

# NEUTRON STAR MODELS

A. G. W. CAMERON

Atomic Energy of Canada Limited, Chalk River, Ontario, Canada

*Received June 17, 1959*

## ABSTRACT

Previous models of neutron stars were constructed with the assumption that the equation of state of a neutron gas is that of non-interacting Fermi gas. Such models have a maximum observable mass of about 0.7 solar mass. In fact, the potential energy of a neutron gas depends on the density; this introduces additional terms into the equation of state. A revised equation of state has been derived which makes use of a mean nuclear potential recently given by T. H. R. Skyrme. Twenty neutron star models have been constructed by integrating the general relativistic equations of hydrostatic equilibrium of the neutron gas. The results show that there is an upper limit to the observable mass of about 2 solar masses; the corresponding upper limit to the proper mass is about 3 solar masses. There is a lower limit to each of these masses of about 0.05 solar mass, below which the neutron star is unstable against transformation into an iron star. The radii of these neutron stars lie in the range 7–9 km. A qualitative discussion of the effects of transformation of neutrons into hyperons at very high densities is given.

## INTRODUCTION

About twenty years ago there was considerable interest in neutron stars, owing to the incomplete knowledge at that time of nuclear-energy-generation processes and to the fact that neutron stars and stars with neutron cores appeared to be attractive theoretical possibilities. In particular, it was suggested that the sudden enormous release of energy in a supernova explosion might result from the formation of a neutron star (Baade and Zwicky 1938; Zwicky 1938, 1939).

With the discovery of hydrogen-to-helium conversion processes and other mechanisms of nuclear-energy generation, together with studies of stellar evolution and white dwarf star models, it became generally believed that white dwarf stars were the inevitable end point of stellar evolution, to be reached after the exhaustion of nuclear-energy sources and the ejection of some mass. The ejection of mass was necessary because the white dwarf stars have a maximum allowable mass of 1.44 solar masses (Chandrasekhar 1939) or of only 1.21 solar masses when one takes into account the electrostatic spin-orbit effect (Rudkjöbing 1952). The latter limit is smaller than the minimum stellar mass for which it appears there has been time for the star to evolve away from the main sequence. It has been supposed that the mass loss may proceed by coronal evaporation or by ejection in nova and supernova explosions. It is certainly probable that most stars follow an evolutionary path of this kind.

As a result of these ideas the study of neutron stars has been generally neglected. Wheeler (1958) has discussed such stars as hypothetical objects which pose interesting and fundamental problems in general relativity and gravitation theory and in high-energy physics. Apparently, only Zwicky (1958) has continued to believe that neutron stars are formed in supernova explosions.

As a result of an examination of the physics of supernova explosions and of the formation of the elements, the writer has concluded that neutron stars are probable products of the supernova process. A brief summary of this research has been prepared (Cameron 1959); a much fuller account will be published later. As a preliminary to this account it is convenient to describe some new work relating to neutron star models.

## PREVIOUS WORK ON DEGENERATE STARS

In calculating models of completely degenerate stars, it is customary to assume that the temperature is zero, since the equation of state of the gas is very insensitive to the

temperature. An extensive amount of work has been done on white dwarf stars, in which the electrons are degenerate (Chandrasekhar 1939; Schatzman 1958). White dwarf models for a relativistic Fermi gas of electrons were obtained by Chandrasekhar (1939). The relation between the central density and the mass of these models is plotted in Figure 1 for a gas containing 2 atomic-mass units per electron. Rudkjöbing (1952) showed that Chandrasekhar's limiting mass of 1.44 solar masses was too large because of the existence of a relativistic "spin-orbit" effect produced by the strong radial electric field which must be set up in white dwarfs. The modifications of the models produced by this effect are also shown in Figure 1.

