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AN ATTEMPT TO OBSERVE NEUTRAL HYDROGEN BETWEEN THE GALAXIES

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ABSTRACT

The 21-cm absorption by a neutral hydrogen gas between the galaxies has been searched for by means of observations of the extragalactic radio source Cygnus A. The negative results yield an upper limit on the opacity of 0.0075. This result implies that if a homogeneous distribution of neutral hydrogen is expanding with the universe, it has a density-temperature ratio less than 8.1×10^{-7} cm^{-3} deg^{-1}. The temperature referred to is the excitation temperature of the hydrogen hyperfine levels; it is shown in the following paper (Field 1959) that it is *not* simply equal to the gas kinetic temperature but can be calculated theoretically.

I. INTRODUCTION

Discovery of diffuse matter in the space between the galaxies would be of great interest in several branches of astrophysics. We shall briefly review here some previous attempts to establish the existence of diffuse intergalactic matter. One line of reasoning proceeds from the possibility of observing in the velocity-distance relation the effects of deceleration of the expansion of the universe by its self-gravitation. If such decelerations are observed, as indeed has been tentatively claimed by Humason, Mayall, and Sandage (1956), a mean density of matter may be derived; if, in turn, the density so derived cannot be explained by the estimated mass of galaxies, there is reason to postulate the existence of either stellar or diffuse matter in intergalactic space. Although the deceleration data are obtainable only after rather uncertain corrections and the masses of galaxies needed in the estimate of the contribution of galactic matter are uncertain, it seems that an over-all "cosmological" density of 3×10^{-28} gm cm^{-3} is indicated, with galaxies failing to account for the total by at least a factor of 100.

In this calculation we have used the distance scale defined by $H^{-1} = 5.4 \times 10^9$ years (Humason *et al.* 1956). The cosmological density varies as the inverse square of the distance scale. On the other hand, if weight is given to masses of distant galaxies determined by dynamical means, the same is true of the smoothed-out nebular density. Hence the fraction of total mass attributable to galaxies is independent of scale.

A second approach arises from considerations of possible broad-band extinction of the light of galaxies by intergalactic matter, perhaps in the form of dust, perhaps of free electrons. Dufay (1957), on the basis of galaxy counts, concludes that such extinction is less than $0^m.001$ mpc^{-1} on the new distance scale. This leads to the conclusion that if dust is present in the same ratio to gas as in the interstellar space and if it has similar optical properties, the density of the intergalactic medium is less than 10^{-30} gm cm^{-3}. This result is quantitatively borne out by the lack, within observational error, of the reddening of distant galaxies beyond that due to red shifting of the intrinsic spectrum (Whitford 1956). Dufay's upper limit on extinction also implies that the density of ionized matter in intergalactic space is less than 2.4×10^{-27} gm cm^{-3} simply because of the minimal scattering by free electrons which would otherwise be observed.

Zwicky (1957), on the other hand, finds evidence for dust, at least in clusters, in the fact that distant clusters seem rarer behind near ones than in other regions. He does not give quantitative estimates of the amounts of dust, but in any case one might argue that the space within clusters is not typical of intergalactic space in general.

Whitford (1954) pointed out that a density of intergalactic matter greater than 5×10^{-28} gm cm^{-3} is ruled out by the absence of broad intergalactic Ca II absorption lines in the spectra of galaxies, on the assumptions that $T_K = 50°$ K, the radiation field does not differ in quality from that in our Galaxy, the matter is neutral, and the abundances normal. See the following paper for further discussion of this point; we conclude there that if the medium is ionized, the upper limit is reduced to 4×10^{-30} gm cm^{-3} $\times$ $T_K^{1/3}$.

In summary, then, there is some evidence from deceleration for densities as large as 3×10^{-28} gm cm^{-3}. Such densities of diffuse intergalactic matter are not ruled out by the lack of obscuration or selective extinction by dust, since dust would not be expected at such low densities. The lack of absorption lines would not rule out such densities unless the medium were H II; even then, the abundance of Ca relative to H could be lower than usual by a factor large enough to make it invisible. We conclude that if the deceleration effect is real and if the mass in galaxies is insufficient to account for it, we must invoke either (*a*) stars between the galaxies; (*b*) fairly high concentrations of diffuse matter, neutral or ionized, which will rarely be encountered optically; or (*c*) a smooth distribution of neutral or ionized diffuse matter.

In the light of this background, one should examine alternative *c*. A density of 3×10^{-28} gm cm^{-3} corresponds to a density of hydrogen atoms, 1.8×10^{-4} cm^{-3}, provided that the neutral matter is primarily hydrogen, as seems plausible by analogy with the interstellar medium. How might one detect such an amount of hydrogen? The 21-cm line offers a possibility.

