THE RATE OF ACCRETION OF MATTER BY STARS

W. H. McCrea

(Received 1953 January 1)

Summary

It being generally agreed that the present very luminous stars have existed as such for only a small fraction of the past life of the Galaxy, it becomes important to decide whether they are produced from previously existing stars by the accretion of interstellar matter, in accordance with the views of Hoyle and others, or are the result of some continuing process of star formation. It is emphasized that accretion on the scale required need be only a rarely occurring phenomenon. Even so, the quantitative requirements as usually discussed do present difficulties. It is here found, however, that accretion on this scale would take place, if at all, only after a star has been reduced almost to rest relative to surrounding interstellar matter as a result of the retardation produced by such matter. This and other effects that have to be considered in a full discussion of the problem reduce the requirements to ones that seem likely to be found realized in actuality. The main purpose of the present work is to estimate these requirements; the frequency of their realization will be discussed elsewhere. The accretion theory has naturally to account not only for the mass-increment but also for its production at a rate sufficient to compensate the transmutation of hydrogen in the stars concerned. It is shown that no additional difficulty is likely to arise in this way.

1. *Introduction*

1.1. *The problem.*—It is well known* that, according to accepted ideas concerning energy generation, the most luminous stars can maintain their observed luminosities for only a small fraction of the life-time of the Galaxy. It is also well known† that the present existence of such stars could be explained by a sufficiently high rate of accretion of material from interstellar space. The question is whether the required rate is realized in the actually occurring conditions.

The outcome of an elementary quantitative inspection is indeed to suggest that the conditions required for the success of the accretion theory are far removed from anything like normal conditions for stars and interstellar matter. The theory has consequently not gained general acceptance.‡

As the case is usually presented, however, it overlooks the fact that the processes envisaged by the accretion theory of this particular problem must occur only in exceptional conditions. The accretion of a large amount of their material in recent astronomical times is required by the nature of the case to be the experience of only a tiny fraction of all the stars. Moreover, if this large accretion does occur, the mechanism is apparently rather different from that assumed in the elementary treatment. A closer inspection of the theory shows that the conditions required are not so unlikely as those previously thought to be necessary.

* O. Struve, *Stellar evolution*, p. 112, Princeton, 1950.

† F. Hoyle, *The nature of the universe*, Oxford 1950.

‡ See, for example, Struve, *op. cit.*, p. 113; J. L. Greenstein, *Astrophysics* (ed. J. A. Hynek), pp. 591–2, New York, 1951.

The issue is of fundamental significance. The comparatively recent formation of the very luminous stars is not itself in question; the problem concerns the process of their formation. If the accretion theory is correct, then the present existence of these stars in comparatively small numbers is the result of an already understood physical process affecting, on the scale concerned, only these particular stars. The problem of the evolution of the majority of all stars is not essentially involved. On the other hand, any different explanation would, in the first place, require some process of star formation that has not yet been thought of. In the second place, this process has to result in newly made stars, since the growth of previously existing stars is accretion. But we should not expect a process of making new stars to produce only stars of the largest masses. Therefore such an alternative explanation must presumably have far-reaching consequences concerning the evolution of the stars in general. This shows the importance of trying to decide whether accretion provides an adequate explanation. It is put forward as the justification for the rather lengthy discussion that follows.

1.2. *Apparent difficulties of the accretion theory.*—Current criticism of the accretion theory is based mainly upon quantitative difficulties regarding its applicability rather than upon matters of physical principle. Consequently, the difficulties are best illustrated by quoting particular figures.

Consider, for instance, a star* of luminosity 10^6 times that of the Sun. Such a star has mass about 43 times the mass of the Sun. The luminosity requires the transmutation of hydrogen at the rate of about 6×10^{20} g/s. If the star does not acquire hydrogen from outside, even if it starts as almost pure hydrogen and all of this is available for energy generation, its life as a normal star can therefore be only about 5 million years [$\doteqdot$(43 solar masses $\div 6 \times 10^{20}$ g) seconds]. But, according to the accretion hypothesis, the star has not always had its present large mass; this has been acquired from interstellar matter consisting largely of hydrogen. In order for this hypothesis to be effective, we should conclude on a simple view of the problem that such matter must therefore be accreted at least at the rate of 6×10^{20} g/s. Now, according to the usual accretion formula (equation (1) below), the density of interstellar hydrogen required to achieve this rate must be at least about 1 000 U^3 atoms/cm^3, where U km/s is the speed of the star through the interstellar gas. Using the same formula, the time taken to produce a star of large mass starting with a mass of, say, two solar masses in a cloud of this density is about 10^8 years. During this time the star would travel 100 U parsecs.

A speed of 1 km/s would certainly be an exceptionally low speed for a star relative to any surrounding interstellar cloud. Yet, according to these figures, even this low speed demands a cloud about 100 parsecs thick with a density of about 1 000 atoms/cm^3. This would mean a mass for the cloud, if roughly spherical, of about 10^7 solar masses. It is not known† whether clouds exist of one-tenth of this mass. More particularly, a cloud or cloud-complex of 100 parsecs extent is rare‡, while the most familiar estimate § of the maximum density in interstellar clouds is only about 100 atoms/cm^3. Further, since the life of the star after it has ceased to accrete must be less than the 5 million years mentioned above, when observed it cannot be as much as 5 parsecs away from the cloud if its speed is only 1 km/s.

* Such a star is AO Cas A in Table I; the derivation of the figures quoted in the present section is implicit in some of the work that follows.

† L. Spitzer and M. Schwarzschild, *Ap. J.*, **114**, 385–397, 1951.

‡ W. H. McCrea, *The Observatory*, **70**, 100, 1950.

§ B. Strömgren, *Ap. J.*, **108**, 242–275, 1948; but see Section 3.1.

Thus the star would be observed in or very close to the exceptional region of nebulosity described. Such a special association is not observed for every massive star, although it is true that such stars are associated in a general way with regions of relatively high density of interstellar material. Finally, owing to the occurrence of U^3 in the density needed for a given rate of accretion, if U is much more than about 1 km/s, the conditions required for an amount of accretion that would be significant for the present problem are apparently beyond the bounds of possible realization. Yet we have to recall that the mean peculiar velocities of massive stars* are of the order of 10 km/s and the mean peculiar velocities of interstellar clouds† are probably about 5 km/s, so that the mean relative velocities of stars and clouds are also of the order of magnitude 10 km/s. Anything like a Maxwellian distribution of such relative velocities would allow a velocity of less than 1 km/s in only about one case in a thousand.

