

STELLAR GROUPS, IV. THE GROOMBRIDGE 1830 GROUP OF HIGH VELOCITY STARS AND ITS RELATION TO THE GLOBULAR CLUSTERS

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Summary

The available proper motions and radial velocity data have been used to establish the existence of a moving group of subdwarfs (Groombridge 1830 group) which includes RR Lyrae. On the basis of the relationship between the observed ultra-violet excess and displacement below the normal main sequence, the subdwarfs in the Groombridge 1830 group are identified with main sequence stars in the globular clusters. This identification gives a modulus of $m-M=14^m.2$ for the globular cluster M13 with the result that $M_V \sim +0^m.5$ for the 2 RR Lyrae variables and $M_V = -2^m.3$ for the brightest stars in the cluster. RR Lyrae itself, for which we derive $M_V \sim +0^m.8$ from the moving cluster parallax, is shown to obey the period-amplitude relation for the variables in M3 and to be reddened by $0^m.05$ with respect to those variables. By equating the luminosity of RR Lyrae to the mean of the variables in M3 we obtain a modulus of $m-M=15^m.0$ for the cluster. We have not derived this modulus in the logical way of fitting the M3 main sequence to the main sequence of the Groombridge 1830 group because the colour observations of the M3 main sequence stars may contain a systematic error.

Because the presence of RR Lyrae variables in stellar groups may provide the only accurate calibration of the luminosities of these stars, it is important to make a systematic search for such groups. We have tabulated the available information for (1) all stars classified as subdwarfs on the basis of their spectral features, (2) stars that are probably extreme subdwarfs, on the basis of their observed ultra-violet excesses, although classified as normal main sequence objects, and (3) stars that may be globular cluster giants, subgiants or horizontal branch stars. Space motions of objects in these three categories have been computed from parallaxes derived by fitting them to the $(M_V, B-V)$ -diagram of M13, which was calibrated by the Groombridge 1830 group. It is shown that there is a high probability that several of these stars are travelling together in space and that, when additional material becomes available, several groups similar to the Groombridge 1830 group will be found. Also, the space motions of 4 RR Lyrae variables with relatively well-determined proper motions are identified with those for some nearby subdwarfs with a resulting $M_V \sim +0^m.6$ for the four variables.

In previous papers of this series (Eggen 1958 a, b, c; Papers I, II and III) several groups of stars have been discussed on the basis of the apices and total amounts of the stellar motions. Aside from the interesting dynamical questions raised by the existence of these groups (Woolley 1959), the resulting $(M_V, B-V)$ -diagrams are of importance in supplementing the available material from galactic clusters. This is especially true if stellar groups can be found that contain stars for which a luminosity calibration is not available from

other sources. Of particular interest in this connection are the field RR Lyrae variables and subdwarfs that might be identified with globular cluster stars. In this paper we will discuss (1) the evidence for the existence of a group of very high velocity subdwarfs moving together in space and containing RR Lyrae (Groombridge 1830 group), (2) the use of this group in obtaining distance moduli for globular clusters by the identification of the main sequences of the clusters with those of the group stars, and (3) the possible existence of other similar groups.

1. *The Groombridge 1830 group.*—Among the stars known to be within 20 parsecs of the Sun, and for which radial velocities and proper motions are available, the object with by far the largest space motion is the subdwarf Groombridge 1830 (HR 4550). The relevant data are as follows:

$$\begin{aligned}\mu &= 7''.042/\text{yr}, & U &= -263 \text{ km/sec}, \\ \rho &= -98 \text{ km/sec}, & V &= -151 \text{ km/sec}, \\ \pi &= 0''.116, & W &= -22 \text{ km/sec},\end{aligned}$$

where the velocity vectors, relative to the Sun, are directed as follows: U , away from the galactic centre ($l=148^\circ$, $b=0^\circ$); V , in the direction of galactic rotation ($l=58^\circ$, $b=0^\circ$); W , toward the north galactic pole ($b=90^\circ$). Taking the velocity of the Sun, relative to the stars within 20 parsecs (Woolley 1958) as $U(\odot) = -11$, $V(\odot) = +17$ and $W(\odot) = +7$ km/sec, the space motion of Groombridge 1830 relative to the mean of these nearby stars becomes

$$\begin{aligned}U + U(\odot) &= -274 \text{ km/sec}, \\ V + V(\odot) &= -134 \text{ km/sec}, \\ W + W(\odot) &= -15 \text{ km/sec}.\end{aligned}$$

A search through a catalogue of 700 high velocity stars yielded the following stars which appear to share the large space motion of Groombridge 1830; the stars were chosen by the convergent point (i.e. moving cluster) method described in Papers I and II:

Star	θ_0	$\Delta\theta \sin \lambda$	μ	ρ km/sec	Q	$V_t=285$ $\Delta\rho \quad \pi_g$	$=300 \text{ km/sec}$ $\Delta\rho \quad \pi_g$
+72°94	3°09	+ 5	0''.25	-266	d	-6 0''.010	+ 7 0''.0095
Gmb. 1830	145	0	7''.042	-98	b	-6 0''.126	0 0''.116
HD 163810	209	0	0''.82	+192	c	-8 0''.020	-18 0''.018
RR Lyr	210	+ 4	0''.24	-72	a	-3 0''.004	-4 0''.0035
HD 219617	203	-11	1''.30	+15	var	-3 0''.0225	-4 0''.0205

The values of $\Delta\theta$ were formed from the observed position angles of the proper motions, θ_0 , and the position angles computed from the assumption that the stars are moving with Groombridge 1830 toward $A=16^h 19^m$ and $D=-54^\circ 4'$; λ is the angular distance between the star and the point (A, D). The observed radial velocities, ρ , and their quality, Q , were taken from R. E. Wilson's catalogue and the computed values were obtained from the expression $V_t \cos \lambda$, with two values for V_t , the total velocity referred to the Sun, determined from two values for the parallax of Groombridge 1830; 285 km/sec for a parallax of 0''.126 and 300 km/sec for a parallax of 0''.116. The group parallaxes, π_g , were computed from the expression $\pi_g = 4.74 \mu / V_t \sin \lambda$.

The proper motion and radial velocity determinations for the group members are in Table I where the uncertainties in some of the data are apparent; the

proper motions of $+72^\circ 94$, HD 163810 and RR Lyrae, and, to a lesser extent the radial velocity of $+72^\circ 94$, HD 163810 and HD 219617, could be greatly improved by additional observations.

TABLE I

*Proper motion and radial velocity determinations
for members of the Groombridge 1830 group*

Star	μ_α	PE	μ_δ	PE	S*	ρ	S*
$+72^\circ 94$	-0.240	0.010	$+0.218$	0.010	G	-266	Md (2)
	-0.195	10	$+0.157$	10	G		
Gmb. 1830	3.995	1	-5.800	1	GC	-99	W (11)
						-97	L (5)
						-98	
HD 163810	-0.473	11	-0.673	12	Y	+187	W (3)
	-0.321	9	-0.746	6	C	+198	Md (2)
						+192	
RR Lyr	-0.122	8	-0.188	6	GC	-70	Md (300)
	-0.121	8	-0.224	8	S	-74	W (20)
						-72	
HD 219617	-0.512	8	-1.186	7	GC	+8	W (8)
	-0.513	0.014	-1.200	0.010	Y	+23	Md (2)
	-0.512		-1.190				

* GC=General Catalogue, Y=Yale Zone Catalogues, C=Cape Observatory, G=Greenwich Observatory, and S=Sproul Observatory. The sources of the radial velocities are Md=McDonald, W=Mount Wilson, and L=Lick Observatories; the observatory abbreviations are followed, in parentheses, by the number of plates.

The space motions of the group members have been computed from the parallaxes derived above on the basis of the group motion and are listed in Table II. In those cases where there is a large difference between two values of the observed proper motion or radial velocity, space motions have been computed for both determinations to demonstrate the sensitivity of the three components of the space velocity to variations in the observed data. For example, because of its apparent place in the sky, the large observed difference in the distribution of the total proper motion of HD 163810 between the two coordinates is reflected almost entirely in the value of the *W*-velocity. From the spread of values of the space velocities in Table II it can be seen that, within the uncertainties in the data, the members of the Groombridge 1830 group can be considered to have isoperiodic galactic orbits (Woolley 1958) and, therefore, to form a kinematical group.

