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DIFFUSION IN THE SUN

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ABSTRACT

The gradients of pressure and temperature in the sun set up an electrostatic field that maintains electrical neutrality to a high degree of approximation. The gradients and the field cause the heavier ions to diffuse downward. The rate of this diffusion is estimated for different depths, on the basis of a model (III) obtained by Weymann. The diffusive drainage of the heavier elements from the convection zone reduces their abundances relative to hydrogen in the sun's atmosphere. It also affects their relative abundances among themselves. The reduction is naturally greater, the shallower the convection region. The low abundance of lithium in the sun's atmosphere is taken to indicate that the convective region extends to a depth at which its destruction by nuclear reactions can occur—namely, down to three-tenths of the solar radius. For this depth the reduction of the abundance of the heavier elements brought about by diffusion during 4.5×10^9 years is about 10 per cent for silicon, 12 per cent for iron, and about 25 per cent for lead. The diffusion will modify the concentration of these elements in the region below, but only by a small percentage.

I. INTRODUCTION

The chemical composition of the primordial matter of the sun is thought to have been uniform or nearly so. Subsequently, the composition in the sun's central core has certainly been modified by nuclear reactions. There the helium content has been increased at the expense of the hydrogen content, but it is thought that the ratios of the heavier elements among themselves (from oxygen onward) have not been appreciably modified by nuclear reactions (see, e.g., Burbidge, Burbidge, Fowler, and Hoyle 1957). Nevertheless, the heavier elements tend to diffuse downward, reducing the relative amount in the outer layers and altering their ratios to one another. This diffusive separation in ordinary stars has been generally supposed to be inappreciable, however. This conclusion was reached long ago by Chapman (1917, 1922), Eddington (1926), and Rosseland (1931). It has recently become clear (Chapman 1958) that diffusive separation of multiply charged ions in an excess of ionized hydrogen, especially by thermal diffusion, can be decidedly more powerful than was formerly supposed. Hence we examined its influence in gaseous nebulae (Aller and Chapman 1958) and concluded that thermal diffusion cannot produce any significant change in them during their probable lifetimes. Here we consider diffusion in the sun.

We conclude that there it may have brought about an appreciable reduction of the proportion of the heavier elements in the sun's atmosphere. The amount of the reduction and of the changes in the ratios of the heavier elements among themselves depends greatly on the depth of the sun's convective zone. At present, this quantity is uncertain, and consequently so also are the estimates of the diffusive changes of composition in the solar atmosphere.

II. THE EQUATION OF DIFFUSION

The general equation of diffusion in a binary gas mixture (Chapman and Cowling 1939, 1952, p. 266) is

$$w_1 - w_2 = -D_{12}\left[\frac{1}{c_1 c_2}\frac{\partial c_1}{\partial r} + \frac{m_2 - m_1}{c_1 m_1 + c_2 m_2}\frac{1}{p}\frac{\partial p}{\partial r} - \frac{m_1 m_2 (F_1 - F_2)}{kT(c_1 m_1 + c_2 m_2)} + \frac{\alpha_{12}}{T}\frac{\partial T}{\partial r}\right]. \tag{2.1}$$

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Here the properties of the gas are taken to vary in only one direction, radially, in the application of this equation to the sun. The symbols have the following meanings:

$w_1 - w_2$ = The velocity of diffusion of constituents (1) relative to constituent (2) in the r-direction,
D_{12} = The coefficient of diffusion,
m_1, m_2 = The masses of particles 1, 2,
c_1, c_2 = Their relative concentrations, namely, n_1/n and n_2/n, where $n = n_1 + n_2$, and n_1, n_2 are the number densities,
p, T = The pressure and absolute temperature,
α_{12} = The thermal diffusion factor (Chapman and Cowling 1952, p. 399),
k = Boltzmann's constant,
F_1, F_2 = The forces per unit mass on the particles 1, 2. As they are equal in the case of gravitational forces, these make no contribution to the third term on the right of equation (2.1).

Equation (2.1) will here be applied to a mixture of particles for which n_1/n_2 is large, i.e., the second constituent is rare. In this instance, $c_1 = 1$ approximately. Although the mass ratio, W, given by

$$W = \frac{m_2}{m_1}, \tag{2.2}$$

may be considerable (e.g., over 200 for lead ions in hydrogen), we suppose that $c_2 m_2$ is small compared with $c_1 m_1$. Also, since

$$c_1 + c_2 = 1 , \tag{2.3}$$

it follows that

$$\frac{\partial c_1}{\partial r} = -\frac{\partial c_2}{\partial r}. \tag{2.4}$$

If the gas is at rest, $c_1 w_1 + c_2 w_2 = 0$ or, approximately, $w_1 = 0$, and w_2 denotes the diffusive speed of constituent 2 relative to the general mass.