The opacity due to a uniform distribution of intergalactic hydrogen expanding with the galaxies and absorbing locally in a narrow spectral line is easily found. The effect of the expansion is to correlate the distance of an absorbing atom with its velocity, and hence frequency of absorption, by

$$\nu = \nu_0 \left(1 - \frac{H r}{c}\right), \tag{1}$$

so that

$$\frac{d\nu}{d r} = -\frac{H\nu_0}{c}.$$

Thus, if there are n hydrogen atoms per cubic centimeter, the number of atoms per square centimeter per frequency interval is

$$\frac{d N}{d\nu} = n \frac{d r}{d\nu} = \frac{n c}{\nu_0 H}. \tag{2}$$

This formula applies for $\nu < \nu_0$ in every direction, for all frequencies greater than $0.6 \times \nu_0$, for which the expansion has been verified. The opacity then follows, for the 21-cm line, from

$$\tau = \frac{2.64 \times 10^{-15}}{T_S} \frac{d N}{d\nu}, \tag{3}$$

given by Wild (1952). When we put in $\nu_0 = 1420.4 \times 10^6$ sec^{-1} and $H^{-1} = 5.4 \times 10^9$ years, we obtain

$$\tau = 9.29 \times 10^3 \frac{n}{T_S}. \tag{4}$$

In this equation the density, n, is assumed everywhere equal; the hydrogen streams with a uniform velocity gradient, H. The atoms are almost equally divided between the upper and lower hyperfine states, so the effective opacity contains the factor, $1 - \exp(-h\nu/kT_S) \simeq h\nu/kT_S$, to account for stimulated emission, where T_S is the "spin temperature," or excitation temperature, of the hyperfine levels. Now two types of observation might be imagined to detect the hydrogen. Since the gas has temperature T_S and opacity τ_ν, there should be an excess brightness temperature,

$$\Delta T_\nu = T_S (1 - e^{-\tau_\nu}) \simeq T_S \tau_\nu = 9290n , \tag{5}$$

in every direction, for frequencies less than ν_0, compared with those greater than ν_0. Inserting the value of density (1.8×10^{-4} cm^{-3}) needed to give decelerations, a temperature of 1.67° results. On the other hand, if a sufficiently distant source of antenna temperature, T_A, is available, there should be a fraction absorption,

$$\frac{T_A - T_A e^{-\tau_\nu}}{T_A} \simeq \tau_\nu = 9290 \frac{n}{T_S}, \tag{6}$$

which, expressed as the difference in intensity for $\nu > \nu_0$ and $\nu < \nu_0$, in degrees, would be

$$\Delta T_\nu = 9290n \frac{T_A}{T_S}. \tag{7}$$

Notice that the absorption extends over the interval $\nu_S < \nu < \nu_0$, where ν_S is the frequency of the hydrogen line in the source. Comparison of equations (5) and (7) reveals that if $T_A < T_S$, the emission observation gives the larger ΔT_ν, while, if the opposite is true, the absorption observation is better.

At this point we come to a fundamental difficulty which will be discussed at greater length in the following paper. What is the value of T_S? There is, in fact, a favorable source, Cygnus A, which has an antenna temperature of about 100° K at the Harvard 60-foot telescope. Is T_S greater or less than 100°? At the outset of this work, it seemed that T_S would be in the neighborhood of 1° K, which would make the absorption experiment much more favorable. Qualifications of this view will be considered in the following paper.

In any case, there is another advantage to the absorption experiment. One may compare, at frequencies in and out of the absorption, Cyg A and Cass A, which presumably suffers no effect because it is galactic. But in the emission case, one must establish a temperature change of 1.67° as one turns to lower frequencies, looking at the sky. The receiver noise will change with frequency much more than that, so again arrangements must be made to compare the sky with something having no effect—such as the ground. This probably introduces more difficulties than the comparison with Cass A.

For this reason we have attempted to detect the absorption of 21-cm radiation from Cyg A. Since the sensitivity of the receiver used was about 1° K, it was felt that an opacity ($= \Delta T_\nu / T_A$) of about 0.01 could be established if it existed. According to equation (6), then, n/T_S as little as 10^{-6} could be detected.

