ON MAGNITUDE SYSTEMS*

HAROLD L. JOHNSON
Yerkes and McDonald Observatories
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ABSTRACT

The relations between several accurate systems of magnitudes and colors are discussed. It is shown that the inclusion of radiation to the violet of 3800 A in the measurements makes the relations between different color systems nonlinear and, in most cases, multivalued. The North Polar Sequence is shown to be inadequate as a standard region, because of the absence of blue, supergiant, heavily reddened, and red dwarf stars.

During the course of work, the first portion of which is reported in an earlier paper,[1] it became apparent that comparisons with, and transformations to, other accurate photometric series are much more difficult to make with sufficient accuracy than has been supposed. The first indication of these difficulties came at the time the NPS transformations given in Paper I were derived. The difference in color index between the stars NPS 6 and NPS 10 is $C_6 - C_{10} = -0.06$, according to Stebbins, Whitford, and Johnson,[2] and $C_6 - C_{10} = -0.12$, according to my recent measures.[1] It is impossible that either series of measures could be in error by the necessary amount; Stebbins, Whitford, and Johnson have a total of 19 observations on each of the two stars, while my McDonald values are the results of 13 observations on each star. The difference between the two series is certainly real and must be explained. Further discordances of this type will be noted among the other NPS stars, particularly for NPS 19.

The only significant difference between the filters used in the two investigations is the ultraviolet cutoff of the blue filter, which occurs at about 3550 A for the observations of Stebbins, Whitford, and Johnson and at about 3800 A for my McDonald work. It seems unbelievable, at first glance, that this small change could produce significant differences, but, as will be shown below, it does.

A comparison between Eggen's[3] colors for the near-by stars and the McDonald values is shown in Figure 1. The value on the International System is $P_0 - V$, according to the linear transformation equations given in Paper I; the significance of the subscript 0 on P is discussed later.

The straight line is the transformation equation given by Eggen:[4] $P - V = -0.10 + 1.10C_p^E$. It is evident that a straight line is not an adequate transformation between the two color systems.

Comparisons between Eggen's[3] photographic magnitudes, Pg_p^E, and the McDonald values[1] are shown in Figure 2. The magnitude from the linear transformation to the NPS given in Paper I is P_0; the color index, $B - V = C_y + 1.040$.[5] Figure 1, *a*, can be interpreted as indicating that a large scale difference, amounting to nearly 0.3 mag. in 10 mag., exists between the two photoelectric magnitude systems. That such a scale error cannot exist in my work will be shown in a later paper of this series; it is *not* proposed that

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[1] H. L. Johnson and W. W. Morgan, *Ap. J.*, **114**, 522, 1951. Hereafter, this paper is referred to as "Paper I."

[2] *Ap. J.*, **112**, 469, 1950.

[3] *Ap. J.*, **111**, 65, 81, 414, 1950; **112**, 141, 1950; **113**, 657, 663, 1951.

[4] *Ap. J.*, **114**, 141, 1951.

[5] B is a magnitude obtained directly from the blue filter. The zero point of $B - V$ has been set at spectral type A0; this point will be discussed in detail in a later paper of this series.

such a scale error exists in Eggen's work. Since all the bright stars in the comparison are blue and all the faint ones are red, the scale error can be interpreted as a color equation between the two systems, as in Figure 2, *b*. It will be shown that the latter is the correct interpretation.

The ultraviolet measures, made with the filter described in Paper I, make possible a series of tests of the effect of the inclusion of the region beyond 3800 A in the blue filter. Four blue filters, containing different amounts of ultraviolet, were formed by taking the observed blue deflection and adding to it a constant, *a*, times the ultraviolet deflection.