In Paper II an attempt was made to estimate, by the use of a synthetic convergent point, the number of spurious group members that might be present in groups of various velocities as a result of the method of selection. It was estimated that two or three spurious members would be included for groups

TABLE II

Space motions for members of the Groombridge 1830 group

μ	θ	ρ	$U+U(\odot)$	$V+V(\odot)$	$W+W(\odot)$	π	V_B	$(B-V)$	M_V	Sp.
<i>(+72°94)</i>										
"	°			km/sec		"	^m	^m	^m	
0.32	312	-266	-294	-110	+39	0.0085	10.03	+0.44	+4.7-	sdF2
0.32	312	-266	-276	-129	+22	0.0105			+5.2	
0.25	309	-266	-265	-127	-4	0.0085				
0.25	309	-266	-253	-139	-15	0.0105				
<i>(Gmb. 1830)</i>										
7.042	145	-98	-288	-139	-13	0.110	6.44	+0.74	+6.6-	sdG8
7.042	145	-98	-268	-127	-20	0.120			+7.0	
7.042	145	-98	-247	-116	-26	0.130				
<i>(HD 163810*)</i>										
0.82	215	192	-255	-140	+37	0.018	9.62	+0.625	+5.9-	sdF8
0.82	215	192	-247	-122	+29	0.020			+6.1	
0.82	203	192	-253	-136	-3	0.018				
0.82	203	192	-247	-117	-2	0.020				
<i>(RR Lyrae†)</i>										
0.225	213	-72	-246	-127	+18	0.004	7.81	+0.38	+0.5-	...
0.225	213	-72	-282	-138	+21	0.0035			+1.1	
0.255	208	-72	-250	-134	-7	0.0045				
0.255	208	-72	-283	-134	-11	0.004				
<i>(HD 219617†)</i>										
1.300	206	8	-235	-178	-33	0.0205	8.17	+0.47	+4.7-	sdF8
1.300	206	23	-238	-173	-46	0.0205			+4.9	
1.300	206	8	-215	-160	-28	0.0225				
1.300	206	23	-225	-156	-40	0.0225				

* ADS 10938, $\Delta m = 1^m$.

† Mean magnitude and colour, obtained by planimetering the light curves converted into intensity units, are listed. No correction has been made for absorption.

‡ ADS 16655, $\Delta m = 0$. The luminosity was corrected for components of equal magnitude and colour before plotting in Fig. 1.

with a total space motion of 100 km/sec. The same material (FK₃) indicates that no spurious members will be included in groups with total space motions in excess of 150 km/sec, but because the stars in the FK₃, which were tested by the synthetic convergent point ($\delta = -90^\circ$) have generally very low velocities compared to the subdwarfs, this result may not be applicable to the present case. On the other hand, the fact that relatively few stars have space motions near 300 km/sec makes it highly unlikely that spurious motions of this size will simultaneously (1) result from placing the stars on a definite sequence in the (M_V , $B-V$)-diagram, (2) have vectors U , V and W similar to those for Groombridge 1830, and (3) represent stars that have the same ultra-violet excess as Groombridge 1830 (Section 3). For example, there are 33 F-, G- and K-type subdwarfs in Tables VI and VII that have ultra-violet excesses comparable with those of Groombridge 1830 and the globular cluster stars. Only seven of these—the first three stars in Table VI, the second in Table VII and the three members of the Groombridge 1830 group for which $U-B$ values are available—are inbound toward the galactic centre with velocities greater than 200 km/sec. Since the values of $V+V(\odot)$ and $W+W(\odot)$ for these seven stars range, respectively, from -120 to -450 km/sec and from -140 to $+60$ km/sec, it seems unlikely that nearly fifty per cent of the stars would accidentally satisfy the requirements for membership in the Groombridge 1830 group. More quantitative statistical tests for spurious members of high velocity groups will be possible when extensive astrometric, photometric and spectroscopic observational programmes for subdwarfs, which are now in operation, are completed.

The magnitudes and colours, which were observed in either the (P , V)_E-system or, by Roman (1955), in the (B , V)-system, are listed in Table II together with the spectral types, also by Roman, and the range in luminosity resulting from the range of parallax shown in the table. The colours on the (B , V)-system are used here since our ultimate aim is to compare the group stars with globular cluster stars and the latter have mostly been observed on that system. The colours and luminosities of the group members are shown in Fig. 1 where the Pleiades-Hyades main sequence is also plotted. The estimated uncertainties in the luminosities of the group members, based on the extreme range of the parallaxes shown in Table II, are indicated by the length of the lines in Fig. 1. The trigonometric parallaxes of the group members, other than Groombridge 1830, are small and offer little check on the derived luminosities. The individual values are as follows; the observatory abbreviations and the weights, in parentheses, are those given by Jenkins (1952):

Star	π_t (unit = 0".001)
+72°94	0 M(6)
Gmb. 1830	111 A(28), 107 M(8), 164 St(3) — mean = 116
HD 163810	26 M(7), 11 Y(12), 31 C(7) — mean = 20
RR Lyr	11 A(28), 12 Yk(6), 5 W(7), 25 S(7) — mean = 9
HD 219617	14 M(8), 39 Y(12), 43 D(4), 0 C(7) — mean = 24.

2. *The double stars.*—Fig. 1 shows that members of the Groombridge 1830 group are undoubtedly subdwarfs lying 1^m.5 below the Pleiades-Hyades main sequence in the (M_V , $B-V$)-diagram. The question of the existence of these and other subdwarfs in the (M_{Bol} , $\log T_e$)-diagram is discussed elsewhere in

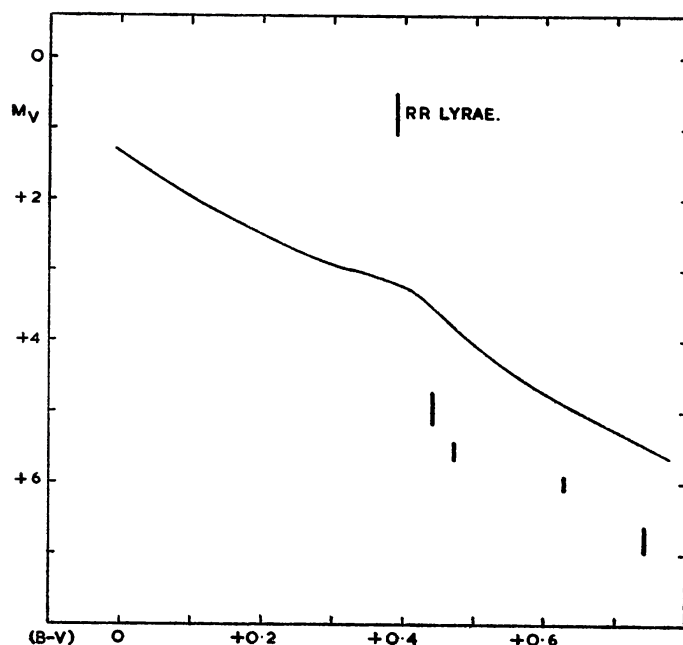


FIG. 1.—The $(M_V, B-V)$ -diagram for members of the Groombridge 1830 group. The length of the vertical lines indicates the range of luminosities for the various values of the parallax given in Table II. The continuous curve represents the Pleiades-Hyades main sequence.

Monthly Notices (Sandage and Eggen 1959). From the equations developed in that discussion it is apparent that if subdwarfs do exist in the $(M_{\text{Bol}}, \log T_e)$ -diagram, they will violate the mass-luminosity relation for normal dwarfs and it is, therefore, very important that masses of subdwarfs be independently determined. As pointed out previously (Eggen 1956) there are very few known or suspected subdwarfs in binary systems. The best mass determination available is that for 85 Pegasi A, but since the derived mass depends upon the trigonometric parallax, there is necessarily considerable uncertainty in the value. On the other hand, membership of visual binaries in stellar groups permits the use of the group motion parallax in deriving the mass. Representative observations of the two binaries in the Groombridge 1830 group, ADS 10938 and 16644, are shown in Table III. Very few observations of ADS 10938 have been made since its discovery in 1900; the three listed in Table III show an angular motion of $1^\circ/\text{year}$ and a fixed separation near $0''.5$ but orbital elements obviously cannot yet be derived. The observations of ADS 16644 extend back to 1876 and more than one-half an orbital period may have elapsed since then, but unfortunately this system is seen nearly edge-on (inclination near 90°) and any orbital elements must depend mainly on the observed separations. The two components of the system are of nearly equal magnitude and are indistinguishable, but a double reversal of quadrant (180° of motion) has been assumed to have occurred between 1902 and 1929. The apparent orbit is nearly rectilinear with one end as yet unobserved. Using the observations to 1930, Parenago (1946) derived the value of $P=145$ years and $a=0''.52$ from the observed rate of change of the observed separations and the assumption that van den Bos's observations in 1928 define one end of the rectilinear apparent orbit. These elements do not satisfy later observations and it doubtful whether even preliminary elements can be derived until it can be shown from observations made now that apastron has

TABLE III

Observations of ADS 10938 and 16644

ADS 10938					ADS 16644				
<i>t</i>	PA	Sep.	Obs.	<i>n</i>	<i>t</i>	PA	Sep.	Obs.	<i>n</i>
	°	"				°	"		
1900·47	218	0·48	Hu	3	1876·28	42	0·83	D	3
1923·36	198	0·47	VB	2	1891·59	49	0·63	β	3
1947·83	172	0·51	VB	4	1898·66	48	0·58	β	2
					1901·16	42	0·58	β	2
					1901·73	47	0·63	Doo	2
					1927·00	Too close		B	...
					1927·80	180	0·15	B	2
					1929·60	252	0·15	V	2
					1932·33	Too close		B	3
					1937·53	224	0·26	B	4
					1942·56	226	0·34	B	4
					1943·55	229	0·47	V	3
					1944·84	223	0·40	VB	2
					1948·64	222	0·44	VB	2
					1949·85	224	0·42	B	2
					1955·62	226	0·72	Wor	4
					1957·90	223	0·68	Cou	2

been reached. An attempt to find the extreme range of periods and semi-major axes that will fit the available observations indicated that the total mass of the system is less than two solar masses, that is, the two (assumed) equal masses are both less than the mass of the Sun, but this result is very uncertain.