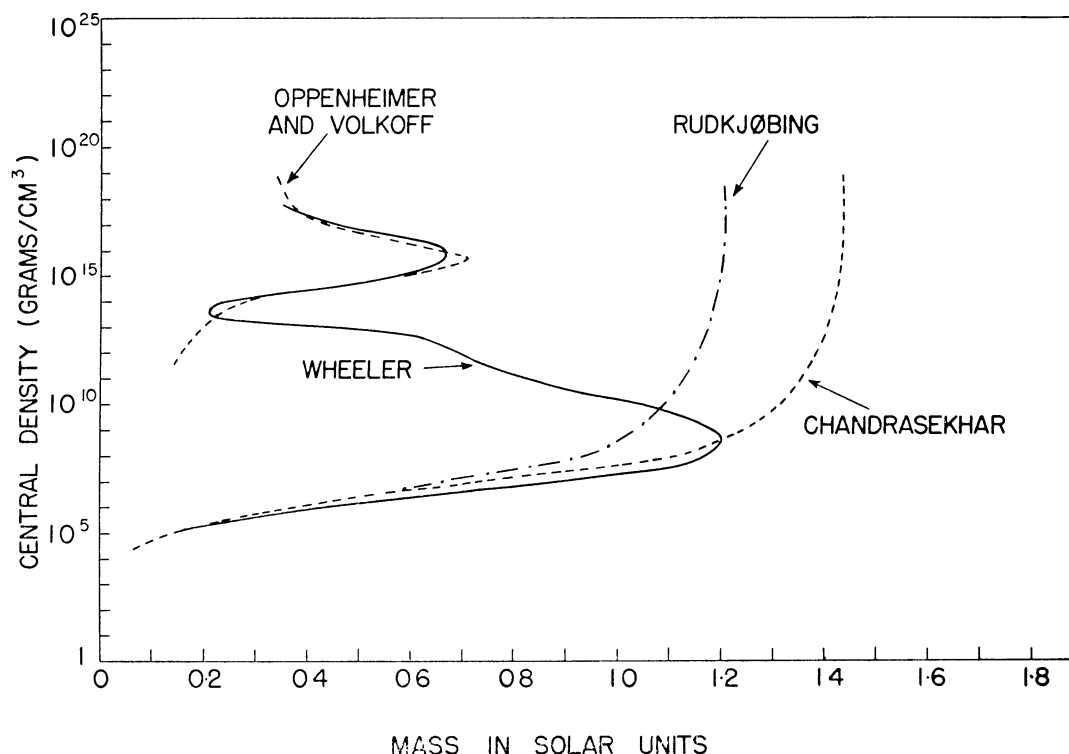


FIG. 1.—The relation between central density and the mass of various degenerate star models. Chandrasekhar's curve is for a white dwarf with a mean molecular weight of 2 atomic mass units. Rudkjöbing's curve is the same except for inclusion of the relativistic spin-orbit effect. The curve labeled "Oppenheimer and Volkoff" is for a set of neutron star models. The solid line marked "Wheeler" is a set of models computed with a generalized equation of state.

However, the limiting configurations calculated in this way cannot be closely approached. When the central density becomes very high, the Fermi level of the electrons rises until electron capture in the near-by nuclei takes place. These nuclei are driven to the neutron-rich side of the valley of beta stability. When the central density reaches  $2 \times 10^{11}$  gm/cm<sup>3</sup> (at zero temperature), conversion of the nuclei to neutrons takes place (Cameron 1959), accompanied by a rapid collapse of the stellar core. At high temperatures the critical density is considerably smaller. It is evident that any star that has not shed enough mass to form a stable white dwarf configuration with a central density smaller than this critical one must suffer this collapse when the critical density is reached, and the energy released by the collapse will be sufficient to expel the outer envelope. The remaining core will consist mostly of neutrons with a small content of electrons and protons. The process of formation of the neutron star may release more gravitational

potential energy than the total of all the nuclear conversion energy which was radiated away during the entire previous history of the star. The neutron star is therefore enormously hot. The very high thermal conductivity of the degenerate neutron gas, together with the small dimensions of the neutron star itself, insures that the surface layers of the star will be maintained at a temperature very close to that of the center. This will allow the surface layers to evaporate until the temperature has reached a reasonably low value. Subject to some further conditions to be discussed below, the structure of the neutron star should then be well approximated by that of a cold degenerate neutron gas.

For reasons of simplicity, most early discussions of neutron stars were carried out with the assumption that they were spheres of incompressible fluid. Oppenheimer and Volkoff (1939) took the important step of calculating neutron star models, using the equation of state for a relativistic Fermi gas of neutrons. The relation between the central density and the observable mass of their models is shown in Figure 1. They found that there was an upper limit to the masses of neutron stars of 0.71 solar mass, and they raised an important philosophic question regarding the behavior of a neutron gas containing more than the number of particles corresponding to this upper limit, for which there is no solution to the general relativistic equations of hydrostatic equilibrium. This same question prompted the writer's present investigation of neutron star models, since it appeared that a stellar core composed of iron would have to grow to the order of a solar mass before the electron degenerate "barrier" could be surmounted and the collapse could take place. It would then become a question of the dynamics of the collapse whether or not enough mass would be ejected to bring the mass of the neutron core below the Oppenheimer-Volkoff limit.

Also shown in Figure 1 is the relation between the central density and the mass for a series of models which Wheeler (1958) has obtained, using an equation of state for a gas at zero temperature whose composition changes with density in such a way as always to be in the state of lowest energy. This gas is composed of  $\text{Fe}^{56}$  in the white dwarf region; when the density becomes about  $10^8 \text{ gm/cm}^3$ , the gas changes to heavier neutron-rich nuclei, and at still higher densities it becomes composed predominantly of neutrons. It may be seen that this series of models passes through all the phases of the successively more highly condensed systems described above. Wheeler also discusses the problem raised by a contracting star which is sufficiently massive to surmount the electron degeneracy barrier and which would arrive in the region of nuclear densities with a mass above the Oppenheimer-Volkoff limit. If such a star is unable to lose its excess mass, then its behavior cannot be predicted on the basis of existing knowledge of general relativity and high-energy physics.

#### THE EQUATIONS OF EQUILIBRIUM

The most general static line element exhibiting spherical symmetry may be expressed in the following form (Tolman 1934):

$$dl^2 = - e^{\lambda(r)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + e^{\nu(r)} dt^2. \quad (1)$$

From this line element and with the assumption that the matter supports no transverse stresses and has no mass motion, Oppenheimer and Volkoff (1939) have shown that the general relativistic equations of hydrostatic equilibrium are

$$\frac{dM(r)}{dr} = 4\pi \rho(P) r^2, \quad (2)$$

$$\frac{dP(r)}{dr} = - \frac{[\rho(P) + c^{-2}P] G [M(r) + 4\pi c^{-2}P(r) r^3]}{r [r - 2c^{-2}GM(r)]}. \quad (3)$$

Here  $P$  and  $\rho$  are the pressure and the macroscopic energy density measured in proper co-ordinates. If  $R$  is the radius of the star, then  $M(R)$  is the observable mass of the star which would be determined by its gravitational effect on a distant test particle. This differs from the proper mass, which is the mass the star would have if its particles were dispersed to infinity by the gravitational binding energy in mass units.