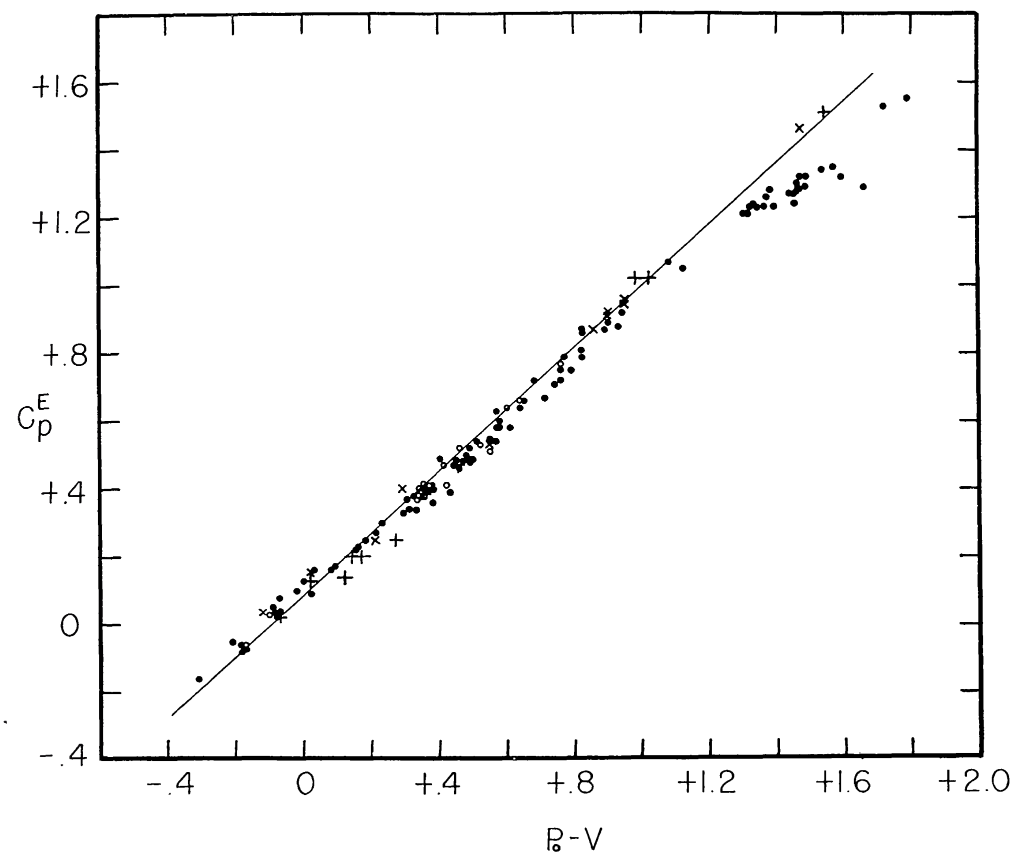

FIG. 1.—Eggen's colors, C_p^E, versus $P_o - V$. The symbols are defined as follows. *filled circles:* luminosity class V; *open circles:* luminosity class IV; *crosses:* luminosity class III; *plus signs:* NPS stars.

The filters thus obtained are illustrated in Figure 3. The procedure is legitimate, since the response of the photocell is linear and its output is strictly proportional to the area under the response-curve.

The magnitudes and colors obtained in each case have been transformed by linear relations to the NPS. The photographic magnitudes so obtained are designated P_0, $P_{0.5}$, $P_{1.0}$, and $P_{2.7}$, according to the value of *a*. The relations are:

$$P_0 - V = +0.96 + 1.09 C_0 , \quad (1)$$

$$P_{0.5} - V = +1.06 + 1.00 C_{0.5} , \quad (2)$$

$$P_{1.0} - V = +1.12 + 0.93 C_{1.0} , \quad (3)$$

$$P_{2.7} - V = +1.29 + 0.79 C_{2.7} . \quad (4)$$

The quality of the fit to the NPS may be estimated from the values given in Table 1 (upper half); in no case is the fit unsatisfactory, although it is better in some cases than in others. The color indices on the International System are given for a number of other stars in Table 1 (lower half); it is evident that the values obtained for intrinsically blue, or high-luminosity, stars depend very strongly upon the amount of ultraviolet included in the blue filter.

V has no color equation to the International photovisual magnitudes in the NPS. Table 2 shows that the effective wave lengths for $P_{0.5}$ agree very well with those given by

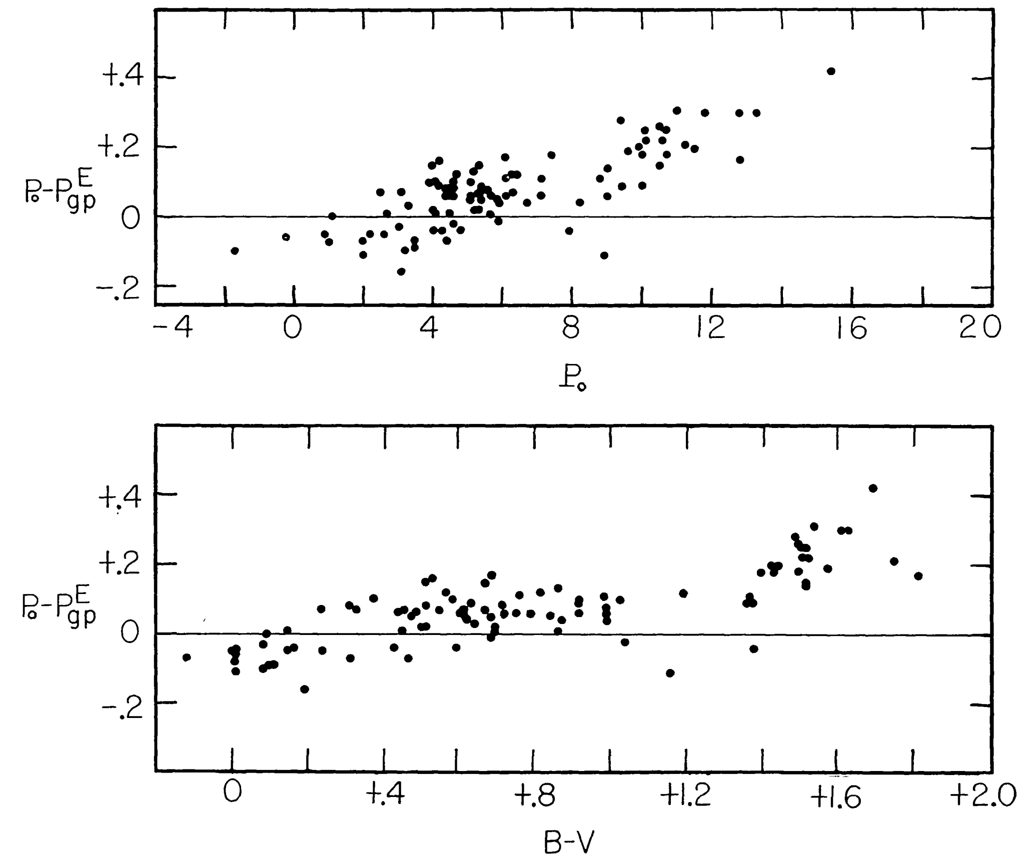

FIG. 2.—Comparisons of P_o with Pg_p^E, as a function of P_o and of $B - V$

Seares[6] for IPg, for 3000° K and 11,000° K. Table 3 shows that the intrinsic colors for $a = 0.5$ agree very well with Seares's[6] values on the International System. It appears, therefore, that $P_{0.5}$ corresponds closely to the photographic magnitudes on the International System.