We suppose also that the forces are purely gravitational and electrical (due to a field of intensity E). If the particles have charges e_p, Ze_p, then

$$F_1 - F_2 = \left(\frac{1}{m_1} - \frac{Z}{m_2}\right) E e_p , \tag{2.5}$$

where e_p denotes the charge of a proton. With these approximations, in the present special instance equation (2.1) may be replaced by

$$w_2 = D_{12}\left(-\frac{1}{c_2}\frac{\partial c_2}{\partial r} + \frac{W-1}{p}\frac{\partial p}{\partial r} - \frac{W-Z}{kT} E e_p + \frac{\alpha_{12}}{T}\frac{\partial T}{\partial r}\right). \tag{2.6}$$

The solar gas is not a binary mixture. For our purposes it must be considered as including at least three kinds of particles: (1) protons, (2) ions of another kind in smaller number, and (3) electrons. As the gas must be almost exactly neutral,

$$n_3 \, (\text{or } n_e) = n_1 + Z n_2 .$$

In a gas consisting solely of electrons and positive ions of one kind only, gradients of pressure and temperature tend to produce a gradient of electron concentration. Any such gradient, however, is held down to a very low value by the opposing influence of the electrostatic field produced by such a gradient. If there is a steady state with no electric current flow, $w_1 - w_2 = 0$. The first term on the right of equation (2.1) also being negligible, equation (2.1) indicates that an electrostatic field, E, will be set up by the gradients of p and T. In the present case, we consider equation (2.1), taking particles (1) to be ions of charge Ze_p and particles (2) to be electrons. The condition of electrical neutrality gives

$$c_1 Z = c_2 \qquad \text{or} \qquad c_i Z = c_e \,. \tag{2.7}$$

Hence, by equation (2.3),

$$c_i = \frac{1}{1+Z} \tag{2.8}$$

and

$$m = c_i m_i \,, \tag{2.9}$$

neglecting m_e in comparison with m_i. Then equation (2.1) leads to the expression

$$\frac{E e_p}{kT} = \frac{\alpha_{ie}}{(1+Z)\,T}\,\frac{\partial T}{\partial r} - \frac{1}{p}\,\frac{\partial p}{\partial r} \,. \tag{2.10}$$

Chapman (1958, p. 356) has shown that, approximately,

$$\alpha_{ie} = \frac{-3\,(1+Z)}{2.6 + 2^{3/2} A/Z} \,, \tag{2.11}$$

where A is approximately 0.4. Hence

$$\frac{E e_p}{kT} = \frac{-3}{2.6 + 2^{3/2} A/Z}\,\frac{1}{T}\,\frac{\partial T}{\partial r} - \frac{1}{p}\,\frac{\partial p}{\partial r} \,. \tag{2.12}$$

If the ions are protons,

$$\frac{E e_p}{kT} = -\frac{0.805}{T}\,\frac{\partial T}{\partial r} - \frac{1}{p}\,\frac{\partial p}{\partial r} \,. \tag{2.13}$$

If the ions have charge $15 e_p$,

$$\frac{E e_p}{kT} = -\frac{1.12}{T}\,\frac{\partial T}{\partial r} - \frac{1}{p}\,\frac{\partial p}{\partial r} \,. \tag{2.14}$$

In a gas composed mainly of protons and electrons with a very small admixture of ions of mass $W m_p$ and charge $Z e_p$, the electric field will have a value departing only slightly from that given by equation (2.13), and the deviation will be toward the value given by equation (2.14). As regards the pressure-gradient term, the fields are the same; the temperature-gradient term is only slightly greater for the ion-electron mixture than for the proton-electron mixture. Hence, very approximately, E will be given by equation (2.13) for this type of proton-ion electron mixture.

This field keeps the gas neutral with no resultant mean motion of the electrons relative to the positively charged part of the gas. There will still remain a tendency for the ions to diffuse relative to the protons. It seems reasonable to use the binary equation (2.6) to give the diffusive speed of ions (2) relative to protons (1), inserting in this equation the electric field given by equation (2.13). This was the procedure adopted by Chapman (1958), who wrote (p. 360), "The electric field in the triple mixture will cause the electrons easily and quickly to adjust their distribution so as to preserve the neutrality of the gas," despite a gradient of relative concentration of the protons and ions. Thus, from equations (2.6) and (2.13), we have

$$w_2 = D_{12}\left[-\frac{1}{c_2}\,\frac{\partial c_2}{\partial r} + \frac{2W - Z - 1}{p}\,\frac{\partial p}{\partial r} + \frac{2.65 f Z^2 + 0.805\,(W - Z)}{T}\,\frac{\partial T}{\partial r}\right] , \tag{2.15}$$

where the value of α_{12} (or α_{pi}) given by Chapman (1958) has been inserted. It is $2.65fZ^2$, where f is approximately 0.96. Hence $2.65f = 2.54$.