The element of proper volume of a spherical shell in the star is  $4\pi e^{\lambda/2} r^2 dr$ . The function of  $e^{\lambda/2}$  is (Oppenheimer and Volkoff 1939)

$$e^{\lambda/2} = \left[ 1 - \frac{2GM(r)}{c^2 r} \right]^{-1/2}. \quad (4)$$

Hence the proper mass,  $M_p$ , and the gravitational binding energy,  $M_B$ , may be found by integrating the following equations:

$$\frac{dM_p(r)}{dr} = 4\pi \rho(r) r^2 \left[ \frac{r}{r - 2c^{-2}GM(r)} \right]^{1/2}, \quad (5)$$

$$\frac{dM_B(r)}{dr} = 4\pi \rho(r) r^2 \left\{ \left[ \frac{r}{r - 2c^{-2}GM(r)} \right]^{1/2} - 1 \right\}. \quad (6)$$

It is evident that the solution of the differential equations (2), (3), (5), and (6) depends only on the equation of state assumed for the neutron gas and on the conditions at the center of the star.

#### THE EQUATION OF STATE

The kinetic energy of a relativistic particle is related to its momentum by the equation

$$E_K = [(m_0 c^2)^2 + p^2 c^2]^{1/2} - m_0 c^2. \quad (7)$$

In a degenerate gas at zero temperature the momentum of the particle at the Fermi level is related to the particle and energy densities by

$$\frac{8\pi}{3} \frac{p_{\max}^3}{h^3} = \frac{N}{V} = \frac{\rho}{m}, \quad (8)$$

where  $N/V$  is the number of particles of mass  $m$  per unit volume.

From these relations it follows that, for small (non-relativistic) energies, the kinetic energy of the Fermi level of a degenerate gas varies as  $\rho^{2/3}$ . In the case of a degenerate electron gas, where the rest mass of the electrons is enormously smaller than the mass supported by each electron through the electrostatic field which is established, the Fermi energy of the electrons varies as  $\rho^{1/3}$  in the relativistic region. In the case of a neutron gas where the Fermi energy may become large compared to the rest energy of the particles in the extreme relativistic region, the Fermi energy varies as  $\rho^{1/4}$ .

Normal nuclear density is approximately  $2.91 \times 10^{14}$  gm/cm<sup>3</sup> (Cameron 1957). At this density the Fermi level of the nucleons is 60 Mev, and hence the nucleons are non-relativistic.

In a degenerate gas of electrons the particles interact only through their electric fields, and the matter is electrically neutral to a high degree of approximation. Hence the only contribution to the pressure of the gas comes from the kinetic energy of the particles. This gives the well-known equation of state for a degenerate electron gas. However, in a degenerate neutron gas there are complicated forces between the neutrons; these give a potential energy for the neutron gas which is a function of the density. Thus there must

be additional terms in the pressure in such a case, since the pressure of a system can be written as

$$P = \left( \frac{\rho}{V} \frac{\partial U}{\partial \rho} \right)_{T=0}, \quad (9)$$

where  $V$  is the volume of the system and  $U$  is its internal energy.

In recent years considerable progress has been made in constructing many-body theories of nuclear matter (see, for example, Eden 1959). These theories are based on a consideration of the scattering of two nucleons in the presence of other nucleons and in the presence of a general potential due to all the rest of the nuclear matter. Recently T. H. R. Skyrme (1959) has constructed a simple three-body effective nuclear potential which represents quite well many features of the more complicated many-body approach. This potential, modified for the case of a pure non-relativistic neutron gas, has been used in developing the equation of state in this paper. The writer is greatly indebted to Dr. Skyrme for providing the modified form of his potential.

According to Skyrme, the energy per cubic centimeter in a neutron gas,  $\epsilon$ , is

$$\epsilon = 7.98 \times 10^9 \rho^{5/3} + 9.79 \times 10^{-6} \rho^{8/3} - 1.381 \times 10^5 \rho^2, \quad (10)$$

where  $\rho$  is the density in grams per cubic centimeter. The internal energy of a system of  $N$  neutrons in a volume  $V/6.025 \times 10^{23} \rho$  is given by

$$U = [7.98 \times 10^9 \rho^{2/3} + 9.79 \times 10^{-6} \rho^{5/3} - 1.381 \times 10^5 \rho] N/6.025 \times 10^{23}. \quad (11)$$

From equation (9), the pressure-density relation becomes

$$P = 5.32 \times 10^9 \rho^{5/3} + 1.632 \times 10^{-5} \rho^{8/3} - 1.381 \times 10^5 \rho^2, \quad (12)$$

where  $P$  is the pressure in dynes per square centimeter.

In this equation the first term represents the kinetic energy of the neutrons, and the other two terms are obtained from the most important contributors to Skyrme's mean effective potential. All these terms would acquire a less rapid dependence on density when the neutrons become relativistic. Such modifications have not been made here, for a variety of reasons which will now be discussed.