II. METHOD OF OBSERVATION

In accordance with what has been said above, the expected absorption "line" in Cyg A has the appearance given in Figure 1. The source frequency, ν_S, is 1342.5 Mc/s as determined by Lilley and McClain (1956) by the local absorption effects in Cyg A itself. This agrees well with the optical recession velocity, 16830 km/sec, of the colliding galaxies associated with Cyg A. The antenna temperature of Cyg A at the 60-foot Harvard radio telescope, T_A, is about 100° K. Note that T_A is achieved outside the frequency interval (1342.5, 1420.4), for only in that interval is there neutral hydrogen receding from us and lying between us and Cyg A. In that interval an absorption $\Delta T_\nu = \tau_\nu T_A$ is shown. There are, at 1342.5 and 1420.4 Mc/s, local effects due to the hydrogen in Cyg A and in our own Galaxy; the former have been studied by Lilley and McClain. Note that T_A is really a function of ν; without further information, we cannot be certain as to what the intrinsic radio-frequency spectrum of Cyg A is in this range. For example, it could conceivably make a dip simulating the absorption effect. We shall assume, however, that it can be represented by a straight line of unknown slope over the 10 per cent frequency interval involved.

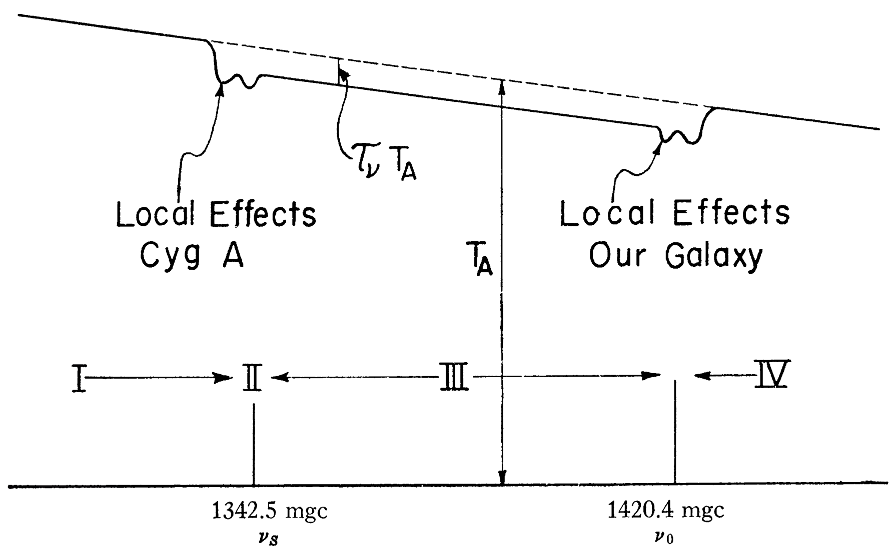

FIG. 1.—Schematic diagram of expected absorption effect, as a ratio of Cygnus A to Cassiopeia A. Groups *I* and *IV* show no absorption; group *III* shows absorption of 21-cm radiation by hydrogen between the galaxies; and group *II* is affected by hydrogen in the Cygnus A source.

A glance at Figure 1 indicates that a conventional comparison-type 21-cm radiometer is unsuitable for detecting broad-band absorption. As it measures the difference of powers in two bands a few megacycles apart, it would effectively differentiate the curve of Figure 1, giving no effect except near the ends of the absorption band. At the same time, it is essential somehow to eliminate the effects of gain fluctuations.

Total power reception was employed in the present experiment. The resulting gain fluctuations lead to two difficulties. First, the zero level of intensity is unknown at any moment. To establish this, the antenna was pointed alternately at Cyg A and a nearby point in the sky. The difference, source − sky, then measures the flux of Cyg A, assuming that there is no difference in sky background at the source and at the comparison point and that the zero level of the radiometer has not changed during the comparison. The second difficulty is the lack of knowledge of the gain of the instrument during the comparison. This is estimated by performing a similar measurement on a standard source presumed to show no effects of intergalactic absorption—Cass A in this case. The ratio of the source-sky differences for Cyg A and Cass A should be a good measure of the Cyg A flux, provided that the Cyg A and Cass A determinations are made within a time short compared to the fluctuations of gain.

A source-sky comparison was made in about 8 minutes, as follows. The antenna was put 20^m west of the source and allowed to drift in R.A. for 2 minutes. It was then moved, at the declination of the source, to the source and until the recorder indicated that approximately 80 per cent peak power had been reached. The source was then allowed to drift for about $2\frac{1}{2}$ minutes through the point of maximum antenna gain, back to the 80 per cent point, whence the antenna was moved again quickly 20^m west of the source, and allowed to drift in R.A. for 2 minutes. The remainder of the time was consumed moving the antenna; the times are typical but not exact.