For the heavier ions in the sun, the co-factors of dp/dr and dT/dr in equation (2.3) are positive, whereas these gradients are negative. Hence the diffusive speeds w_p and w_T, associated with the gradients dp/dr and dT/dr, are negative (i.e., downward).

The value of D_{12} is given approximately (Chapman and Cowling p. 179) in our case by

$$D_{12}=\frac{3\,(2kT)^{5/2}}{16n_1\,(\pi m_1 M_2)^{1/2}Z^2\epsilon^4 A_1(2)}\,. \tag{2.16}$$

Here

$$M_2=\frac{W}{1+W}, \tag{2.17}$$

$$A_1(2)=\log_e(1+x_D^2)=2.303\,\log_{10}(1+x_D^2)\,, \tag{2.18}$$

$$x_D=\frac{4d_D kT}{Ze^2}, \tag{2.19}$$

$$d_D=\text{Debye length}=\left(\frac{kT}{4\pi n_e e^2}\right)^{1/2} \tag{2.20}$$

(Landau 1926; Persico 1926; Bohm and Aller 1946; Spitzer and Routly 1950). Thus

$$x_D^2=\frac{2.73\times 10^8 T^3}{Z^2 n_1}, \tag{2.21}$$

$$D_{12}=1.947\times 10^9 T^{5/2}\,[n_1 M_2 Z^2 A_1(2)]^{-1}\,. \tag{2.22}$$

For silicon ($W = 28$), iron ($W = 56$), and lead ($W = 207$), the factor M_2 differs from unity by less than 2 per cent. This difference will be ignored, i.e., we take $M_2 = 1$. Then, for such elements, D_{12} depends only on n_1, Z, and T.

III. THE REGIONS IN WHICH DIFFUSION CAN OCCUR

Gradients of concentration, pressure, and temperature necessarily produce relative diffusion of the particles of different kinds. In general, the combined effect is to set up a non-uniform distribution of composition. Turbulence and mixing, however, can defeat this tendency. In the sun, such mixing will occur in the convective region (here called "region C"). Hence the composition will be maintained uniform in this region. Its upper surface, of radius R, lies just below the photosphere. Below the inner surface of radius r_0 there is a region in radiative equilibrium. There the sun's rotation is supposed to set up a slow meridional circulation, but the mixing influence of this motion is thought to be negligible (Sweet 1950), so that diffusion can operate in this region.

Because of the pressure gradient (which depends mainly on solar gravity), the heavy particles tend to descend, and the light particles tend to rise. The downward increase in temperature enhances this tendency; hence region C tends to lose its heavier constituents across its lower surface.

The diffusive speeds w_p and w_T for a few typical ions will be calculated from our equations at various levels, regardless of the extent to which mixing may occur at these levels. The part of the diffusive speed that depends on the concentration gradient will be separately considered (Secs. X, XI); this term is initially zero if the composition of the primordial matter was uniform.

In computing the diffusive speed, w, the presence of helium is ignored. Thus the solar gas is treated as if it consists only of atomic hydrogen with a small admixture of heavier elements. The gas is supposed to be fully ionized in the sense that there are no electrically

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neutral particles in the region considered. The heavier atoms are highly ionized but not fully stripped of their electrons. Each kind of atom has a constant W, but its charge Ze is a function of the radial distance r from the sun's center.

IV. THE SOLAR MODEL

Our numerical illustrations are based on a particular solar model, namely, Model III given by Weymann (1957). It has the following properties: The proportions (by mass) of hydrogen, helium, and heavier elements are, respectively: $X = 0.74$, $Y = 0.24$, and $(1 - X - Y) = Z = 0.02$. Also $r_0/R = 0.847$ (hence the depth of the inner surface of region C is 107000 km). The temperature T_0 and density ρ_0 at this depth are $T_0 = 1000000^\circ$ K, $\rho_0 = 0.019$ gm cm^{-3}. The electron and proton number densities are thus related:

$$\frac{n_e}{\rho} = \frac{1+X}{2m_1} = 5.25 \times 10^{23}\,; \qquad \frac{n_1}{\rho} = 4.4 \times 10^{23}\,; \qquad n_e = 1.19 n_1 . \tag{4.1}$$

Weymann's values of n_e and T and of the logarithmic gradients of p and T at various levels r/R in the sun are given in Table 1. The sixth row gives $(4d_D kT/\epsilon^2)^2$. This quantity is $Z^2 x_D^2$, which is used later in calculating $A_1(2)$ and D_{12}.

