

THE SYNTHESIS OF LARGE RADIO TELESCOPES

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Summary

Many investigations in radio astronomy are limited by the resolving power which can be achieved by conventional methods of aerial construction.

A new method of obtaining increased resolving power has been developed, which has been applied to the construction of both "pencil-beam" systems and interferometers. In this method two aerials are arranged so that their relative position may be altered to occupy successively the whole area of a much larger equivalent aerial. By combining mathematically the information derived from these different positions, it is possible to obtain a resolving power equal to that of the large equivalent aerial. Since the combination of the individual records may be done with different phase relationships, it is possible, without extra observations, to "scan" the synthesized aerial over an appreciable solid angle; because of this the total observing time of a synthesized instrument is of the same order as that of a conventional instrument.

An interferometric system designed for the study of radio stars has been built which has an equivalent area for resolution of $8 \cdot 10^5$ sq. ft as well as a "pencil-beam" system with an equivalent area of $3 \cdot 10^6$ sq. ft. The sensitivity of both systems corresponds to a "collecting area" of about $2 \cdot 10^5$ sq. ft.

1. *Introduction.*—The operation of a radio telescope is limited by two dominating factors. One is the sensitivity, which specifies the smallest radio flux which may be detected, and the other is the angular resolving power. The relative importance of these factors is strongly dependent upon the type of investigation undertaken and for this reason specialized instruments are often developed for particular applications. In studies of the solar radio emission, for example, where high resolving power is needed at metre wave-lengths the primary requirement is that of angular resolution; the use of conventional aerials such as paraboloids is clearly impracticable. Even if structures large enough to give the required resolution were mechanically feasible their energy-collecting area would be far in excess of that needed to detect the relatively intense solar radio emission. Instruments have therefore been developed in which high angular resolution has been obtained by dispersing small aerials over a wide area of ground. Such systems include the diffraction grating type of array (Christiansen 1953; Blum, Boischot and Ginat 1958) and the interferometer of variable spacing (Stanier 1950, Machin 1951, O'Brien 1953); these two methods achieve the same result but in the former case all the aerials are present simultaneously, while in the latter case two aerials only are moved, successively, to the different positions of the grating.

Similar techniques have been developed for the study of the distribution of the galactic emission. Conventional systems have been successfully employed for the short wave-lengths, but in many problems, particularly those involving the distribution of radio "brightness" at high galactic latitudes, it is important to make observations at wave-lengths of several metres; by dispersing the available collecting area it is again possible to obtain much larger resolving powers with adequate sensitivity. Two systems have been used, one of which employs a pair of mutually perpendicular thin apertures (Mills and Little 1953); the other utilizes a single thin aperture in conjunction with a small movable aerial (Blythe 1957). An earlier version of the latter instrument was used by Scheuer and Ryle (1953) to obtain increased resolving power in one coordinate for studying the distribution of radio emission across the galactic plane.

The observation of radio stars presents a more complex problem in which three main approaches have been made. (a) The use of conventional systems of moderate size at a wave-length sufficiently short to provide the required angular resolution (Piddington and Trent 1956; Seeger, Westerhout and Conway 1957; Roman and Yaplee 1958). These observations have been characterized by very small signal/noise ratios, and in consequence long integration times are used and extensive surveys have not been possible. (b) The use of conventional aerials of larger collecting area at longer wave-lengths (Hanbury Brown and Hazard 1953; Ryle and Hewish 1955); the signal/noise ratio of such systems has usually been large and the observations have been limited by resolution. (c) The use of "unfilled" systems such as the Mills Cross (Mills and Little 1953) at a long wave-length. The angular resolution of such a system may be very large but the collecting area is smaller.

If any major increase in the depth of radio star surveys is to be made it is necessary to construct systems with a collecting area exceeding 50 000 ft² capable of operating at shorter wave-lengths or to use, at long wave-lengths, "unfilled" systems having larger collecting areas than the Mills Cross.

Since the flux density of the sources falls with wave-length and since the difficulties of constructing an aerial of given physical size get very much greater if it is to operate at short wave-lengths, it is possible that even with the new receiver techniques now being developed (Adler 1958, Giordmaine *et al.* 1958) it may be difficult to attain a sufficient improvement by using conventional systems at short wave-lengths. The alternative approach, which utilizes a long wave-length and correspondingly large collecting areas, appears to offer greater possibilities.