3. *Identification of the subdwarf group members with globular cluster stars.*—In addition to the dynamical questions raised by the possible existence of a moving group of subdwarfs near the Sun, the existence of RR Lyrae stars in this and possibly other stellar groups (Section 5) provides a powerful method of calibrating the luminosities of the cluster type variables themselves. Furthermore, since the Groombridge 1830 group shows that subdwarfs and cluster variables are related, it seems probable that the globular cluster main sequences should be compared to the subdwarfs rather than to the ordinary dwarfs near the Sun. In the past this has been a question of some importance because, as Fig. 1 indicates, an error of more than a magnitude can result in the luminosities of the globular cluster stars if the cluster main sequences are fitted to the Pleiades–Hyades main sequence rather than to the subdwarfs. In this section we use the main sequence of the Groombridge 1830 group (subdwarfs) to compare with the main sequence of M13 and discuss the resulting luminosities of the M13 stars. A similar discussion of the available M3 data is given in Section 4 but in this case greater reliance is placed on the absolute magnitude of RR Lyrae itself as determined from the assumption that it is moving with the Groombridge 1830 group than on available observations of the M3 main sequence stars. Justification for comparing RR Lyrae with the M3 variables comes from the similarity of the characteristics of these stars (Section 4). The divergence of the M3 main sequence from that of the Groombridge 1830 group, which results from this comparison (Fig. 8), is interpreted as an error of about $0^m.1$ in the colours of the faint stars in M3.

The most reliable data for faint globular cluster stars are probably those published by Baum, Hiltner, Johnson and Sandage (1959) for M13. Earlier

results for M13 by Arp and Johnson (1955) provided accurate three-colour observations to visual magnitude 17.5 and the later results carried this to magnitude 20.5 in three colours and magnitude 22 in two colours. The two colour indices, $U-B$ and $B-V$, obtained from more than one photoelectric observation by Arp and Johnson for giant and subgiant members of M13 are shown in Fig. 2 as open circles. The filled circles in the figure represent the photoelectric colours obtained for main sequence stars in the cluster by Baum, Hiltner, Johnson and Sandage. The continuous curve represents the relationship between $U-B$ and $B-V$ for the Hyades stars (Sandage and Eggen 1959). The following values of $U-B$ are also available for three subdwarf members of the Groombridge 1830 group:

Star	$B-V$ m	$U-B$ m
Gmb. 1830	+0.74	-0.17
HD 163810	+0.625	-0.06
HD 219617	+0.47	-0.20

these are shown in Fig. 2 as crossed circles. The faint companion to HD 163810 probably has little effect on the colour of the system: the equal components of HD 219617 probably have identical colours.

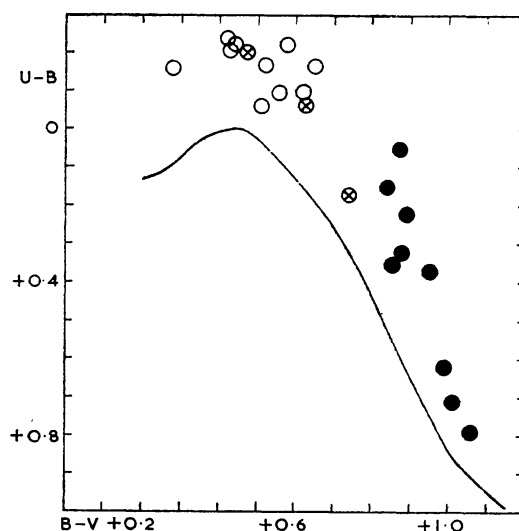


FIG. 2.—Three colour diagram for the giant and subgiant members of M13 (filled circles), main sequence members of M13 (open circles) and Groombridge 1830 group members (crossed circles). The continuous curve represents the $(U-B, B-V)$ -relation for the Hyades main sequence stars.

It is evident from Fig. 2 that the observed ultra-violet excess of the Groombridge 1830 group members and the M13 stars is very similar. Since we have shown elsewhere (Sandage and Eggen 1959) that, at least to a first approximation, displacement below a standard main sequence in the $(M_V, B-V)$ -diagram is directly correlated with the observed ultra-violet excess, we will fit the main sequence of M13 to that defined by the Groombridge 1830 stars. The result of this procedure is shown in Fig. 3. It has been assumed that M13 is free from reddening since the bluest stars are observed to be bluer than any stars known in the Galaxy. Arp (1955) and Arp and Johnson (1955) have derived a reddening of $0^m.16$ and $0^m.12$, respectively, but we believe that the procedures used in these derivations are probably unsatisfactory.

The filled circles in Fig. 3 represent mean points from the photographic observations by Baum, Hiltner, Johnson and Sandage (1959) for luminosities fainter than $M_V = +2^m$ and from those of Arp and Johnson to $+3^m$; the open circles represent the individual photoelectric results for cluster members observed

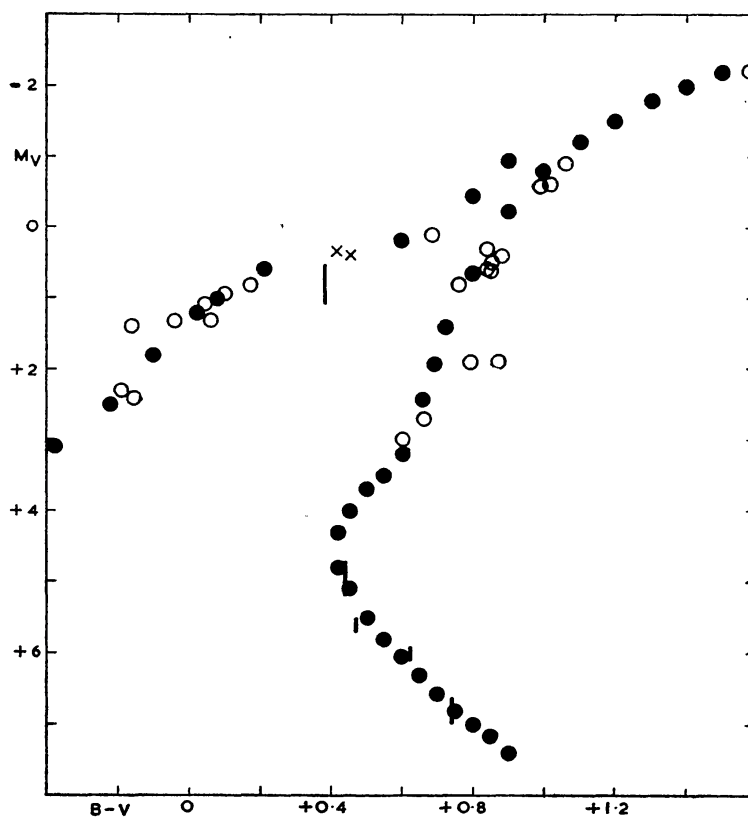


FIG. 3.—The $(M_V, B-V)$ -diagram for M_{13} (circles) and the Groombridge 1830 group (vertical lines). The filled circles represent photographic mean points and the open circles individual photoelectric results. The two crosses represent the RR Lyrae variables in M_{13} . The modulus of M_{13} was determined as $m-M=14.2$ by fitting the main sequence of the cluster to the subdwarf members of the Groombridge 1830 group. M_{13} is assumed to be unreddened.

by Arp and Johnson. A mean, colour-luminosity relation for the stars fainter than $M_V = +5^m$ in Fig. 3 is tabulated in Table IV based on a distance modulus of $14^m.2$ obtained from the above procedure.

TABLE IV
Mean colour-luminosity relation for the main sequence of M_{13}

$(B-V)$	$(P-V)_E$	M_V
m	m	m
+0.45	+0.565	+5.1
+0.50	+0.605	+5.5
+0.55	+0.655	+5.8
+0.60	+0.705	+6.05
+0.65	+0.75	+6.3
+0.70	+0.80	+6.55
+0.75	+0.85	+6.8
+0.80	+0.90	+7.0
+0.85	+0.945	+7.15
+0.90	+0.99	+7.4

20*

One consequence of the above procedure is that the brightest stars in M_{13} are at $M_V = -2^m.3$. This value should be compared with the usually adopted one of -3^m which is based on (1) the observed fact that in most globular clusters the brightest giants are three magnitudes brighter than the horizontal branch (RR Lyrae variables) and (2) the assumption that this branch is at $M_V = 0^m.0$. The value of $M_V = -3^m$ for the bright end of the globular cluster giant sequence has been used in many estimates of galactic and extra-galactic distances and therefore, if the procedure in fitting the main sequence of M_{13} to that of the Groombridge 1830 group is correct, these distances will need a substantial revision.

