OCCULTATIONS AND LUNAR MOUNTAINS

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Abstract. Possibilities for stellar and selenographic research using photoelectric observations of occultations are suggested. The geometrical and selenographic problems of the distortion of occultation curves of stars of large angular diameter by mountain formations on the lunar limb are discussed. Tables for the rapid calculation of occultation curves are given. A method is discussed for the statistical study of slopes on the lunar limb of an extent too small to be visible, and some observational results are given. The accuracy of measurement of diffraction patterns observed at occultations of stars of negligible angular diameter is discussed. A statistical correction to observed angular diameters of large stars for mountain formations on the limb is suggested.

1. *Introduction.* During recent years a considerable amount of work has been done on the photoelectric observation of lunar occultations. This work has been directed to various ends: Whitford (1939, 1946) determined the sizes of stars of small angular diameter by studying the diffraction patterns observed. In this he made use of the theoretical apparatus provided by J. D. Williams (1939). Diercks and Hunger (1952) have investigated the effect of irregularities on the lunar limb on the diffraction patterns of small stars. The accurate timing of occultations has been undertaken by O'Keefe and his associates (1952) for geodetic purposes. Occultation timing by photoelectric methods has also been done on a small scale by Butler (1951) and in a semi-routine manner at the Cape Observatory (1951 *et seq.*). Measures of the angular diameter of Antares have been made by Evans and his associates (1951–53), by Cousins and Guelke (1953), and by the Union Observatory staff (1955), all in South Africa.

From this work has emerged the realization that it is possible to infer, from photoelectric traces of occultations of "point" stars, information about slopes on the lunar limb which are too small to be observed directly. In the case of stars such as Antares, which show a disc of appreciable size, there remains the problem of the extent to which their occultation curves are liable to distortion by irregularity of the lunar limb. This can be tested to some extent by observing the same occultation from different stations so that different parts of the lunar limb are involved. This was done in South Africa for two of the recent occultations of Antares and, within the limits of observational uncertainty, no effect of the lunar limb was detected. The series of Aldebaran occultations which begins about 1959 provides another opportunity for detecting such effects. These occultations will be most readily visible in the northern hemisphere.

There is another interesting possibility with regard to the Aldebaran occultations which can best be exploited in North America. Suppose that an occultation of Aldebaran were observed from N. Lat. 6° at P.A. 135°; then, if the Moon's declination were constant the angle of incidence of the star on the lunar limb would be 45°. If the same occultation were observed from N. Lat. 26°, it would take place at P.A. 45° and the incidence of the star on the limb would be at 45°, but in the opposite sense. In other words the directions of advance of the lunar limb over the star disc at the two stations would be at right angles to each other, and the angular diameter of the star could thus be measured in two directions at right angles. It is, of course, not to be supposed that the foregoing figures can be secured exactly. The important point is that by cooperation of two observatories with a latitude difference of about 20 degrees (not necessarily very close in longitude) star diameters nearly at right angles can be measured.

2. *Distortion of occultation curves: geometry.* The problem of the distortion of occultation curves of large stars by formations on the lunar limb involves two considerations: first, the liability to distortion of the curve by some lunar formation of given size and shape; second, the liability of occurrence on the lunar limb of formations of various kinds.

The first is a purely geometrical problem. The following mathematical formulation simplifies the calculation of any given case. In what follows, diffraction, which for a disc of angular diameter $0\rlap{.}''040$ never produces a change from the geometrical curve by as much as 2 per cent, will be neglected. For Aldebaran, with an expected angular diameter near $0\rlap{.}''02$ there will be a moderate diffraction effect to be removed.

The stellar disc is assumed circular in outline and axes are taken so that its center moves relative to the lunar surface along the y-axis. The shape of the lunar limb is defined by a function $h(x)$, where h is the height above some datum fixed on the Moon, the height of the center of the star disc above the same datum having the value y. Let $\phi(p, x)\,dx$ be the intensity contributed by a strip of stellar surface of width dx at position x when obscured to a height p above the lower horizontal tangent to the star disc (see Figure 1). The word "horizontal" means horizontal in the figure. The lunar horizon may be chosen in any direction we please. If the radius of the star projected on the lunar limb be R, then