In this paper an account is given of a new principle in the design of large radio telescopes. The method, which has already been used successfully by Blythe (1957), makes use of two aerials arranged successively in different configurations to provide information equivalent to that obtained from the use of an aerial of much greater physical size. Apart from a considerable economy of structure this method avoids some of the difficulties associated with the physical achievement of a graded excitation of amplitude and phase which is required in the case of large extended arrays such as the Mills Cross. Besides allowing greater collecting areas to be realized, the shape of the reception pattern can be adjusted, by computation alone, to suit different types of observation. The method necessarily involves considerable computation but this does not present a serious problem with the large electronic computers now available.

2. *The principle of aperture synthesis.*—It is well known that a simple interferometer, consisting of a pair of non-directional receiving elements connected to a receiver, gives a response proportional to one Fourier component of the two-dimensional distribution of radio brightness across the sky. By taking measurements in which both the spacing and the orientation of the interferometer are varied it is possible to determine the two-dimensional transform of the brightness distribution within certain limits and hence, by Fourier inversion, to derive the brightness distribution as observed with a specified resolving power (O'Brien 1953). Aperture synthesis may be regarded as a logical extension of this method but it will here be discussed in a somewhat different, but essentially equivalent, way which may give a more direct understanding of the method.

Consider, for example, the operation of a receiving aperture such as a large two-dimensional array, or paraboloidal reflector. The signal which is delivered to a receiver from such an aerial may be regarded as the vector addition of the currents induced in each portion of the aperture. In the case of a paraboloid this vector addition is achieved, automatically, at the focal point, while for an array the same result is achieved by connecting all the elements to the receiver in the same phase. When the direction of an incident wave does not coincide with the normal to the aperture plane the induced current in the elements of the aperture will suffer a progressive phase shift and vector addition then gives rise to the usual directional properties of the aperture. In the case of an array, it is possible to add the currents with their phases suitably adjusted, and in this way to vary the direction of the principal response of the aperture; this feature is of great importance in large aerial systems where physical tilting of the aperture plane may be impossible.

Now if it were possible to measure, separately, the current induced at each portion of the aperture by moving a small aerial successively across it, vector addition of the currents would give, for a constant source, exactly the same result as that obtained by using the complete large aperture. This process is not possible using a single receiving element since measurement of the phase of the induced current raises difficulties. By using two small aerials arranged as an interferometer, however, the relative phases are readily obtained and the successive determination of the current in this way forms the basis of the synthesis method.

(a) *The synthesis of a single aperture.*—Consider a single rectangular aperture such as that shown in Fig. 1. The resultant current induced in the aperture by an incident plane wave is $\sum I_n e^{i\phi_n}$ where ϕ_n is the phase of the wave at the position of the n th element. The power P delivered to a receiver is then given by

$$P \propto \sum I_n e^{i\phi_n} \sum I_n e^{-i\phi_n} \propto \sum I_n^2 + \sum I_m I_n \cos(\phi_m - \phi_n). \quad (1)$$

If all the elements are of the same size, the first term in expression (1) is simply N times the power derived from a single element; the terms of type $I_m I_n \cos(\phi_m - \phi_n)$ are just the output obtained when a phase-switching receiver is connected to the elements m and n through equal cables (Ryle 1952). A single measurement of the power induced in one element, suitably combined with the outputs from a phase-switching receiver connected to a pair of elements, arranged successively to cover all possible combinations of positions, then gives a result

exactly equivalent to using the whole large aperture. By performing the addition with the value of $(\phi_m - \phi_n)$ adjusted to give a progressive phase shift across the aperture it is possible to vary the direction of the maximum response of the aperture in a manner analogous to tilting the aperture plane. It is thus possible, without the necessity for further observations, to scan the synthesized aperture through an angle which is only limited by the directional properties of the small element. To perform the addition with modified values of $(\phi_m - \phi_n)$ it is also necessary to measure terms of the type $I_m I_n \sin(\phi_m - \phi_n)$, and these are obtained by connecting the pair of elements in phase quadrature to the phase-switching receiver.