The sky point, always west of the sources, on which the internal receiver noise was recorded, was chosen for freedom from continuum radiation at 21 cm. Repeated drift-curves failed to establish measurable continuum gradients (probably $< 0\overset{\circ}{.}5$ per hour R.A.) along the parallels of declination passing through the sources, 20^m west of the sources (although Cyg X, which extends east of Cyg A, would give fairly large gradients). We are therefore insured against errors in base line resulting from casual setting of the antenna on the sky points; $0\overset{\circ}{.}1$ error in T_A is the maximum ascribable to this cause.

The antenna was never moving while powers on and off the source were being recorded. This minimized fluctuations due to possible mechanical movement of parts of the IF amplifier, mounted on the antenna. However, the antenna was slewed quickly between off and on readings to minimize the time in which the gain could change. The declinations of both sources on the setting dials were checked constantly to within $1'$ by tracking in R.A. and finding the maximum north-south beam response.

Side lobes could give trouble because they might be favorably oriented toward the ground at some settings, while at others the ground effect would be smaller, thus giving an artificial gradient of output over the sky which might be falsely ascribed to source-sky differences. However, on the days of best gain stability, no effects of moving the antenna were ever noted. Furthermore, it is considered unlikely that the 100° or so entering the side lobes would change more than a fraction of a degree in moving the antenna at high altitudes (usually $>50°$), which were involved in a sky-source comparison. The sun was never less than 75° from the nearest source (Cass A) and was usually below the horizon during these measures.

The radiometer used was the conventional Harvard 21-cm receiver (Lilley 1954) modified in certain respects. First, the "signal" band was turned off, and the output of the comparison band was recorded. This makes the receiver a total power receiver, as mentioned above, and broadens the reception band to reduce noise fluctuations. Second, no automatic volume control was used, as this constitutes a control of the comparison-band gain depending on input, which would tend to erase any time-varying input. Third, a narrow-band filter in the form of a single stub tuner was employed at the input to the crystal mixer, to reject frequencies from the antenna in the image band. Finally, the recorder was used with minimum integration time, about 1 second, minimizing distortions of fluctuations and source drift-curves which might otherwise have occurred.

Used in the way described, the receiver accepts a band which, from Lilley (1955), is seen to be 2.55 Mc/s wide, centered on the local oscillator (LO) frequency $\pm$ 29.55 Mc/s. In Table 1 is given a list of the LO settings, together with the signal-band frequency, taken to be that frequency for which the filter was tuned for maximum reception (usually close to 100 per cent). Also given is the "Nominal Signal Frequency" defined as LO $\pm$ 30 Mc/s, for ease of identification. Table 1 also shows the fraction of the received power which would be within the frequency interval in which absorption may be expected (the "absorption interval"), should there be 100 per cent image acceptance; those labeled "$\frac{1}{2}$" would have 50 per cent of their reception in a band 29.55 Mc/s on the other side of LO frequency and out of the "absorption interval."

