1954MNRAS.114..524R

TEMPERATURE AND TURBULENCE IN THE CHROMOSPHERE

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(Received 1954 September 4)

Summary

Measures of chromospheric line profiles in high resolution spectrograms obtained at the total solar eclipse of 1952 February 25 have been used to investigate further the question of temperature and turbulence in the solar chromosphere. In trying to deduce a distribution of line-of-sight velocities, arising from either temperature motions or turbulence, two problems are encountered: (*a*) to estimate the chromospheric heights to which the spectra refer, and (*b*) to correct the measures for broadening by self-absorption and Stark effect. Heights have been estimated partly from a comparison of line intensity ratios with those measured by Dr J. Houtgast on slitless spectrograms obtained at the same eclipse, and partly from the time of exposures, using the motion of the Moon relative to the Sun. Broadening by self-absorption has been estimated in the case of metallic lines by an empirical method. The corrected line widths, together with others of He I, suggest an approximately linear increase of line-of-sight velocity with height, from $\xi_0 \simeq 2 \cdot 5$ km/sec at the base of the chromosphere, to $\xi_0 \simeq 16$ km/sec at height 2600 km. Taking into account the uncertainty in effective height, these results agree with those obtained in 1940.

The hydrogen Balmer lines have been examined in more detail. The earlier members of the series are broadened by self-absorption, while later members are broadened by Stark effect. Both kinds of broadening were first estimated by an empirical method, which led to the conclusion that the temperature of the lower chromosphere does not exceed 10 000 deg. K. The hydrogen line widths were then computed more elaborately by a theoretical method, which showed that the observations could be fitted almost as well by a temperature 6 000 deg. as by 10 000 deg. It seems unlikely that the present data will yield any closer estimate of temperature than this.

At the total solar eclipse of 1940 October 1 line widths in the chromospheric spectrum were measured which seemed to indicate two important results: (*a*) the turbulence in the low chromosphere is small, the average (horizontal) line-of-sight velocity being about 1·8 km/sec, (*b*) the kinetic temperature of the low chromosphere is about 30 000 deg. K.* While no objection has so far been raised to the first of these conclusions, the second has provoked a great deal of discussion and the general trend of opinion today is that such a high temperature is most unlikely. Thus there has been a call for more work, first to re-measure the line profiles to ensure that the 1940 measures contain no substantial error, and then, if they turn out to be correct, to seek some alternative explanation consistent with a temperature not far from that of the photosphere.

With this in mind, observations were made at the total eclipse of 1952 February 25, in Khartoum, with a large grating spectrograph. A preliminary

* R. O. Redman, *M.N.* **102,** 140, 1942 (Paper I).

description of this work has already been given elsewhere* and further details are now in the press.† The more essential points may be repeated here. The spectrograph used the second order of a 6-inch Rowland concave grating, having 15 000 lines per inch, in a Wadsworth mounting. The collimator mirror had a focal length 12 feet, the nominal grating radius was 21 feet. For thermal protection the instrument was installed in a pit in the ground. The slit was set tangential to the solar image near the point either of second or third contact. The solar image had a diameter 52 mm; the slit width was 0·03 mm; scintillation was estimated to give an image spread roughly 2 seconds of arc. Eight photographs were taken at second contact covering the region 5550–6040 A, with dispersion 2·2 A/mm, and seven at third contact covering 3430–4110 A, with dispersion 2·4 A/mm. The average exposure time was about 1·5 seconds, with an interval between the centres of successive exposures about 2·3 seconds. The plates were calibrated with a Hilger rhodium step wedge, and the instrumental profile was determined by means of a water-cooled ^{198}Hg isotope lamp. The spectrograph gives very satisfactory definition. The sum of the Rowland ghosts in the second order is about one per cent of the parent line. The general photometric procedure was in principle the same as in 1940, although almost all the instrumental details were different.

Results from narrow lines are given in Paper III and confirm quite closely the 1940 measures. The assumption that for these lines the observed width, after correction for instrumental broadening, is chiefly due to line-of-sight motion, is supported by the fact that the two spectrum regions yield nearly the same mean velocities. Following Unsöld's notation‡, the velocities ξ are taken to have a distribution given by $\exp -(\xi/\xi_0)^2$, so that the results may be expressed in terms of the parameter ξ_0. The 1940 measures (Paper I) were given in terms of the arithmetical mean velocity $\bar{v}$. The two parameters are connected by $\xi_0 = \bar{v}\sqrt{\pi}$.

In this paper we shall merely quote the results for narrow lines and then concern ourselves mainly with the problem of wider lines, very many of which are known to be broadened by self-absorption. Two results will emerge: (*a*) evidence is found which suggests a considerable increase of velocity with chromospheric height, (*b*) the kinetic temperature in the low chromosphere does not exceed 10 000 deg. K, while 6 000 deg. K may almost equally well be reconciled with the line widths. At present it appears doubtful whether a more accurate kinetic temperature can be deduced from the available line profiles.

Heights.—The practical problem of deducing a velocity distribution from chromospheric line profiles leads to two other questions: (1) the determination of effective heights, (2) the correction of the measured line widths for various distortions, by the instrument, by self-absorption, and in the case of hydrogen by Stark effect.

The assignment of a chromospheric height to observations made with a slit spectrograph is a difficult question to which no entirely satisfactory answer has been found. As explained in Paper III, the only tolerably satisfactory way of estimating the heights at which our ultra-violet (third contact) observations were made is to compare line intensity ratios with those from slitless spectra.

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* R. O. Redman, *Accademia Nazionale dei Lincei, Convegno Volta*, **11**, 72, 1953 (Paper II).

† R. O. Redman, *Vistas in Astronomy*, in press, 1955 (Paper III).

‡ A. Unsöld, *Z. f. Naturforschung*, **7a**, 121, 1952.