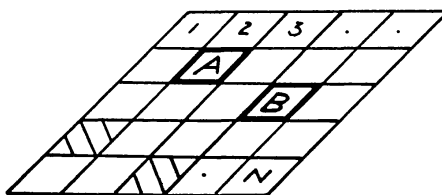


FIG. 1.—*The synthesis of a rectangular aperture.*

The measurements might proceed in the following way: having determined the power induced in a single element, an interferometer is set up with one element *A* fixed at position 1 and the sine and cosine terms are obtained by placing the other element *B* successively in all the remaining positions. Element *A* is then moved to position 2, etc., and the same procedure carried out. Now if this were done it is immediately obvious that many arrangements of *A* and *B*, such as those shaded in Fig. 1 are identical. A procedure for obtaining the different terms with no repetition is indicated in Fig. 2 (*a*) where the element *A* is fixed and element *B* is arranged to cover, once only, all the positions contained in a rectangle of approximately twice the area. It is of course possible to move both elements, in which case all arrangements can be obtained without moving the elements outside a rectangle of side *D*. It may be noted that if a total of *N* observations was required initially, the number now needed with no repetition is $(2N)^{1/2}$.

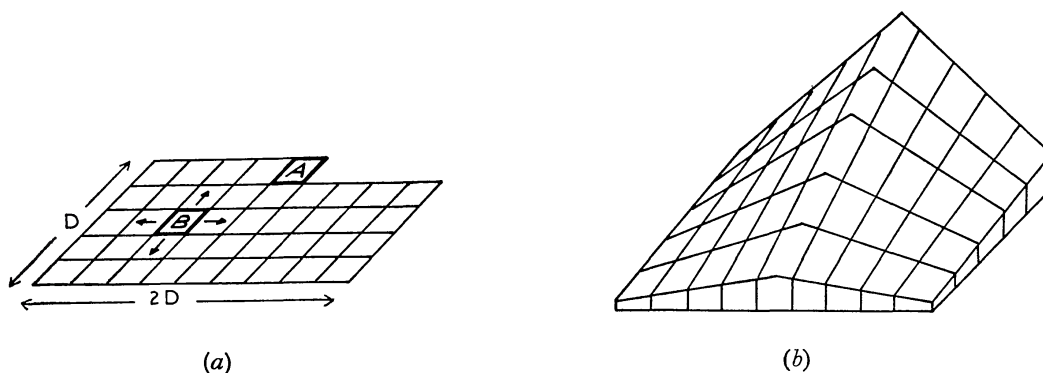


FIG. 2.—(*a*) *One method of synthesizing a rectangular aperture without repeating any element configuration.*

(*b*) *The weighting function for each position of the movable element.*

To obtain results equivalent to using the complete aperture the different terms must now be added with a weighting factor appropriate to the number of repetitions of each configuration of the elements. The form of the weighting function appropriate to each position of the movable element is sketched in Fig. 2(b). Direct addition of the terms without such a tapered weighting function gives a result equivalent to the use of an "optimum" array as discussed by Arsac (1955) and Barber (1958). While such systems give the greatest ratio of information to noise they are sometimes undesirable in practical measurements owing to the large subsidiary maxima inherent in their reception patterns.

When the elements used in the synthesis have a width much greater than one wave-length, addition of the different terms with a progressive phase-shift is equivalent to the use of an aperture having a discontinuous "stepped" phase distribution across it. As in the case of an echelon diffraction grating this gives rise to more than one principal maximum in the reception pattern. This difficulty can be avoided if it is arranged that the different locations of the movable element overlap slightly.

It is not necessary for the elements A and B to have the same shape and there are sometimes practical reasons why it is preferable for one of them to extend over the complete width of the required aperture so that synthesis is only performed in one dimension. A system of this type, which is exactly equivalent to that shown in Fig. 2(a) is shown in Fig. 3(a). When this method is adopted it is necessary to arrange that the current induced in each element of the extended array has the value appropriate to the weighting function shown in Fig. 2(b). The aperture must consequently have a tapered distribution of excitation along its length.