It would have been wise to measure the image acceptance (defined as the per cent

TABLE 1

RECORD OF OBSERVATIONS

No.	Nominal Signal Freq.	Actual Signal Freq.	Local Oscillator Freq.	Date 1957	Image Acceptance (Per Cent)	No. of Observations	Probable Error (×1000)	Statistical Weight (=1 for 0.01 p.e.)	Fraction in Absorption Interval if Acceptance=1	Raw Cyg/Cass	Extinction Correction (×1000)	Corrected Cyg/Cass	Remarks concerning Zero Stability
1	1300	1300.45	1330	8/17	....	6	7.94	1.586	½	0.637	+3.0	0.640	Good
2	1310	1310.45	1340	8/17	=25	19	3.49	8.210	½	.649	0	.649	Good, but radar
3	1320	1320.45	1350	8/14	....	12	7.96	1.578	½	.626	+2.2	.628	Gain changes
4	1330	1330.45	1360	8/14	....	20	4.46	5.027	½	.632	−2.2	.630	Fair, gain changes
5	1340	1340.45	1370	8/11	....	11	1.51	43.858	×	.624	−3.0	.621	Excellent
6	1350	1350.45	1380	8/11	....	6	8.44	1.404	1	.631	+1.7	.633	Good short term, but long term gain changes
7	1360	1360.45	1390	8/12	≪56	12	6.09	2.696	1	.633	−2.7	.630	Poor, many gain changes
8	1370	1370.45	1400	8/12	....	10	4.61	4.706	½	.633	+1.3	.635	Fair
9	1380	1380.45	1410	8/15	≪56	25	3.58	7.803	½	.630	−0.4	.630	Poor, long term due to drifts
10	1390	1390.45	1420	8/16	< 3	16	8.64	1.340	½	.617	+1.9	.619	Frequent gain changes, poor
11	1400	1400.45	1430	8/17	....	12	7.90	1.602	½	.614	−4.4	.610	Fair
12	1410	1409.55	1380	8/10	≪56	13	12.19	.673	1	.621	−1.4	.620	Poor, possibly frequency effect
13	1412.5	1412.05	1382.5	6/20	....	10	3.81	6.887	1	.611	+0.8	.612	Fine
14	1416	1415.55	1386	6/21	....	12	2.00	25.000	1	.633	−1.5	.632	Excellent
15	1417.5	1417.05	1387.5	6/2	....	20	0.98	104.123	1	.621	−0.6	.621	Fine
16	1425	1424.55	1395	5/29	....	22	2.20	20.661	½	.619	−0 9	.618	Fine
17	1427.5	1427.05	1397.5	6/5	....	14	2.72	13.516	½	.624	−1.7	.622	Fine
18	1430	1429.55	1400	6/6	....	18	2.91	11.809	½	.624	−0.4	.624	Correlated drift
19	1432.5	1432.05	1402.5	6/5	....	12	1.77	31.919	½	.624	+1.6	.625	Fair
20	1435	1434.55	1405	7/12	....	14	3.33	9.018	½	.617	+0.4	.617	Fair, fluctuations large
21	1437.5	1437.05	1407.5	7/21	....	9	3.04	10.820	½	.622	+1.0	.623	Good, but some interference
22	1440	1439.55	1410	8/13	....	11	12.68	.622	½	.631	−2.1	.629	Frequent gain changes, poor
23	1450	1449.55	1420	8/13	≪56	11	7.11	1.978	½	.615	+2.6	.618	Frequent gain changes, poor
24	1341.5	1341.95	1371.5	8/18	<10	18	9.63	1.078	×	0.614	−1.8	0.612	Frequent gain changes, poor

received power at the image frequency) for all frequencies, even though there is reason to believe that it would remain nearly the same for all settings. We did not, however, do this but three times. We have noted the three times when the image acceptance was measured with some accuracy in Table 1, at nominal frequencies 1310, 1390, and 1341.5 Mc/s. In only the first case was it at all significant—25 per cent. Four other times a lower accuracy determination showed that the acceptance was less than 56 per cent. We can only conclude that probably the image acceptance was negligible at all but a few of the frequencies used in the measurements. Moreover, 6 other of the measured