A second consequence of the identification of the main sequence of M_{13} with that of the Groombridge 1830 group is that the RR Lyrae variables have $M_V \sim +0^m.5$ instead of $M_V \sim 0^m.0$. Additional evidence on this point will be discussed in the following sections.

4. *RR Lyrae and its relation to the M_3 variables.*—The only cluster for which there are available three colour data for a large number of variables is M_3 (Sandage 1959). The relation between the periods and light amplitudes, B , V , and U , for 33 variables in this cluster is shown in Fig. 4 where the separation

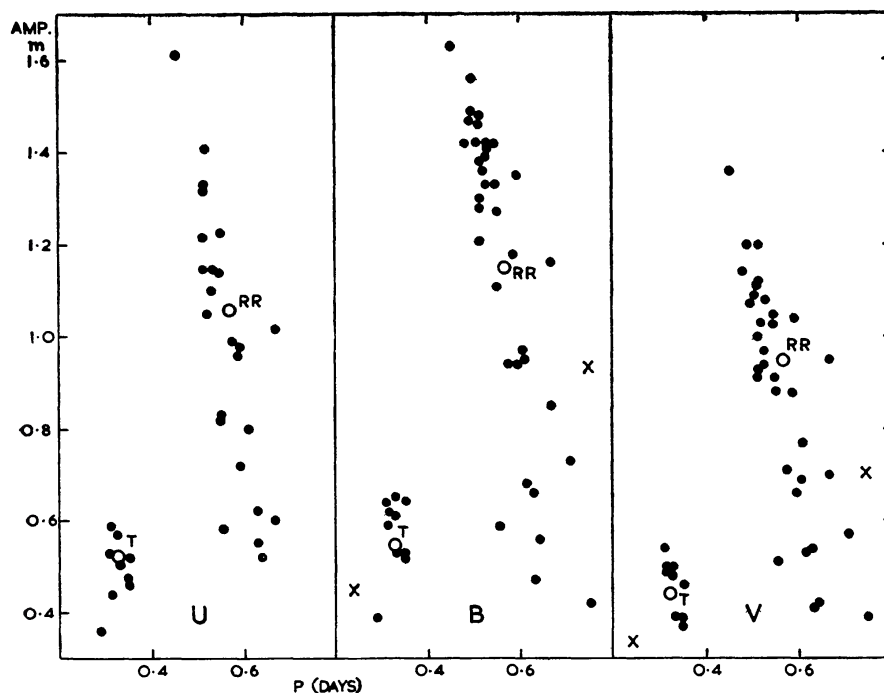


FIG. 4.—The period-colour relationship for the variables in M_3 more distant than $180''$ from the cluster centre. The position of RR Lyrae and T Sextantis and of two variables in M_{13} (crosses) are also shown.

of the Bailey type *a* and type *b* stars (large amplitudes) and the type *c* stars (small amplitudes) is apparent. The three colour results for RR Lyrae (Hardie 1955) and for T Sextantis (Tifft and Smith 1958) are also plotted in the figure and indicate that, in light amplitude at least, these two field variables are very similar to the M_3 stars. The results for RR Lyrae and T Sextantis are as follows:

	$U-B$	$B-V$	A_V	A_B	A_U
RR Lyr	+0.05	+0.38	0.95	1.15	1.08
T Sex	+0.06	+0.24	0.82	0.55	0.44

On the other hand, the available two-colour data for the M_{13} variables (Arp 1955), which are represented in Fig. 4 by crosses, show marked deviations from the mean period-amplitude relations for the M_3 variables. Both of the M_{13} variables stand off, in opposite directions, from the mean period-amplitude relations for the ab and the c type M_3 variables more than any individual variable in M_3 . This result is a particular example of the fact that the period-amplitude relation can vary markedly from cluster to cluster (Belserene 1954). This variation is probably related to the fact that small variations in the mean luminosity of the variables may occur from cluster to cluster (Sandage 1958) and to the great variation in the spectral features observed in field RR Lyrae stars (Iwanowski 1953, Preston 1958). Therefore the field variables probably represent an inhomogeneous group and it is only through the period-amplitude relations discussed above that we can identify RR Lyrae with the variables in M_3 . However, before the final comparison can be made between RR Lyrae and the M_3 variables, the possible reddening of the former must be investigated.

We have assumed that the stars in both M_3 and M_{13} are unaffected by absorption on the basis of their high galactic latitudes and the presence of stars observed to be bluer than any known galactic stars. On the other hand, RR Lyrae lies nearly in the galactic plane ($b = +11^\circ$) and, at a distance of 250 parsecs, may be reddened. The values of $B - V$ and $U - B$ for the variables in M_3 are plotted in Fig. 5 as crosses; non-variable stars on both sides of the variable star "gap" in M_3 are also shown in the figure as dots. Also, the mean values of $U - B$ and $B - V$ for T Sextantis and RR Lyrae, both of which were shown above to be similar to the variables in M_3 , are plotted in Fig. 5 as large

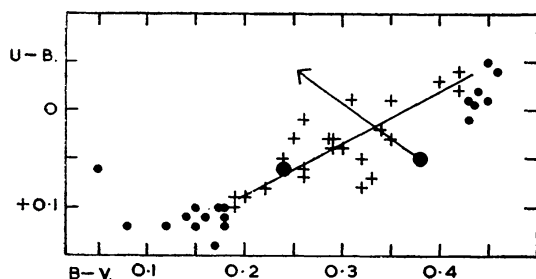


FIG. 5.—Three colour diagram for the M_3 variables in Fig. 4. T Sextantis falls on the mean relation for the cluster stars but RR Lyrae needs to be moved $0^m.04$ in $U-B$ and $0^m.05$ in $B-V$ along the reddening line indicated by the arrow. Non-variable horizontal branch stars are indicated by filled circles.

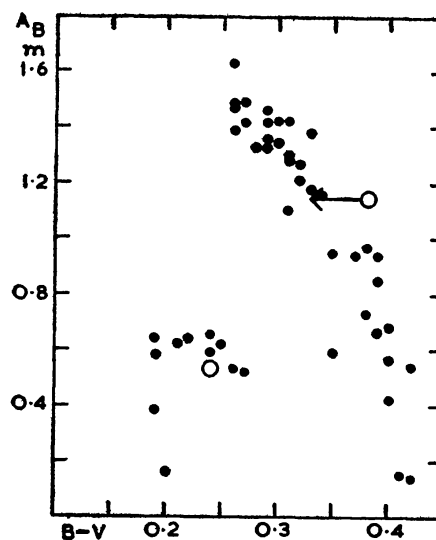


FIG. 6.—Colour-(blue) amplitude relation for the M_3 variables in Fig. 4. T Sextantis (unreddened) and RR Lyrae (with a reddening of $0^m.05$ indicated by an arrow) are plotted as open circles.

filled circles. There appears to be little or no reddening affecting T Sextantis, because it falls on the mean $(U-B, B-V)$ -line defined by the M_3 stars, but RR Lyrae needs to be moved $0^m.05$ in $B-V$ and $0^m.04$ in $U-B$ along the reddening line $(U-B) = 0.74(B-V)$ (Lindholm 1957). The value of

$E_{(B-V)} = +0^m.05$ derived in this way differs considerably from $+0^m.12$ derived previously (Sandage 1956) but, within the framework of the assumptions that RR Lyrae is similar to the M3 variables and that the latter stars are unreddened, the present value is undoubtedly more reliable. The previous determination rests on the questionable assumption that the intrinsic colour of RR Lyrae is the same as that of the supposed unreddened, non-variable stars with which it was compared. The consistency of the result that RR Lyrae is reddened by $+0^m.05$ and the assumption that it is similar to the M3 variables is demonstrated in Fig. 6 which contains the colour (blue)–amplitude relation for the M3 variables. A correction for $+0^m.05$ reddening in this diagram puts RR Lyrae on the mean colour–amplitude relation of the cluster variables.

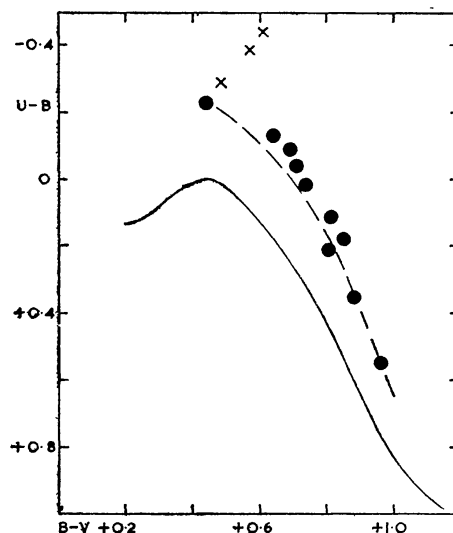


FIG. 7.—Two colour diagram for M3 giants and subgiants (filled circles) and main sequence stars (crosses). The broken curve represents the mean relationship for M13 giants and main sequence stars and the continuous curve that for the Hyades main sequence stars.