What we aim at getting is only an apparent height, measured from the solar limb, and we try to express the physical properties of the chromosphere in terms of what would be seen, integrated along the line of sight, in ideal conditions. What ultimately we require are the physical properties in terms of true heights within the chromosphere, but the derivation of these can be regarded as a separate problem, which is not tackled here.

The apparent heights deduced from line intensity ratios are not very precise for at least two reasons: (*a*) What the *slit* spectrograph records is an indeterminate blend of spectra from an appreciable range in chromospheric height, the degree of mixing depending chiefly on scintillation, guiding and length of exposure (the coelostat having been rated to follow the Moon), (*b*) the structure of many of the spectrum lines changes greatly with height, and what *slitless* spectrographs record is the sum of all the chromosphere not covered by the Moon at the time of exposure. If the seeing is very good and the instrumental resolution high enough, these various line structures tend to be recorded side by side; if the scintillation is great they are all blurred together.

However, despite various difficulties, this method of height determination, i.e. by comparison of line intensity ratios between slit and slitless spectrograms, appears to be the least unsatisfactory for the present case. Using data kindly supplied in advance of publication by Dr J. Houtgast, apparent heights have been estimated as follows:

3rd contact	exposure 1		1200: km
	2		1100
	3 level	1	<50
		2	<100
		3	600

The chief cause of the small differences of effective height between exposures 1 and 2 is probably poor guiding. This arose from a slight mishap with the coelostat drive, which slipped out of gear about 30 seconds before the end of totality. The solar image was recovered only just in time for third contact. The three heights on exposure 3 refer to measures made from different points of the spectrograph slit.

For the yellow observations at second contact, heights have been estimated from the motion of the Moon across the Sun. The last traces of Fraunhofer spectrum are seen on exposure 3 and the narrow lines measured in Paper III are from exposure 4, at a height <500 km. Subsequent exposures show nothing but Na I, D_1 and D_2, and He I D_3. For these the estimated heights were based on the assumption that second contact took place between exposures 3 and 4, and that the guiding was good. In some respects this method is more satisfactory than the use of relative line intensities, but when a rather slow succession of exposures is made, as here, it can be applied only to strong lines which extend to comparatively great heights. The adopted heights are as follows:

2nd contact	exposure 4	200 km
	5	800
	6	1400
	7	2000
	8	2600

Metallic lines.—In the violet and ultra-violet spectrum from low chromospheric levels, say less than 600 km, there is an adequate selection of lines, notably of the singly ionized rare earths, which appear to be little broadened by self-reversal, and these give the line-of-sight velocities already reported and reproduced here in Table I.

TABLE I

Line-of-sight velocities from narrow lines

	Height	$\bar{v}$	ξ_0	Number of lines
ultra-violet	< 50 km	1·5 km/sec	2·7 km/sec	38
	< 100	1·8	3·2	54
	$\simeq 600$	2·2	3·9	27
yellow	< 500	1·2	2·1	18
1940 violet	< 500	1·8	3·1	30

At these same comparatively low levels the strong metallic lines, especially those which are also strong in the Fraunhofer spectrum, are generally considerably broader than the narrowest lines, and often their profiles show clear indication of of self-reversal.* At greater heights, unfortunately, practically the only lines which persist in sufficient strength to be measurable are these same lines which at low levels are known to be affected strongly by self-reversal. True, the optical depth in the line of sight decreases with increasing height, so that the self-absorption decreases and somewhere becomes negligible, but we cannot be sure without further examination that this happens before the line becomes too weak to measure.

A fairly certain answer may be obtained if there are enough lines to construct a curve of growth, for on the first linear part of the curve there should be negligible self-absorption. Construction of an empirical curve of growth from Fe I multiplets showed that most of the easily measurable iron lines on our u.v. spectrograms fall near or on the second section of the curve, so that their profiles are unlikely to give satisfactory line-of-sight velocity distributions without correction. It is possible to compute the necessary corrections on the basis of some model chromosphere, but here we have preferred to use an empirical method, from extrapolation of the observed data.

As many lines as possible of a given element are measured at some given height, and their half-widths (ordinates) plotted against line intensities (abscissae). As expected, the widths increase with growing intensity, as the broadening by self-absorption increases. Extrapolation of the observed curve back to zero intensity should, however, give a " true " line width, free of the effect of self-absorption. The extrapolated curve is assumed to start from the ordinate axis with zero slope. This procedure was first tried for u.v. exposure 3, at the 600 km level, where there are also sufficient rare-earth and other narrow lines to give a "true" half-width without correction. The extrapolated half-widths (corrected for instrumental broadening) for Fe I and Ti II give reasonably accordant results:

600 km	rare earths, etc.	$\bar{v} = 2{\cdot}2$ km/sec	$\xi_0 = 3{\cdot}9$ km/sec
	extrapolated Fe I	2·2	3·9
	extrapolated Ti II	2·7	4·8

At higher levels there were rather few lines measurable, even of Fe I and Ti II, and since exposures 1 and 2 refer to approximately the same height they were combined, with the following result:

1200 km	extrapolated Fe I	$\bar{v} = 5$ km/sec	$\xi_0 = 9$ km/sec
	extrapolated Ti II	5	9

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* Not merely self-absorption, but an actual self-reversal at the centre of the line profile.

The exact coincidence of the Fe and Ti velocities here should be regarded as fortuitous, and not a sign of great accuracy in the result.

In the yellow the only metallic lines visible above 500 km were the *D* lines of sodium. These are very heavily self-reversed in the low chromosphere, in the manner typical of all low-excitation lines which are also strong in the Fraunhofer spectrum. However, at the highest level at which they are measurable, 1400 km, both *D* lines have the same width and we have supposed them to be free of serious self-absorption broadening at this height. (We note, however, that their intensities here are in the ratio 1 : 1·4):

1400 km *D* lines $\bar{v}=5{\cdot}3$ km/sec $\xi_0=9{\cdot}4$ km/sec

As far as these spectrograms are concerned there are no further data to be obtained from metallic lines at greater heights; for such levels a faster spectrograph or longer exposures would be required.