The needed combination of rare occurrences in respect to velocity, cloud-density, and cloud extent are apparently the considerations that lead many astrophysicists to reject the accretion theory. Nevertheless, the entire problem is necessarily one concerning exceptional occurrences‡ since the stars concerned are such a small proportion of all stars (perhaps about one in a million). Moreover, figures such as those quoted are sufficient to show that, in a favourable combination of admittedly rare but not impossible conditions, accretion is at any rate on the verge of being significant. If refinement in the theory and in its application can combine to yield a factor of about 10 in the right direction there would be little doubt that it is significant.

It is the purpose of this paper to call attention to certain considerations that do appear to sway the decision in this direction. Since in the nature of the case we are dealing with marginal considerations, it is not surprising that the work depends upon rather critical and tedious arguments. In this paper we deal with the possibility of the occurrences; elsewhere some attempt will be made to investigate their frequency.

In Section 2 the basic results of the theory of the accretion process are quoted and certain formulae required for their application are derived. In Section 3 the quantitative application of these formulae is discussed. The position is briefly reviewed in Section 4.

2. *Accretion theory*

2.1. *Rate of accretion.*—We consider a star S moving through a cloud of interstellar material of indefinitely large extent in all its dimensions. Apart from the disturbance produced by the star, we treat the cloud as being at rest and having uniform density ρ. We consider only rectilinear motion of the star; we take an axis OX along its path. At epoch t, let the star be at P on OX, where $OP=x$, and let its velocity§ be $U=dx/dt$. Let O be taken so that $x=0$ when $t=0$, and let the values of other parameters at $t=0$ be distinguished by suffix 0. Let M be the mass of S, this being a function of t if accretion takes place.

* Spitzer and Schwarzschild, *loc. cit.*

† A. Blaauw, *B.A.N.*, **11**, 459–473, 1952.

‡ This point has been particularly emphasized by Mr F. Hoyle in various discussions on the subject.

§ In Section 1.2 the velocity U was expressed in km/s; this convention is *not* employed in the general formulae in the rest of the paper.

For cloud material of sufficiently small density and kinetic temperature, Bondi and Hoyle* derived the accretion formula

$$dM/dt = \alpha \,.\, 2\pi\rho G^2 M^2/U^3, \tag{1}$$

where G is the gravitational constant. Here α is a numerical factor which depends upon the way in which the " accretion stream " has been established but is such that $1 < \alpha < 2$. Its value for particular ways of establishing the stream has been discussed by the authors quoted and also by Dodd.† The value $\alpha = 2$ had been given by the earlier treatment of Hoyle and Lyttleton‡ (see § also DM (3.19)).

For cloud material in which the kinetic temperature is not negligible and the density is not necessarily small, Bondi || has recently obtained in the case $U = 0$ the formula

$$dM/dt = \alpha' \,.\, 2\pi\rho G^2 M^2/a^3, \tag{2}$$

where a is the speed of sound in the cloud at infinity. Here α' is again of the order of unity; Bondi's values for various possible types of behaviour of the medium lie in the range $0{\cdot}5 \leqslant \alpha' \leqslant 2{\cdot}24$.

When $U \neq 0$, $a \neq 0$, Bondi conjectures that the formula

$$dM/dt = \alpha \,.\, 2\pi\rho G^2 M^2/(U^2 + a^2)^{3/2}, \tag{3}$$

with $\alpha \doteqdot 1$, should give the order of magnitude of the accretion rate. The solution of the hydrodynamical problem presented by this general case is of extreme difficulty. However, with the aid of an electronic computer, Dodd¶ has made considerable progress with its solution by numerical methods for particular values of U/a when this ratio is greater than unity and when the gas is isothermal. He obtains substantial confirmation of the quantitative validity of Bondi's estimate (3) taking $\alpha \doteqdot 2{\cdot}24$, the value actually indicated by Bondi's treatment for the isothermal case. It is not, of course, suggested that (3) gives accurately the analytical form of the solution of the hydrodynamical problem.

Thus the combination of Bondi's general arguments with Dodd's particular values would permit us to use the formula (3) with considerable confidence. However, we shall see that we require this formula only in a somewhat subsidiary way. If the arguments to be given in this paper are correct, the major part of any actual accretion that occurs is governed by formula (2), and this appears to be on an even firmer footing. Nevertheless, an interesting point arises here. What we actually want is to be justified in using (2) as a valid approximation to the rate of accretion after U has been reduced to a sufficiently small value compared with a. Now, when U is precisely zero, Bondi shows that the value of α' in (2) is strictly not determined by his analysis, but only by certain physical arguments which he uses to supplement the latter. These arguments show that α' should be maximal in a certain sense. The significant point is that Dodd finds the corresponding indeterminacy not to arise in his case of $U \neq 0$. Hence it would be tempting to conclude that it is removed from Bondi's analysis by taking the limit when $U \to 0$. This would not be rigorously justified, since Dodd's work applies only for $U > a$. Nevertheless, we shall deal with cases where initially $U > a$;

* H. Bondi and F. Hoyle, *M.N.*, **104**, 273–282, 1944.
† K. N. Dodd, *M.N.*, **112**, 374, 1952.
‡ F. Hoyle and R. A. Lyttleton, *Proc. Camb. Phil. Soc.*, **35**, 405, 1939.
§ K. N. Dodd and W. H. McCrea, *M.N.*, **112**, 205–214, 1952; this paper will be quoted as "DM".
|| H. Bondi, *M.N.*, **112**, 195–204, 1952.
¶ K. N. Dodd, *Proc. Cambridge Phil. Soc.*, **49**, 1953.

by virtue of Dodd's work, no indeterminacy is involved here; hence, the problem being fully defined physically, no indeterminacy can arise at any later stage and, if we appeal to physical continuity (where the mathematical continuity is unavailable in the absence of calculations for $0<U<a$), we get back to formula (2) with Bondi's maximal value of α'. In other words, Dodd's work gives additional justification for using (2) with Bondi's values for α' in the circumstances in which we need to do so.

2.2. *Retarding force.*—We have to consider the force F by which the cloud retards the motion of the star. Suppose, first, that the cloud consists of particles (atoms or grains) having negligible thermal motions and moving independently of each other when disturbed by the gravitational attraction of the star. Then the force produced by all the particles whose initial undisturbed positions lie within distance s from OX is

$$F=-2\pi\rho\frac{G^2M^2}{U^2}\ln\left(1+\frac{s^2U^4}{G^2M^2}\right) \tag{4}$$

acting along OX; this is proved in DM (3.6).

Under the conditions stated in DM the formula (4) is exact. No difficulty arises merely from the fact that in DM the motion relative to the star was considered, while here it is considered relative to the cloud. However, here we shall be concerned also with the variation of the relative velocity. Some correction would be needed if this variation is considerable during the time taken by a cloud-particle to traverse the significant portion of its trajectory; for present purposes we neglect this refinement.

