DISTRIBUTION OF PRE-MAIN-SEQUENCE STARS IN THE H-R DIAGRAM

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ABSTRACT

The distribution of pre-main-sequence stars on the H-R diagram is derived by assuming a constant rate of star formation and by using a mass-radius-luminosity relation derived from the evolutionary tracks which have been computed by Henyey, LeLevier, and Levée for the gravitationally contracting stage. The distributions of these stars with respect to the effective temperature have been numerically evaluated in two cases: (1) all stars have the same mass, and (2) their masses are distributed in accordance with Salpeter's empirical law. The results indicate that most of these stars are located far away from the main sequence in the infrared and thus suggest the importance of the search for infrared stars, in order to understand the evolution of pre-main-sequence stars.

The distributions of those stars, such as T Tauri and dMe stars, which are near the main sequence but nevertheless are still contracting, have been studied. Also, the discrepancy between the locations in the H-R diagram of the pre-main-sequence stars observed in NGC 2264 and those expected from the contraction theory is discussed.

I. THE MASS-RADIUS-LUMINOSITY RELATION OF THE PRE-MAIN-SEQUENCE STARS

In order to derive the distribution function of pre-main-sequence stars in the H-R diagram, we have first to know the mass-radius-luminosity relation of these stars. If the opacity of the stars follows a modified Kramers' law, the mass ($\mathfrak{M}$)-radius(R)-luminosity (L) relation is given by

$$L=\frac{\mathfrak{M}^{\alpha}}{R^{\beta}}. \tag{1}$$

For the sake of convenience, all quantities of a star in equation (1) and elsewhere in this paper are expressed in units of the values of the corresponding quantities in the sun. Now Henyey, LeLevier, and Levée (1955) have computed a series of evolutionary tracks of stars in the gravitationally contracting stage. From their diagram we have found that, for each contracting star, the relation between the luminosity and the effective temperature, T, may be represented by

$$L\propto T^{1\ 13}, \tag{2}$$

from which it can be immediately found that $\beta = 0.79$, and the corresponding value of α is 5.8, according to the results of calculations based on the modified Kramers' law (e.g., Schwarzschild 1958). If, instead of relying on this law of opacity, we use the result of direct calculation by Henyey, LeLevier, and Levée, we have $\alpha = 5.4$, provided that the relation between the radii and the masses of the main-sequence stars given by Russell and Moore (1940) is assumed. On the other hand, if we calibrate α by the aid of the empirical mass-luminosity relation and temperature-luminosity relation of the main-sequence stars, we obtain $\alpha = 4.7$. In the following calculation we shall consistently use

$$\alpha=5.4 \qquad \text{and} \qquad \beta=0.79 . \tag{3}$$

Actually, as we shall see later, our conclusion does not depend on the exact value of α, although it is very sensitive to the value of β.

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II. DISTRIBUTION OF THE PRE-MAIN-SEQUENCE STARS

If a star undergoes homologous contraction, we have

$$L dt = -\frac{\mathfrak{M}^2 dR}{R^2}, \tag{4}$$

where time, t, is expressed in the unit of the Helmholtz-Kelvin scale for the sun (e.g., Chandrasekhar 1939). The assumption of homologous contraction is valid when the star is not very near to the main sequence, i.e., at the evolutionary stages in which we are interested. If we now assume that stars are being formed at a constant rate, the number of stars formed is proportional to the time interval, dt, elapsed. Thus, from equations (1) and (4), we derive

$$dt = -\frac{\mathfrak{M}^{2-\alpha} dR}{R^{2-\beta}}. \tag{5}$$

At the same time, it can be easily seen that, in the contracting stage,

$$R = \mathfrak{M}^{\alpha/(2+\beta)} T^{-4/(2+\beta)}. \tag{6}$$

Thus, if all stars should have the same mass, the normalized distribution function $f(T)$ of the pre-main-sequence stars with respect to T follows directly from equations (5) and (6):

$$f(T)\, dT = \frac{4(1-\beta)}{2+\beta} T_0^{-4(1-\beta)/(2+\beta)} T^{-(5\beta-2)/(2+\beta)} dT, \tag{7}$$

where T_0 is the effective temperature when the star has reached the main sequence.

