SUPRATHERMAL PARTICLE GENERATION IN THE SOLAR CORONA*

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ABSTRACT

It is shown that hydromagnetic waves propagating outward through the solar corona will convert all but a small portion of their energy into suprathermal particles It is suggested that this is the source, of the 3×10^{28} ergs/sec necessary to maintain the 2×10^6° K solar corona with its continued expansion into the solar wind The temperature of the solar corona will then correspond to an ion thermal velocity of the same order of magnitude as the hydromagnetic wave velocity.

I. INTRODUCTION

We shall begin our discussion of the heating of the solar chromosphere and corona with the now generally accepted hypothesis that the energy is transported upward from the photosphere by pressure waves (Biermann 1948; Schwarzschild 1948) and/or hydromagnetic waves (Alfvén 1947, 1950). It is estimated (van de Hulst 1953) that perhaps 3×10^4 ergs/cm² sec (or 1.8×10^{27} ergs/sec over the entire sun) is required to maintain the quiet corona; a rather greater quantity of energy is needed to maintain the denser chromosphere. It has been pointed out that both the spicules (Thomas 1948, 1950) and the pressure waves from the granules (Biermann 1948; Schirmer 1950) may account for the heating of the chromosphere. The corona has proved somewhat more difficult because of the tendency of acoustical waves to dissipate in (and thereby heat only) the lower chromosphere; what is more, hydromagnetic waves which will propagate through the chromosphere do not dissipate in the corona at all. This dilemma may have been resolved by the recent work of Piddington (1956) and Cowling (1956, 1957), who have pointed out that in the chromospheric H I–H II transition region the comparable numbers of neutral and ionized atoms lead to dissipation of hydromagnetic waves far in excess of that in the dense un-ionized photosphere or in the tenuous ionized corona. Their calculations would indicate that the dissipation in the transition region can readily account for the heating of the quiescent corona.

However, the continued 500–1500 km/sec outward expansion of solar gas observed in interplanetary space by Biermann (1951, 1952, 1957) reopens the question of coronal heating. Biermann suggests a quiet-day outward velocity of 500 km/sec and a density of 10^2 hydrogen atoms/cm³ at the orbit of the earth, which represents a solar expenditure of about 3×10^{28} ergs/sec. It has been pointed out (Parker 1958*a*) that the simplest explanation of this *solar wind* observed by Biermann is the hydrodynamic expansion of the solar corona; no decrease of the $\sim 2 \times 10^6$° K coronal temperature has been observed out to at least 2 solar radii (Roberts and Billings 1958), and, if such temperatures extend to 5 or 10 radii, the observed solar wind velocity and density follow automatically from the conventional hydrodynamic equations. Thus we seem to be faced with the problem of supplying an additional heating of 3×10^{28} ergs/sec to the necessary static coronal 10^{27} ergs/sec and supplying it not in the chromospheric H I–H II transition region but throughout the entire depth of the solar corona where the outward expansion occurs. Thus the question is whether there is any mechanism which can dissipate hydromagnetic waves in the tenuous, fully ionized solar corona.

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It is the purpose of this paper to demonstrate that suprathermal particle generation constitutes just such a mechanism and is evidently responsible for most of the heating in the expanding corona.

II. SUPRATHERMAL PARTICLES

The basis for suprathermal particle generation is not new. It involves the fact that high-speed charged particles in a violently agitated plasma carrying magnetic fields will undergo Fermi acceleration (Fermi 1949). The mean free path of an ion with mass M, charge e, and velocity w in a plasma of N ions and N electrons per unit volume and ion thermal velocity, u_p, or temperature, T, is (Spitzer 1956; Parker 1957)

$$\lambda = \frac{3M^2 w^6}{32\pi N e^4 u_p^2 \ln \Lambda} = \frac{6.21 \times 10^{-21} w^6}{TN \ln \Lambda},$$

where

$$\Lambda = \left(\frac{3}{2\,e^3}\right)\left(\frac{k^3 T^3}{\pi N}\right)^{1/2} = \frac{1.24 \times 10^4 T^{3/2}}{N^{1/2}},$$

and the numerical forms applying to hydrogen. If the distance between reflections from moving magnetic fields is somewhat less than λ, then obviously a net Fermi acceleration is the result. Since $1/\lambda$ decreases as the minus sixth power of the ion velocity, the slowing-down which it represents immediately becomes negligible, once a net Fermi acceleration has occurred. Ginsburg (1953) discussed just this acceleration of cosmic-ray particles in the solar corona quite some time ago. It has also been pointed out that thermal ions can be accelerated to cosmic-ray energies in the solar chromosphere during the time of a solar flare (Parker 1957).