Figure 4, a and b, shows comparisons of $P_{0.5}$ and $P_{1.0}$, respectively, with Eggen's[3] photographic magnitudes, Pg_p^E. It is evident that increasing the ultraviolet content of the blue filter to $a = 1.0$ results in much better agreement between Eggen's photographic magnitudes and mine. Figure 4, b, shows little color equation between the two systems. The addition of somewhat more ultraviolet might make the agreement even better for the red dwarfs, and calculations show that the color equation disappears completely for $a \simeq 1.5$. It is evident that the entire amount of the scale divergence appearing

[6] *Ap. J.*, **98**, 261, 1943.

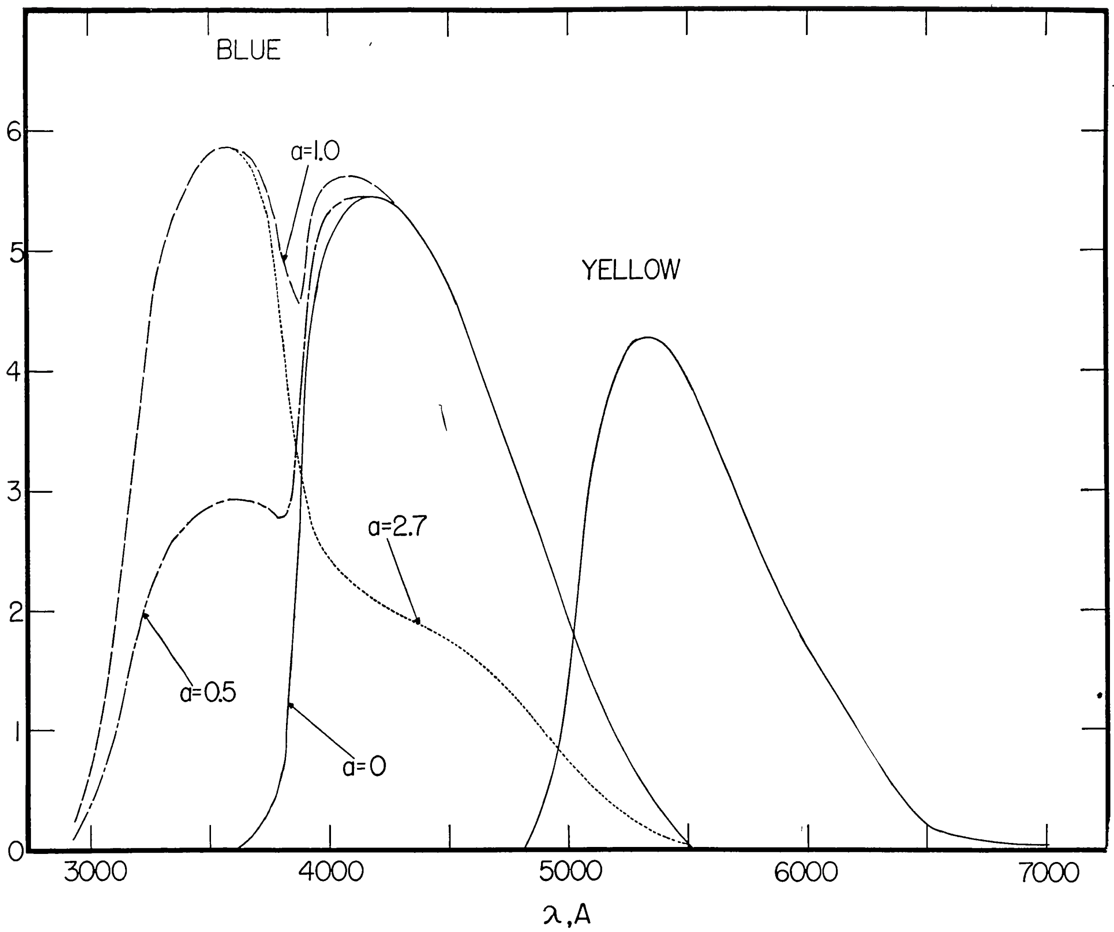

FIG. 3.—The response of the photometer to equal intensity of radiation at all wave lengths. The four blue filters are described in detail in the text.

