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INHOMOGENEOUS STELLAR MODELS. VI. AN IMPROVED SOLAR MODEL WITH THE CARBON CYCLE INCLUDED

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ABSTRACT

An improved solar model is computed in which the effect of the carbon cycle is taken into account and an improved run of chemical inhomogeneity used. The results differ only slightly from those obtained previously. The core of the sun is found not to be convective.

INTRODUCTION

A recent model of the solar interior by Schwarzschild, Howard, and Härm (1957), referred to hereafter as "Paper V," indicated that the central temperature was approaching the point where the carbon cycle would contribute significantly to the energy generation in the innermost portion of the sun. In the present paper the effect of the carbon cycle upon the structure of the interior is taken into account. In addition, an improved run of chemical composition is used.

RUN OF CHEMICAL COMPOSITION

The run of chemical composition was obtained by averaging two burning rates computed from a model of the initial homogeneous sun and an approximate model of the present sun of Paper V. The use of such an average rather than just the initial burning rate, as employed in Paper V, should give a substantial improvement in the run of the chemical composition.

The models selected were model VIII of the homogeneous series and model III of the inhomogeneous series of Paper V. The energy-generation laws were taken to be

$$\epsilon_{pp} = 9.5 \times 10^{-30} X^2 \rho T^4 \tag{1}$$

for the proton-proton reaction and

$$\epsilon_{cc} = 6.3 \times 10^{-143} X X_{CN} T^{20} \tag{2}$$

for the carbon cycle. A T^4 law was used for the proton-proton reaction rather than the usual $T^{4.5}$ law, since it seemed to represent the energy generation more accurately in the relevant temperature ranges.

Specification of the mass and chemical composition enables one to convert the run of the non-dimensional variables, t and p, given in the models of Paper V to the physical variable T and ρ, from which the rate of conversion of hydrogen into helium may be computed. The mass was, of course, taken to be that of the present sun. The composition assumed was $X_e = 0.80$ and $Z = 0.015$, with $X_{CN} = Z/3$. In the case of the inhomogeneous model this composition refers to the envelope only. For the models chosen, the assumed composition corresponds to luminosities for the initial and present sun, respectively, of 59 and 94 per cent of the present observed value. The inaccuracy indicated by the second value (which should, of course, be 100 per cent) is certainly less than that introduced by the uncertainties of the cross-sections.

The average of the two conversion rates, when multiplied by a time scale, taken here

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to be 4.5×10^9 years, yields the variation in hydrogen content throughout the sun. Figure 1 shows the results of the calculations. For comparison, the run of chemical composition used in the computation of the inhomogeneous models of Paper V is also shown.

The somewhat higher hydrogen concentration in the present paper may be attributed chiefly to the fact that different assumptions were made in the computation of the burning rates in the two papers. In Paper V an assumed luminosity rather than an assumed chemical composition was used to compute burning rates based upon the homogeneous models. The present observed luminosity was the one assumed, and, since the sun has

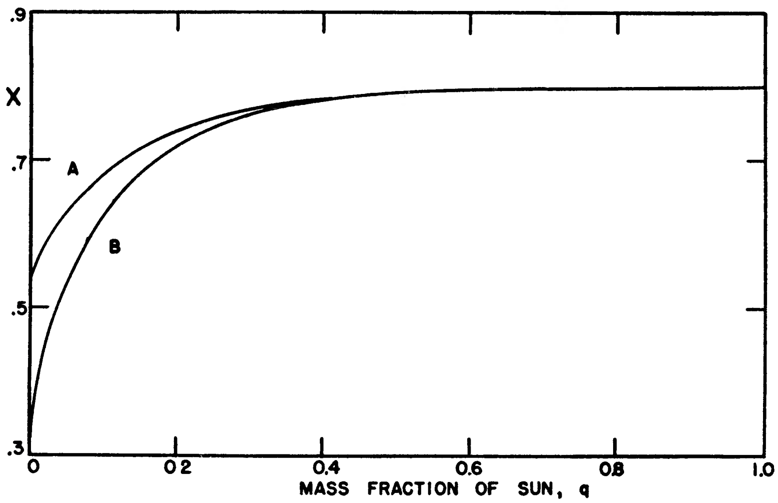

FIG 1 —Distribution of hydrogen in the present sun. Curve A is the distribution used in the present paper; curve B the distribution used in Paper V.