However, though it has been known for some time that ultra-high-speed charged particles can be accelerated by the Fermi mechanism in a field-bearing plasma, it has only recently been pointed out that Fermi acceleration results in copious numbers of ions being raised just a little above thermal energies, so that a strongly non-Maxwellian ion velocity distribution is a generally occurring and dynamically important phenomenon (Parker 1958*b*; Parker and Tidman 1958*a*, *b*). The ion velocity distribution in tenuous, agitated plasmas is generally non-Maxwellian, possessing a high-velocity, *suprathermal* tail, which may fall off as slowly as a small inverse power of the ion velocity and which may contain a large portion of all the ions present. In extreme cases the suprathermal particle energy may greatly exceed that of the thermal particles, thereby grossly modifying the dynamical properties of the gas as a whole (Parker 1958*c*).

The generation of suprathermal particles is evidently responsible for the effective viscosity of the solar wind as it blows past the geomagnetic field and for the resulting primary auroral proton spectrum (Parker 1958*b*). The generation of suprathermal particles in interstellar space is, of course, responsible for the galactic cosmic-ray field and thus is the dynamical process which maintains the effective speed of sound in the interstellar and galactic halo gas near the mean turbulent velocity, the *Mach one effect* (Parker 1958*c*). The generation of suprathermal particles is evidently responsible for the production of neutrons in laboratory deuterium plasmas (Colgate 1957) and for the general heating of the deuterium ions (Parker and Tidman 1958*a*); we are proposing that the solar corona is similarly heated.

To treat the suprathermal particle generation quantitatively, we have used (Parker and Tidman 1958*a*, *b*) the Fokker-Planck or Langevin equation for the random walk of the velocity, c, of an individual particle undergoing collisions. We let $F\,(c,\,t)\,dc$

represent the number of ions per unit volume in the velocity interval $(c, c + dc)$ at time t. If N is the total number of ions per unit volume, we have

$$N = \int_0^\infty d\,cF(c, t)\,.$$

Then the Fokker-Planck equation is (Chandrasekhar 1943)

$$\frac{\partial F}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial c^2}[\langle (\Delta c)^2\rangle F] - \frac{\partial}{\partial c}[\langle \Delta c\rangle F]\,, \tag{1}$$

where $\langle \Delta c\rangle$ and $\langle (\Delta c)^2\rangle$ are the mean and mean-square changes in particle speed per unit time due to all collisions and interactions.

In a quiescent ionized gas the only collisions are due to the Coulomb forces, and it has been shown that the time rate of change of F can be written approximately as

$$\left(\frac{\partial F}{\partial t}\right)_c \cong a\frac{\partial}{\partial c}\left[\frac{1}{c}\frac{\partial}{\partial c}\left(\frac{F}{c^2}\right) + \frac{M}{kT}\frac{F}{c^2}\right], \tag{2}$$

where

$$a = \frac{4\pi N kT e^4 \ln \Lambda}{M^3}$$

$$= 2.01 \times 10^{19} NT \ln \Lambda$$

for hydrogen. Note that a Maxwellian distribution is a stationary solution of relation (2).