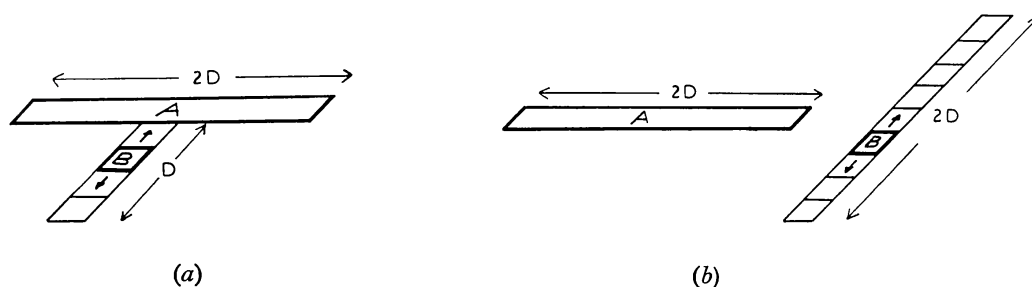


FIG. 3.—(a) One-dimensional synthesis of a single aperture.
(b) One-dimensional synthesis of an interferometer.

(b) *The synthesis of an interferometer.*—For many investigations it is preferable to use an interferometer rather than a single receiving aperture and the method of synthesis may be applied in exactly the same way. By an obvious extension of the preceding arguments it is readily shown that an interferometer comprising two large apertures of side D may be synthesized as shown in Fig. 3(b). In the case of a phase-switching interferometer, however, there is no term corresponding to $\sum I_n^2$ in the output of the receiver and thus it is not necessary in the synthesis to measure the power received by a single element.

3. *Observation time and signal-to-noise ratio.*—Before describing practical applications of the synthesis method it is necessary to consider how it compares with conventional methods in regard to the observation time and signal to noise

ratio. Since a synthesis using small elements requires many different measurements it might be expected that the observing time is necessarily longer. The time required for the repeated observations is, however, almost exactly compensated by the increased scanning rate which may be employed. It is useful to define a quantity which gives a direct comparison of the performance of synthesized and conventional instruments having the same angular resolution. In principle, the observation of a given area of sky can, by suitably arranging the scanning rate and the receiver time constant, be made to occupy a certain interval of time. We can therefore define the "efficiency" of a synthesized instrument as the ratio of its absolute sensitivity (i.e. signal/noise ratio) as compared with that of a conventional instrument of the same resolving power when the same observing time is allowed in each case. The efficiencies of various types of synthesized instrument will now be discussed.

(a) *Scanning rate and observation time.*—During a survey a radio telescope is generally scanned continuously on one coordinate, either by steering the instrument or by using the Earth's rotation, and a given region of the sky is investigated strip by strip. For our purpose it is more convenient to imagine that the scanning is performed in discrete intervals in both coordinates, the instrument being directed towards a certain point in the sky for the required integration time and then moved instantaneously to the next point. The angular distance between adjacent points in the scanning lattice depends upon the resolving power of the instrument and Bracewell (1956) has shown that the greatest interval allowed when the aerial is a rectangular aperture of sides a and b is given by $\lambda/2a$ and $\lambda/2b$ in the respective coordinates, where λ is the wave-length. No additional information is gained by using a finer scanning lattice but information is lost if a coarser lattice is adopted.

Suppose, now, that a given solid angle Ω of the sky is scanned using a square aperture of side D . The total observing time is given by $4\Omega D^2\tau/\lambda^2$ where τ is the integration time of the receiver. If the sky is scanned using the equivalent synthesized instrument in which the small elements are square apertures of side d , the time required to scan the solid angle for each of the arrangements is $4\Omega d^2\tau/\lambda^2$ where it is assumed that the receiver time constant is the same as before. The number of different arrangements is approximately $2D^2/d^2$ and the total observing time for the same receiver time constant is therefore $8\Omega D^2\tau/\lambda^2$. This is seen to be only twice that required had the complete aperture been used and it is important to notice that it is independent of the size of the small elements.