TABLE 1

COLORS OF STARS ON THE INTERNATIONAL SYSTEM

Star	$P_0 - V$	$P_{1.0} - V$	$P_{1.5} - V$	$P_{2.7} - V$	Sp.	C_p(SWJ)
NPS 6	+0.02	+0.03	+0.04	+0.06	....	+0.06
NPS 10	+0.14	+0.13	+0.13	+0.13	....	+0.12
NPS 16	+0.35	+0.33	+0.33	+0.31	....	+0.32
NPS 19	+0.45	+0.43	+0.42	+0.40	....	+0.41
NPS 2r	+1.54	+1.55	+1.55	+1.56	....	+1.56
NPS 4r	+0.98	+1.01	+1.01	+1.02	....	+1.02
NPS 8r	+1.02	+1.03	+1.03	+1.03	....	+1.02
α Lyr	−0.17	−0.16	−0.14	−0.11	A0 V	....
α Aql	+ .08	+ .08	+ .09	+0.10	A7 IV, V	....
α Cyg	− .07	− .11	− .13	−0.13	A2 I*a*	....
9 Cep	+ .15	+ .04	− .02	−0.10	B2 I*b*	....
λ Cep	+ .10	− .06	− .14	−0.24	O6f	....
β Ori	− .20	− .33	− .38	−0.44	B8 I*a*	....
τ CMa	− .34	− .54	− .62	−0.70	O9 III	....
HD 14633	− .41	− .64	− .72	−0.80	O8	....
ι Ori	− .45	− .68	− .77	−0.84	O9 III	....
υ Ori	− .47	− .70	− .78	−0.86	B0 V	....
BD+28°4211	−0.56	−0.83	−0.93	−1.01	Op	....

in Figure 2, *a*, is due to ultraviolet beyond 3800 A included in Eggen's blue filter but excluded from mine. Furthermore, the systematic discordances apparent in Figure 1 can be explained in this same manner.

Figure 4, *c*, shows a comparison of V with Eggen's[3] photovisual magnitudes, Pv_p^E. The zero-point correction at $C_{\rm int} = 0$, $+0.07$ mag., is just the value one would expect from Figure 4, *b*, and Eggen's linear transformation to the NPS. The slightly nonlinear relation between V and Pv_p^E probably is caused by his linear NPS color transformation reflecting a small amount of the ultraviolet difficulties into his visual magnitudes. Such a reflection can occur only if the effective wave length for the yellow magnitudes differs significantly from that for *IPv;* this is the case for Eggen's[3] yellow magnitudes, but not for mine.[1]

Figure 5 illustrates the relation between $P_{1.0} - V$ and $P_0 - V$. The straight line represents the linear transformation, based upon the NPS, between the two color systems.

TABLE 2

EFFECTIVE WAVE LENGTHS FOR $P_{0.5}$

Temp. °K	$P_{0.5}$ (A)	IPg (Seares) (A)
3,000.	4590	4560
11,000.	4210	4240

TABLE 3

COMPARISON OF $P_{0.5}-V$ WITH $C_{\rm int}$

Sp.	$P_{0.5}-V$	$C_{\rm int}$ (Seares)	Sp.	$P_{0.5}-V$	$C_{\rm int}$ (Seares)
B3.	−0.51	(−0.50)	A5.	0.00	−0.01
B5.	− .42	− .42	F0.	+ .15	+ .12
A0.	− .17	− .17	F5.	+ .27	+ .27
A2.	−0.10	−0.06	G0 V. .	+0.44	+0.45

Figure 5 should be compared with Figure 1, since, as has been shown above, $P_{1.0}$ is fairly close to Eggen's Pg_p. If we restrict our attention to $P_0 - V < +1.2$, the same nonlinear relationship appears in both figures, for the giants (*crosses*), subgiants (*open circles*), and dwarfs (*filled circles*).[7] The relation between the two systems is not only nonlinear but *multivalued*, since the supergiants (*triangles*) and reddened stars deviate greatly from the relation defined by the lower-luminosity, unreddened stars. The use of a linear transformation between the data making up either Figure 1 or Figure 5 will result in systematic errors much larger than the accidental observational errors.

Figures 1 and 5 differ significantly for $P_0 - V > +1.2$. Figure 1 shows a large systematic deviation for the M dwarfs; Figure 5 exhibits a smaller deviation, but in the same direction. The explanation may be that the contribution of the ultraviolet beyond

[7] The smaller scatter in Fig. 5 is due principally to the fact that the ordinate and abscissa are independent only to the extent of the ultraviolet present in $P_{1.0}$, but not present in P_0. The figure therefore exhibits the effect of the ultraviolet much more clearly than if entirely independent observations were used.

3800 A to Eggen's blue magnitudes is an even greater fraction of the total than in the case of $P_{1.0}$.

Figure 6 gives a comparison between P_0 and $P_{0.5}$. Since it has been shown above that $P_{0.5}$ is fairly close to *IPg*, it is evident that the linear transformation from the blue magnitudes, m_p (or B), to the NPS (see Paper I) is inadequate. Such a linear transformation is only a crude approximation for $B - V > 0$ and is useless for blue stars, supergiants,