Helium.—The only He I lines measurable on our spectrograms are 5876 A (D_3), 4026 A, and on one exposure 3819 A. A few other helium lines could be expected in the 3500–4000 A region, but at the fairly low levels where they might otherwise be measurable they are concealed by blends with other lines. Throughout the violet and ultra-violet spectrum, blending is generally severe at low levels, especially below 100 km, and owing to the intrinsic width of the lines an increase of instrumental resolving power can bring only partial amelioration.

All three of the measured He I lines are multiple. D_3 consists of a close unresolved doublet with a partially resolved fainter companion, the others contain a single line with a partially resolved fainter companion. In each case the fainter companion has, as far as possible, been removed from the measures. After this correction, and after correction for instrumental broadening, we get:

Height	Line		
600 km	4026 A	$\bar{v}=6{\cdot}2$ km/sec	$\xi_0=11{\cdot}0$ km/sec
1100	4026	6·6	11·7
1100	3819	6·6	11·7
1400	5876	6·0	10·7
2000	5876	7·5	13·3
2600	5876	8·9	15·7

On the 1940 spectrograms only the 4471 A He I line was measured, without correction for the faint companion, giving $\bar{v}=8$, $\xi_0=14$ km/sec at some uncertain height probably less than 1500 km. It was remarked at the time that this line width was probably a little too great.

Variation of ξ_0 *with height.*—If now we plot all our results for ξ_0 against height, including those of Paper III, we find an almost linear increase from about $\xi_0=2{\cdot}2$ km/sec, at the base of the chromosphere, to 16 km/sec at 2500 km (Fig. 1). No definite explanation can be offered for the two seriously discordant points, from He I 4026 and 3819 A, but the figures for all the He I lines have to be accepted with some reserve. In the first place no examination has been made of possible widening by self-absorption, but if comparable with self-absorption in hydrogen lines of the same intensity, the effect is quite weak for 4026 and 3819 A at 1100 km. In the case of D_3 nothing can be said except that the line is weakening from exposure 6 to 8, but at the same time the measured line width actually increases; that is, if there is really an appreciable amount of self-absorption broadening (which must diminish with height), the velocity increase with height is still greater than that just deduced.

There is another complication in the case of helium. As already explained, the heights we have tried to assign to the measures are those at which the spectrograph slit would have to be placed to give the observed line widths in perfect observing conditions (no scintillation, etc). These apparent heights are not the same as heights in the chromosphere itself. For instance, were there an exponential intensity distribution with height, with no self-absorption, half the scale height would need to be added to the apparent height to give the true mean height in the chromosphere. With helium the relation between intensity and height is certainly more complicated than this. Many observers have noted that at low levels He I lines tend to behave differently from most others, in that their intensities at first *increase* with apparent height, a behaviour which is presumably closely associated with the general increase of spectrum excitation with height. A simple exponential law does not represent the facts adequately. The conversion of behaviour with regard to apparent height to behaviour in terms of true height can be made sufficiently well only with slitless spectra and we do not attempt it here. Fig. 1 ignores the correction from apparent height to true height, and we have to remember that the relation between these heights is probably less simple for He I than for any other element.

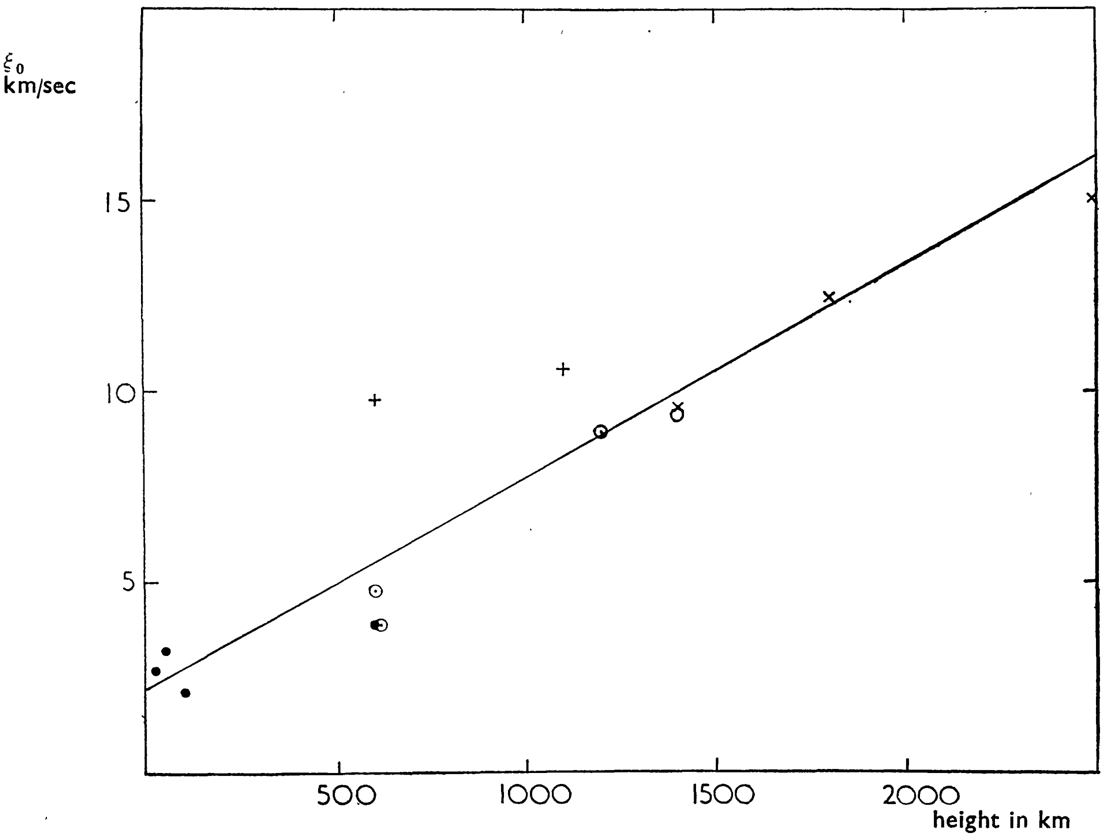

Fig. 1.—● "*narrow lines*" *in u.v., rare earths, etc.* *in yellow, mainly Fe* I.
⊙ *extrapolated Fe* I, *Ti* II.
○ *Na* I *D lines.*
+ *He* I 4026, 3819.
× *He* I D_3.