In an agitated plasma containing magnetic fields there is a Fermi interaction, in addition to the Coulomb forces, which contributes to $\partial F/\partial t$. The simplest model for the Fermi interaction occurs when the hydromagnetic disturbances in the plasma have sharp fronts, as they will if of sufficiently large amplitude (discussed in Sec. IV). Then we may imagine the magnetic fields carried by the sharp-crested plasma motions to reflect the ions much as would hard elastic spheres. It can be shown that the contribution to the time rate of change $\partial F/\partial t$ follows from equation (1) with

$$\langle \Delta c\rangle = \frac{V^2}{L}, \qquad \langle (\Delta c)^2\rangle = \frac{2\,V^2 c}{3L}, \tag{3}$$

where V^2 is the mean-square sphere velocity and L is the mean free path for the collision of an ion with a sphere. We obtain the Fermi contribution

$$\left(\frac{\partial F}{\partial t}\right)_F = \frac{V^2}{3L}\left(c\frac{\partial^2 F}{\partial c^2} - \frac{\partial F}{\partial c}\right), \tag{4}$$

which obviously prevails over the Coulomb interaction of relation (2) when c is in excess of c_0, with

$$c_0 = \left(\frac{3La}{V^2}\right)^{1/4},$$

obtained by comparing the coefficients of $\partial^2 F/\partial c^2$ in relations (2) and (4).

On the other hand, if the plasma motions are not sufficiently violent as to develop sharp fronts, the hard-sphere model does not apply. In place of a sphere velocity, V, we would write something like $\langle \partial V/\partial s\rangle\, c/\Omega$, where $\langle \partial V/\partial s\rangle$ represents a mean velocity gradient in the plasma motions, and the Larmor radius, c/Ω (where Ω is the mean cyclotron frequency), represents the distance the ion is able to penetrate across these continuous gradients. Then in place of equations (3) we would have

$$\langle \Delta c \rangle = \gamma c^2 , \qquad \langle (\Delta c)^2 \rangle = \frac{2\gamma c^3}{3},$$

where

$$\gamma = \frac{\langle \partial V / \partial s \rangle^2}{\Omega^2 L}.$$

We obtain

$$\left(\frac{\partial F}{\partial t}\right)_F = \frac{\gamma}{3}\left(c^3 \frac{\partial^2 F}{\partial c^2} + 3 c^2 \frac{\partial F}{\partial c}\right), \tag{5}$$

which obviously prevails over the Coulomb interaction of relation (2) when c is in excess of c_1, with

$$c_1 = \left(\frac{3\alpha}{\gamma}\right)^{1/6}.$$

The simplest treatment of the problem of calculating $F(c, t)$ takes advantage of the sharp decrease in the Coulomb interaction across the *critical velocity* c_0 or c_1. It allows us to suppose to good approximation that $\partial F/\partial t$ is determined entirely by the Coulomb interaction of relation (2) when $c < c_0, c_1$, and entirely by the Fermi interaction of equation (4) or (5) when $c > c_0, c_1$.

Now it sometimes happens, as in the case of laboratory attempts at thermonuclear power generation in electrical discharges, that c_0 is rather less than the mean ion thermal velocity. Then, obviously, most of the ions experience Fermi acceleration, and the net effect is largely an over-all warming of the plasma ions. On the other hand, in a relatively denser plasma the slowing-down by Coulomb interaction is so effective that c_0 is rather larger than the thermal velocity, as seems to be the case in the chromospheric levels of a solar flare. Then only a few of the ions, composing the high-velocity tail of the initial Maxwellian distribution, experience Fermi acceleration. These ions are quickly accelerated to form a high-velocity non-Maxwellian suprathermal particle tail, and the whole process is limited by the rate at which the Coulomb interactions below $c = c_0$ or c_1 can push ions across the critical velocity into the Fermi range. We have shown that this flux of ions across c_0 is

$$\begin{aligned} R(c_0) &= \left(16\pi^2 e^4 \frac{N^2}{M^2}\right) \ln \Lambda \left(\frac{M}{2\pi k T}\right)^{3/2} \exp\left(-\frac{M c_0^2}{2kT}\right) \\ &= 0.26 \ln \Lambda \left(\frac{N^2}{T^{3/2}}\right) \exp\left(-0.60 \times 10^{-8} \frac{c_0^2}{T}\right) \end{aligned} \tag{6}$$

ions/cm^3 sec for hydrogen. As we shall soon see, we have both cases in the solar corona, with c_0 greater than the thermal velocity throughout the dense inner corona and c_0 less than the thermal velocity in the tenuous outer corona.