Skyrme's equation of state, equation (12), is plotted in Figure 2. Also plotted in the figure is the first term of equation (12), which gives the kinetic-energy contribution to the pressure-density relation. It may be seen that at low densities there is less pressure than is contributed by the kinetic energy alone, whereas at densities above  $10^{15}$  gm/cm<sup>3</sup> the kinetic energy gives only a small contribution and the second term of equation (12) gives the major contribution. However, the range of validity of Skyrme's equation of state is not expected to extend beyond about  $10^{15}$  gm/cm<sup>3</sup>, since four-body and higher terms may be expected to make significant contributions. These terms would tend to make the pressure-density relation even steeper, since it appears likely that internucleon potentials possess hard cores. Such additional dependence of the pressure on density would offset the lesser dependence of the relativistic forms of the terms in equation (12).

Gammel and Thaler (1957) have fitted the proton-proton interaction at high energies by a potential which goes abruptly to infinity at a hard-core distance of  $0.4 \times 10^{-13}$  cm. The effect of this short-range repulsion on the equation of state can be taken into account by assuming that there is a limiting density at which the neutrons are closely packed hard spheres with  $0.4 \times 10^{-13}$  cm between the centers of the closest neighbors. Allowing for the fact that 26 per cent of the volume will be interstitial, this limiting density is  $3.7 \times 10^{16}$  gm/cm<sup>3</sup>. At this limiting density the neutron gas would achieve the physically impossible condition of incompressibility. It is nevertheless of interest to investigate what effects such a hard-core modification to the equation of state would have on neutron star models. The modification is also plotted in Figure 2.

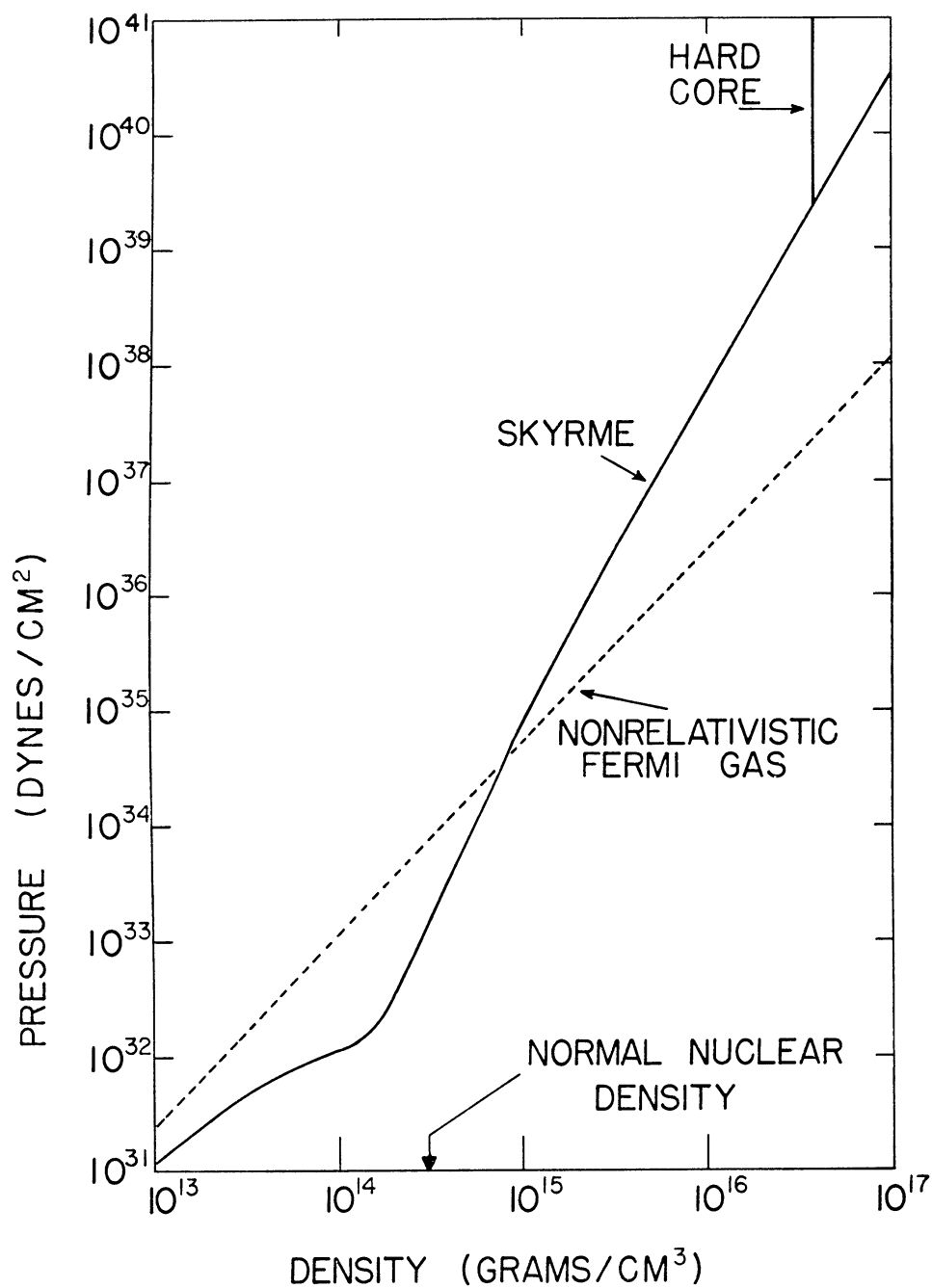


FIG. 2.—Skyrme's equation of state, equation (12). The hard-core modification is shown at the upper right, and the dashed line is the equation of state for a non-relativistic Fermi gas of neutrons, which is also the first term in equation (12).