The application will be to a star S in the presence of a typical distribution of other stars. These will determine a "cut-off" value for s. In fact, we have to assign a sphere of influence to S such that the motion of the cloud-particles may be considered to be affected by S, and S alone, only when the particles are within this sphere. It can be shown* that (4) gives a good approximation for the force if s in the formula is put equal to the radius of this sphere. Now we shall be interested in those members of the stellar distribution for which F is most appreciable. Elementary properties of particle-orbits under an inverse-square law of force show that, for producing a deflexion of the relative motion in excess of some stated amount, the effective target-area of a star is proportional to U^{-4}. Hence there is comparatively little effect from the presence of stars with values of U larger than those in which we are interested. We conclude that the significant approximation for s is about half the mean separation of neighbours amongst these particular stars, the presence of other, faster-moving, stars being ignored.

For the values of the parameters concerned in our work we shall have $s^2U^4/G^2M^2 \gg 1$. Then the logarithm in (4) is effectively the logarithm of this quantity. Its values for some typical cases are given later in Table III. Since it varies so much more slowly than the other factors in F, we shall now write as a sufficiently good approximation to (4)

$$F=-\beta\,.\,2\pi\rho G^2M^2/U^2. \tag{5}$$

In any particular application we shall treat β as a constant. In principle, we should evaluate β as a suitable average of the logarithm for the range of values of the parameters occurring, but we shall see that in practice we may use its value at one point in the range.

* This approximation has been studied by Mr Dodd in work not yet published.

The formula (4) was obtained in DM for the simplest possible conditions in the cloud. But it was shown to agree essentially with that got by Bondi and Hoyle (*loc. cit.*) for rather more realistic conditions. These authors also used the approximate form (5). However, for their immediate purposes, they did not have to give much attention to the determination of the cut-off value of s.

In the extreme case where the motion of the cloud is hydrodynamical, no direct estimate of the force has yet been achieved. But, since we know that the passage in this case makes little difference to the calculated accretion rate, except for small values of U, we have ground for supposing that it also makes little difference to the calculated force. So far as the consequences of the force are concerned, the case of $U > a$ is the important one, and this is the one to which the foregoing argument particularly applies.

2.3. *Other forces.*—The force expressed by (4) or (5) arises because the star deflects the motion of particles of the cloud in its vicinity. But the star and the cloud-particles are acted upon also by the gravitational forces produced by all the other matter present. Nevertheless, such forces will not in themselves have a first-order effect upon the *relative motion* of the star and the neighbouring part of the cloud. It is true that they could have such an effect if, in the case of the cloud-material, they were opposed by a pressure gradient. This, however, is unlikely to occur in practice, since the cloud is presumably somewhat "broken" into patches or filaments, as is seen commonly to be the case with interstellar clouds that happen to be directly visible. Such a system would not be in a condition to establish a pressure gradient of the required character. We may conclude that the relative motion of the star and the portion of the cloud in which it is immersed at any epoch is controlled by the retarding force discussed in 2.2, and that we may ignore other forces so far as their relative motion is concerned.

More correctly, we may conclude that there can be cases to which this treatment applies and we may assert that these are the cases to which we shall restrict ourselves. At the same time, it would be desirable to examine more fully the possible effects of gravitational fields, though this will not be done here.

2.4. *Equation of motion.*—The equation of motion of the star relative to the cloud is

$$d(MU)/dt = F. \tag{6}$$

This expresses the law of conservation of momentum for the system when, for the reasons stated in 2.3, we assume F to be the only force affecting the relative motion.

We first consider the motion under the conditions in which the formulae (1), (5) provide sufficiently good approximations for dM/dt and F. This means that $U > a$, though we expect the approximation to be adequate almost down to $U = a$.

From (1), (5), (6) we then obtain

$$dM/dt = \alpha \,.\, \sigma M^2/U^3, \tag{7}$$

$$dU/dt = -(\alpha+\beta)\sigma M/U^2 \tag{8}$$

and, by definition,

$$dx/dt = U. \tag{9}$$

Here we have written

$$\sigma = 2\pi\rho G^2 \tag{10}$$

and we shall also write

$$\kappa = \alpha/(\alpha+\beta). \tag{11}$$

The system (7), (8), (9) has the following integrals which are readily verified. They give the unique solutions satisfying the initial conditions stated in Section 2.1.

$$M/M_0 = (U_0/U)^{\kappa}, \tag{12}$$

$$t = \frac{U_0^{3+\kappa} - U^{3+\kappa}}{(4\alpha + 3\beta)\sigma M_0 U_0^{\kappa}}, \tag{13}$$

$$x = \frac{U_0^{4+\kappa} - U^{4+\kappa}}{(5\alpha + 4\beta)\sigma M_0 U_0^{\kappa}}. \tag{14}$$

In applications, κ is a small positive fraction, while U_0/a is most likely to be in the range about 2 to 5. As we have said, our results should be sufficiently good approximations almost down to $U=a$. Therefore, the smallest value of U for which these results are to be used will give $U^3 \ll U_0^3$ and $U^4 \ll U_0^4$. Therefore, if we substitute this value of U in (13), (14) we obtain $t=t_1$, $x=x_1$, approximately, where

$$t_1 = U_0^3/(4\alpha + 3\beta)\sigma M_0, \tag{15}$$

$$x_1 = U_0^4/(5\alpha + 4\beta)\sigma M_0. \tag{16}$$

In any case, these are not underestimates of the values of t, x. The quantities t_1, x_1 are independent of the particular value of U which has been employed. The physical explanation is evident from the forms of (13), (14); they show that, owing to the steep increase in the effective resistance expressed by (8) with decreasing U, most of the time and distance are required to produce a small initial reduction of U. Once any appreciable reduction has been achieved, it does not take relatively much longer to complete the reduction.

This has another consequence. Since, over most of the time and distance given by (13), (14), U does not differ much from U_0, the appropriate estimate of β to be used in these expressions is simply the entry in Table III for $U=U_0$, and not a value between this and one for a small value of U.

Further, if the force merely remains of the same order of magnitude from a value of U a little above a to a value a little below a, the time and distance required for this further reduction of U are small compared with t_1, x_1. This is easily checked. Hence t_1, x_1 given by (15), (16) are sufficiently good estimates of the time and distance in which U is reduced from U_0 to a value somewhat less than $U=a$. This conclusion is valid apart from any detailed knowledge of the precise manner in which the equations (7), (8) may have to be modified in the neighbourhood of $U=a$.

When U is less than a, we can use Bondi's simple accretion formula (2). We do not require the more general formula (3) for actual application, but merely to assure ourselves that (2) is a good approximation as soon as U^2 is sufficiently small compared with a^2.