III. PHYSICAL CONDITIONS IN THE CORONA

Consider those physical parameters of the solar corona which are relevant to suprathermal particle generation. For the coronal density we shall use the numbers given by van de Hulst (1953, p. 262) at solar maximum (at solar minimum the densities are somewhat lower, which would enhance the suprathermal particle effects). Whenever a coronal temperature is needed, we shall put $T = 2 \times 10^6 \,^\circ$ K. We shall take the general solar magnetic field to be $B_0 \cong 1$ gauss at the photosphere (Babcock and Babcock

1953) and suppose that it drops off with radial distance r from the sun as r^{-2} (Parker 1958d, e),

$$B(r) = B_0 \left(\frac{a}{r}\right)^2 , \tag{7}$$

where a is the radius of the photosphere[1] (if we were to suppose that $B[r] \propto r^{-3}$ as in a dipole, we would enhance the suprathermal particle generation).

We shall suppose that the corona is agitated from below by hydromagnetic waves in $B(r)$, which have a velocity of propagation

$$\begin{aligned} C(r) &= \frac{B(r)}{[4\pi M N(r)]^{1/2}} \\ &= 2.19 \times 10^{11} \frac{B(r)}{N^{1/2}(r)} \text{ cm/sec} \end{aligned} \tag{8}$$

TABLE 1

Radial Distance (r/a)	Magnetic Field (B/B)	Relative Wave Amplitude ($N^{1/4}/B$)	Hydromagnetic Wave Velocity (km/sec) (C)	Mean Free Path for 220 km/sec Proton in Units of 104 km (λ)	Critical Velocity c_0 (km/sec)		
					Eq (9)	Eq (10)	Eq (11)
1 00	1 00	141	110	0 039	385	607	568
1 10	0 83	121	144	0 098	327	460	422
1 20	0 69	131	181	0 22	283	360	325
1 40	0 51	136	234	0 78	227	251	230
2 00	0 25	164	331	5 6	146	124	125
3 00	0 11	213	435	50	91	59	68
4 00	0 062	278	460	174	67	12	15

and a total energy content at least as large as the needed 3×10^{28} ergs/sec. Thus, if the wave length is L_0 where the field is B_0 and the density N_0, we have a wave length $L(r)$ elsewhere, where

$$L(r) = L_0 \left[\frac{B(r)}{B_0}\right] \left[\frac{N_0}{N(r)}\right]^{1/2} .$$

Now both the relative magnetic amplitude, $\Delta B/B$, and the relative velocity amplitude, $\Delta V/C$, of a hydromagnetic wave vary as $N^{1/4}(r)/B(r)$ (Parker 1955). We see from the third column in Table 1 that both amplitudes increase with increasing r. Therefore, we need not worry that the diverging magnetic field or increasing wave length will smooth out the hydromagnetic waves with increasing r and thereby render suprathermal particle generation impotent in the outer corona.

We shall make three independent estimates of the characteristic plasma velocity, V. If we suppose simply that we have hydromagnetic shock waves, then we might put V comparable to the thermal velocity,

$$V^2 \cong \frac{3kT}{M} .$$

It follows immediately from equation (5) that the critical velocity is

$$c_0^4 \cong 2.42 \times 10^{11} \left(\frac{L_0 N_0^{1/2}}{B_0}\right) B N^{1/2} \ln \Lambda . \tag{9}$$

[1] This radial field is suggested both by the solar wind and by the observed radial coronal streamers

On the other hand, we may suppose that V is produced at the base of the chromosphere by, say, spicules of velocity V_0 and varies with r according to the usual velocity-amplitude relation,

$$V(r) = V_0\left[\frac{N_0}{N(r)}\right]^{1/4},$$

which we note is independent of $B(r)$. Then from equation (5) we have the critical velocity

$$c_0^4 = 6.03 \times 10^{19}\left(\frac{L_0}{V_0^2}\right)\left(\frac{B}{B_0}\right) NT \ln \Lambda . \tag{10}$$