Another reason why the writer has not taken into account complications inherent in using a relativistic equation of state is that no such things as pure neutron stars can be expected to exist. The neutrons must always be contaminated with some protons and sometimes with other kinds of nucleons (hyperons or heavy mesons). The detailed argument for this statement is as follows:

At extremely low densities the Fermi level of the neutrons would be negligible, and the neutrons would all decay to protons and electrons. In order to prevent this decay from proceeding, the neutrons must be contaminated with sufficient protons and electrons so that the Fermi energy of the electrons is equal to the decay energy of the neutrons, or 0.782 Mev. The partial density of the proton-electron gas is then  $10^7$  gm/cm<sup>3</sup>.

However, this is merely a minimum partial density of the proton-electron gas. In fact, the sum of the Fermi energies of the electrons and the protons must be equal to the Fermi energy of the neutrons plus 0.782 Mev at all times, when the neutron gas is in its lowest energy state. Suppose this were not so and that the Fermi energy of the neutrons was larger than required by the above equality. Then two neutrons at the top of the Fermi sea could make a collision in which each was transformed into a proton and electron, thus conserving both energy and momentum and preserving the Pauli exclusion principle. Hence, when the neutron density is  $10^{15}$  gm/cm<sup>3</sup>, the partial density of the protons and electrons must become  $10^{13}$  gm/cm<sup>3</sup>.

When the difference between the neutron and proton Fermi energies becomes equal to the rest energy of the mu meson, or 106 Mev, then neutrons can be transformed into protons and negative mu mesons:

$$n \rightarrow p + \mu^- + \bar{\nu}.$$

Once again, at the threshold of this process it is necessary to conserve momentum by having a collision take place between two neutrons at the top of the Fermi sea, each of which makes the transformation. The threshold density for the transformation is just under  $10^{15}$  gm/cm<sup>3</sup>. At higher densities the sum of the proton plus mu meson Fermi levels plus 106 Mev will be equal to the neutron Fermi level when the gas is in its lowest energy state.

When the density continues to increase, both protons and neutrons can be transformed into various kinds of hyperons. In every case the threshold for the process occurs when the difference in Fermi energies of the initial and final particles is equal to the threshold energy for creation of the hyperon. The gas attains its lowest energy state when the difference in Fermi energies of the particles appearing on each side of each transformation equation is equal to the reaction energy of the transformation. At the threshold it is always necessary to conserve momentum by arranging to have collisions take place in which two nucleons or hyperons at the top of the appropriate Fermi sea make the new transformation. The hyperon production equations are as follows:

$$\begin{aligned} n + n &\rightarrow n + n + 2\pi^0 \rightarrow \Lambda^0 + \Lambda^0 - 2(177) \text{ Mev}, \\ p + p &\rightarrow p + p + 2\pi^0 \rightarrow \Sigma^+ + \Sigma^+ - 2(251) \text{ Mev}, \\ \Lambda^0 + \Lambda^0 &\rightarrow \Lambda^0 + \Lambda^0 + \pi^0 \rightarrow \Sigma^0 + \Sigma^0 - 2(77) \text{ Mev}, \\ n + n &\rightarrow n + p + \pi^- \rightarrow p + \Sigma^- - 256 \text{ Mev}, \\ n + \Lambda^0 &\rightarrow p + \pi^- + \Lambda^0 \rightarrow p + \Xi^- - 204 \text{ Mev}, \\ \Lambda^0 + \Lambda^0 &\rightarrow \Lambda^0 + \Lambda^0 + 2\pi^0 \rightarrow \Xi^0 + \Xi^0 - 2(195) \text{ Mev}. \end{aligned}$$

In each transformation the collision creates one or two virtual pi mesons, which are then reabsorbed to form the final products. The threshold creation energy for each transformation is shown on the right-hand side, shown multiplied by 2 when the hyperons are created in pairs.

Thus it is apparent that in the density range  $10^{15}$ – $10^{17}$  gm/cm<sup>3</sup> the neutrons are largely transformed into protons and hyperons. All these particles are nucleons which interact strongly through nuclear forces. The creation of these new particles depresses the Fermi level of the neutrons far below the value to be expected for a pure neutron gas of the same density. Nevertheless, one may expect that the mean nuclear potential provided by all the particles is qualitatively the same as for a pure neutron gas. The terms in this potential give the major contribution to the pressure-density relation in this density range. These terms will have a slightly reduced dependence on density because of the depressed Fermi levels of the particles (Skyrme 1959), but again one can expect that higher-order terms will come in as one approaches the hard cores of the internucleonic potentials at high densities.