TABLE 1

WEYMANN'S SOLAR MODEL III

	r/R									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.86	0.9
$n_\epsilon = 10^{23}\times$	448	190	67.5	21.7	6.79	2.12	0.65	0.186	0.088	0.050
$T = 10^6$ °K$\times$	12.6	9.35	6.65	4.74	3.42	2.50	1.80	1.27	0.885	0.605
$dp/pdr = 10^{-10}\times$	1.207	1.82	2.42	2.18	2.15	2.18	2.24	2.44	3.05	4.05
$(1/T)(dT/dr) = 10^{-10}\times$	0.362	0.482	0.494	0.470	0.471	0.445	0.477	0.587	1.26	1.59
$(4d_D kT/\epsilon^2)^2 = 10^4\times$	1.23	1.17	1.19	1.33	1.61	2.02	2.45	3.00	2.14	1.21

V. THE IONIC CHARGES Ze

The degree of ionization of the atoms increases with the depth below the photosphere. The value of the average charge Ze for each ion in the higher layers can be calculated by the usual ionization equation, inserting the appropriate ionization potential (Moore 1958). The approximate theory due to Strömgren (1932, 1933) can be used for the deeper layers. It estimates that a nucleus of atomic number z has

$$2n^2 \left\{ 1 + \frac{G(T)}{n_\epsilon} \exp \left[\frac{\chi_n(z)}{kT} \right] \right\}^{-1}$$

bound electrons of principal quantum number n. Here

$$\chi_n(z) = 2\pi^2 \epsilon^4 m_\epsilon \frac{(z - \sigma^2)}{n^2 h^2}$$

and

$$G(T) = 2\left(2\pi m_\epsilon \frac{kT}{h^2}\right)^{3/2} = 4.82 \times 10^{15} T^{3/2} ,$$

where σ is the number of screening electrons bound in shells of lower quantum number. The number of electrons in the $K, L, \ldots,$ shells are successively calculated to determine Z.

For silicon, iron, and lead at various depths in Weymann's Model III, the approximate values thus obtained for Z are given in Table 2.

VI. THE DIFFUSIVE SPEEDS w_p, w_T

Using equations (2.15), (2.18), (2.21), and (2.22) and the values given in Tables 1 and 2, we have calculated the numbers $A_1(2)$, the coefficients of diffusion, D_{12}, and the diffusive speeds, w_p, and w_T, at various depths for silicon, iron, and lead. The results are given in Tables 3, 4, and 5. In each table the last row gives $w_p + w_T$.

The number $A(2)$ in these three tables has only a small range, from about 3.8 to 5.7. The diffusion coefficient, D_{12}, in general increases upward and at each level is greatest for silicon and least for lead. In almost all instances the pressure diffusive speed, w_p, is less than w_T. The combined speed, w_p and w_T, for silicon is a little less than for iron, and for lead (over the range r/R greater than 0.7) it is from 40 to 60 per cent greater than for silicon.

TABLE 2

IONIC CHARGES AT VARIOUS LEVELS

	Z	r/R					
		0.1–0.4	0.5, 0.6	0.7	0.8	0.86	0.9
Silicon.	14	16	15	11	10.4	9	7
Iron.	26			15	15	13.6	10
Lead.	82			21	21	21	17

TABLE 3

THE DIFFUSIVE SPEEDS w_p, w_T FOR SILICON; $W=28$

	r/R									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.86	0.9
A_1 (2).	4.49	4.43	4.45	4.56	4.76	4.96	5.35	5.65	5.60	5.54
$D_{12}=10^{-3}\times$.	45	51	61	79	117	151	240	370	429	484
w_p (cm sec^{-1}) $=10^{-10}\times$	2.3	4.0	6.4	7.4	10.8	14.1	23.5	40.3	60	94
w_T (cm sec^{-1}) $=10^{-10}\times$	6.2	9.4	11.4	14.2	21.0	25.5	37.0	63.0	120	109
$w_p+w_{pT}=10^{-10}\times$.	8.5	13.4	17.8	21.6	31.8	39.6	60.5	103.3	180	203