The case of uniform mass for all stars has, of course, little practical significance. However, it is intended here to illustrate by this simple case the important role that β plays in the distribution function of the pre-main-sequence stars. If $\beta = 0$, i.e., the star contracts with a constant luminosity, we have, from equation (7),

$$f(T) \propto T,$$

which shows that the pre-main-sequence stars are distributed most heavily at places near the main sequence in the H-R diagram. This has been the usual understanding, namely, that most contracting stars should be located near the main sequence. Such an understanding is no longer valid if β is appreciably greater than zero. For example, if $\beta = 0.4$, the distribution function $f(T)$ is independent of T, and when $\beta > 0.4$, $f(T)$ increases with decreasing T. For $\beta = 0.79$, the percentages of gravitationally contracting stars between different temperature intervals according to equation (7) is given in Table 1. It is obvious that the stars are dominantly distributed in the range of low

TABLE 1

DISTRIBUTION WITH RESPECT TO EFFECTIVE TEMPERATURE OF PRE-MAIN-SEQUENCE STARS OF UNIFORM MASS

Temperature Range (T_0)	Percentage	Temperature Range (T_0)	Percentage
0–0 1	49 2	0 5–0 6	4 6
0 1– 2	11 7	6–0 7 . . .	4 2
2– 3	8 1	7–0 8 . .	3 8
3– 4	6 4	8–0 9	3 4
0 4–0 5 . .	5 4	0 9–1 0	3.2

temperatures. When β reaches unity, the contraction time diverges. It is no longer permissible to consider the contraction of a star from an infinite extent. Then the distribution function will depend critically on the initial configuration that we have to assume when the star was first isolated from the rest of the interstellar medium.

Actually, the masses of stars are not of the same value. Salpeter (1955) has established that the distribution function of stellar masses between 10 ⊙ and 0.4 ⊙ is given by $d\mathfrak{M}/\mathfrak{M}^{1.35}$. This distribution is believed to be valid also in the range between 0.4 ⊙ and 0.05 ⊙ in the immediate solar neighborhood (Huang 1961). Therefore, the number of stars formed in the mass interval $d\mathfrak{M}$ and in the time interval dt is

$$\frac{d\mathfrak{M}\, dt}{\mathfrak{M}^{1.35}}. \tag{8}$$

Substituting dt given by equation (5) in equation (8), we obtain the distribution function of pre-main-sequence stars with respect to $\mathfrak{M}$ and R. For the contracting star, the variables $\mathfrak{M}$ and R are related uniquely to the variables L and T by equation (1) and the following equation:

$$L = R^2 T^4 . \tag{9}$$

Therefore, we can transform the distribution function with respect to $\mathfrak{M}$ and R to that with respect to L and T. In this way we obtain

$$f(L, T)\, dL dT = L^x T^y dL dT , \tag{10}$$

where

$$x = -\frac{2.5\alpha - 1.65 - 0.825\beta}{\alpha} \tag{11}$$

and

$$y = \frac{\alpha - 3.3\beta}{\alpha}. \tag{12}$$

Observationally, as is implied in the recent investigation by Herbig (1960), we are interested in knowing the distribution of pre-main-sequence stars within a certain range of stellar masses. Let us now denote by $T_{0,1}$ and $T_{0,2}$ the effective temperatures of the stars corresponding to the two limiting masses $\mathfrak{M}_1$ and $\mathfrak{M}_2$ when both have reached the main sequence. Thus the distribution function with respect to T of pre-main-sequence stars of masses between $\mathfrak{M}_1$ and $\mathfrak{M}_2$ can be derived by integrating equation (10) with respect to L in a domain bounded by the curve