TABLE 1\*  
CHARACTERISTICS OF NEUTRON STAR MODELS

| $\rho_c$                       | $P_c$                  | $R$   | $M$     | $M_p$   | $M_B$      |
|--------------------------------|------------------------|-------|---------|---------|------------|
| $1 \times 10^{13}$ . . .       | $1.164 \times 10^{31}$ | 27.94 | 0.04424 | 0.04434 | 0.00009819 |
| $3 \times 10^{13}$ . . .       | $4.398 \times 10^{31}$ | 23.26 | 0.04754 | 0.04768 | 0.0001456  |
| $1 \times 10^{14}$ . . .       | $1.168 \times 10^{32}$ | 21.41 | 0.04426 | 0.04441 | 0.0001483  |
| $2 \times 10^{14}$ . . .       | $3.474 \times 10^{32}$ | 20.49 | 0.03608 | 0.03622 | 0.0001209  |
| $3 \times 10^{14}$ . . .       | $1.306 \times 10^{33}$ | 10.40 | 0.03278 | 0.03300 | 0.0002134  |
| $4 \times 10^{14}$ . . .       | $3.632 \times 10^{33}$ | 7.409 | 0.07062 | 0.07157 | 0.0009427  |
| $6 \times 10^{14}$ . . .       | $1.479 \times 10^{34}$ | 7.538 | 0.2295  | 0.2375  | 0.005349   |
| $8 \times 10^{14}$ . . .       | $3.830 \times 10^{34}$ | 8.232 | 0.4701  | 0.5008  | 0.03075    |
| $1 \times 10^{15}$ . . . . .   | $7.830 \times 10^{34}$ | 8.752 | 0.7459  | 0.8212  | 0.07554    |
| $1.5 \times 10^{15}$ . . . . . | $2.750 \times 10^{35}$ | 9.262 | 1.357   | 1.626   | 0.2695     |
| $2 \times 10^{15}$ . . . . .   | $6.528 \times 10^{35}$ | 9.180 | 1.726   | 2.224   | 0.4977     |
| $3 \times 10^{15}$ . . . . .   | $2.144 \times 10^{36}$ | 8.630 | 1.985   | 2.814   | 0.8287     |
| $4 \times 10^{15}$ . . . . .   | $4.906 \times 10^{36}$ | 8.152 | 2.011   | 3.002   | 0.9915     |
| $6 \times 10^{15}$ . . . . .   | $1.548 \times 10^{37}$ | 7.597 | 1.953   | 3.054   | 1.100      |
| $1 \times 10^{16}$ . . . . .   | $6.441 \times 10^{37}$ | 7.205 | 1.869   | 2.998   | 1.129      |
| $3.7 \times 10^{16}$ . . . . . | $2.314 \times 10^{39}$ | 7.001 | 1.804   | 2.924   | 1.120      |
| $1 \times 10^{17}$ . . . . .   | $3.389 \times 10^{40}$ | 6.994 | 1.801   | 2.919   | 1.119      |
| $1 \times 10^{18}$ . . . . .   | $1.619 \times 10^{43}$ | 6.994 | 1.801   | 2.919   | 1.119      |
| $1 \times 10^{19}$ . . . . .   | $7.562 \times 10^{45}$ | 6.994 | 1.801   | 2.919   | 1.119      |
| $3.7 \times 10^{16}$ . . . . . | $7.562 \times 10^{45}$ | 6.994 | 1.801   | 2.919   | 1.119      |

\*  $\rho_c$  is the central density in grams per cubic centimeter;  $P_c$  is the central pressure in dynes per square centimeter. The radius  $R$  is in kilometers. The observable mass,  $M$ , the proper mass,  $M_p$ , and the gravitational binding energy,  $M_B$  are in solar mass units. The last line of the table contains the results for the hard-core model.

There is thus a great deal of uncertainty about the equation of state of a nucleon gas at very high densities. Equation (12) has been used far outside its range of validity because various effects which cause a departure from it have opposing tendencies and because it has been desired to obtain a qualitative prediction about the properties of the resulting neutron or nucleon star models.

#### THE NEUTRON STAR MODELS

The equilibrium equations (2), (3), (5), and (6) have been integrated numerically with the aid of an electronic computer, starting with various assumptions about the central density. Nineteen models were constructed using equation (12) as the equation of state. One model was constructed with the "hard-core" assumption that the gas abruptly becomes incompressible at a density of  $3.7 \times 10^{16}$  gm/cm<sup>3</sup> and obeys equation (12) at smaller densities; in this case the integration was started with the highest central pressure of those which had been used in the other nineteen models.

The principal characteristic features of the resulting models are given in Table 1. The radius of the models is defined to be the distance from the center to the point at which

the density has fallen to  $10^8$  gm/cm<sup>2</sup>. At this point the scale height of the atmosphere varied from about 20 meters in the models with smallest central density to about 1 cm in the models with the largest central density.

The observable and the proper masses of the models are plotted as functions of the central density in Figure 3. It may be seen that the maximum observable mass has increased from the low value of Oppenheimer and Volkoff (1939) to about 2 solar masses.

The corresponding maximum proper mass is about 3 solar masses. These values are only qualitatively reliable because the appropriate central densities lie in the vicinity of  $4 \times 10^{15}$  gm/cm<sup>3</sup>, where the equation of state is unreliable. Nevertheless, it appears to be most probable that the maximum allowable observable mass of a neutron star is larger than the maximum allowable mass of a white dwarf star composed of helium or heavier elements.

The variation of density with radial distance is plotted in Figure 4 for five of the models. It may be noted that, in all except the models with the highest central densities, the density remains remarkably constant until very near the surface and that the radii