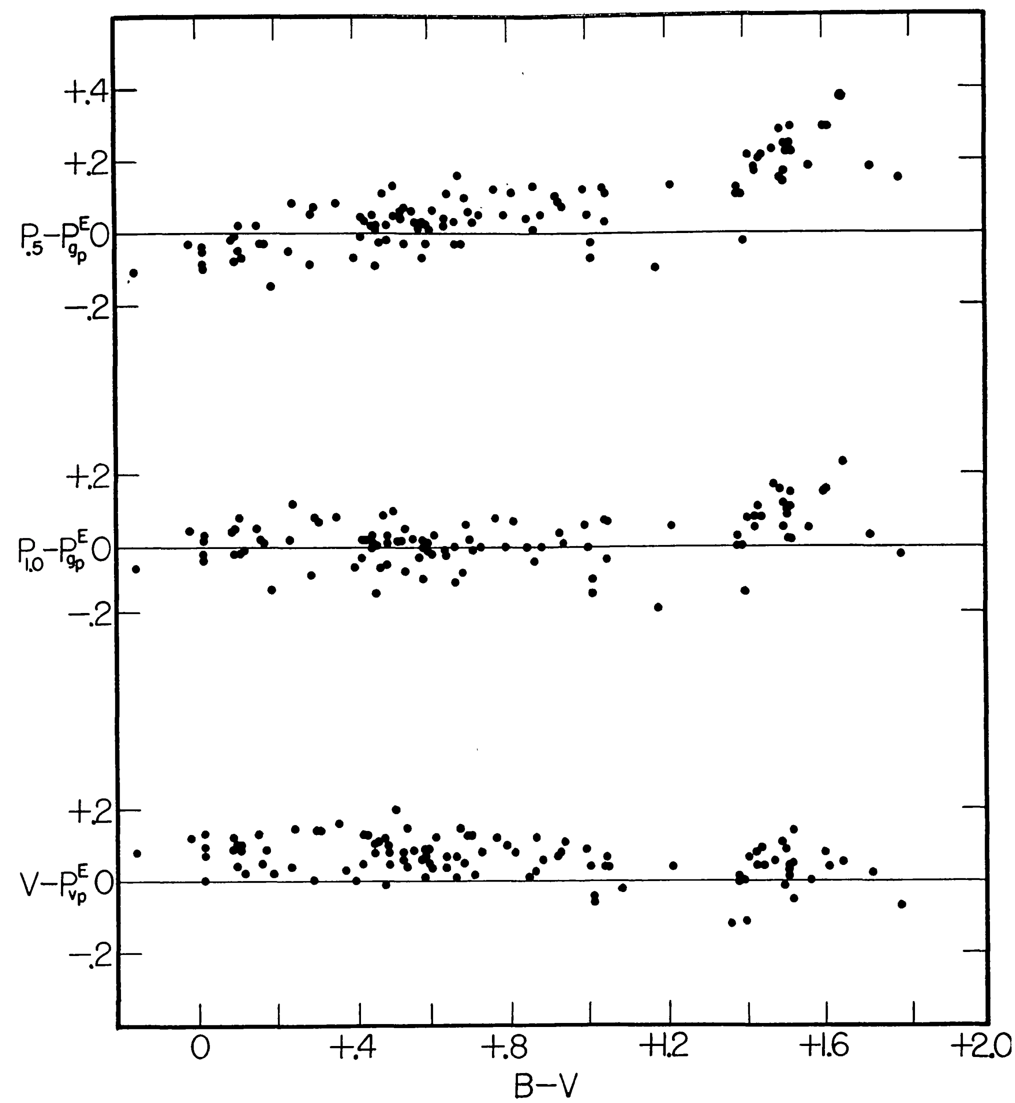

FIG. 4.—Comparisons of $P_{0.5}$ and $P_{1.0}$ with Eggen's Pg_p^E and a comparison of V with Eggen's Pv_p^E

and reddened stars. A comparison of Eggen's Pg_p^E with $P_{0.5}$ is very similar—practically the only difference is a change of sign of the ordinate. It is very clear that *neither* series of photoelectric observations can be transformed to the NPS system, *using the stars available there*, with sufficient accuracy to warrant the transformation. The same statement can be made with regard to most other series of photoelectric observations.

The quality of the transformation possible between two systems both of which discard the ultraviolet beyond 3800 A is shown by a comparison between $B - V$ and Stebbins and Whitford's[8] six-color $V - G$, illustrated in Figure 7. The relation is not exactly linear, although it is nearly so. It should be noted, especially, that the supergiants (*triangles*) and reddened stars fall upon the same sequence as do the other stars. This is in marked contrast with Figures 5 and 6.

The relation between $B - V$ and the six-color $V - I$[8] is shown in Figure 8. Even a *large* change in the yellow filter has only a minor effect on the quality of transformation.

An even more stringent test of the transformability of $B - V$ to other systems not