The above discussion shows that the period–light amplitude relation is the same for RR Lyrae and the M3 variables, and it would therefore appear logical that their absolute magnitudes might also be the same. This could be verified by fitting the main sequence of M3 to that of the Groombridge 1830 group and comparing the resulting mean absolute magnitude of the cluster variables with that of RR Lyrae derived on the assumption that it is a member of the group. However, as mentioned above, with the exception of M13, the photometric observations of faint stars in globular clusters are not of high precision because the photographic interpolation usually depends on very few photoelectrically calibrated standard stars and it is difficult, in the photographic interpolation method, to control the colour indices at faint light levels with an accuracy greater than $0^m.1$. This is particularly true of M3 (Johnson and Sandage 1956) where only 8 photoelectric standards were observed fainter than $V=19^m$ whereas 23 such standards were obtained for the investigation of M13. Because an error of only $0^m.1$ in the colour indices of the faint M3 stars will cause an error of $0^m.4$ in the position of the main sequence, the M3 data are considered to be of insufficient accuracy to compare main sequences. A further indication of possible difficulties in the M3 photometry is the anomalous $U-B$ values derived for M3

main sequence stars shown in Fig. 7. The accurate M_{13} data show that the main sequence stars have an excess of about $\delta(U-B)=0^m.2$ whereas the observed M_3 values rise to $0^m.4$ to $0^m.5$.

Therefore, instead of fitting the main sequence of M_3 to that of the Groombridge 1830 group, we have fitted the mean magnitude of the M_3 variables to that of RR Lyrae in Table II, after correction for the absorption discussed above. The resulting $(M_V, B-V)$ -diagram for M_3 is shown in Fig. 8 which also contains the colours and luminosities of the Groombridge 1830 group members; the colour and magnitude of RR Lyrae have been corrected for absorption before plotting in Fig. 8.

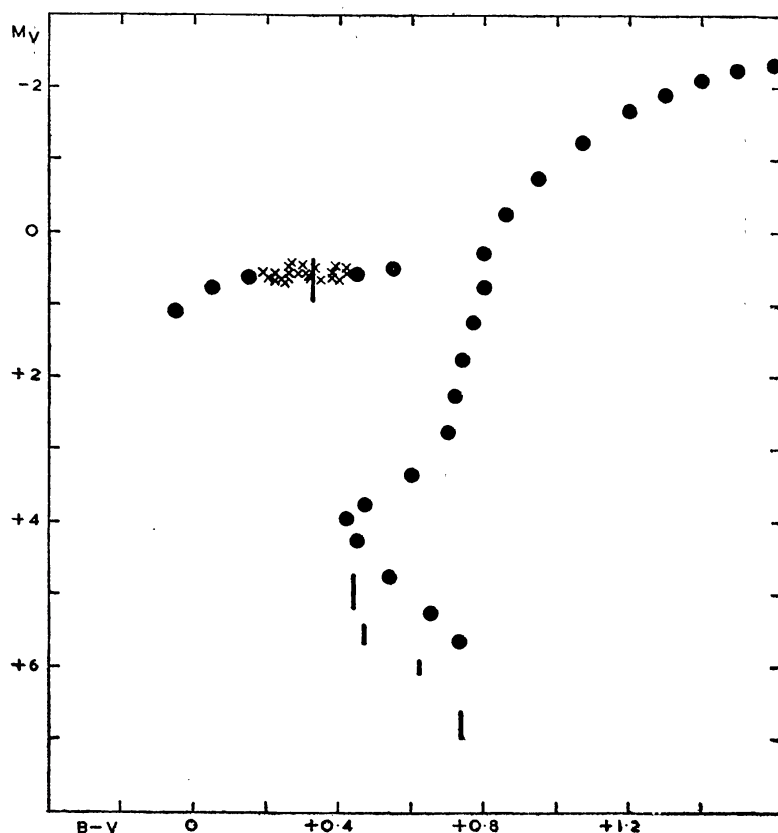


FIG. 8.— $(M_V, B-V)$ -diagram for M_3 . The modulus of $15^m.0$ was derived from a comparison of the variables with RR Lyrae. The vertical lines represent members of the Groombridge 1830 group. The mean colour and luminosities of representative variables in M_3 are plotted as crosses.

The $(M_V, B-V)$ -diagrams for M_{13} (shaded) and M_3 (unshaded) are compared in Fig. 9 where the two variables in M_{13} are shown as crosses and the variable star gap in M_3 by broken lines; the blue extension of the horizontal branch in M_3 , which is populated by only a few stars, is shown as a thin line. Figs. 8 and 9 show that the stars in M_3 fainter than $V=19^m$ do indeed systematically deviate from the main sequence of the Groombridge 1830 group and of M_{13} . This deviation is not considered to be real however, because of a possible error in the measured colour indices of the faint M_3 stars. These same stars, fainter than $V=19^m$, also show an anomalous ultra-violet excess (Fig. 7) which may be caused by the same error.

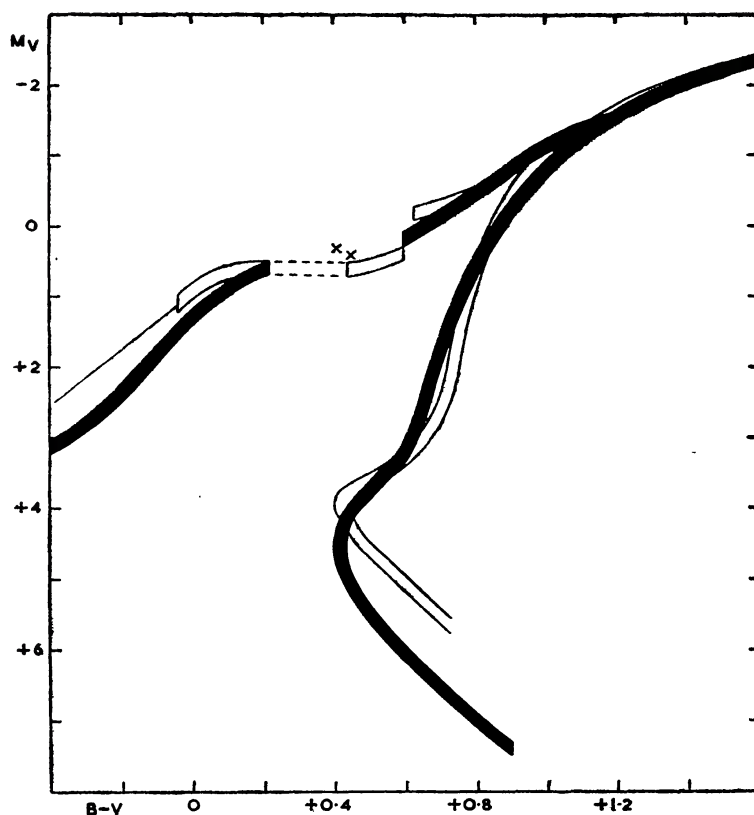


FIG. 9.—A comparison of the $(M_V, B-V)$ -diagrams for M_{13} (shaded) and M_3 (unshaded). The two crosses represent the variables in M_{13} . The systematic deviation of the M_3 main sequence from that of M_{13} in this figure and from that of the Groombridge 1830 group in Fig. 8, is believed to be due to an error in the observed colours.

5. *Other possible groups.*—The above discussion of the fitting of the main sequences of globular clusters to that of the Groombridge 1830 group is intended as an illustration of a procedure that, if it can be extended to other stellar groups, might well afford the only definitive method of calibrating the luminosity of the globular cluster stars. It is therefore important that a search be made for other groups and in this section the data available for such a search are discussed.

Stars have been labelled “subdwarfs” on the basis of a variety of criteria; for example, velocity, position in the HR diagram, the appearance of the spectrum, and the spectral type-colour relationship. The interrelations of some of these criteria are discussed elsewhere in *Monthly Notices* (Sandage and Eggen 1959) and we will deal here only with subdwarfs so classified by the appearance of their spectra, with some additional stars included on the basis of an observed ultra-violet excess in their colours. Stars for which radial velocity observations are available and which, as far as can be determined, have been classified as subdwarfs on the basis of their spectral features, are listed in Table V. The table contains the following information:

Wilson No.: The number in R. E. Wilson’s (1953) catalogue of radial velocities. A few stars, not in that catalogue, are identified by their HD or BD numbers.

m_v : The visual magnitude taken from Wilson’s catalogue except for stars observed photoelectrically, which are indicated by two decimal magnitudes.

Sp.: The spectral type assigned by the observer mentioned in the sixth column. Other classifications are noted at the end of the table.