$$L = T^{\gamma} , \tag{13}$$

which defines the main sequence, and the following two lines:

$$L = T_{0,2}^{2\alpha\gamma/n(2+\beta)} T^{4\beta/(4+\beta)} \tag{14}$$

$$L = T_{0,1}^{2\alpha\gamma/n(2+\beta)} T^{4\beta/(4+\beta)} , \tag{15}$$

which represent, respectively, the evolutionary tracks of stars of masses $\mathfrak{M}_1$ and $\mathfrak{M}_2$ in the L-T diagram. The constant, n, in equations (14) and (15) is the power of $\mathfrak{M}$ in the mass-luminosity relation for the main-sequence stars, i.e.,

$$L = \mathfrak{M}^n .$$

Consistent with Henyey, LeLevier, and Levée's calculation, we have $n = 4.6$. Using Keenan and Morgan's (1951) temperature and luminosity calibration and Kuiper's (1938) bolometric correction, we find that, for the entire range of the main-sequence stars from B0 to K5, $\gamma = 7.0$. Now we can perform the integration as stated and derive the required distribution function as follows:

$$f(T)\, dT = \frac{a}{x+1} [T_{0,2}^{2\alpha\gamma(x+1)/n(2+\beta)} T^{-(5\beta-2)/(2+\beta)} - T^{y+\gamma(x+1)}]\, dT\,, \qquad T_{0,2} \geq T \geq T_{0,1} \tag{16}$$

and

$$f(T)\, dT = \frac{a}{x+1} [T_{0,2}^{2\alpha\gamma(x+1)/n(2+\beta)} - T_{0,1}^{2\alpha\gamma(x+1)/n(2+\beta)}]\, T^{-(5\beta-2)/(2+\beta)} dT\,, \; T_{0,1} \geq T \geq 0\,, \tag{17}$$

where x and y are given by equations (11) and (12) and a is a normalizing factor determined by the following equation:

$$a\, [T_{0,2}^{\gamma(x+1)+y+1} - T_{0,1}^{\gamma(x+1)+y+1}] \left[\frac{2+\beta}{4(1-\beta)} - \frac{1}{y+1+\gamma(x+1)}\right] = x+1\,. \tag{18}$$

In deriving equations (16)–(18), a simple power relation between the mass and the radius of the main-sequence star, i.e., $\mathfrak{M} = R^{\delta}$, has been assumed. Hence, among the various constants, there exist two relations,

$$\alpha = n + \beta\,\delta \qquad \text{and} \qquad \gamma = \frac{4n}{n - 2\,\delta}\,,$$

which are used in obtaining these equations. Also, it can be easily seen that

$$\frac{2\alpha\gamma(x+1)}{n(2+\beta)} - \frac{5\beta-2}{2+\beta} = y + \gamma(x+1)\,,$$

so that $f(T_{0,2}) = 0$, as it should.

It is apparent from equation (17) that the distribution of stars in the range between $T_{0,1}$ and 0 is the same as that given by equation (7) for the case of a uniform mass

TABLE 2

DISTRIBUTION WITH RESPECT TO EFFECTIVE TEMPERATURE OF PRE-MAIN-SEQUENCE STARS OF MASSES ACCORDING TO SALPETER'S EMPIRICAL LAW