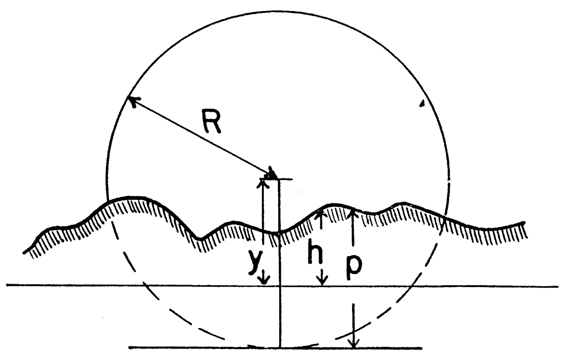

Figure 1. Occultation geometry.

$$p = h(x) - (y - R), \qquad (1)$$

and the total illumination from the unobscured parts of the star is

$$G(y, h) = \int_{-R}^{+R} \phi\{h(x) - \overline{y - R}, x\}\,dx. \qquad (2)$$

The occultation curve is given by the relation between $G(y, h)$ and y.

This integral has several important properties which are especially useful in reducing the almost infinite variety of possible mountain formations to a few manageable cases. The proofs of the following propositions are immediate and are left to the reader:

(i) Whatever the law of darkening to the limb of the star disc, provided it be symmetrical, the mountain formation may be reflected, wholly or in part, about the line of incidence of the center of the star, without producing any change in $G(y, h)$.

(ii) If the shape of the formation be inverted, i.e. if $h(x)$ is replaced by $c - h(x)$, the directions of both axes of the occultation curve are reversed. This follows because $\phi(p, x)$ has the property

$$\phi(p, x) + \phi(2R - p, x) = 2\phi(R, x). \qquad (3)$$

If an occultation curve has the property that it is unchanged when the directions of both axes are reversed, it is said to have standard symmetry. If complete reflection of a mountain formation followed by inversion leaves the mountain formation unchanged, that is, if it is centrally symmetrical about some point, then all occultation curves at any angle of incidence, for any size of star, and for any symmetrical law of darkening have this property of standard symmetry, provided the line of incidence passes through the center of symmetry of the mountain formation.

It has already been remarked that the lunar vertical can be chosen in any direction we please. Any given case of the incidence of a star on a geometrical formation is open to a wide variety of physical interpretations. The case of a horizontal straight line, $h(x) = 0$, may refer either (a) to normal incidence of the star on a smooth level horizon at the vertex of the relative motion of the Moon against the stars, or (b) to incidence on a smooth slope inclined at an angle θ to the horizontal, this slope being located on the lunar limb at a position angle θ away from the vertex. The rate of travel of the star center along the y-axis is the same in the two cases, and is the same as the rate of travel of the Moon's center against the stars. The occultation curves are the same in the two cases, both as functions of y and as functions of time. If we take as the standard case the occultation of a star by a smooth level horizon, then in case (b) the observed occultation curve will be contracted in time in the ratio $\cos\theta : 1$ compared with the curve which would have been obtained if the slope had not been present.

We call such a factor of simple extension or contraction of an occultation curve a scale factor. If the true diameter of a star is D, and it is incident on a part of the lunar limb where there is a formation imposing a scale factor A on the occultation curve, then the diameter inferred will be AD.

In addition to a simple elongation or contraction of an occultation curve there may also be distortion. We define distortion to mean the maximum residual difference shown by a given occultation curve after the standard curve elongated by some scale factor has been fitted to it as well as possible. These definitions are in accord with the natural method of study of occultation curves. The standard curve is plotted on such a scale that the straight-line part of the observed occultation curve is matched. If the fit is satis-