Helium line widths have been corrected for an assumed T=6000 deg.

Thus although the evidence for an increase of ξ_0 with height is fairly convincing, the linear relation of Fig. 1 should not be taken too readily at face value. The points have been taken from a variety of lines, and the height question in particular cannot yet be regarded as properly settled. The evidence, taken together with what follows on the hydrogen lines, suggests that the velocity increase is chiefly one of turbulence, rather than of temperature, although more observations are certainly needed before there can be a really satisfactory separation of turbulent from thermal motions. The idea of a turbulence increasing with height has been favoured by some theorists, especially Unsöld.*

Hydrogen.—There are no hydrogen lines on the plate exposed at second contact, but the third contact photographs cover the whole Balmer series from Hδ to the series limit. Hδ itself was heavily overexposed, and not measurable on any exposure; others of the earlier and stronger series members could be measured only at the higher levels. The measured half-widths, i.e. the total widths at half the maximum intensity, are in Table II. Gaps in the measures are due to blends, which are particularly troublesome at low levels, to overexposure of lower members of the series, or underexposure of higher members. In cases marked * the whole line could not be measured because of blends and the half-width was obtained by assuming a symmetrical line. The heights were estimated as before.

Three specimen hydrogen profiles are given in Fig. 2, the points being single readings from one exposure. Lines near $n=30$ are affected by the Balmer continuum, which is just measurable on exposure 3, at height 600 km. At lower levels the Balmer continuum is concealed by a general continuum; at higher levels it is too weak to appear on these spectrograms. When finding the half-widths the Balmer continuum has not been removed from the measures, but has been regarded as a part of the lines themselves. (It becomes important only for lines with $n>25$.) The behaviour of line width with n is more regular when the Balmer continuum is left in than when it is removed.

A first inspection of the hydrogen line widths shows a tendency to increase as one approaches the series limit, which qualitatively is what is to be expected from Stark effect. In passing we may note that the lines do not overlap near the series limit, as far as these measures go. In the opposite direction, as one goes to stronger lines there is ultimately also a systematic widening, which is fairly certainly due to self-absorption. All lines now measured are substantially narrower than the widths for Hβ, Hγ and Hδ ($n=4$, 5, 6) on the 1940 spectrograms, which tends to support the suggestion that all the earlier and stronger members of the Balmer series are considerably broadened by self-absorption when observed at fairly low chromospheric levels.

We suppose now that the measured hydrogen line widths are due to (*a*) instrumental broadening, (*b*) Stark effect, (*c*) self-absorption, (*d*) Doppler effect. Of these (*a*) has already been removed and we wish chiefly to find (*d*). There is a weakness in the raw material, inevitable in the present case, and not easily avoided altogether in eclipse spectroscopy of this kind, that because of overexposure the accuracy of measurement falls off with the strongest lines, on which we mostly rely for estimating the effect of self-absorption, and because of underexposure it falls off also with the weakest lines, from which we deduce the Stark effect. Ideally, the measures should cover a considerably greater range than is possible on a single photographic exposure.

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* A. Unsöld, *loc. cit.*

TABLE II

Half-widths of chromospheric hydrogen lines

Blended lines marked* have been assumed symmetrical; other blended lines have been omitted. Uncertain measures are marked with a colon. The half-width is the total line width at half the maximum intensity.

	n	Total intensity	Measured half-width	h.w. corrected for instrumental distortion
Height 100 *km*	13*	1360	0·374 A	0·349 A
	14*	1085	·345	·321
	15	865	·328	·304
	16	765	·333	·308
	17*	548	·333	·308
	18	458	·347	·323
	19*	402	·313	·287
	20*	316	·308	·282
	21	265 :	·318 :	·294 :
Height 600 *km*	15	1375	0·342	0·318
	16	1190	·335	·311
	17	955	·318	·294
	18	815	·328	·304
	19	725	·340	·316
	20	615	·364	·340
	21	508	·342	·318
	22	452 :	...	...
	23	345	·364	·340
	24	...	...	...
	25	...	...	...
	26	263	·422	·398
	27	207	·422	·398
	28	...	...	...
	29	...	·443	·419
	30	...	...	...
	31	...	·472	·451
Height 1100 *km*	13	1144	0·345	0·321
	14	960	·357	·333
	15	736	·328	·304
	16	599	·335	·311
	17	488	·340	·316
	18	354	·316	·289
	19	331	·352	·328
	20	257	·330	·306
	21	227	·366	·342
	22	192	·410	·386
	23	162	·398	·374
	24	...	·378	·354
	25	...	·427	·402
	26	...	·424	·400
Height 1200 *km*	7	1905	0·506	0·484
	8*	1635	·458	·436
	9	1190	·393	·369
	10	900	·345	·321
	11	655	·333	·308
	12	519	·308	·282
	13	380	·292	·265
	14	318	·323	·299
	15	275	·308	·282
	16	187	·296	·270
	17	156	·337	·313
	18	140	·342	·318
	19	91	·337	·313
	20	106	·419	·395

Empirical reduction of hydrogen line widths.—The measures cover $n=7$ to 31, but not all on one exposure or at one level. There are not enough observations to determine the variation of Stark effect with height, so that we have taken all the measures together, assuming they refer to some average height between 600 and 1200 km. The Stark effect has been taken to give a line width proportional to $n(n-1)$. This is a good approximation for large n, and is amply accurate for our present needs. We shall find that for $n<15$ the Stark effect makes a very small contribution to the total line width; for $n>25$ it dominates the line profile. The self-absorption broadening has been taken to vary only with the measured total line intensity, irrespective of height. What is left after Stark effect and self-absorption broadening have been removed is taken to be the Doppler profile, which is of course proportional to wave-length, although the range of variation with the lines measured here is less than ten per cent.