Temperature Range ($T_{0,2}$)	Percentage	Temperature Range ($T_{0,2}$)	Percentage
0–0 1... .	57 9	0 5–0 6	3 2
0 1– 2	13 8	6–0 7	0 9
2– .3	9 6	7–0 8 .	0 3
3– .4	7 6	8–0 9	0.1
0 4–0 5 .	6 5	0 9–1 0	0

for all stars. Since α does not enter into the power of T in equation (17), the relative distribution of stars at any two temperatures below $T_{0,1}$ is independent of the values of α. However, the absolute distribution as given by equations (16) and (17) depends weakly on α. We have computed, according to equations (3), (16), and (17), the case with $T_{0,2} = 2$ (11460° K) and $T_{0,1} = 1$ (5730° K). The results are given in Table 2. Here we see that the number of stars with temperatures between $T_{0,1}$ and $T_{0,2}$ is very small. Since equation (4) may not be a good approximation at stages near the main sequence, the actual percentage of stars listed in Table 2 in this range indicates only the order of magnitude. But the conclusion that not many pre-main-sequence stars of very early spectral type will be there is inescapable, according to the present theory of gravitational contraction of stars.

Herbig (1960) has estimated the ratio of the total number of pre-main-sequence stars to the total number of all stars by equating it to the ratio of the time scale of gravitational contraction to that of hydrogen-burning in the core. In this way he is able to derive the total number of the pre-main-sequence stars. The present investigation gives the percentages with which these pre-main-sequence stars would distribute among different spectral types. It is apparent here that if the star is actually formed from a gaseous condensation of a huge volume, there will be far more infrared contracting stars than those near the main sequence. Here we see the importance of a search for infrared stars, especially in the Orion Nebula, where new stars are being formed. There Pişmiş (1954) has found faint infrared stars forming a kind of extended halo around the small Trapezium stars, but no numerical data are yet available for comparison.

III. DISTRIBUTION OF T TAURI AND dMe STARS

Finally, let us calculate the distribution of T Tauri stars and dMe stars from the present considerations. According to the current view (Herbig 1957, 1958), both kinds of stars are in the stage of gravitational contraction. On the other hand, they must be

TABLE 3

RELATIVE NUMBERS OF T TAURI AND dMe STARS ACCORDING TO CONTRACTION THEORY

Sp Range	Temperature Range	Relative No.
G0–G5	1 048–0 946	1
G5–K0	0 964–0 894	2 7
K0–K5	0 894–0 768	31
K5–M0	0 768–0 629	539
M0–M5	0 629–0 489	18800

near the main sequence because of the dwarf characteristics of their spectra. Consequently, we may estimate their frequency of occurrence with respect to T by counting the number of stars within a strip on the L-T diagram bounded by the line defining the main sequence and a parallel line at a distance dL above the main sequence. The required frequency distribution with respect to T can be easily obtained by substituting equation (13) into equation (10) and by dropping dL. It is given by

$$\phi(T)\, dT^{\gamma x+y} dT \;. \tag{19}$$

The function thus obtained gives only an order-of-magnitude estimate because, first, equation (4) does not represent a good approximation when a star is near to the main sequence and, second, both T Tauri and dMe stars may not be infinitesimally near the main sequence.

With the values of γ, α, and β, given before, we find

$$\gamma x + y = -14 \;.$$

Hence the number of T Tauri and dMe stars should increase rapidly with decreasing temperature. Table 3 gives the relative distribution of these stars according to equation (19). Extrapolating Table 3 to infrared stars, we will expect the relative number to be extremely large. Unfortunately, these stars would be difficult to observe because they are not only infrared but also very faint. As regards dMe stars, Haro (1953, 1961) has carried out an extensive study in the Orion Nebula, but his statistical data are not refined enough to compare with the results of the present calculations. Otherwise this would be a good test of the theory that T Tauri and dMe stars are indeed in the stages of gravitational contraction just before reaching the main sequence.

IV. PRE-MAIN-SEQUENCE STARS IN NEW CLUSTERS

The distributions of the gravitationally contracting stars in new clusters such as those observed by Walker (1956, 1957) and by others (Johnson 1957; Parenago 1954) definitely disagree with the one given in Table 3. This discrepancy is understandable. First, the rate of star formation is not likely to be uniform in cluster. Second, while the Salpeter distribution function is good for stars in the galaxy at large, it may not be applied to a single galactic cluster. Indeed, it has been found that the luminosity function of many galactic clusters shows a maximum at one magnitude or another and then declines at fainter magnitudes (Roberts 1958). Therefore, we cannot help but conclude that the stars now present in a galactic cluster do not follow the same distribution of masses as is found in the galaxy at large. Since we do not have a reliable distribution function with respect to the masses of stars in these new clusters, we are unable to discuss the distribution function with respect to luminosities and temperatures. However, we can still study their positions in the H-R diagram, which are the subject of the present section.