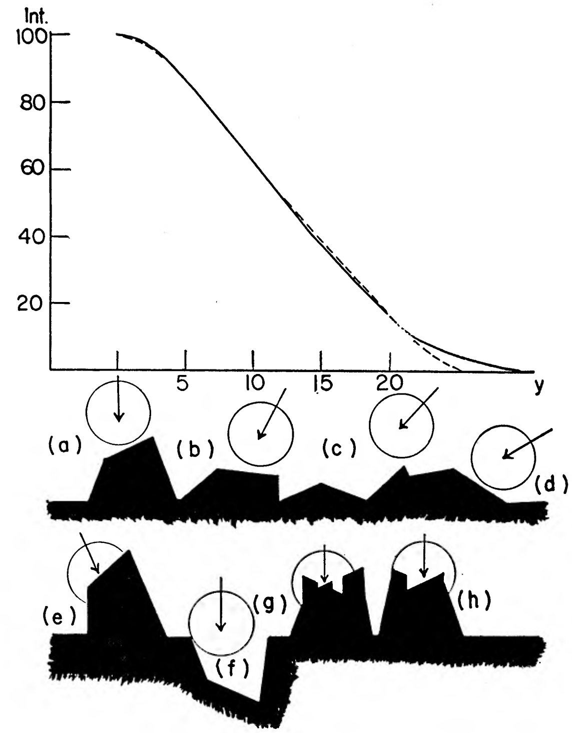

Figure 2. Occultation circumstances producing occultation curves having common shape.

factory, the diameter is inferred at once. If there is any residual difference—the distortion in our sense—the problem which it presents has to be solved, and the distortion attributed either to mountains on the limb, or to a departure from circularity of the star disc.

The transformations outlined above, and the various possibilities of interpretation, permit a single computed case to be applied to a wide variety of different circumstances. The case shown in Figure 2 illustrates an example. All the mountains shown produce the same curve which is drawn in the upper part of the figure. It has a distortion of $3\frac{1}{2}$ per cent from the dotted standard curve. The coordinate y is measured in units of tenths of stellar radius, so that for normal incidence, case (a), the scale factor is 1.28. Cases (g) and (h) are produced by interchange and the scale factor is still 1.28. In cases (b) and (e) where the example is interpreted as referring to incidence at 30°, the scale factor is 1.15. In case (c), for incidence at 45°, the scale factor is 0.90, and in case (d), for incidence at 60°, it is 0.64. Case (f) shows normal incidence on the inverse formation: for this case the directions of the time and intensity axes must be reversed; the scale factor is 1.28. This by no means exhausts the cases which can be developed from this example.

3. *The function* $\phi(p, x)$. For the case of uniform surface brightness, taking $R = 1$ and the total light from the star as 100 units,

$$\frac{\pi}{100}\phi_u(p, x) = 2(1 - x^2)^{\frac{1}{2}} \quad \text{for} \quad p < 1 - (1 - x^2)^{\frac{1}{2}};$$

$$(1 - x^2)^{\frac{1}{2}} + 1 - p \quad \text{for} \quad 1 - (1 - x^2)^{\frac{1}{2}} < p < 1 + (1 - x^2)^{\frac{1}{2}};$$

$$0 \quad \text{for} \quad p > 1 + (1 - x^2)^{\frac{1}{2}}.$$

TABLE I. $\phi(p, x)$ FOR THE CASE OF UNIFORM STELLAR DISC

$p \backslash x$	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
0	636	630	616	596	568	532	484	421	335	199
.1	605	601	595	585	568	532	484	421	335	199
.2	573	569	563	553	539	520	484	421	335	199
.3	541	538	531	521	507	489	465	421	335	199
.4	509	506	499	489	475	457	433	402	335	199
.5	477	474	467	457	443	425	401	370	327	199
.6	445	442	435	426	412	393	369	338	295	199
.7	414	410	404	394	380	361	337	306	263	195
.8	382	379	372	362	348	330	306	274	231	163
.9	350	347	340	330	316	298	274	243	200	131
1.0	318	315	308	298	284	266	242	211	168	099
1.1	286	283	276	266	252	234	210	179	136	068
1.2	254	251	244	235	221	202	178	147	104	036
1.3	223	219	213	203	189	170	146	115	072	004
1.4	191	188	181	171	157	139	115	083	040	000
1.5	159	156	149	139	125	107	083	052	009	000
1.6	127	124	117	107	093	075	051	020	000	000
1.7	095	092	085	075	061	043	019	000	000	000
1.8	063	060	054	044	030	011	000	000	000	000
1.9	032	028	022	012	000	000	000	000	000	000
2.0	000	000	000	000	000	000	000	000	000	000

In units of one-tenth per cent of the total light from the star the function $\phi_u(p, x)/10$ is given in Table I.