We shall now disregard any increase of M during the time t_1 and, on account of what has just been said, we shall conclude that at the end of this interval M starts to increase in accordance with (2), that is,

$$dM/dt = \alpha' \sigma M^2/a^3 \qquad (M=M_0 \text{ when } t=t_1). \tag{17}$$

The main purpose of neglecting any accretion during t_1 is to avoid over-estimating the total accretion. But we notice that, for the range of values mentioned above, this is actually a good approximation. For, assuming that (12) is even approximately valid down to $U \doteqdot a$, it will leave $M/M_0 \doteqdot 1$ on account of the smallness of κ.

The integral of (17) is

$$t - t_1 = \frac{a^3}{\alpha' \sigma}\left(\frac{1}{M_0} - \frac{1}{M}\right). \tag{18}$$

Thus M will have become large compared with M_0 at time t_2 where

$$t_2 - t_1 = a^3/\alpha' \sigma M_0. \tag{19}$$

Since (18) implies that M would become infinite in this time, it is clear that at some stage equation (17) must cease to apply. But the order of increase of M with which we are concerned in practice will remove only a small percentage of the cloud material and so we may conclude that we shall be working within the range of applicability of the equations. A simple property to notice in (18) is that the time required for a very large increase of M is not much more than that for a moderate increase (e.g. a five-fold increase takes only about 60 per cent longer than a two-fold increase).

The derivation has been framed so as to make it clear that we have nowhere had to use a force-formula for very small values of U. But, without going into details, it is evident that the star will be brought effectively to rest relative to the cloud at about time t_1 and that x_1 therefore expresses effectively the whole displacement of the star through the cloud. The significant time for the whole process would be the greater of t_1 and $t_2 - t_1$, but we cannot generalize as to which of these will turn out to be the greater in actual applications.

To sum up: If at any epoch a star is moving through a cloud with a given speed exceeding the speed of sound in the medium, then it will be brought effectively to rest in the cloud in a distance x_1, and its mass will have increased to a considerable multiple of its initial value in a time t_2. These quantities are given by (15), (16), (19) and depend essentially only upon the force-formula for the initial speed and upon the accretion formula for a star at rest. The effective limitation upon the applicability of the results to an actual case is that the cloud should extend over a distance greater than x_1 and should remain in existence for a time greater than t_2.

2.5. *Comparison with simple treatment.*—The simplest treatment of accretion, such as that upon which the figures quoted in the introduction are based, disregards the retarding effect of the cloud. Hence, for a star moving through the cloud at speed U_0, the rate of accretion cannot exceed that given by (1) with $U = U_0$. By analogy with (19), the total time t_2^*, say, required for a large increase in M is then not less than

$$t_2^* = U_0^3/\alpha \sigma M_0 \tag{20}$$

during which the star travels a distance x_2^*, where

$$x_2^* = U_0 t_2^*. \tag{21}$$

Using (15), (16), (19) these give

$$\frac{t_2}{t_2^*} = \frac{\alpha}{4\alpha + 3\beta} + \frac{\alpha}{\alpha'}\left(\frac{a}{U_0}\right)^3, \tag{22}$$

$$\frac{x_1}{x_2^*} = \frac{\alpha}{5\alpha + 4\beta}. \tag{23}$$

It will be noted that (22), (23) are independent of ρ, M_0 and that (23) is independent also of U_0.

Remembering that x_1 is effectively the whole displacement required by the treatment in 2.4, and noting that the numerical values in Section 3 will show that the fraction (23) is less than $\frac{1}{20}$, we see the extent of the difference resulting from the more detailed treatment. A reduction of the linear size of the cloud required by this factor 20 means a reduction in its total mass by a factor of nearly 10 000. This alone would reduce the conditions described in the introduction to something less difficult to find realized in practice.

It will be found also that the fraction in (22) is about $\frac{1}{10}$ in actual cases, so that the times required are also considerably reduced. One way in which this is important appears from the next section.

The effect of our more detailed treatment is to show that retardation takes precedence over accretion, i.e. that it destroys the relative velocity before a significant increase of mass occurs. This is always assuming that no other forces affect the relative motion; if they do, then we may indeed in some cases get back to conditions in which the elementary treatment gives more significant results.

2.6. *Accretion and hydrogen consumption.*—In testing the accretion hypothesis of the production of massive stars, we have not only to see whether stars of the right mass can be formed but also to see whether they can be formed sufficiently rapidly. That is to say, we must now find the condition upon the rate of accretion that ensures its adequately compensating the transmutation of hydrogen within the stars concerned. We again consider accretion within a cloud of effectively uniform density.

At epoch t, let M be the mass of the star under consideration, let hM be its hydrogen content, and let L be its luminosity expressed as the rate of consumption of hydrogen. It will be convenient to depart somewhat from the notation of 2.4 and to let the accretion start at $t=0$, denoting quantities evaluated at that epoch by suffix 0. Also for the moment we shall write the accretion formula (17) as

$$dM/dt=\nu M^n \qquad (\nu,\ n \text{ constant}). \tag{24}$$

For the purpose of the present section, which is to estimate the minimum necessary rate of accretion, we assume that the accreted material is effectively all hydrogen. Finally, we shall assume an empirical mass-luminosity law of the form

$$L=\lambda M^p \qquad (\lambda,\ p \text{ constant}) \tag{25}$$

provided $h \neq 0$.

If all the stars concerned follow approximately the same course, we may take it that the effect of such factors as the rate of mixing of the accreted hydrogen, and the dependence of L upon h as well as M, are automatically taken account of by the assumed relation (25). However, we must obviously restrict the validity of the equations to values of M, t for which $0<h<1$.

Under the conditions stated, the variation of the hydrogen-content is expressed by

$$\frac{d}{dt}(hM)=-\lambda M^p+\frac{dM}{dt}. \tag{26}$$

Equations (24), (26) have the integrals

$$\nu(p-n+1)[(1-h)M-(1-h_0)M_0]=\lambda[M^{p-n+1}-M_0^{p-n+1}], \tag{27}$$

$$t=\frac{1}{\nu(n-1)}\left[\frac{1}{M_0^{n-1}}-\frac{1}{M^{n-1}}\right]. \tag{28}$$

We shall now consider certain approximations derived from (27), (28). We bear in mind that the values ultimately to be substituted for n, p are

$$n=2, \qquad p \doteqdot 3{\cdot}5. \tag{29}$$

The value of n is that in (17); the value of p is an accepted empirical estimate.*

Suppose a normal star of mass M_1 is produced by accretion starting with the mass M_0 and suppose M_1 is fairly large compared with M_0, say $M_1 > 4M_0$. Let $\nu = \nu_1$ determine the smallest rate of accretion that will suffice, and $\mathscr{T}_1$ be the corresponding time required. From (27), this is seen to be the case for which $h_0 = 1$, and $h=0$ when $M=M_1$. This is obvious physically, for the smallest rate of accretion must be that which starts with the star of mass M_0 consisting of almost pure hydrogen and produces the star of mass M_1 only just before it is on the point of hydrogen-exhaustion.