To calculate the observing time when synthesis is performed in only one dimension, consider the system shown in Fig. 4(a). If one of the elements is a rectangle of sides d_1 and d_2 , and the other an extended aperture of width d_1 and length $2D$, then the scanning intervals are $\lambda/2d_1$ and $\lambda/2D$ *. The total number of arrangements is approximately D/d_1 and the observing time is therefore $4\Omega D^2\tau/\lambda^2$. The observing time is thus half that needed when both elements were small and the same as if a complete aperture had been used. It is also of interest to consider the system shown in Fig. 4(b) which is similar to an aerial constructed by Mills except that one arm of the cross is omitted. Although no

* The extended aperture of length $2D$, when used with the appropriate tapered excitation is equivalent, when scanning, to a uniform aperture of length D . More exactly, the scanning interval is $\lambda/2D + d_2$ but we assume $D \gg d_2$.

synthesis is required such a system yields the same (non-repetitive) information as do the synthesized apertures under discussion and it will be shown that it has approximately the same efficiency. We note here that the scanning interval is $\lambda/2D$, as for the complete aperture, so that the observation time is again $4\Omega D^2\tau/\lambda^2$.

It should be noted that this analysis assumes that the bandwidth of the receiver is not too great and that the area of sky under investigation is large enough to contain more than one coarse scanning interval $\lambda/2d$. The effect of a finite bandwidth is to narrow the reception pattern of the individual elements at large spacings (Ryle and Vonberg 1948), and this may necessitate the use of a finer scanning lattice. In the case of a single aperture it may be shown that the effect is negligible provided that $\Delta f/f$ is appreciably less than d/D ; for an interferometer the ratio $\Delta f/f$ must be less than the ratio of d to the maximum element spacing.

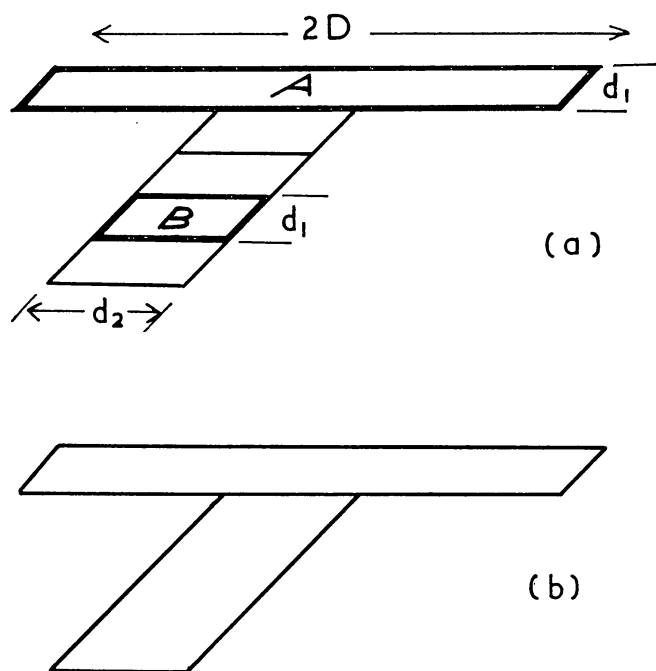


FIG. 4.—(a) The synthesis of a rectangular aperture. (b) A similar system which may be compared to the Mills Cross. It can be regarded as a synthesis in which all configurations are present simultaneously.

(b) *The efficiency of synthesized systems.*—To derive the efficiency of a synthesized aperture we note that the signal obtained with a complete aperture of side D is directly proportional to its area D^2 . If, for an integration time τ , the root mean square receiver noise is n then the signal/noise ratio is proportional to D^2/n .

When the aperture is synthesized using small elements of area d^2 the signal obtained by adding the $2D^2/d^2$ different observations with appropriate weights is proportional to D^2 . Since the receiver noise is unrelated for the different observations the resultant noise is $2^{1/2}nD/3d$ where weighted addition has been performed. The signal/noise ratio is thus given by $3Dd/2^{1/2}n$. Since however, the observing time for a fixed receiver time constant was twice that required when using a conventional aperture, the efficiency is given by $3d/2D$.