TABLE 4

THE DIFFUSIVE SPEEDS w_p, w_T FOR IRON; $W=56$

	r/R									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.86	0.9
A_1 (2).	3.91	3.85	3.86	3.98	4.30	4.52	4.70	4.92	4.79	4.82
$D_{12}=10^{-3}\times$.	29	33	40	51	76	106	146	204	241	274
w_p (cm/sec) $=10^{-10}\times$. . .	3.3	5.8	9.2	10.6	15.9	22.5	31.5	48.0	72	113
w_T (cm/sec) $=10^{-10}\times$. . .	6.9	10.6	12.9	15.8	21.9	28.6	42.2	72.6	154	127
$w_p+w_T=10^{-10}\times$.	10.2	16.4	21.1	26.4	37.8	51.1	73.7	121	226	240

VII. UNCERTAINTIES IN THE PRESSURE AND TEMPERATURE DIFFUSIVE SPEEDS

In Section II the ion-proton diffusion in the solar gas was treated by methods (Chapman 1958) based on the binary diffusion equation. A more cumbersome method of treatment could be adopted, using the full equations of diffusion for a mixture of three kinds of particles. Hellund (1940) and Hirschfelder (1954) and his colleagues have given the general equations for such a mixture. Their treatment can be simplified in the case of the solar gas because the ratios m_e/m_1, m_e/m_2, and n_2/n_1 are small. Burgers (private communication) has treated the triple solar-gas mixture by approximating to the full set of three diffusion equations in another way. He obtains rather different factors in the p, T diffusion terms in equation (2.15). Instead of $2W - Z - 1$ in the dp/dr term, he obtains $W - Z/2$, or approximately half the value given in equation (2.15). Instead of $[2.56Z^2 + 0.80(W-Z)]$ in the dT/dr term, he obtains $[3.45Z^2 - 0.8Z - 1]$.

TABLE 5

THE DIFFUSIVE SPEEDS w_p, w_T FOR LEAD; $W = 207$

	r/R			
	0.7	0.8	0.86	0.9
A_1 (2)	4.06	4.25	3.93	3.78
$D_{12} = 10^{-3}\times$	87	120	112	120
w_p (cm sec^{-1}) $= 10^{-10}\times$	76	115	134	193
w_T (cm sec^{-1}) $= 10^{-10}\times$	53	90	181	171
$w_p + w_T = 10^{-10}\times$	129	205	315	364

TABLE 6

RATIOS OF DIFFUSION FACTORS AND SPEEDS

(Burgers' Values Divided by Chapman's Values for $r/R = 0.8$)

	Silicon $W=28$; $Z=10$	Iron $W=56$; $Z=15$	Lead $W=207$; $Z=21$
Ratio of factors in w_p	0.51	0.51	0.50
Ratio of factors in w_T	1.24	1.27	1.18
w_p (Burgers)	17.80	23.80	57.00
w_T (Burgers)	78.00	92.00	106.00
Ratio of $w_p + w_T$(Burgers' values)/(Chapman's value)	0.93	0.95	0.80

Table 6 gives the ratio of these factors obtained by Burgers to those obtained by Chapman, in some typical cases associated with Tables 3–5, namely, for $r/R = 0.8$. It also gives the corresponding (Burgers) values of w_p, w_T, and $w_p + w_T$ and their ratios to those obtained from equation (2.15).

The use of Burgers' formula would enhance the estimate of w_T and about halve that of w_p, but the value of the combined speed, $w_p + w_T$, given by his formula is not much less than that given by equation (2.15) in this illustrative example.

The discrepancy between the two formulae for w_2 will be further examined, but, as far as diffusion in the sun over the range of r/R here considered is concerned, the results would be little altered if Burgers' value were found to be the better approximation.

VIII. THE PROPORTIONATE DEPLETION OF THE CONVECTIVE REGION

The total mass $M(r)$ of the sun in the region above the level r is given by

$$M(r) = \int_r^R 4\pi r^2 \rho(r)\, dr = 4\pi \rho(r)\, r^2 h(r)\,. \tag{8.1}$$

The corresponding formula for a constituent (2) is

$$M_2(r) = \int_r^R 4\pi r^2 \rho_2(r)\, dr = 4\pi \rho_2(r)\, r^2 h(r)\,. \tag{8.2}$$

The value of $h(r)$ has been determined by numerical integration for Weymann's solar Model III, using his values of ρ, for $r/R = 0.7$, 0.8, and 0.86 with the following results:

r/R	0.7	0.8	0.86
$h(r)/R$	0.0929	0.0762	0.0574

Weymann's model has a convective region C whose inner boundary ($r = r_0$) is at $r_0/R = 0.847$. Here, for illustration, we shall use values of $h(r)$ calculated from his model at $r/R = 0.7$, 0.8, and 0.86 as if they applied to convective regions down to one or another of these depths. This procedure seems adequate in the rough reassessment here attempted of the importance of diffusion in the sun.