From (27), (28) we then have approximately

$$\nu_1 = \frac{\lambda M_1^{p-n}}{p-n+1}, \tag{30}$$

$$\mathscr{T}_1 = \frac{1}{\nu_1 (n-1) M_0^{n-1}}. \tag{31}$$

On account of the neglect of the terms in M_0 in the right-hand members of (27), (28), these are not underestimated. If L_1 is the final luminosity in accordance with (25), we may write (30), (31) in the form

$$\nu_1 = \frac{L_1}{(p-n+1) M_1^{\,n}}, \tag{32}$$

$$\mathscr{T}_1 = \frac{p-n+1}{n-1} \left(\frac{M_1}{M_0}\right)^{n-1} \frac{M_1}{L_1}. \tag{33}$$

Part of the object in retaining algebraic symbols for n, p is to show that the way in which L_1 occurs in (32), (33) does not depend upon the accident of the numerical values of n, p. In applications, if the observed value of L_1 is used, the results therefore do not depend critically upon the exponent in the mass-luminosity law.

Inserting the values (29), we have finally

$$\nu_1 = 0{\cdot}4 L_1 / M_1^{\,2}, \tag{34}$$

$$\mathscr{T}_1 = 2{\cdot}5 (M_1/M_0) M_1/L_1 = 1/(\nu_1 M_0). \tag{35}$$

It may be noted that the smallest rate of accretion that compensates the rate of hydrogen consumption precisely when the final mass M_1 is reached is determined by $\nu = \nu_1{}^*$ where, by (24), (25),

$$\nu_1{}^* = L_1 / M_1^{\,2}. \tag{36}$$

The reason why $\nu_1 < \nu_1{}^*$ is, of course, that the star is able to save some of the hydrogen accreted in the early stages for consumption at the later stages. The formula (36) yields the figure quoted in the introduction. The factor 0·4 in (34) is not in itself of much significance, but it serves at any rate to show that figures such as those used in the introduction actually leave an appreciable margin to spare.

The time $\mathscr{T}_1$ is that required for the slowest permissible rate of accretion to produce the star from an initial mass M_0. This is a quantity of some importance for fixing ideas concerning stellar evolution.

* Struve, *op. cit.*, pp. 23–24.

The possible lifetime of the star considered at fixed mass and luminosity M_1, L_1 is less than $\mathscr{T}_1^*$, where

$$\mathscr{T}_1^* = M_1/L_1. \tag{37}$$

For $\mathscr{T}_1^*$ would be its life were the mass M_1 to consist of almost pure hydrogen (and if we neglect the change of luminosity with hydrogen content at fixed total mass). Therefore, if the star was produced by accretion, *the accretion process must have continued in operation to within less than $\mathscr{T}_1^*$ years ago.* As already noted, it is this quantity $\mathscr{T}_1^*$ that is found for large M_1 to be small compared with the age of the Galaxy.

We can now assert, further, that *this accretion process must have started less than $\mathscr{T}_1$ years ago* or, at any rate, the star must have been of mass M_0 less than $\mathscr{T}_1$ years ago. This does not preclude the star having attained mass M_0 in some earlier spell of accretion. Figures given later show, for example, that a star of present mass 40 solar masses must have been of mass 10 less than 10^8 years ago. Such figures verify the fact that we are concerned with processes which, if they operate at all in the manner under investigation, must do so under "recent" conditions in the Galaxy.

2.7. *Binary stars.*—In the case of a close binary composed of stars of about equal mass M, the rate of accretion for the system as a whole is that for a single star of mass $2M$. This rate is four times the rate for a single star of mass M, since the rate varies as the square of the mass; hence the rate for each star is double what it would be if it alone were present. If the star considered in 2.6 has a close companion of similar mass, we have therefore in place of (34), (35)

$$\nu_1 = 0{\cdot}2 L_1/M_1^2, \tag{38}$$

$$\mathscr{T}_1 = 2{\cdot}5(M_1/M_0)M_1/L_1 = 1/(2\nu_1 M_0), \tag{39}$$

where, as before, ν stands for the coefficient of M^2 in (17).

3. *Applications*

3.1. *Properties of interstellar matter.*—In briefly reviewing some of the observational findings about interstellar matter, we have to recall the fact that the applications in which we are interested are marginal effects, concerned presumably with extreme conditions rather than average ones.

Interstellar matter is parcelled into concentrations which we call *clouds*, though here a cloud may mean also the type of system sometimes called a "cloud-complex". The order of magnitude of the linear dimensions of a cloud may be taken as 10 parsecs, with a very wide spread amongst individual examples.* In a single cloud the density and internal motion must vary from place to place, but presumably the relative motion of neighbouring regions is subsonic.

In the work, for example, of Bates and Spitzer† there seems to be good evidence for the occurrence of densities of 1000 hydrogen atoms/cm^3, despite the fact that these authors fully recognized that this result was unexpected in the light of earlier estimates. Indeed, it seems already to be admitted‡ that values up to 10000 hydrogen atoms/cm^3 can occur. The phenomena studied in this paper, if they occur at all, occur in the spiral arms of the Galaxy to which the stars concerned are apparently confined and which are almost certainly

* McCrea, *loc. cit.*

† D. R. Bates and L. Spitzer, *Ap. J.*, **113**, 441–463, 1951.

‡ Greenstein, *loc. cit.*, p. 557.

the regions of the Galaxy in which interstellar matter attains its greatest densities. It seems fair to say that as yet there is no available estimate of these greatest densities. Some of the criticisms of the accretion theory have arisen from the estimate quoted in the introduction; subsequent work makes this appear either to be too low or else to apply to types of cloud that are not typical for our purposes.

Estimates* of the gas-kinetic temperature of the interstellar material in regions where it is not heated locally by very hot stars lie mainly in the range 30 deg. K to 100 deg. K, though both lower and higher values have been suggested by particular items of evidence. It is not yet known whether the actual temperature varies much from one cloud to another.