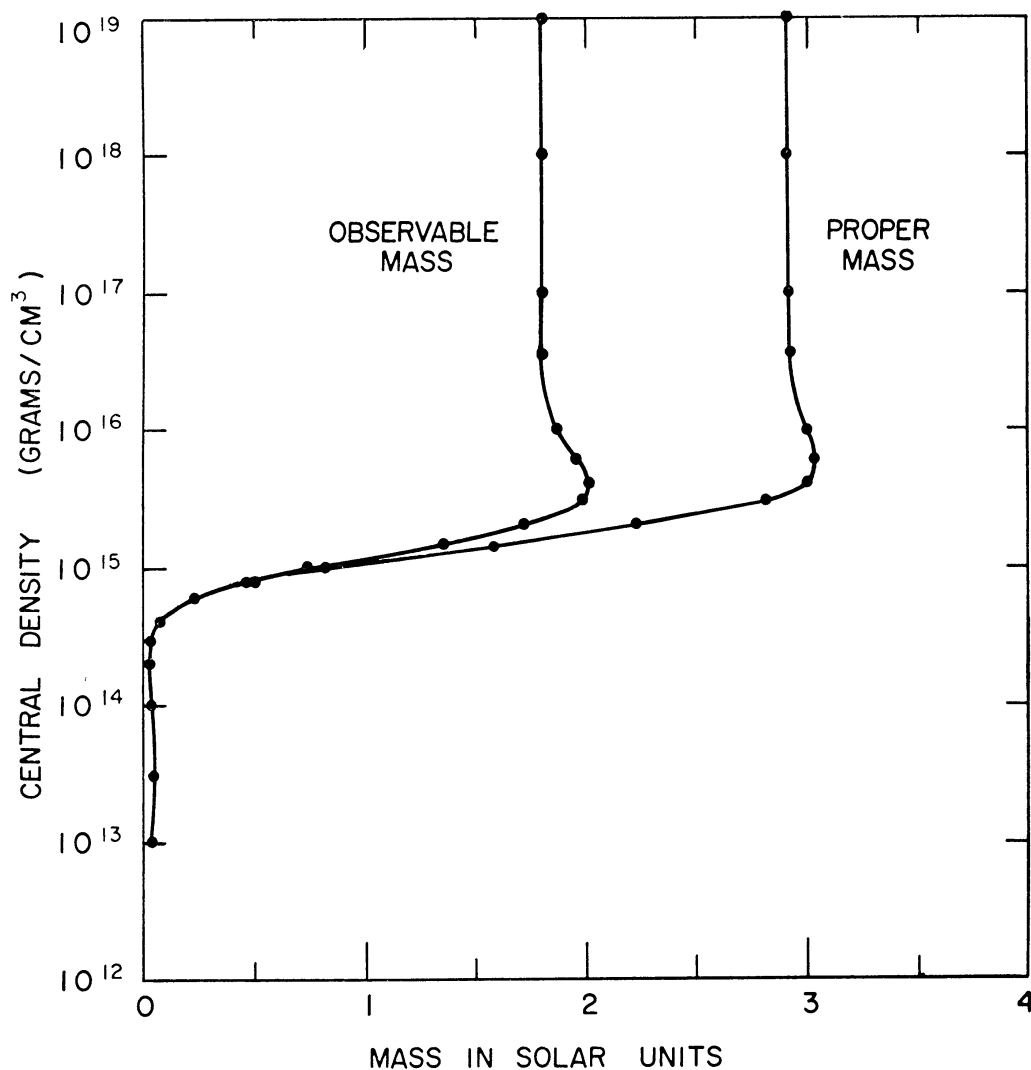


FIG. 3.—The observable and proper masses of the models listed in Table 1

of the models are very nearly the same. In the models with the highest central densities, the density varies as a constant inverse power of the radial distance,

$$\rho = \frac{8 \times 10^{19}}{r^{3/4}}. \quad (13)$$

This relation is no longer followed very close to the center of the star. However, as one assumes higher and higher central densities, equation (13) becomes valid at smaller and smaller distances from the center, and only at the center is there any appreciable difference in the resulting models. These differences thus involve smaller and smaller quanti-

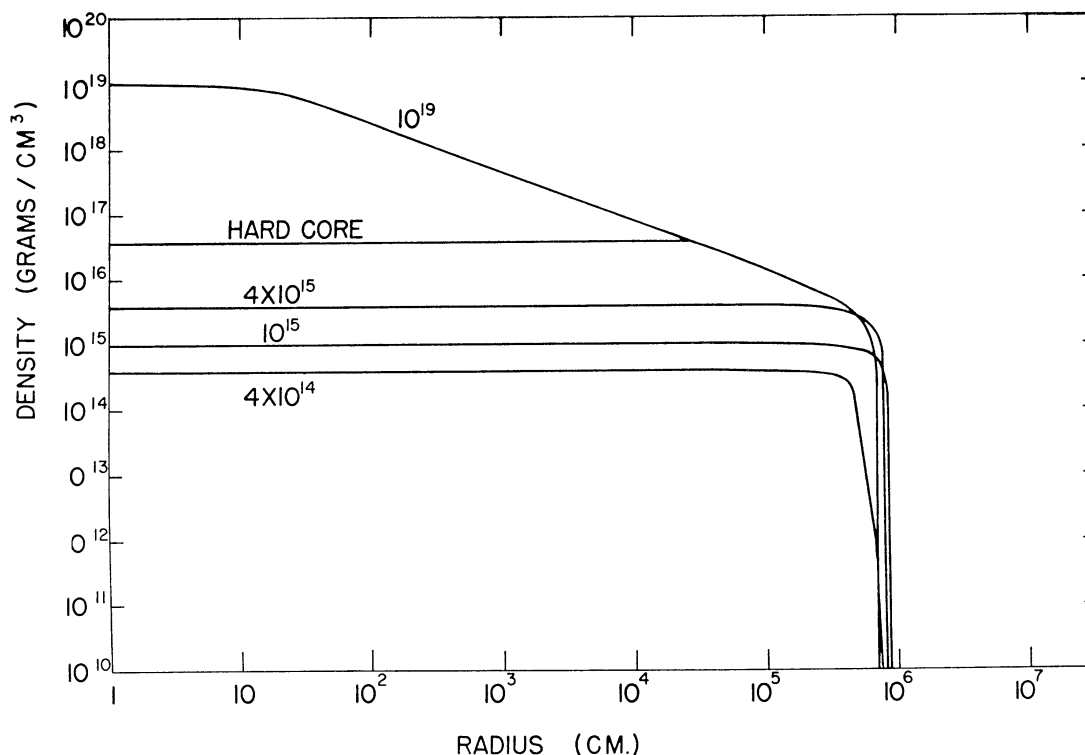


FIG. 4.—The relation between density and radial distance of some of the models given in Table 1. The curves are labeled with the central density of the model.