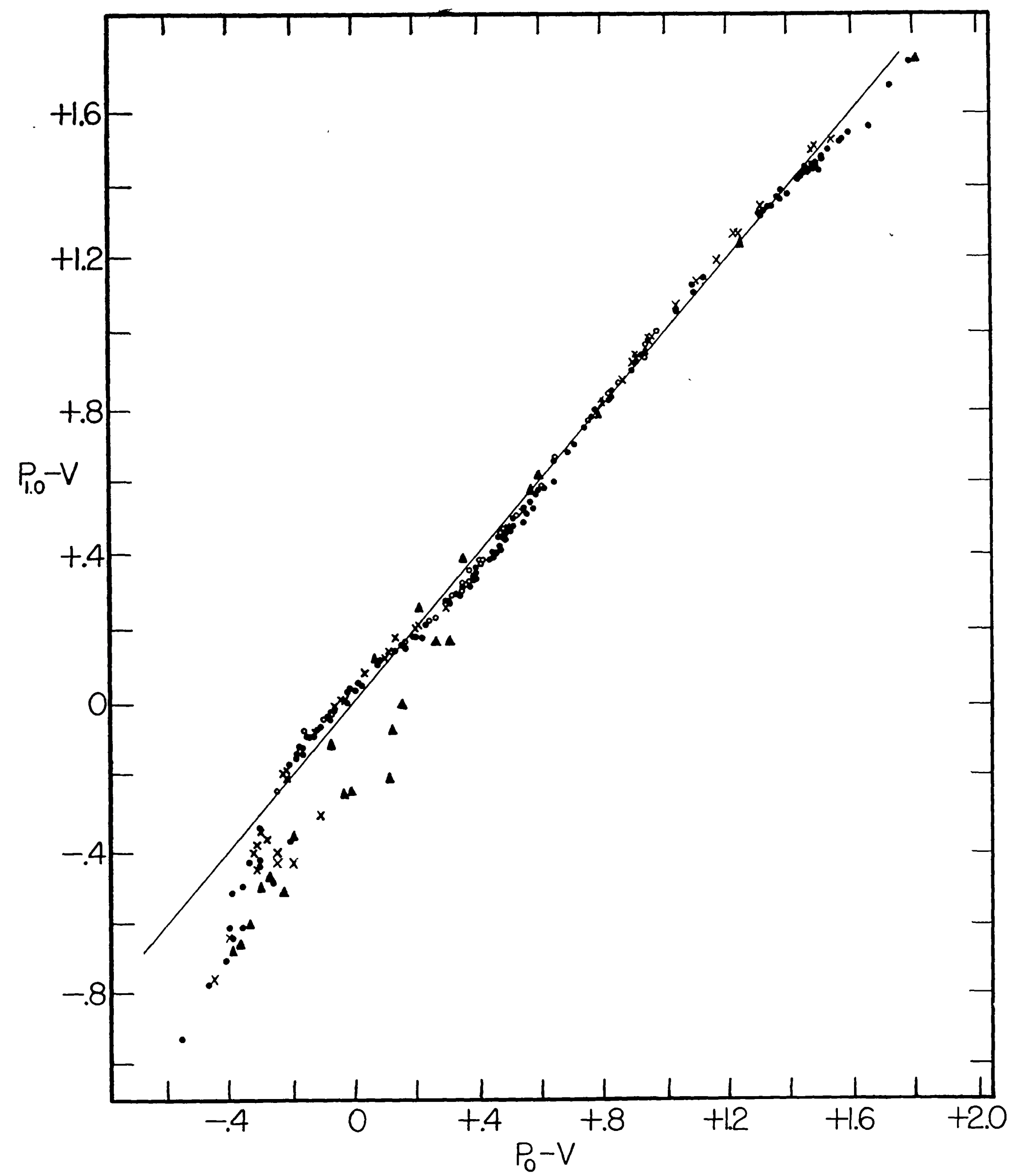

FIG. 5.—Comparison between two color systems, $P_{1.0} - V$ and $P_0 - V$, both on the International System according to the NPS stars. The symbols are defined as follows: *filled circles:* luminosity class V; *open circles:* luminosity class IV; *crosses:* luminosity class III; *triangles:* luminosity classes I*a*, I*b*, and II.

[8] *Ap. J.*, **102**, 318, 1945.

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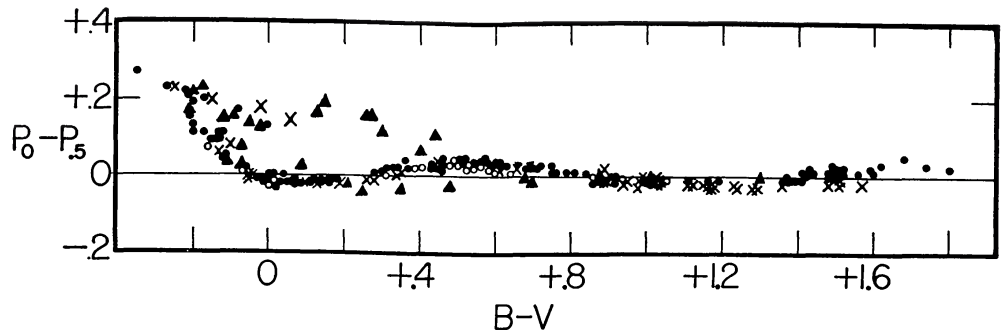

FIG. 6.—Comparison of P_0 with $P_{0.5}$ as a function of $B - V$. The symbols are the same as for Fig. 5