ρ , Q : The radial velocity and its quality, a , b , c , d , taken from Wilson's catalogue. A few values not in that catalogue, or improved by more recent observations, are indicated by the observer; R=Roman (1955), E=Evans (1958), G=Greenstein (1954).

Ob.: The authority for the classification listed in the third column; P=Popper (1942, 1943), J=Joy (1947), R=Roman (1955), M=Münch (1944), E=Evans (1958) and G=Greenstein (1954).

μ , θ : The total annual proper motion and its position angle, taken from various sources.

$B-V$, $U-B$: Colours observed by Roman (1955) on the (U, B, V) -system. A few values, given in parentheses, have been (1) converted from the $(P, V)_E$ -system when no $U-B$ is listed, (2) observed at Herstmonceux with the 20-inch reflector or at the Cape Observatory with the 18-inch reflector, or (3) observed by the Cape observers (Stoy 1958) with the McClean refractor—in the latter case the $(U-B)_C$ on the refractor system is given in the notes to the table.

This list of subdwarfs is undoubtedly very incomplete, even among stars brighter than visual magnitude 10 and radial velocity, astrometric and photometric observations are now in progress to increase the number of known subdwarfs as well as to strengthen the data in Table V.

On the assumption, discussed elsewhere (Sandage and Eggen 1959), that displacement below the main sequence is dependent upon the ultra-violet excess, we have computed space motions for those stars in Table V whose $(U-B)$ and $(B-V)$ colours place them on or near the M13 (and Groombridge 1830 group) line in Fig. 2. The parallaxes used in these computations were derived by fitting stars to the $(M_V, B-V)$ -diagram in Fig. 3. In addition to the number in Wilson's radial velocity catalogue, or BD/HD number, Table VI includes (1) the values of the two components of the proper motion, (2) the radial velocity, (3) the adopted parallax and (4) the three components of the space velocity freed of the solar motion with respect to the nearby stars (cf. Section 1).

There are several objects in Roman's (1955) catalogue of high velocity stars that, although not noted there as subdwarfs, have an ultra-violet excess similar to the globular cluster and Groombridge 1830 group members. A few of these stars are globular cluster giants. Others, although spectroscopically classified as luminosity class V, may be subdwarfs. Some of these stars are listed in Table VII where, in addition to the information supplied for the stars in Table VI, we have included the magnitudes, colours and spectral types; the parallaxes were obtained by fitting the giant or main sequence of the $(M_V, B-V)$ -diagram in Fig. 3. Two A-type stars, which show an ultra-violet *deficiency*, are included in Table VII. The parallaxes of these stars have been computed on the assumption that they belong to the horizontal branch of the $(M_V, B-V)$ -diagram for globular clusters since a similar deficiency has been observed for A-type stars in a globular cluster (Sandage and Walker 1955). Although there may be a range of $0^m.3$ in luminosity of the horizontal branch from cluster to cluster (Sandage 1958), we have fitted these A-type stars to that of M13 in Fig. 3.

In Table VIII we have gathered together the stars in Tables VI and VII that appear to have similar space motions. It should be remembered that in

TABLE V

Stars classified as subdwarfs on the basis of their spectral features

Wilson No.	m_v	Sp.	ρ	Q	Ob.	μ	θ	$B-V$	$U-B$
91*	12.0	sdF4	-100	G	G	0.25	79	...	...
390	10.27	sdA9	-114	d	P	0.35	72	+0.36	-0.16
646*	5.20	G5 VI	-97	a	R	3.76	115	+0.70	+0.11
987	10.08	sdF2	-266	d	P	0.32	...	(+0.44)	...
HD 16031	9.75	sdF4	+63	R	R	0.17	153	+0.45	-0.22
1735	8.06	sdF7	-139	R	R	0.82	194	+0.46	-0.24
1800	10.5	sdK5	-167	b	J	1.36	156	...	...
1828	10.18	sdA8	+154	c	P	0.3:	140:	(+0.46)	(-0.25:)
1892	10.8	sdF3	-33	d	P	0.45	141	...	...
1904	10.6	sdK5	-173	c	J	1.58	135	...	...
HD 21543	8.22	G2 V-VI	+59	R	R	0.41	120	+0.62	+0.03
1919	9.7	sdF9	-160	c	P	1.63	131	...	...
2420	9.22	sdF5	+339	c	R	0.54	122	+0.44	-0.19
2851	6.97	G1 V-VI	+27	b	R	0.66	146	+0.60	+0.23
3314	11.5	sdF6	+543	c	J	0.45	126	...	...
3562	11.9	sdF7	-4	c	P	0.60	170	...	...
3668	10.8	sdF9	+12	c	P	0.43	165	...	...
4005	8.08	sdGo	+167	b	J	0.08	225	+0.86	+0.22
4400	10.0	sdF2	+189	c	J	0.56	175	...	...
5058	11.6	sdG2	+89	c	P	0.62	141	...	...
5240*	8.34	G2 VI	-240	c	R	1.97	158	+0.60	-0.12
5494	9.69	sdF4	+59	c	P	0.66	182	+0.48	-0.19
5730	9.59	sdF6	+203	c	R	0.62	142	(+0.42)	(-0.25)
5757	11.5	sdF8	-5	c	P	0.43	180	...	...
25°1981	9.35	sdF2	+47	R	R	0.35	197	+0.30	-0.12
5784*	8.74	sdGo	-168	b	J	0.09	149	+0.97	+0.53
5793	10.10	sdA5	+26	c	P	0.37	247	(+0.38)	(-0.24)
5815	10.8	sdG6	+276	c	P	0.64	155	...	...
5908	11.5	sdF7	+126	d	P	...	...	...	...
5909	9.8	sdF3	+25	c	J	0.79	138	...	...
5951	11.8	sdF5	-51	c	P	0.47	180	...	...
+9°2190	10.5	sdA5	+265	G	G	...	...	...	...
6236*	11.0	sdA7p	-59	c	P	0.54	160	...	...
6237	10.2	sdF5	-19	c	P	...	...	...	...
6294*	8.24	sdA4	-18	b	J	0.88	154	+0.41	-0.25
6296	10.6	sdA8	-91	c	G	0.18	180	...	...
6704	10.3	sdF5	+74	c	J	0.83	165	(+0.49)	...
6728	12.6	sdGo	-128	d	P	0.38	118	...	...
6734	8.23	sdF8	+62	b	R	0.55	210	+0.47	-0.19
6869	10.8	sdG3	+68	c	P	0.82	230	...	...
6880	9.8	sdF3	-188	c	P	0.52	166	...	...
7017	11.3	sdGo	-108	c	P	0.60	171	...	...
7117	10.70	sdF5	+186	c	P	0.52	266	(+0.53)	...
7131	9.5	sdGo	+61	d	P	1.02	240	...	...
7170	6.49	G8 VI	-98	b	R	7.04	145	+0.74	0.17
7266	10.1	sdA7	+56	c	P	0.34	226	...	...
7296	10.9	sdF2	+98	d	P	0.43	180	...	...
7308	10.18	F2 V-VI	+95	c	R	0.46	203	+0.48	-0.18
7464	9.68	sdF4	+159	c	P	0.48	180	+0.44	-0.25
7547	10.5	sdF6	-40	e	P	0.35	283	...	...
7567	11.7	sdA6	-57	c	P	0.35	250	...	...
7913	9.8	sdK5	+126	c	P	0.63	252	...	...
7923	12.8	sdA8	+252	d	P	...	...	...	...
7932	8.79	sdF4	+143	d	P	0.76	277	(+0.42)	...
7951	11.6	sdF2	-34	d	P	0.18	279	...	...

TABLE V (cont.)