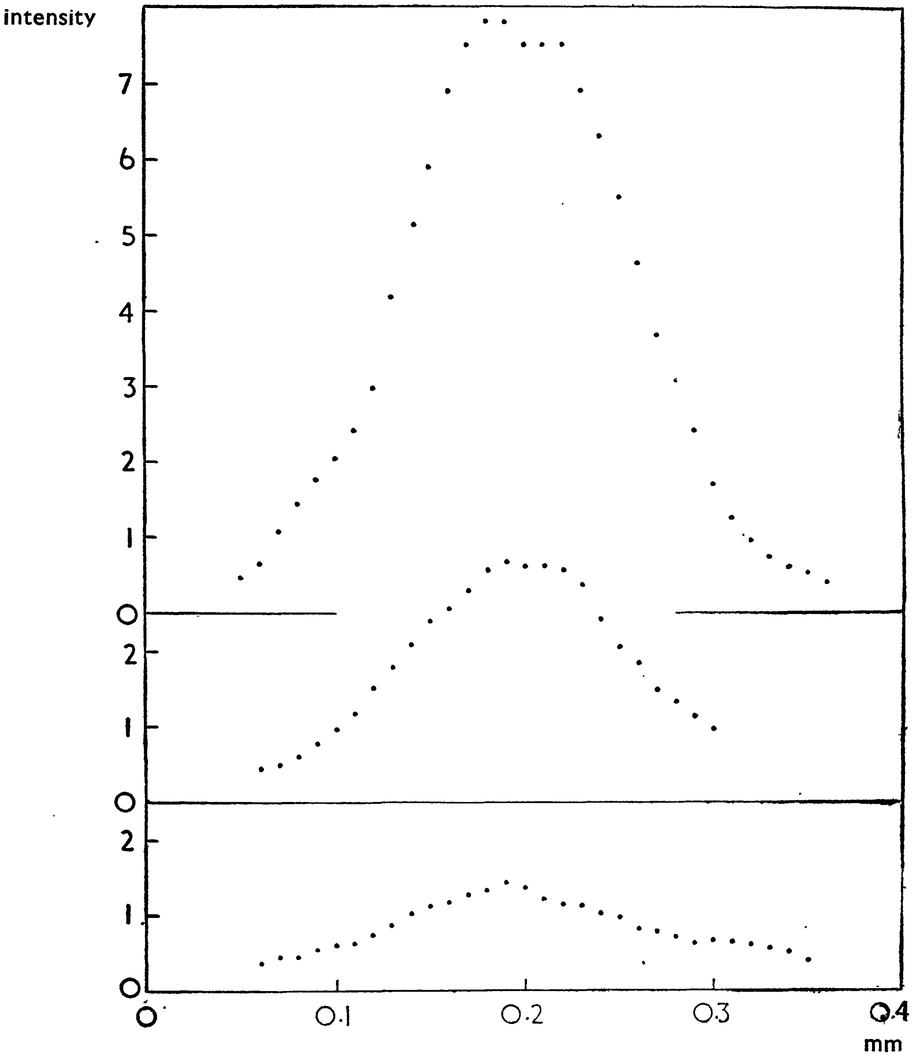

FIG. 2.—*Measured profiles (single readings) of H16, H21 and H26 at height* ≃ 600 *km.* 1 *mm*=2·4 *A.*

With these considerations in mind, empirical values for the Stark broadening and self-absorption broadening were estimated by graphical means and the results are in Table III. It was assumed that for this preliminary examination

TABLE III A

Doppler half-widths of hydrogen lines, empirical reduction

n	100 km	600 km	1100 km	1200 km
7				0·253 A
8				·282
9				·308
10				·289
11				·289
12				·270
13	0·258 A		0·260 A	·253
14	·265		·289	·287
15	·265	0·214 A	·275	·268
16	·275	·236	·284	·253
17	·277	·241	·289	·294
18	·292	·255	·258	·294
19	·246	·268	·292	·282
20	·234	·292	·263	(·364)
21	·236 :	·260	·294	
22		...	·337	
23		·265	·313	
24		...	·275	
25		...	·321	
26		·296	·301	
27		·275		
28		...		
29		·260		
30		...		
31		·248 :		

TABLE III B

Stark half-widths from empirical reduction

n	h.w.
7	0·017 A
10	·036
15	·085
20	·154
25	·243
30	·352

TABLE III C

Corrections for self-absorption from empirical reduction

(to be subtracted from the observed half-width)

I	Correction
250	0·002 A
500	·007
750	·018
1000	·036
1250	·068
1500	·121
1750	·187
1905	·233

Stark and Doppler widths could be combined as Gaussian errors. The self-absorption was estimated as a correction to be subtracted from the measured half-width.

After removing Stark and self-absorption broadening the mean Doppler half-width is 0·276 A at a mean wave-length 3699 A. In obtaining this value a few of the strongest and weakest lines were rejected. This width gives a kinetic temperature 11 000 deg., if no allowance is made for turbulence. The correction for turbulence, following the earlier part of the paper, will depend on the assumed height. If we take this to be 850 km, with a ξ_0 (turbulence) $= 7$, the corrected kinetic temperature is about 10 000 deg. K.

Theoretical calculation of hydrogen line widths.—This treatment assumes that for present purposes we can regard the chromosphere as a layer of gas with a given constant temperature and pressure at all points along any particular line of sight. The Stark profile of a hydrogen Balmer line has been taken from Verweij, as quoted by Miss van Dien.* The profile is $U(\beta)$, where β is the ratio of the field strength to a standard field $F_0 = 2{\cdot}61 N^{2/3} e$. N is the number of charged particles per cm^3 and e the charge on the electron. β has a certain probability distribution, a property of the inter-particle fields in the gas.