Finally, we note that the necessary energy flux into the corona is 3×10^{28} ergs/sec, or 5×10^5 ergs/cm^2 sec over the 6×10^{22} cm^2 solar surface. The energy intensity carried by strong hydromagnetic waves of amplitude ΔB is of the order of $[(\Delta B)^2/8\pi]C$ and, if this is to equal or exceed 5×10^5 ergs/cm^2 sec at the base of the corona, where $N \cong 4 \times 10^8$ hydrogen ions/cm^3, we have that ΔB must exceed 1.1 gauss. Thus ΔB is comparable to B_0, and we have hydromagnetic waves of large amplitude. Since the relative amplitude of the hydromagnetic wave has been shown to increase with r, we have waves of large amplitude for all r. Thus we might wish to equate V to C, yielding

$$c_0^4 = 2.51 \times 10^3 L_0 N^{3/2} N_0^{1/2} \frac{\ln \Lambda}{B_0 B} . \tag{11}$$

We shall put N_0 equal to its value of 4×10^8/cm^3 at the base of the corona. We shall put $L_0 = 10^4$ km, since wave lengths much greater than the scale height are reflected and become merely a radial oscillation of the solar atmosphere. We shall put V_0 equal to 90 km/sec in equation (10), corresponding to spicule velocities (Thomas 1948, 1950). Since ln Λ varies from 21.2 at the bottom of the corona to 25.5 at four solar radii, we shall put $\ln \Lambda \cong 23$ throughout. The resulting critical velocities are given in the sixth, seventh, and eighth columns of Table 1; the differences between the columns represent an estimate of at least some of the uncertainties in our order-of-magnitude calculations.

We note that beyond 1.4 solar radii the critical velocity c_0 is rather less than the 220 km/sec thermal velocity of the 2×10^{6}° K coronal temperature. Hence all the particles soon experience Fermi acceleration, and the net effect is a general warming of the plasma.

Closer to the sun than 1.4 solar radii, the critical velocity is rather in excess of the 220 km/sec thermal velocity. Hence the ion velocity distribution will be Maxwellian at thermal velocities, with a non-Maxwellian distribution extending upward from c_0.

IV. CORONAL HEATING MODEL

Consider the yield of suprathermal particles in the inner corona where c_0 is somewhat greater than the thermal velocity. We use the round numbers $L = 10^4$ km, $V_0 = 100$ km/sec, $N = 3 \times 10^8$ hydrogen atoms/cm^3, $T = 2 \times 10^{6}$° K ($\ln \Lambda \cong 23$) as typical of the region. Then we have $c_0 = 540$ km/sec, and from equation (6) a suprathermal particle yield of $R_0 = 3.4 \times 10^4$ ions/cm^3 sec. Over a coronal depth of 10^5 km (1.00–1.14 solar radii) the total yield from the 6×10^{22} cm solar surface is 3.4×10^{13} gm/sec, which is sufficient to account for the quiet-day solar wind of 2.3×10^{13} gm/sec. On the other hand, if we did not have hydromagnetic waves with sharp fronts, we would obtain $c_1 = 3.1 \times 10^4$ km/sec as the critical velocity, with the trivial yield of $R_1 = 10^{-12000}$ ions/cm^3 sec. Clearly, then, our suprathermal particle coronal model must be based upon a discussion of the coronal hydromagnetic wave properties.

We have postulated that hydromagnetic waves in the amount 3×10^{28} ergs/sec are propagating along the general $\sim$1-gauss solar magnetic field B up into the corona. We have shown that the amplitude ΔB of these waves must be at least 1 guass in order to carry the necessary energy. We note that the magnetic pressure $(\Delta B)^2/8\pi$ of the wave is $\sim$0.04 dynes/cm² and so is comparable to the gas pressure, $NkT \sim 0.08$ dynes/cm² at the base of the corona. It has been shown elsewhere (Petschek 1957; Parker 1958*c*) that both longitudinal and transverse hydromagnetic waves of large amplitude ($\Delta B \gtrsim B$) and large compressibility [$(\Delta B)^2/8\pi \gtrsim NkT$] will develop angular profiles. The sharpness of the angle is comparable to the Larmor radius of the thermal ions ($\sim 2 \times 10^3$ cm). Therefore, we expect the hydromagnetic waves to develop sharp crests as they enter the corona; the hard-sphere model ($c_0 = 540$ km/sec) should be not unrepresentative of the suprathermal particle generation. Even the continuous gradient model yields similar results, $c_1 = 390$ km/sec, if we put $\langle \partial V/\partial s \rangle = V/l$, where l is the $\sim 2 \times 10^3$ cm thickness of the wave front.