Stars classified as subdwarfs on the basis of their spectral features

Wilson No.	m_v	Sp.	ρ	Q	Ob.	μ	θ	$B-V$	$U-B$
8236	10.04	sdF3	-162	c	P	0.54	161	+0.42	-0.21
8246	11.6	sdG3	+16	d	P	0.88	164	...	...
8249	6.20	sdF5p	-22	c	J	0.21	290	+0.90	+0.38
8296	9.7	sdF7	+126	c	J	0.50	215	...	...
8554	10.46	sdG2	-138	d	J	0.87	270	+0.70	+0.11
8603	9.74	sdF4	+34	c	R	0.35	185	+0.42	-0.22
8691*	8.5	sdA9	+167	c	P	0.75	229	...	...
8771*	9.06	Ko VI	+292	c	P	3.68	196	+0.80	0.15
9069	7.20	sdF5	-171	b	R	1.19	254	+0.48:	-0.10:
9236	10.0	sdF5	-157	c	P	0.51	213	...	...
9401	11.4	sdG2	+167	d	P	0.44	183	...	...
9840*	9.13	sdG1	-51	c	P	0.29	229	+0.67	+0.10
+2°3375	9.95	sdF5	-389	R	R	0.37	281	+0.44	-0.20
10287	6.98	sdA2	-363	b	R	0.08	218	+0.16	+0.10
10289	10.8	sdF8	+91	c	M	0.44	148	...	...
10367	11.0	sdF8	-216	c	J	0.60	181	...	...
10404*	9.62	sdF8	+191	c	M	0.83	214	0.62	-0.06
11060	11.4	sdA5	-68	G	G	0.26	184	...	...
HD 181743*	9.64	sdF	+18	E	E	0.82	180	(+0.46)	...
11949	10.20	sdF7	-172	c	R	0.53	183	+0.48	-0.12
+26°3578	9.36	sdF4	-128	R	R	0.17	180	+0.37	-0.23
12730	8.5	sdGo	-172	c	M	1.21	153	...	...
12754	10.6	sdFo	-319	c	M	...	...	...	...
13327	7.38	F9 VI	-45	b	R	0.90	187	+0.51	-0.09
13543*	10.52	sdF	+103	d	M	0.48	188	(+0.40)	...
13782	11.7	sdF6	-67	c	P	...	...	...	...
+17°4708	9.47	sdF6	-296	R	R	0.55	88	+0.43	-0.21
14122*	14.4	sdK5	-157	d	J	1.68	164	...	...
14539	10.00	sdG2	-113	b	P	1.29	189	(+0.48)	...
14568	10.62	sdF6	-32	d	P	0.57	129	(+0.42)	...
14598*	8.17	sdF8	+10	b	R	1.29	203	+0.47	-0.20
14704	10.5	sdFo	-100	d	P	0.47	85	...	...
14884*	7.0v	sdF2	-15	c	J	0.90	163	...	...
15042	8.92	sdA8	+50	d	M	0.34	204	(+0.33)	(-0.20)

Notes to Table V

91	LDS 1A.
646	μ Cas.
4005	Roman (1955) classifies this star as G2 III and states: "All lines are very weak".
5240	Roman (1955): "very weak lines".
5784	Roman (1955) assigns the classification G5 IV and states: "very weak lines and strong CH".
6236	Spectroscopic binary with a range of 83 km/sec.
6294	Roman (1955): sdF2.
8554	Roman (1955): G6 V.
8691	Astrometric binary, $P \sim 1300^d$.
8771	A companion, 300" distant, has common proper motion and velocity.
9840	Roman (1955): G2 V.
10404	ADS 10938.
HD 181743	$(U-B)_C = +1^m.49$, which is roughly equivalent to $U-B = -0^m.17$.
13543	$(U-B)_C = +1^m.45$, which is roughly equivalent to $U-B = -0^m.20$.
14122	Spectroscopic binary with a range of 37 km/sec.
14598	ADS 16644.
14884	SX Phe.
15042	ADS 17173.

TABLE VI

Space motions of stars in Table V with observed ultra-violet excesses comparable to that of the M13 and Groombridge 1830 group stars

Wilson No.	μ_{α}	μ_{δ}	ρ	π_{pt}	$U + U(\odot)$	$V + V(\odot)$	$W + W(\odot)$
	"	"					
8771	-1.012	-3.544	+300	0.037	-282	-450	-43
5240	+0.725	-1.836	-240	0.035	-273	-211	-87
8236	+0.104	-0.438	-162	0.008	-204	-177	-140
1735	-0.203	-0.805	-139	0.026	-163	-103	-57
6294	+0.370	-0.790	-18	0.019	-160	-135	0
8603	-0.031	-0.347	+34	0.010	-130	-100	+29
7464	-0.012	-0.473	+159	0.012	-121	-195	+65
5784	+0.049	-0.082	-168	0.0017	-120	-272	+66
5730	+0.373	-0.488	+203	0.012	-125	-270	+67
13327	-0.112	-0.900	-45	0.034	-82	-86	-40
HD 16031	+0.078	-0.155	+63	0.012	+3	-53	-53
7308	-0.184	-0.430	+95	0.011	-33	-202	+45
6734	-0.268	-0.465	+62	0.025	+21	-98	+26
HD 181743	-0.077	-0.818	+18	0.013	+21	-275	-44
HD 21543	+0.354	-0.204	+59	0.0395	+37	-37	-21
4005	+0.051	-0.056	+167	0.003	+38	-184	+6
8249	-0.203	-0.062	-22	0.0055	+91	-132	+24
5494	-0.010	-0.648	+59	0.0135	+100	-187	+42
10287	-0.050	-0.064	-363	0.006	+160	-285	-125
+ 17°4708	+0.496	+0.071	-296	0.0135	+198	-250	+77
+ 2°3375	-0.368	+0.069	-389	0.011	+338	-213	+52
2420	+0.455	-0.283	+339	0.014	+344	-109	-57

each case the space motion was computed on the assumption that the stars could be fitted to the dwarf (or giant) sequence of the $(M_V, B-V)$ -diagram for M13 and the Groombridge 1830 group. This assumption is based on the observed ultra-violet excess in the colours of the stars. When more photometric and radial velocity material becomes available for subdwarfs, we believe that additional stars will undoubtedly be found to be members of these possible groups. In support of this view we include in brackets in Table VIII a few stars from Table V for which photoelectric, photometric data are not available. For these stars we have tried several parallaxes, guided by a rough spectral type-colour relationship, until one was found that gave space motion vectors similar to those for some star in Tables VI and VII. It should be emphasized that this method is exactly equivalent to the convergent point method described in Papers I and II since, for an individual star, a necessary and sufficient condition for group membership is that the components of the space motion be identical with those of the group. An illustration of the accuracy of the parallax which can be obtained by this method, on the assumption of group membership, is given below for Wilson Nos. 6294 (HD 84937) and 5909 ($-3^\circ 25'25''$). This illustration, as well as the discussion of the Groombridge 1830 stars given in Table II, also shows the sensitivity of the computed space motions to the accuracy of the fundamental data.

It should be noted that, although we have stated that group membership implies that the space vectors of the group stars are identical, this may not be precisely true. Deviations from this requirement can occur, in U and V , of 1 to 2 km/sec/100 pc as a result of small differences in the eccentricities of isoperiodic

Wilson No. 6294				No. 5909			
"	"	GC		"	"	Cape	
$\mu_\alpha = +0.380$	$\mu_\delta = -0.790$	Y		$\mu_\alpha = +0.33$	$\mu_\delta = -0.58$		
$= +0.359$	$= -0.791$						
$+0.370$	-0.790			$\rho = +20$	W (6)		
				$= +40$	Md (2)		
$\rho = -18.1$	W (5)						
$= -15.3$	Md (1)			$+25$			
-17.6							
π	$U+U(\odot)$	$V+V(\odot)$	$W+W(\odot)$	π	$U+U(\odot)$	$V+V(\odot)$	$W+W(\odot)$
"				"			
0.018	-168	-147	+10	0.010	-241	-202	-5
[0.019	-160	-135	0]	0.014	-172	-145	+1
0.020	-153	-128	-10	[0.015	-159	-135	+2]
				0.016	-148	-128	+4

orbits of widely separated stars (Woolley 1958). Also, if W is not tightly coupled with U and V , we would expect its value for the group members to be correlated with the height of the stars above or below the galactic plane because of the differences in the gravitational field at different Z -distances (Woolley 1957). However, an evaluation of this effect for the Z -distances involved here shows that extreme differences of only 10 to 15 km/sec in W will be encountered and in this preliminary investigation they will be ignored.

One of the most important uses of the possible groups discussed above is in calibrating the luminosities of field RR Lyrae variables. As is evident in Table I, even the available data for members of the Groombridge 1830 group are uncertain and this is true of subdwarfs and RR Lyrae variables in general. There are not more than a dozen RR Lyrae variables with well determined annual proper motions in excess of $0''.05$ and, perhaps, another dozen with relatively large but insufficiently observed motions. The same situation exists for the radial velocities which in many cases depend upon only one or two observations and often have a probable error as large as 25 km/sec. There is also an additional uncertainty in the radial velocities of the RR Lyrae variables caused by the uncertainty in the phase of the light variation.