For convenience $U(\beta)$ is represented by a Voigt profile, $V(a_s, \Delta\lambda_s)$, where the parameters a_s, $\Delta\lambda_s$ can be regarded as a fictitious damping width and Doppler width respectively. a_s is expressed in units of $\Delta\lambda_s$. By trial and error a satisfactory fit for $U(\beta)$ was found with $a_s = 0{\cdot}4$, the quality of the representation being shown in Table IV. Here v is the distance from the centre of the line, expressed as a fraction of the (fictitious) Doppler width; $V(v)$ is the Voigt profile, taken from Hjerting's computations (his Table 4).† The ratio of β to v has been chosen so that $V(v)$ fits $U(\beta)$ at $v = 1$. The fifth column in Table IV gives the Voigt profile, and the sixth gives the Stark profile, each in units of its central value.

The $\Delta\lambda_m = s_n \beta F_0$ of Miss van Dien is identified with $\Delta\lambda_s$ when $v = 1$, $\beta = 1{\cdot}35$. Hence $\Delta\lambda_s = 1{\cdot}35 s_n F_0$, where $F_0 = 2{\cdot}61 N^{2/3} e = 1{\cdot}253 \times 10^{-9} \times N^{2/3}$ e.s.u. and, with $\Delta\lambda_s$ measured in angstroms,

$$s_n = 0{\cdot}00256 \left(\frac{n^2}{n^2 - 4}\right)^2 \{n(n-1) + 2\}.$$

Some typical values of $\Delta\lambda_s$ are given in Table V.

We must now combine a real Doppler broadening with the Stark profile, the latter being now expressed as a Voigt profile with a pseudo-damping width a_s and a pseudo-Doppler width $\Delta\lambda_s$. The real Doppler width, following Hjerting's convention and in the customary notation is

$$\Delta\lambda_t = \sqrt{\left(\frac{2kT}{m_H}\right)} \frac{\lambda}{c}.$$

The resulting profile is another Voigt profile, $V(a, \Delta\lambda_a)$, where $\Delta\lambda_a = \sqrt{(\Delta\lambda_t^2 + \Delta\lambda_s^2)}$. If we continue to express the (pseudo-) damping parameter in terms of the (pseudo + real) Doppler width, now $\Delta\lambda_a$, it becomes $a = 0{\cdot}4 \dfrac{\Delta\lambda_s}{\Delta\lambda_a}$. Some figures for the particular case $T = 6000$ deg., $\log N = 11{\cdot}5$, are given in Table VI.

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* Elsa van Dien, *Ap. J.*, **109**, 452, Table I, 1948.

† F. Hjerting, *Ap. J.*, **88**, 514, 1938.

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TABLE IV

v	$V(v)$	β	$U(\beta)$	$V(v)/V(0)$	$U(\beta)/U(0)$
0·0	0·67	0·00	0·287	1·00	1·00
0·5	·57	0·68	·243	0·85	0·85
1·0	·36	**1·35**	·154	·54	·54
1·5	·19	2·02	·082	·28	·29
2·0	·090	2·70	·041	·134	·143
2·5	·049	3·38	·022	·073	·077
3·0	·030	4·05	·014	·045	·049
3·5	0·021	4·72	0·008	0·031	0·028

TABLE V

Values of $\Delta\lambda_s$

	log N=12·0	11·5	11·0
n= 5	0·0135 A	0·00627 A	0·00290 A
10	·0432	·0201	·00931
15	·0951	·0442	·0205
20	·169	·0783	·0363
25	·264	·123	·0569
30	0·381	0·177	0·0820

TABLE VI

	T=6000 deg.		log N=11·5		
n	$\Delta\lambda_s$	$\Delta\lambda_t$	$\Delta\lambda_a$	$\Delta\lambda_s/\Delta\lambda_a$	a
5	0·00627 A	0·1446 A	0·145 A	0·0432	0·017
10	·0201	·1266	·128	·157	·063
15	·0442	·1237	·131	·338	·135
20	·0783	·1227	·146	·537	·215
25	·123	·1223	·173	·711	·284
30	0·177	0·1220	0·215	0·823	0·329

Having obtained the Stark plus Doppler profile for various n, the next step is to compute the effect of self-absorption. For this it is necessary to know the absorption coefficient κ_ν at each point within the hydrogen line. The first stage is to determine κ_0, the value of κ_ν at the line centre, and this we have obtained by way of $B_{12}h\nu$, the total strength of the Balmer line. B_{12} is the Einstein probability of absorption, and may be computed from Menzel and Pekeris' values of A_{21} by the relation

$$B_{12}=\frac{g_2}{g_1}\frac{c^2}{2h\nu^3}A_{21}.$$

Table VII gives an outline of the results.

TABLE VII

n	λ	log A	log B	log $h\nu$	log $Bh\nu$
5	4340 A	6·401	9·512	$\overline{12}$·660	$\bar{2}$·172
10	3798	4·850	8·389	$\overline{12}$·718	$\bar{3}$·107
15	3712	3·962	7·823	$\overline{12}$·728	$\bar{4}$·551
20	3683	3·334	7·425	$\overline{12}$·732	$\bar{4}$·157
25	3669	2·849	7·139	$\overline{12}$·733	$\bar{5}$·872
30	3662	2·452	6·897	$\overline{12}$·734	$\bar{5}$·631

In order to find κ_0 the values of $B_{12}h\nu$ must be divided by a factor depending on the line width. First we remark that Hjerting, whose computations we use, tabulated κ_ν/κ_0^* † as a function of v, the distance from the line centre in units of the Doppler width, and normalized his profiles so that

$$\int_0^\infty \frac{\kappa_\nu}{\kappa_0^*}\,dv=\frac{\sqrt{\pi}}{2}.$$

We now define an effective line width v_{eff}, so that

$$\left(\frac{\kappa_\nu}{\kappa_0^*}\right)_{v=0}.\,v_{\text{eff}}=\int_0^\infty\left(\frac{\kappa_\nu}{\kappa_0^*}\right)dv=\frac{\sqrt{\pi}}{2}.$$