In a cluster it may be a good approximation to assume that all stars were formed at the same instant, rather than assume continuous formation. In such a case all the stars will be distributed at any given instant on a single line in the L-T diagram. The locus of the contracting stars, which moves as time goes on, can be easily written down by integrating equation (5). If (L_1, T_1) represents any point on the locus after a time interval τ since the formation of the stars, we have

$$L_1^{3a/2-2-\beta} = (1-\beta)^{-a}\,\tau^{-a}\,T_1^{2(a-2\beta)} . \qquad (20)$$

Using the values a and β given previously, we obtain

$$\log L_1 = 1.44 \log T_1 - 1.01 \log \tau + \text{const} . \qquad (21)$$

Thus the locus of contracting stars has a constant slope of 1.44 in the $\log L$–$\log T$ diagram at any time, but moves downward as τ increases. This compares with a slope of $\gamma = 7$ (eq. [13]) for the line defining the main sequence and a slope of 1.13 (eq. [2]) for the evolutionary tracks. Thus the slope of the locus of contracting stars should be nearly equal to that of the evolutionary track. Instead, the loci actually observed in new clusters are nearly parallel to the main sequence, although some astronomers (The 1960; Underhill 1960) cast doubt on the membership of those stars which are above the main sequence. This discrepancy has been noticed by Walker (1956) and discussed by Sandage (1958) and by von Hoerner (1960).

In Figure 1 we have illustrated the results of our calculation. The thick line defines the main sequence. The dotted lines are the evolutionary tracks of gravitationally contracting stars of different values of $\log \mathfrak{M}$ as labeled. The thick lines represent the loci at different labeled epochs of those stars which were formed at the same instant. The broken lines are equal-density curves, on which the mean density of the star, i.e., $\mathfrak{M}/R^3$, is constant. The labeled values are expressed in terms of the solar mean density. The area between two curves AB and CD, which are taken directly from Sandage's (1958) paper, is where the pre-main-sequence stars in the new cluster NGC 2264 are located, according to Walker (1956).

The figure illustrates most clearly the disagreement in the locations of the pre-main-sequence stars actually observed and expected from the contracting theory. In a recent paper, however, Varsavsky (1960) has pointed out that, probably because of the influence of emission lines, the $B - V$ color, on which the color-magnitude diagram of a cluster is usually based and from which the area between curves AB and CD in Figure 1 is derived, is not a good measure of the effective temperature of the pre-main-sequence stars. In a group of presumably contracting stars in Taurus, he has found that, for a given spectral type, their color could vary within a range of approximately $0^{m}.8$ and always (with exception of the Ge stars) toward the blue side of the normal relationship. Since the contracting stars in NGC 2264 resemble in many ways those in Taurus, which have been investigated by him, Varsavsky explains the spread of stars in color in the color-magnitude diagram of NGC 2264 as a result of the same kind of color variation as is found in stars in Taurus, and, furthermore, he suggests that the observed