However, we have just pointed out the that hard-sphere generation, R_0, of suprathermal particles is extremely rapid. The sharp angular crest of each wave contains only a very small portion of the total energy of the wave. Therefore, we expect that the equivalent hard-sphere suprathermal particle generation will quickly round the wave crest, and the actual rate of generation will be rather less. But, since the wave is still of large amplitude and highly compressible, it has a strong tendency to resharpen its rounded crest. Obviously, then, a balance will be achieved wherein the rate of suprathermal particle generation is controlled by the rate at which the wave can continually resharpen its crest.

On the other hand, we need not fear that the waves can escape through the corona without giving up most of their energy to suprathermal particle generation. We remember the continued radial increase of relative amplitude, $\Delta B/B \propto N^{1/4}/B$ (third col., Table 1) and compressibility $(\Delta B)^2/8\pi NkT \propto 1/N^{1/2}T$ (T seems to be approximately uniform and N decreases). The amplitude factor $N^{1/4}/B$ increases from 141 at the base of the corona to 278 at 4 solar radii to 1.5×10^5 at the orbit of earth; for a uniform coronal temperature, the compressibility factor may be written $(N_0/N)^{1/2}$ and increases from 1 to 60 at 4 solar radii and to 2000 at the orbit of earth. Hence even waves which are of small amplitude when they enter the bottom of the corona will soon find themselves with huge amplitude and compressibility and generating suprathermal particles with their rapidly regenerated sharp crests.

We conclude that, if there are hydromagnetic waves of intensity $\gtrsim 3 \times 10^{28}$ ergs/sec propagating up into the corona along a more or less radial general solar field, then the necessary coronal heating of 3×10^{28} ergs/sec follows by the mechanism of suprathermal particle generation. The generation occurs over several solar radii and is, then, the source of the solar corona and the solar wind. It is obvious that this phenomenon may be generalized to stars other than the sun, and perhaps to the Galaxy and its halo.

V. GENERAL DYNAMIC RELATIONS

There are several interesting dynamic consequences of suprathermal particle generation of the solar corona. The expected high temperatures and non-Maxwellian velocity distribution may help to explain why such high-excitation lines as 5303 A (Fe XIV) and 6374 A (Fe X) first photographed by Lyot (1944), and the yellow line 5634 A (Ca XV) (Edlén 1942; Pecker, Billings, and Roberts 1954) are found in coronal emission spectra; they seem to have their source in active regions (Roberts 1952; Roberts, Billings, and Dolder 1954; Athay and Roberts 1955; Billings, Hirsch, and Varsavsky 1956). The non-Maxwellian distribution may perhaps be at least partially responsible for the lack of quantitative agreement between coronal temperatures as determined from the ratios of line intensities and as determined by line widths (Billings 1957).

Noting that both the wave amplitude and the wave compressibility are enhanced in cool regions by the increase in N necessary to maintain the static equilibrium pressure, p, we see that suprathermal particle generation is enhanced in cool regions and diminished in hot regions, thereby tending to produce a uniform coronal temperature. The abrupt decrease of the ability of a hydromagnetic wave to grow a sharp front when the compressibility $[(\Delta B)^2/8\pi NkT]$ drops below unity and the immense suprathermal particle generation when the compressibility is large and the front sharp suggest a balance wherein the compressibility is of the order of unity. Then, since the magnetic and velocity amplitudes are related by