Consider the case where an arbitrary addition is made to a given mountain formation. Let

$$h'(x) = h(x) + \Delta(x). \qquad (5)$$

Then

$$G(y, h') = \int_{-1}^{+1} \phi\{h'(x) - (y - 1), x\}\, dx$$

$$= G(y, h) - \int_{-1}^{+1} \Delta(x)\, dx \qquad (6)$$

over the range for which the formula

$$\phi_u(p, x) = \sqrt{1 - x^2} + 1 - p$$

applies. Clearly we must choose $\Delta(x)$ so that it is equal to zero when $x = \pm 1$. Equation (6) shows that if $G(y, h)$ is the occultation curve for a formation $h(x)$, and $G(y, h')$ that for a formation $h(x) + \Delta(x)$, where $\Delta(x)$ is an arbitrary function, zero at $x = \pm 1$, then, except for a displacement, $G(y, h')$ is identical with $G(y, h)$ over that part of the curve for which the arbitrary addition lies wholly in front of the disc. It follows that the addition of small asperities to a smooth formation will produce no effective change in the occultation curve.

For full darkening to the limb according to a cosine law

$$\frac{1}{100}\phi_c(p, x) = \tfrac{3}{4}(1 - x^2) \quad \text{for} \quad p < 1 - (1 - x^2)^{\frac{1}{2}};$$

$$\tfrac{3}{8}(1 - x^2)\left\{1 - \frac{2}{\pi}\left[\sin^{-1}(p - 1)(1 - x^2)^{-\frac{1}{2}} + (p - 1)(1 - x^2)^{-1}(2p - p^2 - x^2)^{\frac{1}{2}}\right]\right\} \qquad (7)$$

$$\text{for} \quad 1 - (1 - x^2)^{\frac{1}{2}} < p < 1 + (1 - x^2)^{\frac{1}{2}};$$

$$0 \quad \text{for} \quad p > 1 + (1 - x^2)^{\frac{1}{2}}.$$

TABLE II. $\phi(p, x)$ FOR FULLY DARKENED DISC

$p \backslash x$	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
0	749	734	704	659	599	524	434	329	209	073
.1	735	721	695	654	599	524	434	329	209	073
.2	711	697	673	636	587	521	434	329	209	073
.3	678	666	644	610	563	503	427	329	209	073
.4	642	630	610	577	534	478	409	323	209	073
.5	602	592	572	541	501	448	383	305	205	073
.6	559	549	531	502	464	415	355	282	194	073
.7	515	506	488	461	424	377	323	255	183	072
.8	469	460	443	418	383	341	288	226	153	064
.9	422	414	398	374	341	302	253	196	129	051
1.0	374	367	352	329	299	262	217	164	104	037
1.1	327	320	306	285	257	222	180	132	079	022
1.2	279	273	260	240	215	183	145	102	055	009
1.3	234	227	215	197	174	146	110	073	026	001
1.4	189	183	172	156	134	108	078	047	014	000
1.5	146	141	131	117	098	075	050	023	004	000
1.6	107	103	093	081	063	045	024	006	000	000
1.7	071	067	060	048	036	020	006	000	000	000
1.8	038	036	030	022	012	003	000	000	000	000
1.9	014	012	008	003	000	000	000	000	000	000
2.0	000	000	000	000	000	000	000	000	000	000

Table II gives values of $\phi_c(p, x)/10$.

4. *Practical calculation.* An examination of Tables I and II will show that for curves accurate to 1 per cent, the integral of equation (2) may be replaced by a sum in which $\phi(p, x)$ is given values extracted from these tables, and $h(x)$, the mountain formation, is defined by 20 numbers. Any given case may be computed by placing a mask cut in the shape of the mountain over the $\phi(p, x)$ table (extended to negative values of x on the left, and extended upwards and downwards by repetition of the top and bottom lines as far as may be necessary), and the function $G(y, h)$ computed by adding across the table the numbers which appear at the edge of the mask for each of its positions. The mask representing the formation must point downwards, and as it is moved downwards over the table the occultation curve for a disappearance is generated. Using the methods of the previous sections one