In the case where a one-dimensional synthesis is used, as in Fig. 4(a), the signal obtained for each arrangement of A and B is proportional to $2(Dd_1d_2)^{1/2}$ since the equivalent area is given by the geometric mean of the two apertures (Ryle 1952). The number of arrangements is D/d_1 and the signal/noise ratio is $D(3d_1d_2)^{1/2}/n$. In this case the observing time was the same as with a conventional aperture and so the efficiency is $(3d_1d_2)^{1/2}/D$. For the system of Fig. 4(b), in which no synthesis is employed, the signal is $D(2d_1d_2)^{1/2}$ and so the signal/noise ratio is $D(2d_1d_2)^{1/2}/n$ and hence the efficiency is $(2d_1d_2)^{1/2}/D$. All the systems shown in Figs. 3 and 4 thus have a comparable efficiency and the maximum possible efficiency in a synthesized aperture is $(d/D)^{1/2}$ which is attained when $d_2 = D$.

While the efficiency of a synthesized aperture is necessarily less than that of a conventional aperture of the same resolving power this should not be regarded as an immediate disadvantage. As mentioned earlier, the efficiency of a conventional aperture is often far higher than is required for a particular investigation so that the same physical aperture, arranged as a synthesized system of adequate efficiency and increased angular resolution, would have provided a much more powerful instrument.

4. The practical application of the method

(a) *General considerations.*—Since systems derived by one- or two-dimensional synthesis have approximately the same performance there might seem to be considerable advantages with a two-dimensional system arising from the economy of physical structure. There are, however, a number of practical disadvantages with two-dimensional synthesis and these are discussed below.

(i) Data handling and computation: for a one-dimensional synthesis involving N arrangements of the elements, the corresponding two-dimensional system would require N^2 arrangements. The computational and data-handling problems are thus enhanced.

(ii) Scanning: the angular scanning rate of a two-dimensional system must be considerably higher than for a one-dimensional system of the same resolving power for a survey to occupy a given observing time (cf. Section 3). It is a great practical convenience to employ the Earth's rotation for scanning in right ascension, but this fixed scanning rate may be too slow for the desired observational programme. Facility for pointing the elements in both coordinates is then necessary which raises practical difficulties which may be avoided in a one-dimensional system.

(iii) Receiver linearity: in a two-dimensional synthesis each intense source will be received over a solid angle λ^2/d^2 , and it may be difficult to ensure a sufficiently linear overall response to allow observation of weak sources within this area. In the case of one-dimensional synthesis the area of sky affected in this way is reduced by the receptivity of the long aerial to λ^2/Dd .

(iv) Bandwidth: a limitation particularly relevant to the synthesis of an interferometer is set by the condition $d/D > \delta f/F$ (cf. Section 3). Since the overall spacing of the synthesized apertures will be several times larger than their individual aperture D , the maximum bandwidth permitted in a two-dimensional synthesis is necessarily smaller than for the corresponding one-dimensional system in which the long aerial runs East–West.

(b) *The design of systems utilizing the Earth's rotation.*—For the reasons outlined above systems adopting one-dimensional synthesis have a number of practical advantages. In current applications long thin apertures running

East-West, with provision for rotation about the long axis, may be constructed relatively economically and so the remaining discussion will be restricted to the design of such systems; it is possible, however, that other problems might reach a simpler solution if two-dimensional methods were employed.

At first sight the restriction of the design to what is essentially a transit instrument, in which the scanning rate is fixed by the Earth's rotation, might be thought to lead either to inadequate sensitivity due to a too short integration time, or to an unnecessarily long observing time. While it is true that the observing time required to survey a given area of sky is determined entirely by the angular resolution of the system and the rotation speed of the Earth, it is, however, possible to adjust the efficiency so that this scanning rate is always the optimum for a particular application. It is assumed that the area of sky involved includes 24^h in right ascension and an angle of at least λ/d in declination.