TABLE 1

DISTRIBUTION OF HYDROGEN IN SUN

Region	$\log (X/X_e)$
$\log q < -4\ 000$	$-0\ 1780$
$-4\ 000 < \log q < -3\ 000$	$+0\ 0042 \log q -\ 1612$
$-3\ 000 < \log q < -2\ 000$	$+\ 0236 \log q -\ 1030$
$-2\ 000 < \log q < -1\ 370$	$+\ 0583 \log q -\ 0336$
$-1\ 370 < \log q < -0\ 540$	$+\ 1177 \log q +\ 0478$
$-0\ 540 < \log q < -0\ 201$	$+0\ 0465 \log q +\ 0093$
$-0\ 201 < \log q$	$0\ 0000$

apparently increased in luminosity, the computed hydrogen consumption was a bit high. Additional smaller differences arise from the use in the present paper of an averaged burning rate, a time scale shorter by 10 per cent, the inclusion of the carbon cycle, and a slightly different form of the energy-generation law.

The run of hydrogen content shown in Figure 1 was represented analytically by fitting seven straight lines to a $\log q$ versus $\log (X/X_e)$ plot, with maximum deviations in this representation of less than 1 per cent. This representation is shown in Table 1. The composition functions, i, j, and l, needed in the integrations are those defined in Paper V and were again computed from the interpolation formula (14) of that paper.

CONSTRUCTION OF MODELS

With the exception of the energy generation, the physics governing the models is exactly that of Paper V. In addition to the modification of the differential equation governing the energy generation, the non-dimensional parameter, D, must be redefined:

$$\frac{df}{dx} = ilDp^2t^2x^2\delta\,, \tag{3}$$

$$\delta - 1 = \left(\frac{l}{i}\right)^{1/2} \frac{6.3\times 10^{-143}Z}{3\times 9.5\times 10^{-30}X_e}\left(\frac{HGM}{kR}\right)^{16} \mu_e^{16}t^{16}\,, \tag{4}$$

$$D = \left(\frac{HG}{k}\right)^4 \frac{9.55\times 10^{-30}}{4\pi} X_e^2\mu_e^4 \frac{M^6}{LR^7}\,. \tag{5}$$

TABLE 2

MATHEMATICAL PROPERTIES OF MODELS

$(n+1)_c$	I 3 26375491	II 3 26375493	III 3 263754946	IV 3 263754947	V 3 263754952
E	4 65	2 89	2 14	2 08	0 97
$\log x_0$	− 1 183	− 1 210	− 1 223	− 1 225	− 1 252
$\log p_0$	+ 2 336	+ 2 442	+ 2 497	+ 2 502	+ 2 610
$\log q_0$	− 1 198	− 1 198	− 1 198	− 1 198	− 1 198
$\log f_0$	+ 0 041	+ 0 041	+ 0 041	+ 0 041	+ 0 041
U_1	0 075	0 039	0 025	0 024	0 008
V_1	12 456	14 694	16 351	16 520	21 961
x_1	0 800	0 830	0 847	0 849	0 886
q_1	0 992	0 997	0 998	0 998	1 000
$\log p_1$	− 1 838	− 2 260	− 2 526	− 2 552	− 3 239
$\log t_1$	− 1 002	− 1 088	− 1 142	− 1 148	− 1 289
f_1	1 000	1 000	1 000	1.000	1 000
$\log p_c$	+ 2 230	+ 2 336	+ 2 391	+ 2 396	+ 2 504
$\log t_c$	− 0 015	+ 0 011	+ 0 025	+ 0 026	+ 0 053
$\log C$	− 5 451	− 5 420	− 5 402	− 5.401	− 5 368
$\log D$	− 1 051	− 1 236	− 1 332	− 1 341	− 1 529

The quantity $\delta - 1$ represents the ratio of energy generated by the carbon cycle to that generated by the proton-proton reaction.

To transform the equations to a form convenient for numerical integration from the center, one follows the procedure used by Osterbrock. As in Paper V, $\log q_0$ becomes a second unknown eigen-value, along with the value of $(n+1)_c$. However, after transforming to the logarithmic variables, the equation for $\delta - 1$ still contains the unknown quantity, t_0. By the use of the four conditions for the constants with zero subscripts, t_0 may be eliminated and equation (4) expressed in terms of q_0, M, R, X_e, and Z. Since q_0 enters here to a high power, it becomes a somewhat more sensitive parameter than before. The observed solar values of M and R and an assumed envelope composition of $X_e = 0.80$ and $Z = 0.015$ were used in equation (4) for all the integrations. Although X_e enters this expression to a rather high power, it is clear physically that the particular value of the envelope composition assumed is not critical for the structure of the interior.