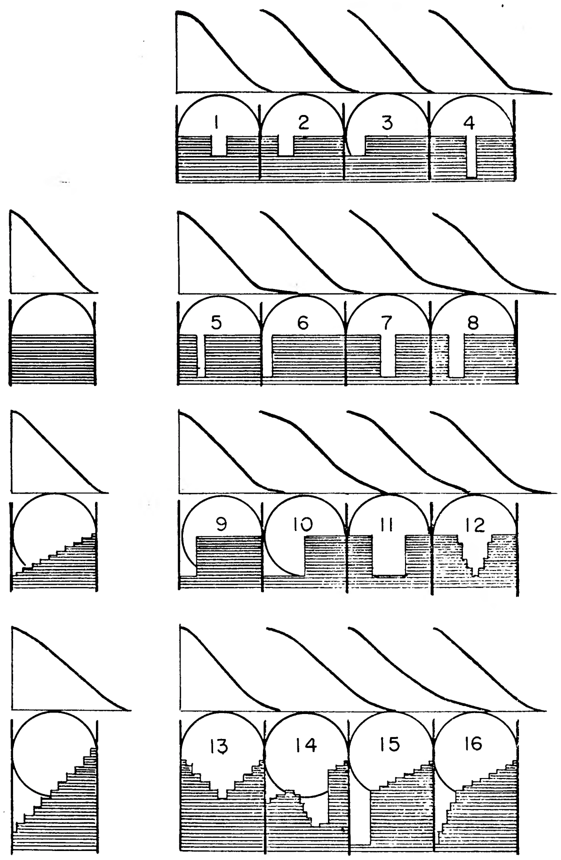

Figure 3. Effect of specialized formations on occultation curve.

finds that each computed case may be extended to numerous other cases equivalent to it.

If h is restricted to the limits 0.0–0.5, there are 3.6×10^{15} distinct formations, but so many of these are equivalent, or can be covered by interpolation, that the whole field may be covered by the computation of a dozen distinct examples. For uniform surface brightness and normal incidence, all scale factors lie in the range 1.00–1.15, and distortion never exceeds 3 per cent. High scale factor and high distortion are mutually exclusive. For incidence at 30°, the range of scale factors is 0.88–1.18. For incidence at 45° the range is 0.81–1.41, and, at 60°, it is 0.70–1.80. The extreme values occur in only a minute number of cases and are accompanied by negligible distortion. At present the discussion is confined to geometrical considerations: certain limitations of a physical kind connected with incidence at high angles will be mentioned later. For a star whose projected disc occupies about 200 feet of lunar limb, the range of h under consideration corresponds to a height range of 50 feet in this base length.

Although not all formations in the range $h=0.0$ to 1.0 have been computed, it seems probable that, for normal incidence, the maximum scale factor will be 1.41, and the maximum distortion 5 per cent. In general, distortion is reduced as the angle of incidence increases. This is a consequence of equation (6), for, as the angle increases, irregularities tend more and more to fall between the positions of first and last contact; compare Figure 3 (12) and (14). Large ranges in scale factor become possible when the angle of incidence reaches 50°. Formations in this group involve a height range of 100 feet in a base of 200 feet.

The search for formations which produce high distortion yields some interesting results. Figure 3 shows a series of examples of such formations for the case of uniform surface brightness. On the left are shown curves of zero distortion, having various values of scale factor, for comparison. In all cases incidence is supposed to be vertically down the page. Distortion is produced by crevasse structures, in which one or both walls are involved, or by complementary pinnacles, for these postpone occultation of part of the disc as compared with the rest. There does not seem to be any other type of structure which can produce high distortion. Inversion of the crevasses to produce pinnacles transfers the distortion to the other end of the occultation curves. Numerous equivalent cases can be developed from these by the processes already described. It is, however, difficult to regard many of the cases shown in Figure 3 as physically possible examples of incidence at large angles, since the formations then involve overhanging rocks.

The case of full darkening to the limb is similar. For normal incidence on a level horizon the occultation curve is identical to within 1 per cent with that for a uniform disc at normal incidence with a scale factor of 0.9 applied. The conclusions of the previous paragraph are not affected by the introduction of limb darkening.

Scale factors markedly different from unity lead to the deduction of wrong diameters for occulted stars if the presence of mountain formations is neglected. Slopes in excess of 15°–20° must be common if errors of this type are to be frequent. Large distortion may lead to the attribution of incorrect shapes to stars. For this to happen crevasses or pinnacles of the order of 50 feet deep or high for a star of angular diameter in the region of 0″.04 must be common, or the situation where the disc glissades along a steep wall of this order of size must be readily possible.