ties of proper mass. It is thus evident that there will be some limiting model with finite central density beyond which further extrapolation of equation (13) toward the center will add less than one nucleon, and hence one does not need to worry about the development of a true singularity at the center.

In the incompressible hard-core case the model has constant density until the pressure falls to the value corresponding to the hard-core density given by equation (12), beyond which the characteristics of the model rapidly become indistinguishable from those of the corresponding model in which no hard-core density was assumed.

#### DISCUSSION

It may be noted that in the models with the smaller central densities the gravitational binding energy,  $M_B$ , is much less than 1 per cent of the proper mass,  $M_p$ . Such models are energetically unstable against transformation into iron white dwarf stars, since the nuclear binding energy of  $\text{Fe}^{56}$  relative to 56 neutrons is 0.0097 of the neutron mass. Only 2 per cent of this nuclear binding energy would be lost in the form of neutrinos

when neutrons are converted to  $\text{Fe}^{56}$ ; the remainder of the nuclear transformation energy would be available (certainly in principle and probably in practice) to expand the neutron star into an iron white dwarf in which the gravitational binding energy is relatively negligible.

Thus from an examination of Table 1 it may be seen that there is a lower limit for the existence of neutron stars of about 0.05 solar mass. It is unlikely that such small neutron stars can be formed in any case.

It has been very gratifying to find that the upper limit on the mass of neutron stars is sufficiently large that no difficulty should arise in their formation by sudden condensation in supernova explosions. The writer believes that neutron stars must be inevitable products of such explosions. Because of the tremendously high thermal conductivity and small dimensions of these stars, they must cool off very quickly after formation, at first by evaporation of mass from the surface and later by radiation from the surface. Thus it may be expected that supernova remnants will have much too low a luminosity for direct observation.

However, the neutron or nucleon gas will approach its lowest energy state only asymptotically, since the lifetimes of the nuclear transformations become longer and longer as the available energy decreases. Thus some low-level activity should take place in neutron stars for a long time. This low-level activity may be the source of the periodic disturbances in the Crab Nebula.

It may also be noted that we are still faced with the basic philosophical dilemma associated with the finite number of nucleons contained in the limiting models for which the general relativistic equations of hydrostatic equilibrium admit a solution. As Wheeler (1958) has noted for similar cases, despite the fact that there are no solutions for larger numbers of nucleons, what is to prevent such nucleons from being added to a neutron star?

The writer is greatly indebted to Dr. T. H. R. Skyrme and to Dr. J. A. Wheeler for stimulating conversations regarding some of the material presented here.

*Note added in proof:* The writer wishes to make a reservation concerning the above conclusion that a neutron star would cool off so rapidly that it must become unobservable. Since no investigation has yet been made of high-temperature neutron stars, it is not known whether a high-temperature non-degenerate envelope composed of heavy elements can be fitted to a neutron core. Such an envelope might be formed at the end of the evaporation stage when the gas at the stellar surface no longer has enough internal energy to expand to infinity. Such an envelope, if it can exist, might well retain a great deal of the internal heat of condensation for appreciable periods of time.

#### REFERENCES

- Baade, W., and Zwicky, F. 1938, *Ap. J.*, **88**, 411.  
 Cameron, A. G. W. 1957, *Canadian J. Phys.*, **35**, 1021.  
 ———. 1959, *Ap. J.*, **129**, 676.  
 Chandrasekhar, S. 1939, *An Introduction to the Study of Stellar Structure* (Chicago: University of Chicago Press).  
 Eden, R. J. 1959, in *Nuclear Reactions*, ed. P. J. Endt and M. Demeur (Amsterdam: North-Holland Publishing Co.).  
 Gammel, J. L., and Thaler, R. M. 1957, *Phys. Rev.*, **107**, 291.  
 Oppenheimer, J. R., and Volkoff, G. M. 1939, *Phys. Rev.*, **55**, 374.  
 Rudkjöbing, M. 1952, *Pub. Københavns Obs.*, No. 160.  
 Schatzman, E. 1958, *White Dwarfs* (Amsterdam: North-Holland Publishing Co.).  
 Skyrme, T. H. R. 1959, *Nuclear Phys.*, **9**, 615.  
 Tolman, R. C. 1934, *Relativity, Thermodynamics, and Cosmology* (Oxford: Clarendon Press).  
 Wheeler, J. A. 1958, paper read at the Solvay Conference, Brussels. Published with B. K. Harrison and M. Wakano, in *La Structure et l'évolution de l'univers* (Brussels: R. Stoops).  
 Zwicky, F. 1938, *Ap. J.*, **88**, 522.  
 ———. 1939, *Phys. Rev.*, **55**, 726.  
 ———. 1958, *Handbuch der Physik* (Berlin: Springer-Verlag), **51**, 760.