The entire series of integrations, including computation of the starting values, was performed on the electronic computer of the Institute for Advanced Study. The radiative solutions thus obtained were fitted to the convective solutions previously obtained by Osterbrock in the usual manner. Table 2 shows the results for these integrations. The final trial value of log q_0, used in the integrations, was -1.20, which differs from the actual value by around 0.002. The errors arising from this discrepancy were found less, however, than those arising in the process of fitting to the convective solutions.

RESULTS

Table 3 summarizes the physical properties of these models for the sun. The data for $X_e = 0.74$ correspond to model III of Table 2. The data for the other X_e values were obtained by interpolation in a *C–D* plot between the other models.

TABLE 3

PHYSICAL PROPERTIES OF SUN

X_e	Y_e	Z	E	x_1	q_1	T_1	ρ_1	T_c	ρ_c
0 80	0 19	0 01	2 05	0 849	0 998	$0\ 94\times10^6$	0 018	$14\ 1\times10^6$	136
74	24	02	2 14	847	998	$1\ 00\times10^6$	019	$14\ 6\times10^6$	134
70	27	03	2 19	846	998	$1\ 04\times10^6$	020	$15\ 1\times10^6$	132
0 60	0 32	0 08	2 33	0 842	0 998	$1\ 17\times10^6$	0 023	$16\ 4\times10^6$	128

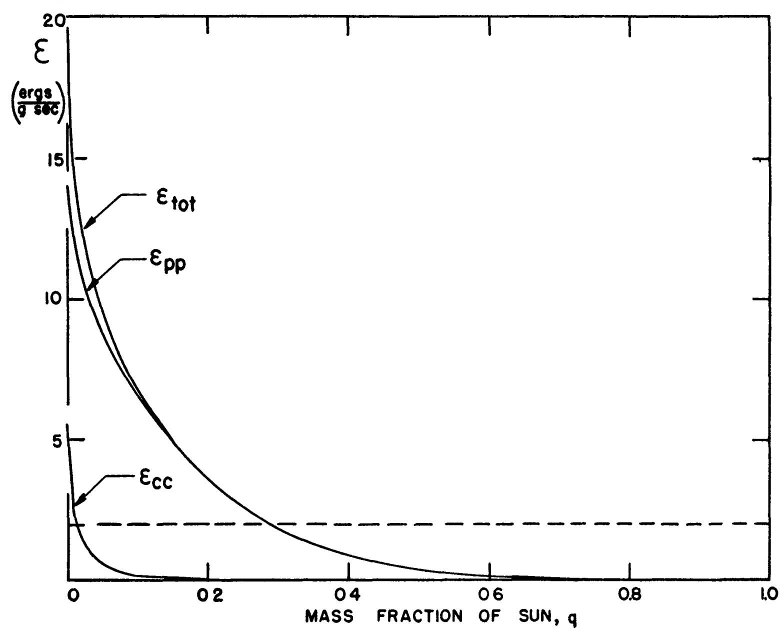

FIG. 2 —Distribution of energy source in present sun based upon model III ϵ_{pp} represents the rate of energy generation by the proton-proton reaction; ϵ_{cc} that due to the carbon cycle ϵ_{tot} gives the total energy-generation rate. The dashed line gives the mean rate for the whole sun

Perhaps the most significant result is the fact that the changes resulting from the modifications introduced in this paper are quite small, so that one may feel some confidence that the models represent reasonably well the over-all structure of the solar interior. In particular, reasonable abundances of the metals are again obtained, the resulting helium content for corresponding metal abundances being a trifle higher than those of Paper V. For the same values of X_e, the central temperatures are lower by roughly three-quarters of a million degrees.

Figure 2 shows the distribution of the energy sources based upon model III. Although lowering the effective polytropic index at the center considerably, the introduction of the carbon cycle is not sufficient to produce a convective core. In fact, in this model, although contributing about 36 per cent of the energy generated at the center, the carbon cycle is responsible for only about 5 per cent of the sun's luminosity.

Regarding the convective envelope, the results again indicate an E value lying between 1 and 10, the limits obtained in investigations of the solar atmosphere. The present E values are slightly lower than those in Paper V. Finally, it should be noted that the temperatures found at the bottom of the convective core are even lower than those obtained before, thus strengthening the conclusion that it does not seem possible to attribute the low abundance of lithium on the surface of the sun to transmutation of lithium occurring at the bottom of the convective envelope in the present phase of solar evolution.

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