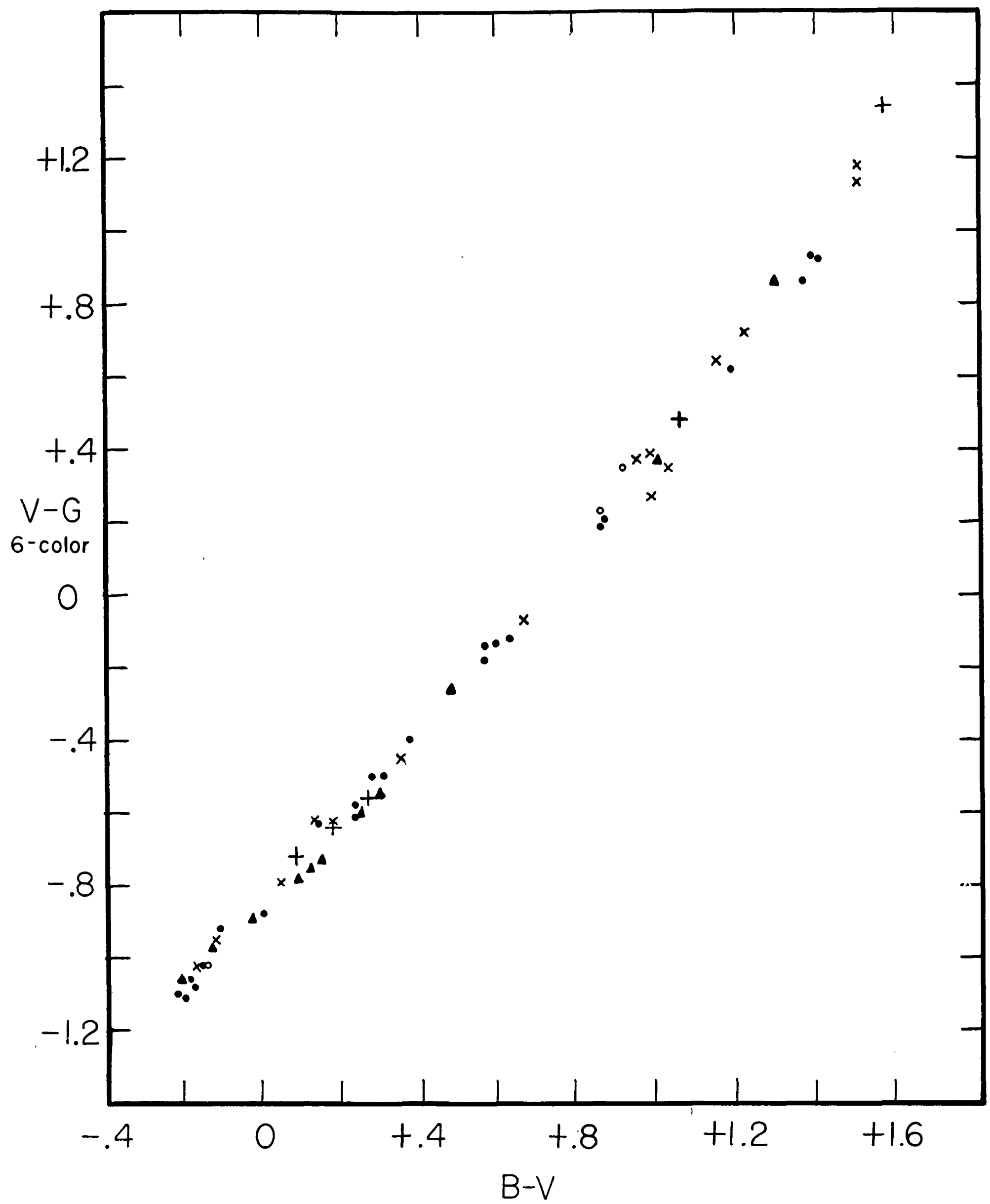

FIG. 7.—Six-color $V - G$ versus $B - V$. The symbols are the same as for Fig. 5, except *plus signs:* NPS stars.

containing ultraviolet is shown in Figure 9, *a*, where the gradients from Greenwich,[9] Mount Stromlo,[10] and Hall,[11] all transformed to the Greenwich system, are compared with $B - V$. The relationship is not linear but is smooth in the region of $B - V = 0$, indicating that the effect of hydrogen absorption on B is small. Figure 9, *b*, shows the relations between the same gradients and $P - V$, as determined from three different filters and linear transformations to the NPS. Curve *a* is the mean relation, $\overline{(P_0 - V)\text{ vs. }G}$; curve *b*, $\overline{(P_{0.5} - V)\text{ vs. }G}$; and curve *c* is the mean relation given by Eggen,[12] transformed by his equation[4] to $P - V$.