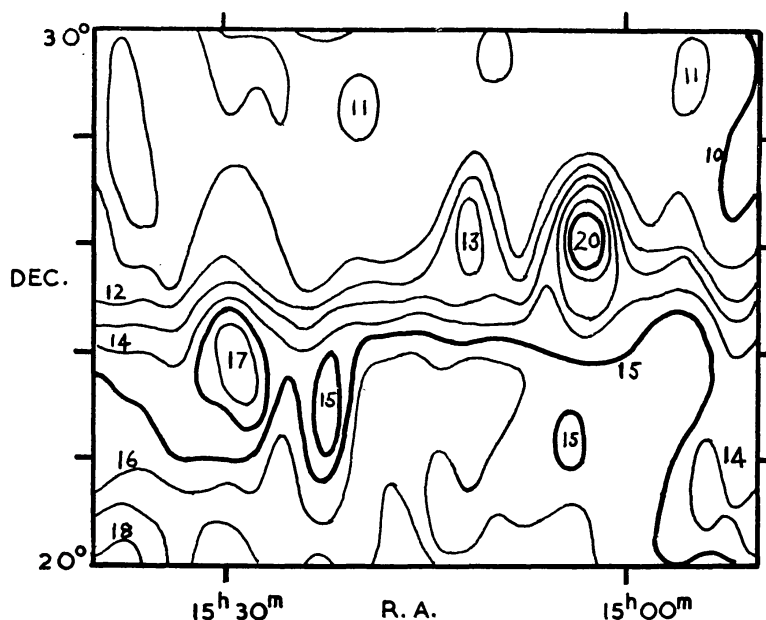


FIG. 5.—Part of a map obtained with the 7.9 m "pencil-beam" system.
The contours are in units of 1000°K.

In Section 3 the efficiency of a one-dimensional system such as that in Fig. 3 was shown to be proportional to the quantity $(d_1 d_2)^{1/2}/D$. Once the efficiency has been determined, the relative values of d_1 and d_2 may be decided by considerations of structural economy provided that their product is constant.

5. The design of two practical systems

(a) *A pencil beam aerial for a wave-length of 7.9 m.*—This instrument was designed to provide a high resolution survey of the general galactic background radiation at a long wave-length so that, in conjunction with other observations at shorter wave-lengths, reliable information would be available on the spectral distribution of the emission from many galactic features.

The original instrument built by Blythe (1957) employed one-dimensional synthesis as in Fig. 4 (a) with an East-West array of length $D=1200$ ft and a movable element of length d_1 =one wave-length. The effective width d_2 was less than the wave-length so that a single series of observations provided a complete map of the accessible sky. However, the presence of ground reflections led to an uncertainty in the receptivity in declination and gave an uncertainty in the overall scaling of brightness at southerly declinations.



The long fixed element of the radio star interferometer.

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In the new instrument, which was designed to give an angular resolution of $0^{\circ}.8 \times 0^{\circ}.8$, a fixed aerial of length 3300 ft and width 40 ft was used so that a well-defined receptivity in declination was achieved at declinations above -10° . This increase in width and the use of a movable element of length 100 ft gave an efficiency of 7 per cent.

A preliminary survey of limited resolving power ($0^{\circ}.8 \times 1^{\circ}.6$), in which spacings of the movable aerial up to 30λ were used, has been carried out and part of the map is shown in Fig. 5.

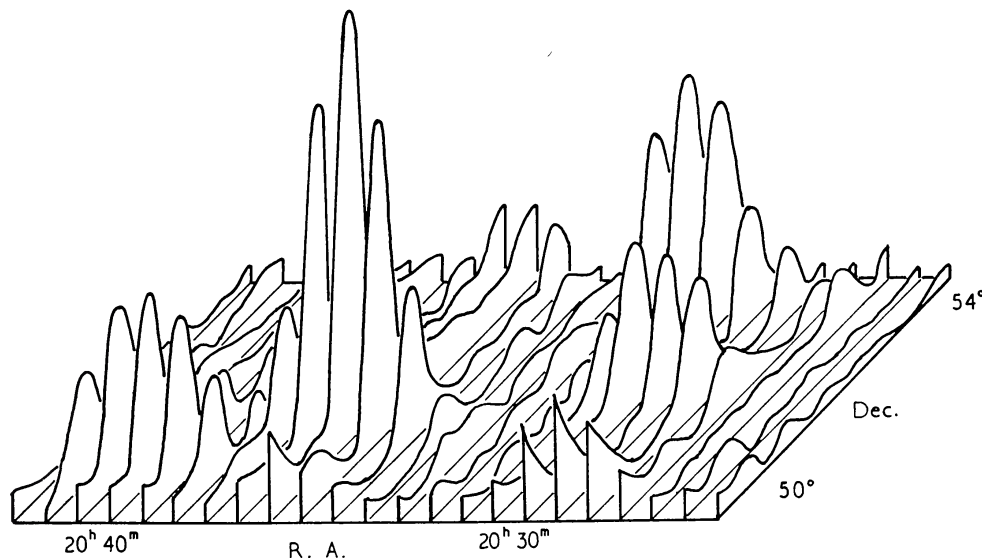


FIG. 6.—A section of the first map obtained with the radio star interferometer. The most intense source at $20^{\text{h}} 37^{\text{m}}$ has a flux density of $11 \times 10^{-26} \text{ w.m.}^{-2} (\text{c/s})^{-1}$.