5. *The selenographic problem.* To visual and photographic observation the Moon appears very rough. Representations of lunar landscapes frequently appear in which every slope depicted is steeper than 60°. Such representations owe much to imagination, for careful examination of the lunar surface at first or last quarter shows that, except on the interior slopes of small craters, shadows only occur near the terminator. The indication is that slopes in excess of 30° are rare. Insufficient allowance seems to have been made for the remarkable degree of exaggeration of the shadow patterns, especially near the terminator. Figure 4(a) reproduces the shadow pattern of a mountain range near the terminator from Loewy and Puiseux (1896). Figure 4(b) shows the actual skyline between the points marked by arrows as seen from point *P*.

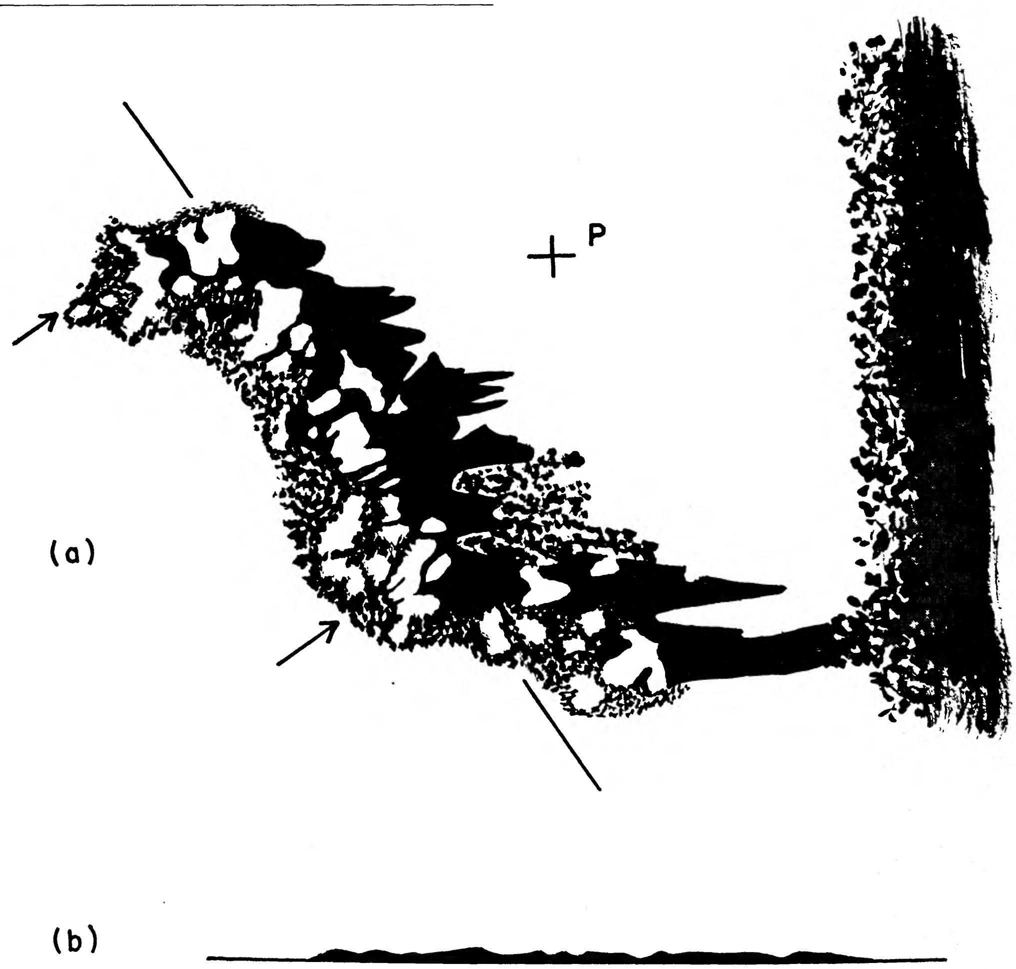

Figure 4a. Shadow pattern of lunar mountain range near terminator.
b. Skyline profile of above range, between arrows, as seen from P.