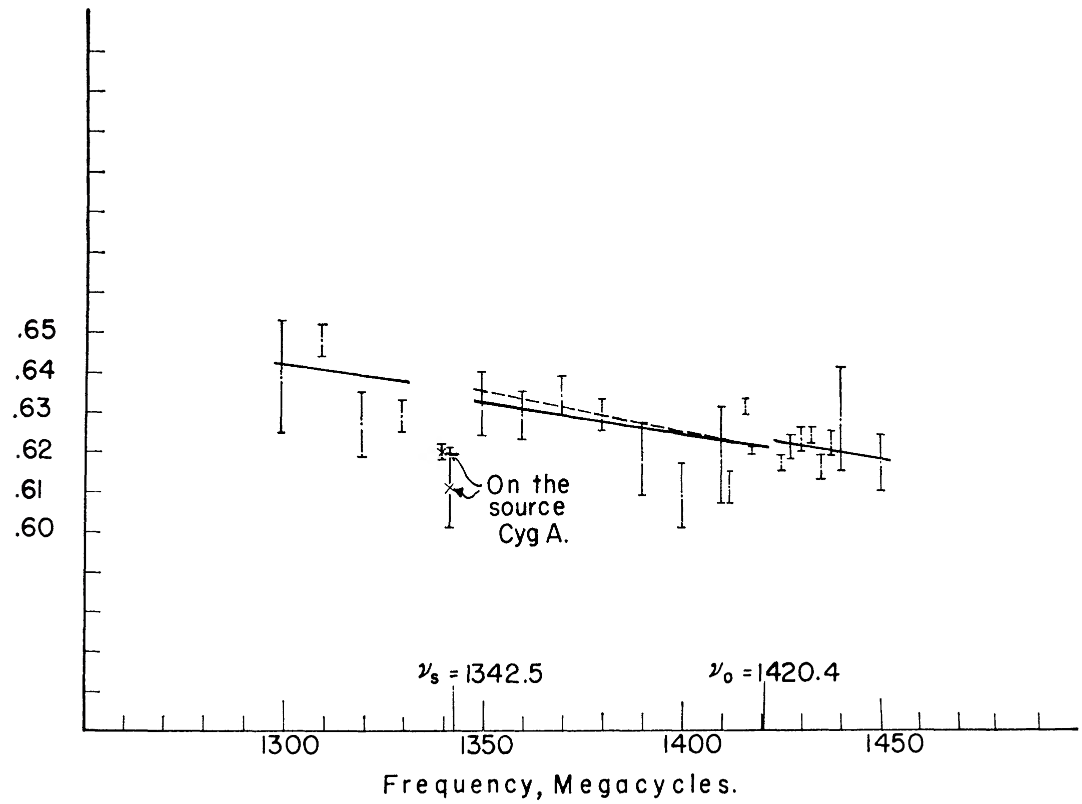

FIG. 2.—The ratio of Cygnus A to Cassiopeia A at various frequencies. The points indicate means at the (nominal) frequencies; vertical lines are probable errors. The outer solid line best represents the external points (groups *I* and *IV*), while the dotted line represents the internal points best. Actual results are in terms of the best internal line of the same slope as the outer solid line. It passes through (1380, 0.6285) and is represented by the inner solid line.

frequencies, making a total of 9 out of the total of 24, should, except for slopes in the spectra, have both signal and image in the absorption interval and hence be independent of the amount of image acceptance. It will be noticed from Figure 2 that there was no tendency for these frequencies to differ in their results from those with possible image contamination.

The LO frequency was set, using the usual 5 and 5.1 Mc/s crystal harmonics as standards; these were checked frequently against WWV to a few cycles. Thus the error from crystal inaccuracy was limited to a few kilocycles per second. But it was not attempted to set the LO closer than 0.1 Mc/s (= 4 per cent of the band width), inasmuch as the automatic frequency-control system which ordinarily keeps the LO frequency constant was not utilized, as it was found that operating at LO frequencies very different from the usual for this equipment resulted in frequency jumps too large for the automatic frequency control (AFC) system to handle. Instead, the frequency was allowed to drift and was checked from time to time. Drifts more than 0.1 Mc/s were very rare, and such stability was judged good enough in view of the 2.55 Mc/s band width.

In accordance with the usual formula, the true thermal-noise component of the output was about 1°. As mentioned previously, the gain fluctuations were more serious and set limits to the accuracy with which a sky-source comparison could be made. Therefore, we attempted to accumulate comparisons until the probable error of a mean was less than 1 per cent.

III. THE OBSERVATIONS

In accordance with the 1 per cent goal, observations were accumulated in blocks of four—two of Cyg A bracketed by two of Cass A. Table 1 gives the frequencies and dates in the first to fifth columns, the image acceptance in the sixth column, and the total number of observations of Cyg A at each frequency in the seventh column. Each source observation was reduced by drawing the best straight line through the sky points and the best straight line of the same slope through the source reading. The difference was averaged for the several records of each source in a block. The ratio r = Cyg A/Cass A was then computed, and this ratio used as the best estimate of the intensity of Cyg A relative to Cass A at the given frequency for that block. The results of all blocks at a given frequency were then averaged; the result (rounded off to three figures) is given in the eleventh column of Table 1. Such a procedure is valid to the extent that the gain may be considered constant over each block; it is felt that this is fairly true because a block was generally measured in 30 minutes.

The probable error of the mean was computed from the formula

$$\text{p.e.} = 0.675 \sqrt{\frac{\Sigma (r_i - \bar{r})^2}{n(n-1)}},$$

where the r_i's are the individual ratios for each block, $\bar{r}$ is the average for the frequency, and n is the number of blocks. A 1 per cent p.e. is about 6.3×10^{-3} units. It is seen from Table 1 that 15 of the 24 measures have p.e.'s less than 1 per cent, with the smallest (No. 15) about 0.16 per cent. Six more are between 1 and 1.5 per cent and 3 are in the range between 1.5 and 2.0 per cent. Unfortunately, however, a glance at Figure 2, which presents the data of Table 1, shows that, with few exceptions, the smallest p.e.'s were obtained in a narrow spectral range near the usual operating frequency of the receiver. This systematic effect is believed to be due to general radar interference leaking into the antenna side lobes. The fourteenth column indicates that the quality of the record deteriorates rapidly as we leave the 30-mc range around 1425 Mc/s.

It is noticed that the frequencies fall into groups according to their position on Figure 1: *I*, Nos. 1–4, which lie definitely beyond the recession frequency of Cyg A, and hence should show no effect; *II*, Nos. 5 and 24, which lie in the range of local effects in Cyg A; *III*, Nos. 6–15, which lie in the absorption interval; and *IV*, Nos. 16–23, which are lower than the laboratory frequency and so should show no effect. The data are weak in four respects: (*a*) there are too few points in group *I*, and their p.e. are too large; (*b*) the points of group *II* have too large p.e.'s, with the exception of three near 1415 Mc/s; (*c*) there are only two points which repeat the Lilley-McClain effect, and (*d*) the range of points at either end is too small.