$(\kappa_\nu/\kappa_0^*)_{v=0}$ is the actual absorption coefficient at the line centre, as tabulated by Hjerting. Then the relative values of the absorption coefficients at the Balmer line centres are $B_{12}h\nu/2\Delta\nu_{\text{eff}}$, where

$$2\Delta\nu_{\text{eff}}=\frac{c}{\lambda^2}2\Delta\lambda_{\text{eff}}=\frac{c}{\lambda^2}\,.\,2v_{\text{eff}}\,.\,\Delta\lambda_a.$$

A summary of the results of this section of the work is in Table VIII.

TABLE VIII

$T=6000$ deg. $\log N=11{\cdot}5$

n	$\Delta\lambda_a$	a	$(\kappa_\nu/\kappa_0^*)_{v=0}$	v_{eff}	$\log 2\Delta\nu_{\text{eff}}$	$\log \dfrac{B_{12}h\nu}{2\Delta\nu_{\text{eff}}}$
5	0·145 A	0·017	0·981	0·903	10·619	$\overline{13}$·553
10	·128	·063	·934	0·948	10·701	$\overline{14}$·406
15	·131	·135	·866	1·024	10·765	$\overline{15}$·786
20	·146	·215	·797	1·112	10·856	$\overline{15}$·301
25	·173	·284	·748	1·184	10·960	$\overline{16}$·912
30	·215	·329	·714	1·241	11·076	$\overline{16}$·555

Suppose now that the optical depth of the chromosphere in the line of sight at the centre of a certain line is τ_0, and is $\tau_{0{\cdot}5}$ at the point where the observed intensity is one-half the central intensity. The intensity distribution within the line, including the effect of self-absorption, is taken to be

$$I\propto(1-\exp[-\tau]),$$

so that

$$\tfrac{1}{2}=\frac{1-\exp(-\tau_{0{\cdot}5})}{1-\exp(-\tau_0)}.$$

The relation between $\log\tau_0$ and $\log\tau_{0{\cdot}5}/\tau_0$ is given in Table IX. τ_0 is proportional to $B_{12}h\nu/2\Delta\nu_{\text{eff}}$ for the particular Balmer line which is in question. We

TABLE IX

$\log\tau_0$	$\log\tau_{0{\cdot}5}/\tau_0$
$-\infty$	$\bar{1}$·699
$\bar{2}$·000	$\bar{1}$·699
$\bar{1}$·000	$\bar{1}$·681
$\bar{1}$·699	$\bar{1}$·642
0·000	$\bar{1}$·580
0·301	$\bar{1}$·453
0·477	$\bar{1}$·332
0·602	$\bar{1}$·228
0·699	$\bar{1}$·137
1·000	$\bar{2}$·839
2·000	$\bar{3}$·839
3·000	$\bar{4}$·839
4·000	$\bar{5}$·839

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† κ_0^* is Hjerting's κ_0, the fictitious absorption coefficient at the centre of the line when the damping is vanishingly small.

proceed by assuming τ_0 for one convenient line of the series, and choose $n = 30$. Table IX then gives $\tau_{0\cdot5}/\tau_0$, and from Hjerting's Table 4, with $a = 0\cdot4\,.\,(\Delta\lambda_s)/(\Delta\lambda_a)$, values of v are chosen so that

$$\left(\frac{\kappa_\nu}{\kappa_0^*}\right)_v \bigg/ \left(\frac{\kappa_\nu}{\kappa_0^*}\right)_{v=0} = \frac{\tau_{0\cdot5}}{\tau_0}.$$

v is the distance from the line centre in units of the Doppler width, so that $2v \times \Delta\lambda_a = 2\Delta\lambda_{\mathrm{cal}}$ should be the half-width of the line as observed.

TABLE X

Values of $2\Delta\lambda_{\mathrm{cal}}$

T	6000 deg.	6000 deg.	6000 deg.	6000 deg.	6000 deg.	8000 deg.
log N	12·0	12·0	11·5	11·5	11·0	11·5
$\tau_0(n=30)$	0·1	0·05	0·1	0·05	0·1	0·05
$n = 5$	0·844 A	0·722 A	0·682 A	0·624 A	0·626 A	0·702 A
10	·528	·446	·412	·346	·358	·388
15	·436	·372	·310	·272	·262	·304
20	·478	·446	·304	·290	·246	·314
25	·628	·624	·364	·352	·256	·378
30	0·872	0·860	0·452	0·448	0·292	0·466

T	8000 deg.	10000 deg.	10000 deg.	10000 deg.	10000 deg.	12000 deg.
log N	11·5	11·5	11·5	11·0	11·0	11·0
$\tau_0(n=30)$	0·03	0·05	0·02	0·05	0·02	0·02
$n = 5$	0·654 A	0·776 A	0·674 A	0·734 A	0·628 A	0·682 A
10	·346	·422	·344	·374	·318	·344
15	·290	·336	·306	·304	·284	·308
20	·314	·342	·340	·294	·294	·318
25	·374	·400	·396	·312	·308	·332
30	0·460	0·484	0·478	0·342	0·334	0·356

Table X contains half-widths calculated in this manner for various combinations of parameters within the ranges

$$\begin{aligned} 6\,000\text{ deg.} &\leqslant T \leqslant 12\,000\text{ deg.},\\ 11\cdot0 &\leqslant \log N \leqslant 12\cdot0,\\ 0\cdot02 &\leqslant \tau_0(n=30) \leqslant 0\cdot1. \end{aligned}$$

For a number of reasons, some of which will be briefly discussed in a moment, a perfect fit with the observations cannot be expected. Fig. 3 shows the observations with two computed curves:

(*a*) $T = 6\,000$ deg. $\log N = 11\cdot5$ $\tau_0(n=30) = 0\cdot1$
(*b*) $10\,000$ deg. $11\cdot5$ $0\cdot02$

Both curves lie fairly near the observations. Of the two, (*b*) appears to be somewhat the better, but it is doubtful whether much significance can be attached to this. The results suggest that an accurate determination of the kinetic temperature by this method is difficult, and could be made only from more comprehensive observations covering most of the Balmer series to $n = 30$ at a number of different heights.