If the convective region C extends down to radius r, then the uniformity of chemical composition implies that, at that value of r,

$$h_2(r) = h(r)\,. \tag{8.3}$$

The rate of loss of mass of a rare ionized constituent (2), across the underside of this region C, by p and T diffusion, is given by

$$\frac{dM_2(r)}{dt} = -4\pi r^2 \rho_2(r)\, w_2(r)\,,$$

where $w_2 = w_p + w_T$. Consequently,

$$\frac{1}{\rho_2(r)} \frac{d\rho_2(r)}{dt} = \frac{w_2(r)}{h(r)} \equiv \frac{1}{t_0}\,, \tag{8.4}$$

and

$$\rho_2(r) \sim e^{-t/t_0}\,. \tag{8.5}$$

In a time t_1 the amount of the constituent (2) in the region C will, so far as p and T diffusion is concerned, be reduced in the ratio

$$\exp\left(-\frac{t_1}{t_0}\right).$$

Table 7 gives for silicon, iron, and lead the values of t_0 and of $\exp(-t_1/t_0)$, taking $t_1 = 4.5 \times 10^9$ years $= 1.42 \times 10^{17}$ seconds, namely, the estimated age of the earth. The percentage loss during this time is also given. The last row gives $\exp(t_1/t_0)$, the correction factor to be applied to the observed abundance ratios of these elements (compared with hydrogen) to obtain the primordial abundances.

As the correction factors differ from one element to another, the present abundance ratios for the heavier elements among themselves also differ from the original ratios. The

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correction factors in the accompanying table are to be applied to the observed abundance ratios for Fe/Si and Pb/Si to obtain the original ratios according to the hypotheses underlying Table 7. These results show that if the lower boundary of the convective region C is at 0.847, namely, the value corresponding to the Weymann Model III, the correction factor is significant for the iron/silicon ratio and considerable for lead/silicon. Weymann's value of r_0, however, is uncertain, since it depends on the adopted mechanistic model for the transport of energy in the convection region. Evidence favoring a lower value of r_0 will next be mentioned.

	r/R				r/R		
	0.7	0.8	0.86		0.7	0.8	0.86
Fe/Si........	1.07	1.05	1.19	Pb/Si........	1.19	1.27	1.54

TABLE 7

	r_0/R Constituent								
	0.70			0.80			0.86		
	Si	Fe	Pb	Si	Fe	Pb	Si	Fe	Pb
$t_0 = 10^{17} \times$	10.7	8.77	5.01	5.13	4.38	2.59	2.22	1.77	127.
Exp $(-t_1/t_0)$............	0.875	0.850	0.754	0.758	0.710	0.578	0.522	0.440	0.328
Per cent loss	12.5	15.0	24.6	24.2	29.0	42.2	47.8	56.0	67.4
Abundance correction....	1.142	1.176	1.327	1.319	1.383	1.730	1.916	2.230	3.065

IX. THE DEPLETION OF LITHIUM

The abundance ratio of lithium to silicon observed in the sun's atmosphere is much less than that determined for the earth and for meteorites. The solar abundance was determined by Greenstein and Richardson (1951) and more recently by Goldberg, Müller, and Aller (1960), while the meteoritic data has been assessed by Suess and Urey (1956). The logarithms of the ratio Li/Si in the two instances are as follows: sun, -6.54; earth and meteorites, -4.0. Thus the sun shows a relative depletion of lithium in the ratio 350.

The most natural explanation of this depletion is that the convective region extends downward to depths where nuclear reactions continuously destroy lithium atoms. As a consequence of the convective mixing, the depletion at those levels would affect the whole region and thus be observable in the sun's atmosphere.

If the temperature is expressed in units of a million degrees, ρ is the density, and X is the fraction of hydrogen by weight, the mean lifetime of a lithium atom is given by (Salpeter 1955; Bonsack 1959)

$$t = 2.63 \times 10^{-19} \frac{T^{2/3}}{X \rho} \exp\left(\frac{84.5}{T^{1/3}}\right) \text{years}.$$

Table 8 gives the values of t (expressed in years) calculated by this expression for several levels. Thus, during the estimated age of the earth of 4.5×10^9 years, the lithium that

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remained constantly at the level $0.7R$ throughout the period would be reduced by only a small amount. The average depletion throughout a convective region extending down to this level would be even less. Hence the notable scarcity of lithium in the sun's atmosphere argues for a still greater depth of the convective region unless it was otherwise destroyed early in the history of the sun.