It is well established that the cloud material is mostly hydrogen gas and our work mainly concerns the so-called H I regions in which this is un-ionized. (When one of the stars concerned reaches a sufficiently great mass and luminosity, the hydrogen in its vicinity will become ionized with a consequent increase in the gas-kinetic temperature. This may be one of the factors that limit the ultimate mass attained, but this effect will not be studied here.) However, since our calculations involve the third power of the speed of sound in the gas, we ought to make allowance for the presence of elements other than hydrogen. Recently estimated relative abundances give for the mean molecular weight μ in terms of $O = 16$, approximately,

$$\mu = 1 \cdot 4 \tag{40}$$

and for the total density approximately 1·6 times the density of hydrogen present. The excess of μ above unity is due almost entirely to helium, of which the abundance is difficult to determine. But the only effect of a revision of the estimate of this quantity would be proportionately to alter the temperatures to which our results apply.†

The interstellar material contains also a small admixture of "dust". The chief rôle in the processes with which we are dealing is to provide a mechanism whereby thermal energy generated in these processes may be sufficiently rapidly dissipated by radiation. This mechanism is explicitly‡ required for the applicability of the accretion formula (1). It is not essential in principle for the applicability of the formula (2), though it affects the values of the parameters involved. According to Bondi's discussion, we should use the value $\alpha' \doteqdot 2 \cdot 24$ if the mechanism is fully operative so as to render the conditions *isothermal.* We should use $\alpha' \doteqdot 0 \cdot 5$ if it is inoperative so that the conditions are adiabatic. Also, the speed of sound at infinity is given in terms of the temperature T at infinity by

$$a^2 = \gamma \mathfrak{R} T/\mu, \tag{41}$$

where $\mathfrak{R}$ is the gas-constant, and $\gamma = 1$ in the isothermal case, $\gamma = \frac{5}{3}$ in the adiabatic case. When Bondi's accretion formula is used, for a given value of T the isothermal rate of accretion is therefore about $9 \cdot 6 [\doteqdot 4 \cdot 48 \times (\frac{5}{3})^{3/2}]$ times the adiabatic rate.

From the way in which the force-formula (5) has been derived, it appears that the mechanism involving the dust has no first-order effect upon the

* Greenstein, *loc. cit.*; Bates and Spitzer, *loc. cit.*; H. I. Ewen and E. M. Purcell, *Nature, Lond.*, **168**, 356, 1951.

† I have used values of relative abundances collated from various sources and kindly supplied to me by Dr F. D. Kahn; the values do not differ greatly from those in several published tables.

‡ Bondi and Hoyle, *loc. cit.*

applicability of this formula nor upon the value of the coefficient β. But, as has just been said, the mechanism is required for the applicability of (1) and so of (7). If it does not operate, we ought therefore to take $\alpha = 0$ in (8), (15), (16). If it operates fully, then α has a value in the interval $1 < \alpha < 2$ as previously stated.

In order to bracket all these possibilities, we shall derive results for the two cases:

$$\text{Case I} \quad \text{Isothermal case} \quad \alpha = 2, \quad \gamma = 1, \quad \alpha' = 2{\cdot}24; \tag{42}$$

$$\text{Case A} \quad \text{Adiabatic case} \quad \alpha = 0, \quad \gamma = \tfrac{5}{3}, \quad \alpha' = 0{\cdot}5. \tag{43}$$

So far as present knowledge of the subject extends, case I is the *optimum* one for all effects associated with accretion. In the sense that we follow Bondi's evaluation of α', as explained at the end of Section 2.1, case A is maximal for strictly adiabatic behaviour. Granting this evaluation, case A then gives the *minimum* effects of the processes here studied because any relaxation of the adiabatic condition renders these processes more efficient.

3.2. *Necessary accretion rates.*—We now consider some actual massive stars and evaluate the smallest accretion rate, determined by the parameter ν_1 in Section 2.6, that would be necessary to account for their existence. For this purpose we take Kuiper's* "selected spectroscopic binaries". The first two stars in his list are not "massive" and so not directly relevant to the present considerations, but they may be retained for comparison. By taking a few actual stars, we possibly gain a more realistic impression than by considering some typical points on the empirical mass-luminosity curve, which would serve as well in principle.

In Table I, the spectral class and the values of $\log \mathscr{M}$, $\log \mathscr{L}$ are reproduced from Kuiper's paper, where $\mathscr{M}$ is the mass M_1 in solar masses and $\mathscr{L}$ the luminosity in solar luminosities. The entries AB give mean values for the two components of the binary concerned.

TABLE I

Kuiper's selected spectroscopic binaries

Star	Spectrum	$\log \mathscr{L}$	$\log \mathscr{M}$	$\mathscr{L}/\mathscr{M}^2$	"Life" $\mathscr{T}_1$* years
Castor C_1	K6+	−1·16	−0·201	0·17	$9{\cdot}1 \times 10^{11}$
C_2		−1·24	−0·247	0·18	$9{\cdot}8 \times 10^{11}$
β Aur A	A1	+1·83	+0·378	11·9	$3{\cdot}5 \times 10^{9}$
B		+1·83	+0·370	12·3	$3{\cdot}5 \times 10^{9}$
μ_1 Sco AB	B3	+3·35	+1·094	14·5	$5{\cdot}5 \times 10^{8}$
V Pup AB	B2	+3·86	+1·265	21·4	$2{\cdot}5 \times 10^{8}$
Y Cyg A	O9	+4·51	+1·240	107	$5{\cdot}4 \times 10^{7}$
B		+4·51	+1·235	110	$5{\cdot}3 \times 10^{7}$
AO Cas A	O8·5	+5·97	+1·634	503	$4{\cdot}6 \times 10^{6}$
B		+5·58	+1·582	261	10×10^{6}
29 C Ma A	O8·5	+5·84	+1·66	331	$6{\cdot}6 \times 10^{6}$
B		+5·39	+1·53	214	14×10^{6}

Since these stars are all close binaries with nearly equal components, the appropriate formulae are (38), (39). Using the fact† that the transmutation of 1 g of hydrogen releases $6{\cdot}4 \times 10^{18}$ ergs, and using the known mass and luminosity of the Sun, it is found that these formulae become

$$\nu_1 \doteqdot 3 \times 10^{-53} \mathscr{L}/\mathscr{M}^2, \tag{38'}$$

$$\mathscr{T}_1 \doteqdot 2{\cdot}7 \times 10^{11} (M_1/M_0) \mathscr{M}/\mathscr{L} \text{ years}, \tag{39'}$$

* G. P. Kuiper, *Ap. J.*, **88**, 472–507, Table 12, 1938.

† See, for example, S. Chandrasekhar, *Astrophysics*, (ed. J. A. Hynek), p. 632, New York, 1951.

while (37) becomes

$$\mathscr{T}_1{}^* \doteqdot 10^{11} \mathscr{M}/\mathscr{L} \text{ years.} \tag{37'}$$

The values of $\mathscr{T}_1{}^*$ are given in the last column. As we have seen, they are upper bounds to the times for which these stars can continue to radiate at their present luminosities without replenishment of their hydrogen. We then note that if we take, say, $M_1/M_0 = 4$ we get from (39), (37) that $\mathscr{T}_1 = 10\mathscr{T}_1{}^*$. So, as explained in Section 2.6, it is seen that *even at the slowest possible rate of growth the O-stars must have acquired most of their mass within the last* 10^8 *years*, or thereabouts.

