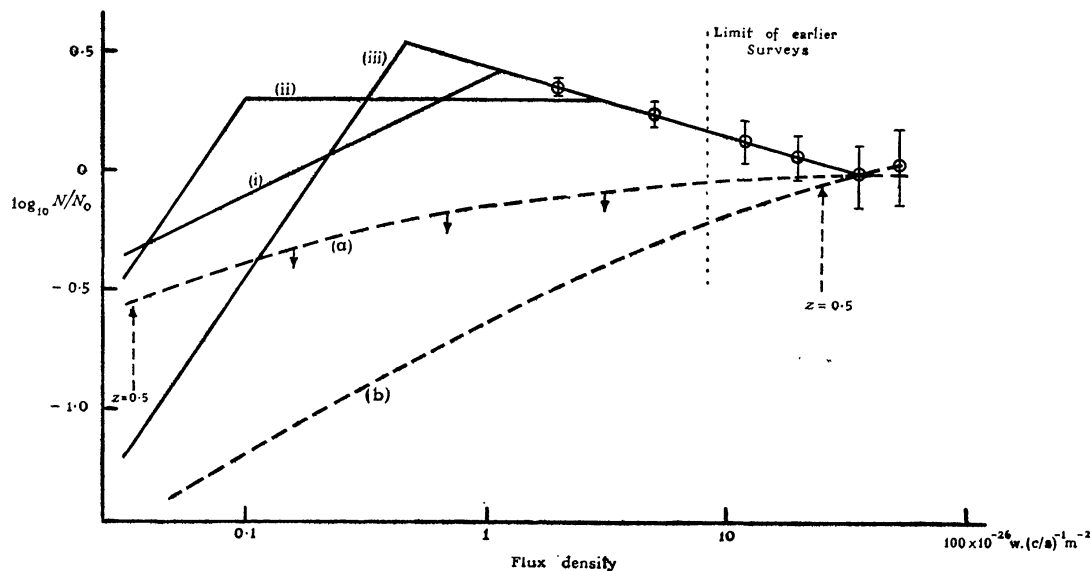


FIG. 6.—Comparison of the observed N/N_0 relationship with that predicted by the steady-state model ($\alpha = -0.8$).

(a) Limiting luminosity function.

(b) Luminosity function similar to that of the identified sources.

The flux density at which the median value of P corresponds to $z=0.5$ for each of the models is indicated. The dotted line corresponds to the useful limit of observation in the earlier surveys (Bennett and Smith 1961).

No attempt has been made here to provide an explanation for the observations in terms of an alternative model and it is evident that the introduction of the evolutionary changes which appear to be necessary will make the selection of a unique model difficult. These observations do, however, appear to provide conclusive evidence against the steady-state model.

We should like to express our thanks to Dr A. Hewish both for useful discussions and for allowing us to use the results of his analysis prior to publication. One of us (R. W. C.) is indebted to the D.S.I.R. for a maintenance allowance.

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1961 January 28.

References

- Ambartsumian, V. A., 1958, *Izv. Akad. Nauk. Armyanski SSR*, **11**, 9.
Allen, L. R., Palmer, H. P., and Rowson, B., 1960, *Nature*, **188**, 731.
Baldwin, J. E., 1955, *M.N.*, **115**, 690.
Bennett, A. S., and Smith, F. G., 1961, *M.N.*, **122**, 71.
Boischoit, A., 1960, 13th General Assembly U.R.S.I., London.
Bolton, J. G., 1960, 13th General Assembly U.R.S.I., London.
Bondi, H., 1952, *Cosmology*, C.U.P.

- Burbidge, G. R., 1958, I.A.U./U.R.S.I. Symposium on Radio Astronomy, Paris, p. 541.
 Burbidge, G. R., and Burbidge, E. M., 1957, *Ap. J.*, **125**, 1.
 Chapman, J. H., 1960, 13th General Assembly U.R.S.I., London.
 Costain, C. H., 1960, *M.N.*, **120**, 248.
 Davidson, W., 1959, *M.N.*, **119**, 665.
 Dewhirst, D. W., 1958, I.A.U./U.R.S.I. Symposium on Radio Astronomy, Paris, p. 507.
 Dewhirst, D. W., 1961 (in preparation).
 Edge, D. O., Scheuer, P. A. G., and Shakeshaft, J. R., 1958, *M.N.*, **118**, 183.
 Edge, D. O., Shakeshaft, J. R., McAdam, W. B., Baldwin, J. E., and Archer, S., 1959, *Mem. R.A.S.*, **68**, 37.
 Elsmore, B., Ryle, M., and Leslie, P. R. R., 1959, *Mem. R.A.S.*, **68**, 61.
 Greenstein, J. L., 1955, I.A.U. Symposium on Radio Astronomy, Jodrell Bank, p. 179.
 Harris, D. E., and Roberts, J. A., 1960, *P.A.S.P.*, **72**, 237.
 Hewish, A., 1961 (in preparation).
 Leslie, P. R. R., 1961 a, *M.N.*, **122**, 51.
 Leslie, P. R. R., 1961 b, *M.N.*, **122** (in press).
 Long, R. J., and Marks, D. R., 1961, *M.N.*, **122**, 61.
 McVittie, G. C., 1957, *Aust. J. Phys.*, **10**, 331.
 Mills, B. Y., 1960, *Aust. J. Phys.*, **13**, 550.
 Mills, B. Y., Slee, O. B., and Hill, E. R., 1958, *Aust. J. Phys.*, **11**, 360.
 Minkowski, R., 1960, Proceedings of 4th Symposium on Mathematical Statistics and Probability, Berkeley.
 Minkowski, R., 1961, *Sky and Telescope*, **21**, 23.
 Priester, W., 1954, *Zeit. f. Astr.*, **34**, 283.
 Priester, W., 1958, *Zeit. f. Astr.*, **46**, 179.
 Ryle, M., 1958, *Proc. Roy. Soc. A.*, **248**, 289.
 Ryle, M., and Scheuer, P. A. G., 1955, *Proc. Roy. Soc. A.*, **230**, 448.
 Sandage, A. R., 1958, *Ap. J.*, **127**, 513.
 Scheuer, P. A. G., 1957, *Proc. Camb. Phil. Soc.*, **53**, 764.
 Scott, P. F., and Ryle, M., 1961, *M.N.*, **122** (in press).
 Scott, P. F., Ryle, M., and Hewish, A., 1961, *M.N.*, **122**, 95.
 Seeger, C. L., 1960, 13th General Assembly U.R.S.I., London.
 Shakeshaft, J. R., 1954, *Phil. Mag.*, **45**, 731.
 Shakeshaft, J. R., Ryle, M., Baldwin, J. E., Elsmore, B., and Thomson, J. H., 1955, *Mem. R.A.S.*, **67**, 106.
 Shklovsky, I. S., 1960, *A. J.*, *USSR*, **37**, 945.
 Tunmer, H. E. C., 1960, *M.N.*, **120**, 360.
 Whitfield, G. R., 1957, *M.N.*, **117**, 680.
 Williams, P. J., Dewhirst, D. W., and Leslie, P. R. R., 1961, *Observatory* (in press).