This experiment involved a retuning of the receiver, which made it useless for usual hydrogen-line work; so, except for the points near 1420 Mc/s, the work was carried out in a single 9-day period. This fact limited the number and accuracy of low-frequency points. Furthermore, it was found that tuning difficulties made operation at frequencies less than 1300 Mc/s or greater than 1450 Mc/s almost impossible. It is hoped to rectify these difficulties in the future, achieving essentially complete coverage over a longer base line, with smaller probable errors.

Finally, we mention corrections made for atmospheric extinction. According to a recent determination by Westerhout (private communication; but see Muller and Westerhout 1957), the extinction is $\alpha = 0.0085 \pm 0.0008$ at the zenith, using e as a base. (Therefore, it is $0^{m}_{.}0092$ or 0.037 db.) The correction to be applied to the data is easily shown to be $\delta = \alpha[\sec z\,(\text{Cyg A}) - \sec z\,(\text{Cass A})]$. This correction is given as an average over the day's run in the twelfth column. It is seen to be less than 0.5 per cent in every case and less than 0.2 per cent in most cases and is usually a small fraction of the p.e. It should be noted, however, that there is a 40 per cent discrepancy between the value of extinction measured by Westerhout and the value (0.0119) computed from theory. However, even if the larger theoretical value should be correct, the extinction corrections would in most cases be smaller than the p.e. The corrected Cyg/Cass ratios are given in the thirteenth column of Table 1.

IV. DISCUSSION AND CONCLUSION

We have mentioned the causes of random errors: noise, gain fluctuations, and, to a limited degree, radar and low-frequency interference. It is believed that the computed probable errors are reliable guides to the magnitudes of these errors. Systematic errors could arise from three sources: (1) Extinction. The correction applied may be in error by 40 per cent, but it is nevertheless small compared to random errors. (2) Ground reflections. These conceivably could always make zenith observations more intense than low-altitude ones. As mentioned previously, it is believed that comparison with a nearby sky point eliminates this effect if it exists, but it is difficult to put quantitative upper limits on the effect. (3) Lack of determinations of image acceptance could, in the worst cases, mean that some observed ratios refer not to the indicated signal band but to the mean of the signal and image bands. Evidence has been quoted against this possibility, in that, although there is a systematic tendency for the low frequencies to give larger ratios, the three points where almost complete image rejection is certain (Nos. 2, 10, and 24) show no tendency to differ from the early results. However, again little can be said quantitatively.

The analysis of the data was carried out by means of least squares. First of all, it is conceded that the ratio of Cyg A to Cass A can be represented by a straight line of non-zero slope, owing to different spectral indices. Hypothesis I is, therefore, that the data in groups *I* and *IV* can be represented by the same straight line; we call this the "external line." The best external line was determined by least squares, weighting the points according to the inverse square of the probable errors. (The weights relative to that of a point of 0.01 probable error are given in the ninth column of Table 1.) The best external line is given a full line in Figure 2. (We note that, for the external line, 6 of the 12 points lie within 1 probable error, while, for the internal line, 6 out of 10 do, in accordance with expectations.)

The predicted value from this "external" straight line is 0.6302 ± 0.0014 standard deviation at approximately the mid-point of the absorption interval, at 1380 Mc/s; the slope is $-(1.64 \pm 0.23 \text{ s.d.}) \times 10^{-4} (\text{Mc/s})^{-1}$. Now we make hypothesis II: that the "internal" points (group *III*) can be represented by a straight line of the same slope as the external line, but of possibly different mean value. Actually, if there were constant opacity in this region, the slopes should be slightly different because we have the same percentage of a larger ratio at low frequencies. However, we ignore this difference, which in any case is small. To test whether the slopes are essentially equal, we compute the best internal straight line of any slope. It was found to have the value 0.6298 ± 0.0012 s.d. at 1380 Mc/s and the slope $-(2.06 \pm 0.64 \text{ s.d.}) \times 10^{-4} (\text{Mc/s})^{-1}$. This line is shown as dotted in Figure 2. It is seen that the slopes of the external and internal lines are equal within their standard deviations. We then computed a best internal line having the same slope as the external line: $-1.64 \times 10^{-4}(\text{Mc/s})^{-1}$. This line has the value 0.6285 ± 0.0012 s.d. at 1380 Mc/s. This value is 0.0017 less than the predicted value from the best external line; it is obvious that this difference is not statistically significant. The difference ranges between 0.00182 and 0.00166 for the slope equal to the mean value ± its probable error. It should be noted, however, that the small standard deviations (about 0.2 per cent) result particularly from the well-determined points in the 25 Mc/s region about 1425 Mc/s, and therefore are perhaps optimistically small.