For the massive stars in Table I, which we shall take to be those with $\mathscr{M} > 10$, the values of $\mathscr{L}/\mathscr{M}^2$ range from about 14 to about 500. From (38′), the range of ν_1 for these particular stars is thus about

$$4 \times 10^{-52} < \nu_1 < 1{\cdot}5 \times 10^{-50}. \tag{44}$$

Also, using (2), (24),

$$\nu = \alpha' \,.\, 2\pi G^2 \rho / a^3 \quad \text{giving} \quad \rho = \nu a^3/(\alpha' \,.\, 2\pi G^2), \tag{45}$$

so that we can express the interval (44) in terms of the corresponding density ρ_1, say. The results are given in Table II for some temperatures of the interstellar material in the range mentioned in Section 3.1. We recall that the formulae for these smallest necessary accretion rates were derived in Section 2.6 on the hypothesis that this material consists effectively only of hydrogen. This hypothesis is, of course, retained in the present section; in particular the values of a are got from (41) with $\mu = 1{\cdot}008$.

TABLE II

Densities for smallest necessary accretion rates

Temperature T	Density in hydrogen atoms/cm³	
	Isothermal case	Adiabatic case
30 deg. K	$0{\cdot}5 < \rho_1 < 18$	$5 < \rho_1 < 170$
50 deg. K	$1 < \rho_1 < 39$	$10 < \rho_1 < 370$
100 deg. K	$3 < \rho_1 < 110$	$28 < \rho_1 < 1050$

There is no intention of suggesting that any of the stars concerned acquired the hydrogen at these smallest rates. Also there is, of course, nothing very special about the particular bounds here found, beyond the fact that the stars in Kuiper's list happen to typify those which present the problem giving rise to the present work.

The significance of the bounds for ρ_1 is that they are all below the maximum hydrogen densities that seem actually to occur according to the figures mentioned in Section 3.1. On general grounds we have to conclude that the phenomena under consideration occur only in clouds where the density is exceptionally high. Those figures indicate that an "exceptionally high" density is more than 1 000 hydrogen atoms/cm³. How much more we have still to infer. At any rate, the accretion that then occurs is much faster than that at almost all the densities in Table II. That is to say, *it is more than what is necessary to compensate hydrogen consumption in the accreting stars.*

3.3. *The values of* M_0, U_0, β, ρ.—The values of $\mathscr{T}_1{}^*$ in Table I show that the stars of greatest fixed mass that can maintain their luminosity for a time of the order of the past life of the Galaxy are those of somewhere about twice the solar mass. On the hypothesis being studied, stars now of mass exceeding about two solar masses must have existed in the past as stars of smaller mass than at present and must have acquired the additional mass by accretion. But all the effects associated with accretion are more pronounced the greater the mass of the star concerned. Hence, other things being equal, the stars that have experienced these effects must originally have been amongst the most massive whose existence does not depend upon these effects. It follows that M_0 corresponding to $\mathcal{M}=2$ is a suitable initial mass for our calculations. However, the effects are enhanced if the stars concerned are members of sufficiently close binary systems. Also we have to recognize the possibility of a star, that has already experienced accretion in one cloud, experiencing further accretion at a later stage. In order to cover such possibilities, we shall give results also for M_0 corresponding to $\mathcal{M}=5$.

As regards the values of U_0 to be considered, we have to take those for which the accretion effects can be significant. Later, we have to discuss the circumstances in which such velocities can occur. We can sufficiently illustrate the results by giving them for $U_0=1, 2, 5$ km/s. The speed of sound in the interstellar gas is in the range about 0·4 to 1 km/s. So we need not give results for less than 1 km/s, since a star with an appreciably lower speed would behave approximately as though it were at rest in the gas.

We require the parameter β only for the evaluation of t_1, x_1 from (15), (16). As explained in relation to these formulae, the effective value of β is approximately the value of $2\ln(sU^2/GM)$ for $U=U_0$. Also, the effective value of s was estimated as being of the order of one-half the mean distance between the stars affected by the processes considered. Were the phenomena to occur in a stellar distribution such as that near the Sun, it will be shown elsewhere that only about one star in a thousand would be affected. The mean distance between such stars would be of the order of 10 parsecs. However, so as not to overestimate the effect, we shall give results for the case of $s=1$ parsec. They are relatively insensitive to the value used. Table III gives the required figures.

TABLE III

Values of $2\ln(sU^2/GM)$ *giving for* $U=U_0$ *the required estimates of* β

[$s=1$ parsec]

U / M	1 km/s	2 km/s	5 km/s
2 solar masses	9·5	12·3	16·0
5 solar masses	7·7	10·5	14·1

A density given by

$$\rho=2{\cdot}5\times10^{-21}\ \mathrm{g/cm^3} \tag{46}$$

is just over 1 000 atoms/cm^3 if the mean molecular weight is given by (40). This is a convenient density to choose so as to make the results realistic in the sense that they will apply to what is normally regarded as a region of high interstellar density. The values of t_1, t_2, x_1 are inversely proportional to ρ, and so they are easily read off for other values of ρ.

3.4. *Values of* t_1, x_1.—Table IV gives the values of t_1, x_1 calculated* from (15), (16) using the values of β in Table III and the value of ρ in (46).

* The theory applies to a cloud extending indefinitely in all directions. If we consider a star that enters a finite cloud from outside, the given values of α, β will not hold good in the initial stages of the motion. The values of t_1, x_1 are then to be measured from the stage at which the star has penetrated sufficiently far for the theory to become applicable.

TABLE IV

Times and distances for reducing a star to rest relative to a cloud of density $2{\cdot}5 \times 10^{-21}$ *g/cm*^{3}

	U_0	1 km/s	2 km/s	5 km/s
Isothermal case ($\alpha=2$)				
$M_0=2$ solar masses	t_1	$3{\cdot}1 \times 10^6$ years	20×10^6 years	260×10^6 years
	x_1	2·4 parsecs	32 parsecs	990 parsecs
$M_0=5$ solar masses	t_1	$1{\cdot}5 \times 10^6$ years	$9{\cdot}3 \times 10^6$ years	110×10^6 years
	x_1	1·2 parsecs	14 parsecs	440 parsecs
Adiabatic case ($\alpha=0$)				
$M_0=2$ solar masses	t_1	$4{\cdot}0 \times 10^6$ years	25×10^6 years	300×10^6 years
	x_1	3·1 parsecs	38 parsecs	1150 parsecs
$M_0=5$ solar masses	t_1	$2{\cdot}0 \times 10^6$ years	12×10^6 years	140×10^6 years
	x_1	1·5 parsecs	18 parsecs	520 parsecs

3.5. *Values of* t_2-t_1.—Table V gives the value of t_2-t_1 calculated* from (19) using the value of ρ in (46) and the value of a determined by (40), (41).