$$\Delta V = \frac{\Delta B}{(4\pi NM)^{1/2}},$$

we have

$$kT \cong \tfrac{1}{2} M (\Delta V)^2 .$$

Thus we expect that the coronal temperature will correspond to thermal velocities of the order of the velocity amplitude ΔV of the generating hydromagnetic waves. This is the familiar *Mach one effect* already encountered in the interstellar and galactic halo regions (Parker 1958*c*), where the suprathermal particle generation maintains the speed of sound at the same order of magnitude as the motions of the generating waves. We have estimated that $\Delta B \cong B$ in the solar corona. Hence $\Delta V \cong C$, and, from our assumption of a general solar field of about 1 gauss, we calculate a $2 \times 10^{6\circ}$ K coronal temperature at 1.4 solar radii, with perhaps higher temperatures farther out. We hesitate to apply this principle to the innermost coronal layers because we are so near the boundary and in such steep density gradients; our numbers would yield about $0.7 \times 10^{6\circ}$ K at the base of the corona, where $C \cong 110$ km/sec.

REFERENCES

Alfvén, H 1947, *M.N* , **107,** 211
——— 1950, *Cosmical Electrodynamics* (Oxford: Clarendon Press)
Athay, R G , and Robert, W O 1955, *Ap J* , **121,** 123
Babcock, H. W., and Babcock, H D 1955, *Ap J* , **121,** 349
Biermann. L 1948, *Zs f. Ap* , **25,** 161.
———. 1951, *ibid* , **29,** 274
——— 1952, *Zs. f Naturforsch.*, **7a,** 127
——— 1957, *Observatory*, **77,** 109
Billings, D E. 1957, *Ap. J* , **125,** 517
Billings, D E , Hirsch, S., and Varsavsky, C 1956, *Ap J* , **123,** 532
Chandrasekhar, S 1943, *Rev Mod. Phys* , **15,** 1.
Colgate, S. 1957, *Lockheed Symposium on Magnetohydrodynamics, Palo Alto, California, December 16, 1957* (to be published)
Cowling, T. G. 1956, *M N* **116,** 114
——— 1957, *Magnetohydrodynamics* (New York: Interscience Publishers), pp 110–112
Edlén, B 1942, *Zs f Ap* , **22,** 30.
Fermi, E. 1949, *Phys. Rev* , **75,** 1169.
Ginsburg, V L (1953), *Doklady Akad Nauk S.S S R* , **92,** 727; Nat Sci Found trans No 207
Hulst, H C. van de 1953, *The Sun*, ed G P Kuiper (Chicago: University of Chicago Press), p 305
Lyot, B. 1944, *Ann d'ap* , **7,** 31
Parker, E. N 1955, *Phys. Rev* , **99,** 241.
——— 1957, *ibid* , **107,** 830
——— 1958*a*, *Ap J* , **128,** 664
——— 1958*b*, *The Physics of Fluids*, **1,** 171
——— 1958*c*, *Phys Rev* , **109,** 1328
——— 1958*d*, *ibid* , p. 1874
——— 1958*e*, *The Plasma in a Magnetic Field*, ed. R K M Landshoff (Stanford, Calif : Stanford University Press), p. 7
Parker, E N , and Tidman, D A 1958*a*, *Phys Rev* (in press)

——— 1958*b*, *Phys Rev.* (in press)
Pecker, C , Billings, D. E , and Roberts, W O 1954, *Ap. J* , **120**, 509
Petschek, H E. 1957, *Third Symposium on Cosmical Gas Dynamics, June 24–28, Smithsonian Institution; Astrophysical Observatory, Cambridge, Massachusetts* (Proceedings published in *Rev Mod. Phys* , **30**, 908–1108, 1958)
Piddington, J. H 1956, *M N* , **116**, 314
Roberts, W O. 1952, *Ap J* , **115**, 488
Roberts, W O , and Billings, D E 1958, private communication
Roberts, W O , Billings, D E , and Dolder, F P 1954, *Ap J* **119**, 120
Schirmer, H. 1950, *Zs. f. Ap* , **27**, 132
Schwarzschild, M. 1948, *Ap J* , **107**, 1.
Spitzer, L. 1956, *The Physics of Fully Ionized Gases* (New York: Interscience Publishers), p. 78.
Thomas, R. N. 1948, *Ap. J* , **108**, 130
———. 1950, *ibid* , **112**, 343.