We have tentatively identified the motions of a few of the field RR Lyrae variables, that have large and relatively well determined motions, with those of subdwarfs. These stars are listed in Table IX where various determinations of the proper motion and radial velocity and the space motions of the variables, for two values of the parallax are given. The mean absolute visual magnitude of the variables, determined by the group parallax method, together with RR Lyrae from the Groombridge 1830 group, is $M_V = +0^m.6$. This value must be considered as preliminary since accurate photometry is not yet available for most of the variables. This preliminary value of $+0^m.6$ should be compared with the usually adopted value of $0^m.0$ determined solely from a statistical discussion of the motions of these stars. It should also be noted that the most recent determination based on these statistics is $+0^m.5$ (Pavlovskaja 1953). But the agreement of $+0^m.6$ and $+0^m.5$ obtained by the two methods should

TABLE VII

Space motions for a few giant, horizontal branch and subdwarf stars in the solar neighbourhood

Wilson No.	μ_{α}	μ_{δ}	ρ	π_{pt}	$U+U(\odot)$	$V+V(\odot)$	$W+W(\odot)$	V	$B-V$	$U-B$	Sp.
6379	+0.144	-0.228	+13	0.0038	-253	-214	+42	^m 7.99	^m +0.12	^m +0.16	A1 V
3785	+0.673	-0.633	-190	0.024	-228	-121	+60	9.32	+0.62	-0.08	Go V
7708	+0.307	-0.792	+144	0.029	-157	-125	+10	8.38	+0.53	-0.11	F8 V
665*	+0.028	+0.046	-245	0.0028	-121	-193	+116	6.73	+1.18	+0.88	K1 III
13660	-0.374	-0.507	-51	0.035	-82	-53	+10	9.41	+0.85	+0.29	Ko V
HD 221170*	+0.011	-0.045	-120	0.002	-69	-207	+89	7.66	+1.13	+0.64	G2 IV
58	+0.334	-0.007	+80	0.0385	-12	-63	-33	8.37	+0.65	-0.07	G2 V
12951	+0.044	-0.436	-30	0.0295	-8	-75	-15	8.21	+0.51	-0.13	F6 V
8433	-0.062	-0.298	-47	0.022	+14	-19	-62	9.33	+0.60	-0.07	G2 V
HD 117880	-0.061	-0.135	-45	0.0025	+21	-190	-185	9.08	+0.04	+0.06	Ao V
2302	+1.745	-1.348	-30	0.050	+38	-190	+25	8.52	+0.88	+0.36	K1 V
4801	-0.499	+0.173	+60	0.0495	+58	-2	-26	7.73	+0.63	-0.01	G2 V
8754	-0.514	-0.058	-60	0.0335	+61	-43	-4	8.27	+0.57	-0.06	F9 V
HD 63410	-0.026	-0.016	+81	0.004	+73	-12	+9	6.78	+0.95	+0.59	G8 III
12767	+0.113	-0.549	-246	0.026	+85	-230	+12	8.36	+0.49	-0.14	F6 V
6038*	-0.147	+0.046	+56	0.008	+93	+36	-12	5.97	+0.86	+0.37	G6 III
HD 5916	+0.106	-0.021	-71	0.0030	+92	-138	+3	6.84	+0.90	+0.45	G8 III-IV
6120*	-0.090	-0.114	+135	0.0055	+99	-128	+26	6.56	+0.94	+0.55	G8 III
1428	+0.400	-0.060	+234	0.010	+200	-164	-134	10.42	+0.47	-0.17	dF1

* 665 Mt Wilson, dG5.

HD 221170 "The colour indicates a much later spectral type, but the strength of the hydrogen lines makes this unlikely. CH is strong; all other lines are weak." (Roman 1955.)

6038 Mt Wilson; sgG3.

6120 Mt Wilson; dG5. Roman (1955): "Strong CH and very weak lines".

not be stressed since the inherent inaccuracies of the traditional statistical parallax method make it inferior to the method demonstrated here.

TABLE VIII

Possible groups

Wilson No.	π	$U+U(\odot)$	$V+V(\odot)$	$W+W(\odot)$	V	$B-V$	Sp.	M_V
	"				^m	^m		^m
6294	0.019	-160	-135	0	8.24	0.41	sdF2	+4.6
7708	0.029	-157	-125	+ 10	8.38	0.53	F8 V	+5.7
[5909	0.015	-159	-135	+ 2	9.8:	...	sdF3	+5.7:]
6728	0.0105	-160	-126	0	12.2:	...	sdGo	+7.3:]
2302	0.050	+ 38	-190	+ 25	8.52	0.88	K1 V	+7.0
4005	0.003	+ 38	-184	+ 6	8.08	0.86	G2 III*	+0.5
HD 5916	0.0030	+ 92	-138	+ 3	6.84	0.90	†	-0.8
8249	0.0055	+ 91	-132	+ 24	6.20	0.90	†	-0.1
6120	0.0055	+ 99	-128	+ 26	6.56	0.94	†	+0.3
5730	0.012	-120	-270	+ 67	9.59	0.43	sdF6	+5.0
5784	0.0017	-120	-272	+ 66	8.74	0.97	‡	-0.2
[5058	0.015	-115	-198	+ 67	11.6:	...	sdG2	+7.5:]
7464	0.012	-121	-195	+ 65	9.68	0.44	sdF4	+5.1
8236	0.0075	-204	-177	-140	10.04	0.42	sdF3	+4.4
[6880	0.0105	-203	-174	-126	9.8:	...	sdF7	+6.0:]
[7017	0.020	- 71	-133	+ 47	11.2:	...	sdGo	+7.7:]
8296	0.015	- 74	-145	+ 50	9.7:	...	sdF7	+5.6:]
10287	0.006	+160	-285	-125	6.98	0.16	sdA2§	+0.9
[10367	0.009	+143	-303	-155	11.0:	...	sdF8	+5.8:]

* 4005. Roman (1955): "All lines are very weak".

† HD 5916 and Wilson Nos. 8249 and 6120. HD 5916 has been classified as G8 III-IV by Roman (1955) and as dG2 at Mt Wilson (Wilson 1953); the observed colour is much too red for either classification. No. 8249 (HD 122563) has been classified Go VI by Roman and sdF5p by Wilson and Joy (1950) who state that the hydrogen and metallic lines are weak; the colour is much too red for either classification. No. 6120 (HD 81192) has been classified as G8 II by Roman, who states, "Strong CH and very weak lines", and as G7 III+ by Keenan and Keller (1953), who state, "CN and metallic lines weak and 4172 slightly strong". The parallaxes used in computing the space motions of all three stars were obtained by fitting the stars to the $(M_V, B-V)$ -diagram of Fig. 3 near the bifurcation of the giant sequence.

‡ 5784. This star (HD 74462) has been classified as G5 IV by Roman with the statement, "very weak lines and strong CH", and as sdGo by Wilson and Joy. The colour is too red for either classification and the space motion has been computed from a parallax derived on the assumption that the star is a globular cluster giant.

§ 10287. This high velocity A-type star (HD 161817) is probably a horizontal branch star. The spectra are described in detail by Slettebak (1952).

TABLE IX

Space motions for a few RR Lyrae variables compared with those for subdwarfs

Star	μ_{α} "	μ_{δ} "	S*	ρ	S*	π	U + U($\odot$)	V + V($\odot$)	W + W($\odot$)	M _V
TU U Ma	-0.076 $\pm$ 6 -0.076 $\pm$ 6	-0.050 $\pm$ 6 -0.042 $\pm$ 7	I I							
	-0.076 $\pm$ 5	-0.047 $\pm$ 5		+105	W (3)	0.0018 0.0013	+155 +193	-182 -255	+54 +35	+0.4
+17°4708	+0.510	+0.071	Y	-296	R (4)	0.0135	+198	-250	+77	+5.1
SU Dra	-0.035 $\pm$ 6 -0.025 $\pm$ 10 -0.044 $\pm$ 10	-0.068 $\pm$ 6 -0.075 $\pm$ 10 -0.071 $\pm$ 11	I I I							
	-0.035 $\pm$ 4	-0.070 $\pm$ 4		-180	W (8)	0.0013 0.0016	-21 -13	-319 -271	-7 -35	+0.8
W 5757	0.00	-0.43	H	-5	Md (2)	0.007	-18	-265	-31	+6.0:
X Ari†	+0.091 $\pm$ 4 +0.080 $\pm$ 6 +0.054 $\pm$ 10	-0.075 $\pm$ 4 -0.093 $\pm$ 6 -0.081 $\pm$ 10	Y I I							
	+0.084 $\pm$ 3	-0.081 $\pm$ 3		-40 -20	W (1) ...	0.0020 0.0019	+1 +11	-261 -275	+24 +10	+0.8
HD 181743	-0.077	-0.818	C	+18	C (4)	0.013	+21	-275	-44	+5.2
W CVn	-0.061 $\pm$ 13 -0.047 $\pm$ 9 -0.020 $\pm$ 13	-0.014 $\pm$ 13 -0.020 $\pm$ 9 -0.008 $\pm$ 13	I I I							
	-0.044 $\pm$ 6	-0.016 $\pm$ 6		+26.5	W (24)	0.0009 0.0008	+95 +126	-185 -200	+120 +127	+0.2
W 7932	-0.749	+0.096	C	+142	C (4)	0.016	+95	-205	+120	+4.8

* I = *Variable Stars*, 9, 1955; Y = Yale; H = Hertzsprung; C = Cape; W = Mt Wilson and Md = McDonald.

† The radial velocity depends upon one Mt Wilson spectrogram. The dependence of the space motion vectors on the radial velocity is illustrated by assuming an alternative value of -20 km/sec.

Although the accuracy of the group parallax value of $M_V = +0.6$ should also not be emphasized at the present time, the potential accuracy of this method is so high that its exploitation promises to provide a definitive calibration of the luminosity of RR Lyrae variables when good astrometric and photometric data are available.

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