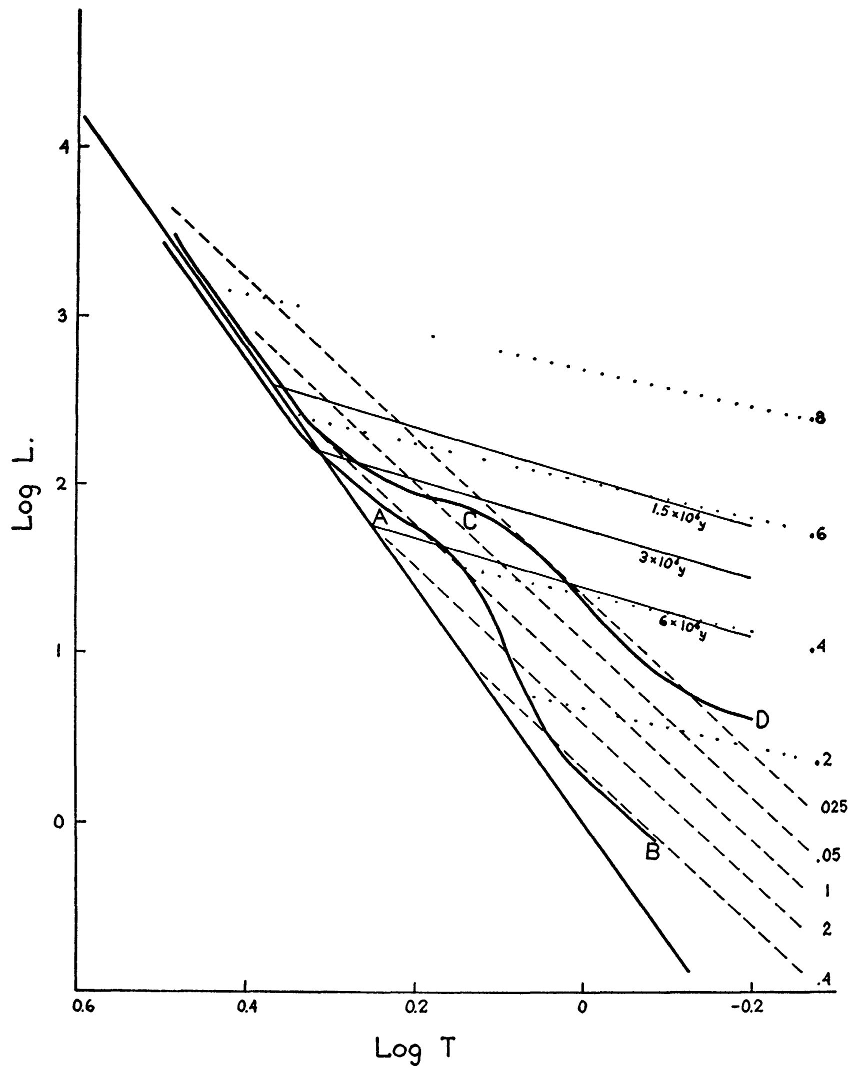

FIG. 1.—The location—an area bounded between two curves AB and CD—of the observed pre-main-sequence stars in NGC 2264 in the log L–log T diagram and the predicted loci on which stars formed simultaneously should be situated at different epochs. The thin solid lines represent these loci, where the accompanying figures indicate the times elapsed since the formation of the stars. The thick, solid line defines the main sequence. Each dotted line represents the evolutionary track during the gravitationally contracting stage. The labeled values are log $\mathfrak{M}$. The broken lines represent the curves of equal mean densities, whose values are given in the unit of the mean solar density.

locus of pre-main-sequence stars in NGC 2264 could be represented by a single curve (instead of an area) which coincides approximately with the curve *CD* in Figure 1. Thereby he concludes that no disagreement exists between the prediction from the conventional theory of gravitational contraction and the observed results in new clusters.

Actually, the curve *CD* is still far from being in agreement with any one of the parallel lines which denote the loci of stars at different epochs. Moreover, the spectral classification of pre-main-sequence stars with their emission lines is known to be very difficult. Therefore, their spectral types, from which Varsavsky has determined their effective temperatures, may be subject to a larger error. To what extent this uncertainty affects the temperature thus determined cannot be estimated here. Consequently, the problem of the discrepancy between the contraction theory and the observations remains open for further investigation.

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