TABLE V

Times for large mass-increment in cloud of density $2{\cdot}5 \times 10^{-21}$ *g/cm*^{3} *and mean molecular weight* 1·4

Temperature T		30 deg. K	50 deg. K	100 deg. K
Isothermal case ($\gamma=1$, $\alpha'=2{\cdot}24$)				
Speed of sound a		0·42 km/s	0·54 km/s	0·77 km/s
$M_0=2$ solar masses	t_2-t_1	$3{\cdot}8 \times 10^6$ years	$8{\cdot}3 \times 10^6$ years	23×10^6 years
$M_0=5$ solar masses	t_2-t_1	$1{\cdot}5 \times 10^6$ years	$3{\cdot}3 \times 10^6$ years	$9{\cdot}4 \times 10^6$ years
Adiabatic case ($\gamma=\frac{5}{3}$, $\alpha'=0{\cdot}5$)				
Speed of sound a		0·54 km/s	0·70 km/s	0·99 km/s
$M_0=2$ solar masses	t_2-t_1	37×10^6 years	80×10^6 years	226×10^6 years
$M_0=5$ solar masses	t_2-t_1	15×10^6 years	32×10^6 years	90×10^6 years

3.6. *Values of* t_2^*, x_2^*.—Table VI gives for comparison the values of t_2^*, x_2^* calculated from (20), (21). In the strictly adiabatic case, these quantities are non-existent.

TABLE VI

Times and distances for large mass-increment of a star moving at uniform speed U_0 *in a cloud of density* $2{\cdot}5 \times 10^{-21}$ *g/cm*^{3}

Isothermal case ($\alpha=2$)	U_0	1 km/s	2 km/s	5 km/s
$M_0=2$ solar masses	t_2^*	57×10^6 years	460×10^6 years	7200×10^6 years
	x_2^*	59 parsecs	940 parsecs	36700 parsecs
$M_0=5$ solar masses	t_2^*	23×10^6 years	180×10^6 years	2900×10^6 years
	x_2^*	23 parsecs	380 parsecs	14700 parsecs

3.7. *Illustrative example.*—These tables could, of course, have been made more extensive, but this is not necessary on account both of the simple character of the formulae and of the inherently approximative character of any likely applications. The only factors in all the formulae that do not follow some simple rule of proportionality are those containing β, but this is a slowly varying quantity and its variation can be ignored in any small interval of the other parameters.

In order to illustrate the use of the figures given and as an example for discussion, consider the following:—A fairly close binary star, each component

* It is to be noted that the time t_2-t_1 is here calculated from the formula for *steady* symmetric accretion. So far as I have succeeded in further examining the problem, the assumption of a steady flow does not lead to any underestimate of the accretion-time.

having initial mass equal to two solar masses, enters a cloud of density 5×10^{-21} g/cm^3 and gas-kinetic temperature 50 deg. K, with initial relative speed 1·5 km/s; sufficient interstellar dust is present to render conditions approximately isothermal.

The effective mass M_0 corresponds to four solar masses. So, having regard to the way in which density, mass and speed enter the various formulae, and using Table IV, the star is brought to rest in time, approximately, $\frac{1}{2} \times \frac{5}{4} \times (1{\cdot}5)^3 \times 1{\cdot}5 \times 10^6 \doteqdot 3{\cdot}2 \times 10^6$ years and distance, approximately, $\frac{1}{2} \times \frac{5}{4} \times (1{\cdot}5)^4 \times 1{\cdot}2 \doteqdot 3{\cdot}8$ parsecs. Using Table V, the mass will have reached a large value after a further time, approximately, $\frac{1}{2} \times \frac{5}{4} \times 3{\cdot}3 \times 10^6 \doteqdot 2 \times 10^6$ years.

Using Table VI, we get corresponding time and distance approximately 5×10^7 years and 75 parsecs.

Thus, according to the full theory, the star would be stopped by the cloud within the moderate distance of about 4 parsecs and would acquire a large mass in a total time of about 5 million years. On the other hand, a simple application of the simplest formulae would have demanded nearly twenty times the distance and ten times the duration.

This is an arbitrarily chosen example. But none of the requirements seems to be excluded by our present knowledge of the properties of interstellar matter, though this knowledge indicates that the requirements are still somewhat extreme. We seem at any rate to have reached the position that, having found all the factors entering into the refined discussion to tend to make it easier for accretion to operate, it would now be rather surprising to discover that the requirements for its successful operation are never quite realized.

4. *Conclusions*

If the theory here presented is correct and applicable then we may provisionally conclude that massive stars are produced from less massive ones by the accretion process. The mechanism is simply that the interstellar material halts the motion of a star through it and then falls into the star. For material of given density and kinetic temperature forming a cloud of given size (which must be big enough to yield an adequate quantity of material), the requirement is simply that the initial relative speed of the star should be sufficiently low. Recognizing that the occurrences must be rare, the sample calculations and observational results mentioned in the paper strongly indicate that all the requirements are to be found realized in the actual Galaxy. It is also noticed that, if the phenomena do occur, the demarcation between the stars affected and those not affected must be quite sharp, owing principally to the sensitivity of the effects to the velocity factors.

The only element of uncertainty on the purely theoretical side concerns the force-formula. The significant densities are such that the flow is "hydrodynamic". We have been able to give arguments in support of the use of the formula in this case, but it has not yet been proved directly. Also, the formula is a "steady-state" formula as derived and some investigation of its use in non-steady conditions would be desirable.

As regards the applicability of the theory, the only physical factor that appears to be ignored is the heating effect of the radiation from the star concerned upon the neighbouring cloud material. On general grounds this is expected at most to enter at some stage, when the star has attained great luminosity, to produce a cut-off effect. Otherwise there seems to be no obstacle in principle to the applicability of the theory. In that case, the main problem is now to endeavour in some way to estimate the actual frequency of occurrence of the processes.

Acknowledgments.—I am indebted to Mr K. N. Dodd for checking the calculation of the tabulated results. This work was done while the author was on leave of absence granted by the Council of Royal Holloway College and was holding a Comyns Berkeley Bye-Fellowship awarded by the Council of Gonville and Caius College.

Gonville and Caius College,
Cambridge:
1952 *November* 27.