It might be objected that these features are much larger than those which are important in occultations. On the other hand, the limit of visual and photographic observation is a purely fortuitous result of the lunar distance and the resolution of optical instruments, so that evidence obtained in this way must be of some value as a general guide. Moreover, the view that the lunar surface is rough to the point of jaggedness only depends on this same evidence, so that, if it should be found that exaggerated inferences have been drawn, this view must be revised.

van Diggelen (1951) has shown that slopes in the maria do not exceed 1 in 40. Atkinson (1953) emphasizes the very important distinction between the roughness of individual lunar features and the relative smoothness of the lunar limb: "Large scale photographs of the Moon, visual observations, and the traces published by Hayn, all agree that although there are some fairly steep places on the limb, there are really relatively few of them, and the number of cases where a slope greater than three or four degrees persists for any appreciable distance is not, in itself, large enough to cause serious anxiety." The extraordinary efficacy of this effect of "saturation" of the limb can easily be verified with sand models. It operates for irregularities of any scale. Weimer (1954) has given traces of the lunar limb showing, even with a vertical scale exaggerated 24 times, very few steep places. It may be objected that, except when the line of vision and the line of illumination are identical, cross shadowing will produce differences between the illuminated limb and the silhouette limb. Fujinami's (1952) list of depressions deeper than $0''.1$ based on eclipse observations is worthy of study in this connection. O'Keefe and Anderson (1952) in their work have, apparently, found it best to ignore the mountains altogether.

Crevasses are especially effective in producing distortion of occultation curves. A true crevasse in a lunar plain, 50 feet deep and 25 feet wide, would have to be six miles in length, perfectly straight, and pointed exactly at the observer, in order to appear on the skyline. Crevasses might be formed as gaps between elevated structures, but if their length is short they will frequently be submerged below the limb by libration, and if their length is great, they must be exactly pointed. If, in a deliberate attempt to design a structure where there is a high liability to occurrence of crevasses on the skyline, we imagine a row of blocks backed by further rows, then, in a random arrangement, those behind will seal the gaps between those in front. The reader is invited to experiment for himself. It is important that the direction of view be tangential to the mean surface. The foregoing arguments break down for lines of sight inclined upwards, but these are not the conditions of view of the lunar limb.

Even if crevasses do exist, they will be effective only for normal incidence. The same structures at inclined incidence produce far less distortion. At high incidence distortion can only be produced by part of the disc glissading down a slope.

6. *Observational investigations*. The correctness of these ideas may be tested in a variety of ways by observation of occultations of point-source stars. Clearly, by combining results from different observatories and results from different parts of the lunar orbit, it is possible to discuss occultations on the same part of the lunar limb at a wide variety of angles of incidence. Observations both by eye and by photoelectric methods are of value.

If slopes of angle θ occur, double disappearances at occultations at angles of incidence greater than $90°-\theta$ are possible. If the eye can be reckoned capable of seeing a reappearance lasting 0.1 second, visual observation should be capable of discovering slopes of the order of $0''.04$ in length, and only a fraction of this in height. Photoelectric observation should be capable of detecting slopes very much smaller than this. The Cape Observatory now has about 40 photoelectric observations of occultations, none of which shows this phenomenon. It is understood that double disappearances have only been observed visually at grazing occultations. This method should also reveal the presence of crevasses and pinnacles on the limb.

The observations for statistical study of the microscopic character of the limb are rather more difficult to obtain, for the occulted stars must be brighter, the telescope preferably larger, and the seeing better than in the previous investigation. Observations from different observatories may readily be combined.

Diercks and Hunger (1952) have investigated the effect which roughness in the lunar limb has on the diffraction pattern produced by the occultation of a point source star. Although roughness tends to blur out the later oscillations of brightness, the spacing of these in time is effectively unchanged. This spacing will be affected, independently of the roughness, by the average gradient of the slope behind which the occultation takes place.

Let f be the scale factor for a given occultation found by comparing the observed curve with the case of normal incidence on a smooth level horizon. Let f' be the scale factor found by comparing the observed curve with the case of incidence at the same angle on a smooth level horizon. If the angle of incidence is θ, $f=f' \sec \theta$. If the skyline slopes at an angle ϕ, $f=\sec(\theta\pm\phi)$, the sign being chosen according to whether the slope increases or reduces the effective angle of incidence. For an observed case we can, by measuring the trace, infer a value ϕ for the inclination of the slope behind which the occultation took place.