X. DIFFUSIVE CHANGES OF COMPOSITION BELOW THE CONVECTIVE REGION

From Tables 3–5 it is possible to calculate whether, according to the Model III distribution of pressure and temperature, the pressure and temperature diffusion below the convective region would initially increase or decrease the concentration of silicon, iron, and lead at the different levels considered. The rate of pressure and temperature diffusive flow across a sphere of radius r is $4\pi r^2 \rho_2 w_2$. If the composition is initially uniform, the concentration-gradient term in the diffusion equation (2.15) will initially be zero. Hence at that time the diffusive flow is due only to the pressure and temperature gradients and the associated electrostatic field. The diffusive speeds due to these causes are given in the last rows of Tables 3–5. Initially, ρ_2/ρ is uniform. Hence the progression of $r^2 \rho w_2$ from level to level will show whether the layers between any pair of adjacent levels is being depleted or enriched. Values of $r^2 \rho w_2/R^2$ are given in Table 9.

TABLE 8

LIFETIME OF LITHIUM

($X=0.74$)

	r/R						
	0.70	0.72	0.74	0.76	0.78	0.80	0.84
$T(10^6\,°\mathrm{K})$.	1.80	1.69	1.58	1.47	1.37	1.27	1.04
$\rho = 10^{-3}\times$	124.00	97.00	76.00	59.00	46.00	35.40	21.20
$\log t$ (yr.)..	11.73	13.47	13.84	15.09	16.03	17.02	19.28

TABLE 9

VALUES OF $r^2 \rho w_2/R^2$

	r/R							
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
ρ....................	85.5	36.4	12.86	4.14	1.295	0.404	0.124	0.0354
Si $r^2\,\rho w_2/R^2 = 10^{-10}\times$...	7.26	19.6	20.6	14.3	10.3	5.74	3.68	2.33
Fe $r^2\,\rho w_2/R^2 = 10^{-10}\times$...	8.72	23.9	24.5	17.5	12.3	7.41	4.48	2.73
Pb $r^2\,\rho w_2/R^2 = 10^{-10}\times$...	....	....	....	....	....	....	7.84	4.63

From Table 9 it is clear that, initially, the elements silicon and iron would tend to accumulate below the level $r/R = 0.3$. The layers above would be depleted. The two values for lead likewise show that the upper layers around $r/R = 0.7$–0.8 would begin to be depleted.

This process would set up concentration gradients that would produce a partial counterdiffusion. The tendency due to $w_p + w_T$ would be retarded and reduced but not reversed. Hence, immediately below the convective region, there would be a reduction in the concentration of the heavier elements. This concentration gradient would there

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somewhat increase the drainage of the heavier elements from the convective region. Hence the changes in their abundances in the solar atmosphere should slightly exceed the values obtained in Section VIII.

XI. CHANGES OF COMPOSITION BELOW THE CONVECTIVE REGION

The mass of a constituent (2) between the radii r_1 and r_2 ($>r_1$) is

$$4\pi \int_{r_1}^{r_2} r^2 \rho_2 d r = \frac{4\pi}{3} (r_2^3 - r_1^3)\, \bar{\rho}_2 \,.$$

The rate of increase in the amount of this constituent in this spherical shell is $4\pi\Delta(\rho_2 r^2 w_2)$ where $\Delta(\rho_2 r^2 w_2)$ denotes the difference (value of $\rho_2 r^2 w_2$ at r_2) — (value of $\rho_2 r^2 w_2$ at r_1). Here w_2 denotes the downward diffusive speed $w_p + w_T$.

Hence the proportionate rate of decrease of constituent (2) in this shell is

$$\frac{3\Delta(\rho_2 r^2 w_2)}{\bar{\rho}_2 (r_2^3 - r_1^3)} \quad \text{or} \quad \frac{3\Delta(\rho\, r^2 w_2)}{\bar{\rho}\, (r_2^3 - r_1^3)}\,, \tag{11.1}$$

where we take ρ_2/ρ to be sensibly uniform, as is initially true.

TABLE 10

PERCENTAGE CHANGE OF COMPOSITION BY DIFFUSION IN 4.5×10⁹ YEARS

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Adopted $\bar{\rho}$......	63	30.5	10.7	3.4	1.07	0.344	0.102	
Per cent change silicon........	1.72	0.106	− 0.976	−1.18	−2.83	−2.88	−4.78	

Table 10, based on Table 9, gives values of the percentage change in the amount of constituent (2) in each shell considered in Table 9, over the time t_1 (the age of the earth) considered in Section VIII. In this table ρ_1/ρ_2 is crudely calculated from the limiting values of ρ at the inner and outer boundaries; $\bar{\rho}$ is their weighted mean, giving triple weight to the value at the inner boundary.