The difficulties may be summarized as follows:

(1) Owing to the large range of light intensity to be covered the present observational accuracy falls off at each end of the measured range of n.

(2) Measures at different heights 600–1200 km have been combined, although it is to be expected that both self-absorption broadening and Stark broadening will decrease with increasing height. The computed half-widths for the earlier members of the series are rather sensitive to the assumed "Balmer optical depth" (expressed as τ_0 at $n=30$), so that they are strongly dependent on height. There are two places at which this fact needs particularly to be borne in mind. In the measures quoted in Table II there is a strong suggestion that those from effective height 1200 km correspond to a substantially smaller τ_0 ($n=30$) than the others, so that to combine them all together is not entirely satisfactory. We should also refer to the 1940 measures, in which the half-width for $n=5$ was 0·53 A. This is smaller than one would expect either from extrapolation of the 1952 measures, or from the computed cases (*a*) or (*b*) above. Satisfactory data are not available for the height at which the 1940 measures were made, although 1500 km was tentatively suggested at the time. No great reliance can be placed on this figure, but it suggests a τ_0 lower than for the 1952 measures, and qualitatively this agrees with the line widths.

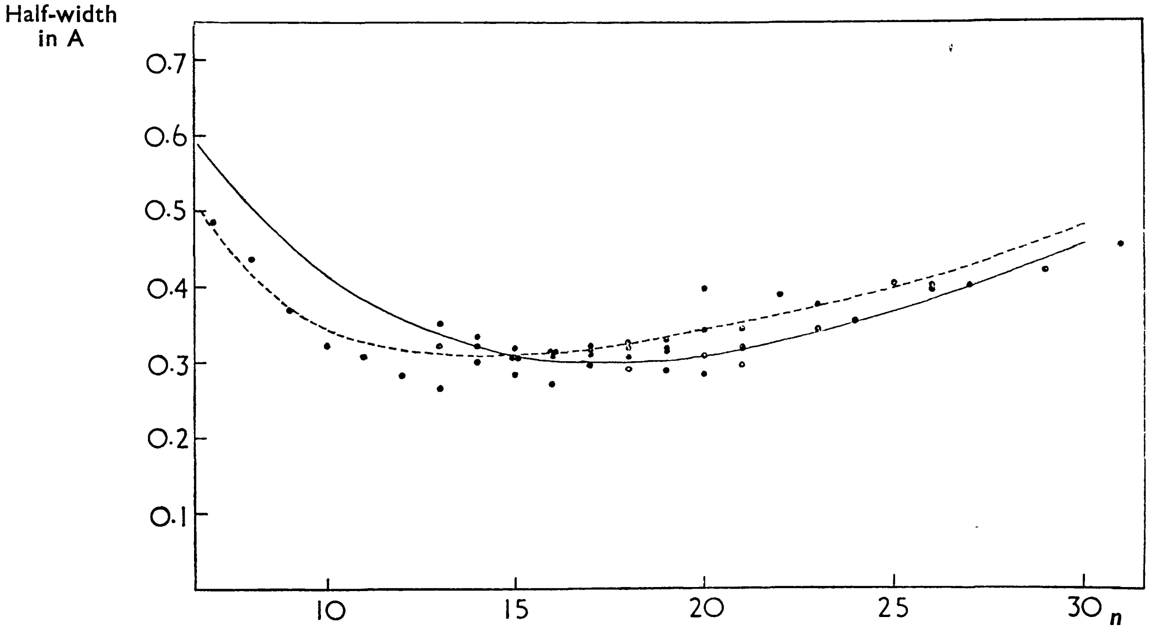

FIG. 3.—*Half-width of hydrogen Balmer lines plotted against n.*
—— *Curve* (*a*), *for* $T=6\,000$ deg. - - - - - - *Curve* (*b*), *for* $T=10\,000$ deg.

(3) We have made no allowance for turbulence in our computations. If the increase with height suggested by Fig. 1 is substantially correct, turbulence is becoming important, even for hydrogen, at a height about 2500 km. The turbulence makes a kinetic temperature determination from metallic lines virtually impossible by present techniques, and not very practicable from helium.

(4) The computations have neglected all variations in the properties of the chromosphere along the line of sight. Even in a static atmosphere, what is observed is an integration over a wide range of physical parameters. If the chromosphere has to be treated as a field of filaments or jets the question becomes forbiddingly complicated.

Since this paper was completed, work by Kawaguchi has appeared* in which he fits theoretically computed hydrogen profiles to the 1940 measures made by one of us. His conclusion with regard to these lines is the same as ours, viz. that it is possible to interpret the observed profiles with a kinetic temperature of 10 000 deg. K or less, if self-absorption is taken into account.

Acknowledgments.—The observations on which this work is based were made by the Cambridge 1952 Eclipse Expedition to Khartoum, which was financed wholly by the Joint Permanent Eclipse Committee. Special thanks are due to Dr D. E. Blackwell and Mr I. Kendall, who gave essential help with the spectrograph. The construction of the spectrograph was due to Mr R. Overhill of the Cambridge Observatories staff.

One of us (Z. S.) is indebted to the I.A.U., through the President of Commission 38, Professor F. J. M. Stratton, for a grant permitting him to spend a year of study in Europe, chiefly at the Cambridge Observatories.

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* I. Kawaguchi, *Publ. Astro. Soc. Japan*, **5**, 137, 1954.