Now consider occultations on random lines at any angle of incidence on the structure shown in Figure 5, where all the slopes are inclined to the horizon at angle ϕ. For incidence at angle θ, $(\theta+\phi<90°)$, the possible values of f are those given above. The probability of occurrence of each value is proportional to the front which the formation presents to incidence at angle θ. These fronts are equal to $A\cos(\theta-\phi)$ for forward slopes and to $B\cos(\theta+\phi)$ for reverse slopes, A and B being the respective total lengths of the two types of slope. If the ends of the whole formation are at the same level, A and B are equal, and the mean value of f for a given θ can easily be shown to be $\sec\theta\sec\phi$. Thus the mean value of f' is $\sec\phi$ as the result of the presence of this mountain formation. The argument holds, and f' is constant, for values of θ up to $90°-\phi$, and for larger values of θ double disappearances can occur. We define a roughness factor, F, as the mean value of f' taken over all values of θ in this range. It may be shown that if there is a length of limb, L, which is subdivided into parts of lengths, L_1, L_2, etc., having, respectively, roughness factors F_1, F_2, etc., then the roughness factor for the whole length is given by $L\cdot F = \sum_n L_n F_n$. The analysis may be applied to a section of a lunar limb occupied by many mountains, or by a single mountain. The size of the irregularities involved is extremely small.

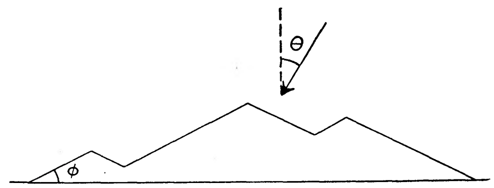

Figure 5. Effect of random limb lines on occultations.

It is, therefore, proposed that photoelectric observations of stars showing good diffraction patterns be obtained, and the traces measured to determine the values of f'. As more are acquired, they may be grouped according to the lunar regions involved, and the values of F for different regions estimated. Results available so far are:

Star	Date	Obs.*	P.A.	θ	f'	Slope
NZC 2268	51 Sept. 7	C	64°	34°	1.06	7°
NZC 2864	51 Nov. 4	C	63	0	1.05	18
NZC 2861	51 Nov. 4	C	10	55	1.06	2
Antares B	52 Apr. 13	P	349	58	1.02	1
Antares B	53 Mar. 8	P	282	7	1.06	14
NZC 2779	54 Nov. 1	C	144	75	0.91	2
					Mean 1.03 = sec 14°	7°

* Observed at Cape (C) or Pretoria (P).

The measurement of even unpromising traces is reproducible to a good degree of accuracy. The practice is to measure on each film the coordinates of the maxima, minima, and passages through the 100 per cent value of as many of the diffraction oscillations as can be clearly distinguished, and in addition to measure a few points on the non-oscillatory part of the trace within the geometrical shadow. Using the observed rate of film travel and the computed rate of advance of the lunar limb, the spacing of the measures of these standard points is compared with the spacing computed from optical theory. The value f' is determined from the percentage change of scale needed to produce superposition of the measured points on the standard set. So far this percentage has always been found small. The need for fitting the observed oscillations to the standard ones so that they shall be in phase fixes this percentage rather well. For an occultation at the lunar vertex the diffraction oscillations approximate to 50 c/s so that the displacement from in-phase to out-of-phase is only $0^s.01$ and the uncertainty should be a good deal less than this.

These studies tentatively suggest a correction to be applied to the observed angular diameters of large stars. If it is found that the occultation curves for such a star observed from different stations are effectively the same, then it may reasonably be inferred that no distortion is present, on the ground that the type of formation necessary to produce distortion of a given kind is very different at different angles of incidence. On the other hand a conspiracy of slopes with only minor degrees of roughness to produce the same erroneous value of diameter at different stations cannot be entirely ruled out. The argument for stars of large diameter is, here, parallel to the argument outlined above for the extension or contraction of diffraction patterns of point stars by slopes on the limb. On the average the measured diameters will be increased by a factor equal to the average value of f'. On the basis of