(b) *A radio star interferometer on a wave-length of 1.7 m.*—A system capable of carrying out a deeper survey of radio sources than those made hitherto (Mills and Slee 1957, Edge *et al.* 1959) was required, and considerable increases of both resolution and sensitivity were necessary. In addition the design was influenced by the importance of obtaining the positions of sources with greater accuracy, particularly in declination, and by the necessity for obtaining good measurements of the angular diameter of the most intense sources.

An interferometric system having an approximately symmetrical primary reception pattern was adopted, in which the resolution in right ascension was provided by a 1450 ft parabolic trough running East–West (as in Plate 7). The required efficiency (about 25 per cent) was decided from an extrapolation of the number versus flux density counts of previous surveys; the ratio of the width d_1 of both aerials, and the length d_2 of the movable aerial were governed (cf. Section 4) by consideration of the relative costs of the parabolic elements and of the railway track supporting the movable aerial (Ryle 1960).

A section of the first map obtained with this instrument is shown in Fig. 6. Some observations to establish accurate positions of 64 of the most intense radio stars have already been described (Elsmore, Ryle and Leslie 1960).

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References

- Adler, R., 1958, *Proc. I.R.E.*, **46**, 1300.
 Arsac, J., 1955, *Opt. Acta.*, **2**, 112.
 Barber, N. F., 1958, *New Z. J. Sci.*, **1**, 35.
 Blum, E. J., Boischot, A., and Ginat, A., 1957, *Ann. d'Astrophys.*, **20**, 115.
 Blythe, J. H., 1957, *M.N.*, **117**, 644.
 Bracewell, R. N., 1956, *Aust. J. Phys.*, **9**, 297.
 Brown, R. Hanbury, and Hazard, C., 1953, *M.N.*, **113**, 123.
 Christiansen, W. N., 1953, *Nature*, **171**, 831.
 Edge, D. O., McAdam, W. B., Shakeshaft, J. R., Baldwin, J. E., and Archer, S., 1959, *Mem. R.A.S.*, **68**, 37.
 Elsmore, B. E., Ryle, M. and Leslie, P. R. R., 1959, *Mem. R.A.S.*, **68**, 61.
 Giordmaine, J. A., Alsop, L. E., Mayer, C. H., and Townes, C. H., 1958, *Proc. I.R.E.*, **47**, 1062.
 Machin, K. E., 1951, *Nature*, **167**, 889.
 Mills, B. Y., and Little, A. G., 1953, *Aust. J. Phys.*, **6**, 272.
 Mills, B. Y. and Slee, O. B., 1957, *Aust. J. Phys.*, **10**, 162.
 O'Brien, P. A., 1953, *M.N.*, **113**, 597.
 Piddington, J. H., and Trent, G. H., 1956, *Aust. J. Phys.*, **9**, 481.
 Roman, N. G., and Yaplee, B. S., 1958, *Proc. I.R.E.*, **46**, 199.
 Ryle, M., 1952, *Proc. Roy. Soc. A*, **211**, 351.
 Ryle, M., 1960, *J.I.E.E.*, **6**, 14.
 Ryle, M., and Hewish, A., 1955, *Mem. R.A.S.*, **67**, 97.
 Ryle, M., and Vonberg, D. D., 1948, *Proc. Roy. Soc. A*, **193**, 98.
 Seeger, C. L., Westerhout, G., and Conway, R. G., 1957, *Ap. J.*, **126**, 585.
 Scheuer, P. A. G., and Ryle, M., 1953, *M.N.*, **113**, 3.
 Stanier, H. M., 1950, *Nature*, **165**, 354.