Calculations show that the change in composition for silicon and iron is not more than about 1 per cent within $r = 0.5R$; higher up, it increases to a few per cent. The small concentration gradient will produce a counterdiffusion speed that will slightly reduce these figures. Thus it would appear that, below the convective region, diffusion in the sun is of little importance.

XII. APPLICATIONS TO STARS OTHER THAN THE SUN

We have made no applications of diffusion theory to models of stars other than the sun. Later-type dwarfs with deeper convection zones will be less affected than the sun, but earlier-type dwarfs with shallower convection zones may show the effects of diffusion if the stars have ages comparable with that of the sun. The heavier elements would tend to sink out of sight into the inner radiative zone, with the result that the metal/hydrogen ratio would steadily decrease with time. Possibly some of the depletion in certain subdwarf stars might be accounted for in this way, although it is difficult to see how diffusion can account for all the depletion. In particular, one would expect a dependence of the abundance on the atomic weight that does not appear to be observed.

In stars of very great age, diffusion effects may become important in the atmospheric layers which can be subjected to spectroscopic analysis. The effects on the deep interior, i.e., on the model of the star, are small and probably can be neglected.

Diffusion in the sun has also been considered by Biermann (1937) and more recently by Wasiutynski (1958). However, our equations and the numerical coefficients we have derived differ from those of these authors; the results are, therefore, not comparable.

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REFERENCES

Bierman, L. 1937, *Astr. Nachr.*, **263**, 185.

Bohm, D., and Aller, L. H. 1947, *Ap. J.*, **105**, 131.

Bonsack, W. K. 1959, *Ap. J.*, **130**, 843.

Burbidge, E. M., Burbidge, G. R., Fowler, W. A., and Hoyle, F. 1957, *Rev. Mod. Phys.*, **29**, 547.

Chapman, S. 1917, *M.N.*, **77**, 539, 540.

———. 1922, *ibid.*, **82**, 292.

———. 1958, *Proc. Phys. Soc.*, **52**, 353. Note the following corrections: on the right of equations (31) and (33), change − to +; in the formulae for D_{12} on page 361, the factor $(\pi m_1 M)^2$ should be $(\pi m_1 M)^{1/2}$.

———. 1958, *Transport Properties in Gases*, ed. A. B. Cambel and J. B. Fenn (Evanston: Northwestern University Press), p. 143.

Chapman, S., and Aller, L. H. 1958, *Ap. J.*, **127**, 797. Note the correction: the numerical factor in equation (1) should be 6.5×10^7. This smaller value reinforces the conclusion therein reached, that in gaseous nebulae thermal diffusion is not important.

Chapman, S., and Cowling, T. G. 1952, *Mathematical Theory of Non-uniform Gases* (Cambridge: Cambridge University Press).

Eddington, A. S. 1926, *Internal Constitution of the Stars* (Cambridge: Cambridge University Press), pp. 276 ff.

Goldberg, L., Müller, E. A., and Aller, L. H. 1960, *Ap. J. Suppl.*, in press.

Greenstein, J. L., and Richardson, R. S. 1951, *Ap. J.*, **113**, 536.

Hellund, E. J. 1940, *Phys. Rev.*, **57**, 319, 328, 737, 743, as given on p. 408 of Chapman and Cowling.

Hirschfelder, J. O., Curtiss, C. F., and Bird, R. B. 1954, *Molecular Theory of Gases* (New York: John Wiley & Sons).

Landau, L. 1936, *Phys. Zs. U.S.S.R.*, **10**, 154.

Moore, C. E. 1958, *National Bureau of Standards Circular*, No. 467.

Persico, E. 1926, *M.N.*, **86**, 93.

Rosseland, S. 1931, *Astrophysik* (Berlin: Springer).

Salpeter, E. E. 1955, *Phys. Rev.*, **97**, 1237.

Spitzer, L., Routly, P. M., and Cohen, R. S. 1950, *Phys. Rev.*, **80**, 230.

Strömgren, B. 1932, *Zs. f. Ap.*, **4**, 118.

———. 1933, *ibid.*, **7**, 222.

Suess, H., and Urey, H. C. 1956, *Rev. Mod. Phys.*, **28**, 53.

Sweet, P. A. 1950, *M.N.*, **110**, 568.

Wasiutynski, J. 1958, *Ann. d'ap.*, **21**, 137, 169.

Weymann, R. 1957, *Ap. J.*, **126**, 208.