We may apply the usual criteria for the difference of two means. The standard deviation of the difference of the means is found to be 0.00181. From this, using the assumption (hypothesis III) that the true internal means at 1380 mc must be less than the true external mean, we find a 95 per cent confidence interval of 0–0.00472 for the difference of the two means. We mean by this that in 95 per cent of the cases an interval of this sort around the observed difference of the two means will contain the true difference. Thus we seem fairly safe in concluding that the difference of the means is less than 0.00472 or that the absorption is less than 0.75 per cent.

We may check these results against other knowledge. First, we have the ratio Cyg/Cass at 3200 Mc/s, 2.25 times the 21-cm frequency, given by Haddock, Mayer, and Sloanaker (1954). Their value is 70/150 = 0.466, while Seeger, Westerhout, and van de

TABLE 2

COMPARISON WITH LILLEY AND MCCLAIN

Point	L and M	F	Diff. (LM−F)	
a) Red mean	0.642	0.633	+0 009	Put on basis of 100 per cent image acceptance
b) Blue mean	.633	.629	+ .004	
c) No. 5, 1340.45 Mc/s	.611	.621	− .010	Put on basis of 0 image acceptance
d) No. 24, 1341.95 Mc/s	0.606	0.613	−0.007	

Hulst (1956) obtained 42.1/56.0 = 0.754 at 400 mc, about 0.28 times the 21-cm frequency. We can compute spectral indices, using our value of 0.622 at 1430 mc. We obtain $\beta = \Delta \log r/\Delta \log \lambda$ from all possible pairs as follows: 400 and 3200 Mc/s:0.23; 1430 and 3200 Mc/s:0.36; 400 and 1430 Mc/s:0.15. There is, therefore, evidence that Cyg A increases faster than Cass A with wave length in this region, but this rate is lower for lower frequencies. Now β may also be found from our slope. It is easy to show that the slope is close to $\beta r_0/\nu_0$, where the zeros refer to 1430 Mc/s. Setting this equal to 1.64 ± 0.23, we find 0.38 ± 0.05 as the best value of β from our data. That this value is closest to that determined from the pair (1430, 3200) may be taken to indicate that the high-frequency β represents the situation better in the neighborhood of 1430 Mc/s. That there is a curious break in the spectra of both sources at about 1000 Mc/s and lower frequencies was pointed out by Roman and Haddock (1956), and thus the 400-mc ratio may have little to do with the present situation.

Another test to which we can submit the present data is a comparison with the data collected by Lilley and McClain in their study of the local effects in Cyg A. They used a similar technique, and their data concerning Cyg/Cass would be directly comparable except for the fact that they used no image rejection. Hence we must do some reduction to find comparable quantities. First of all, they established a "blue mean" around 1329 Mc/s. Their results are given as r = Cyg/Cass in Table 2. Similarly, we may compute what we should have got from our mean data by averaging in the effect of the image band.

In addition, we may, by integrating over the Lilley and McClain absorption-curve, establish what we should have got for the points of group *II* (1340.45 and 1341.95 mc). In this calculation, allowance was again made for their lack of image rejection. Again the results are tabulated in Table 2.

The agreement between these sets of data is obviously poor. Our mean values are consistently less than Lilley and McClain's, while the local effect in Cyg A seems real but is only about half the amount quoted by them. While the red and blue means each differ by some 1 per cent, we find, as Lilley and McClain did, that the blue is less than the red, although by only two-thirds of 1 per cent rather than their $1\frac{1}{2}$ per cent.

Perhaps there is an indication here that the true spectra (including absorption) are not entirely smooth but have fluctuations, such as seem evident from the three points near 1415 Mc/s. In that case, comparisons such as we have made between means over frequency intervals may not be significant. In fact, this remark applies to our whole testing of hypotheses, which are based on straight-line behavior. It may, in fact, be true that, for example, the point at 1400 Mc/s is really 2.5 per cent less than the average. In any case, the present experiment cannot reveal such fluctuations with any certainty. We are left with the conclusion that $\tau < 0.0075$ for a uniform medium. In view of equation (4) we may conclude, then, that, in all probability,

$$\frac{n}{T_S} < 8.1 \times 10^{-7} .$$

The further conclusions which may be drawn from this will be discussed in the following paper.

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