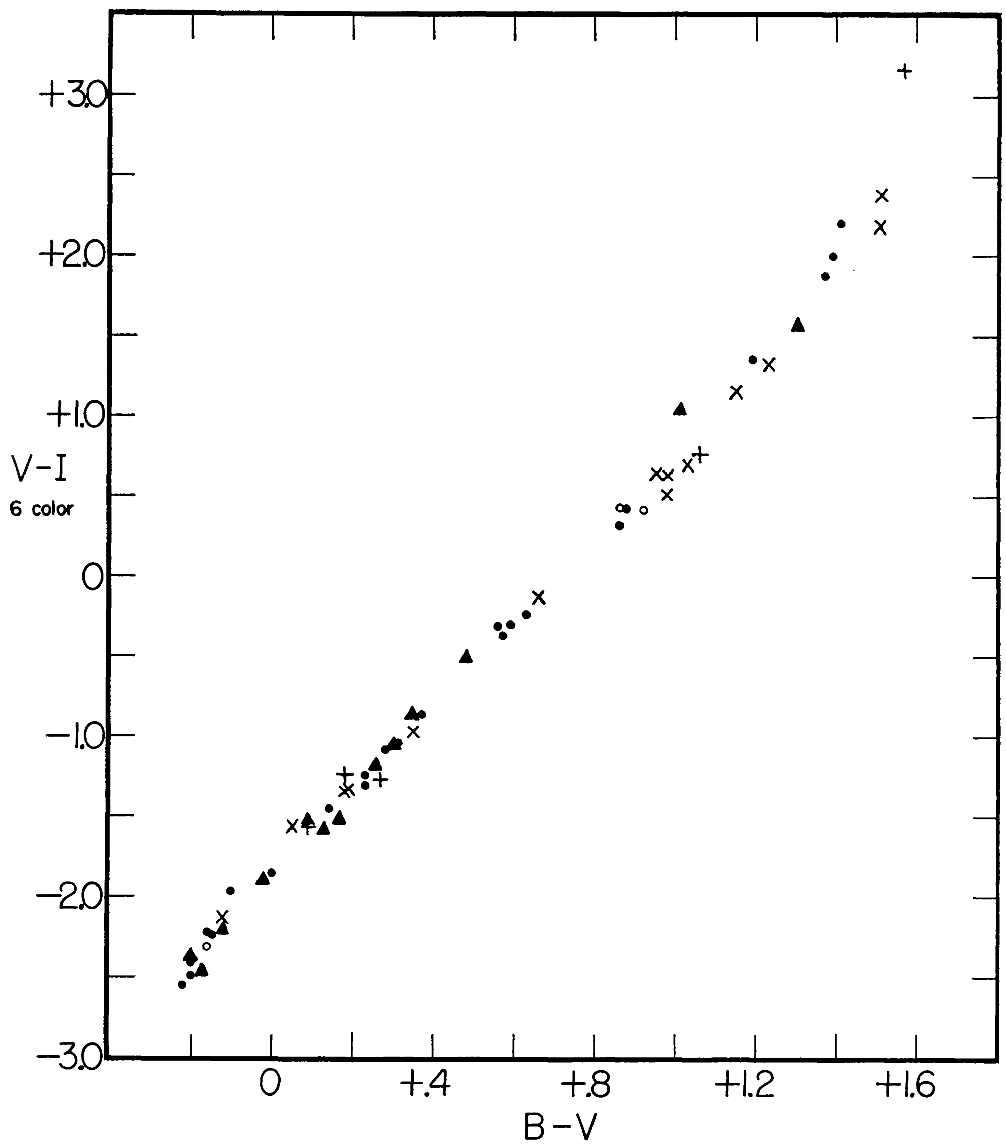

FIG. 8.—Six-color $V - I$ versus $B - V$. The symbols are the same as for Fig. 7

[9] *M.N.*, **100**, 189, 1939.

[10] S. C. B. Gascoigne, *M.N.*, **110**, 15, 1950.

[11] *Ap. J.*, **95**, 231, 1942.

[12] *Ap. J.*, **113**, 663, 1951.

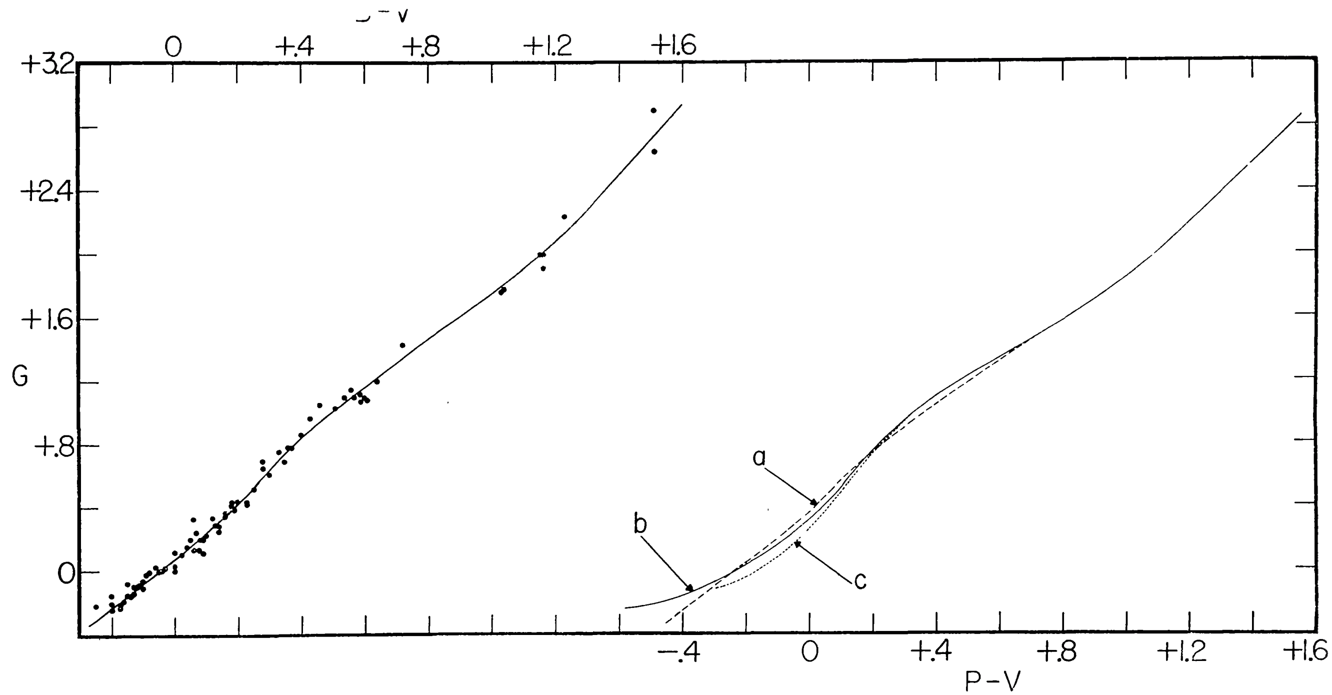

FIG. 9.—*a:* The Greenwich gradients, G, versus $B - V$. *b:* The mean relations between the gradients and three color systems, all based on the NPS. The curves are as follows: *a:* $P_0 - V$ versus G; *b:* $P_{0.5} - V$ versus G; *c:* C_p^E versus G, transformed to $P - V$ by Eggen's transformation.

The data presented above make it possible to draw a number of conclusions about magnitude systems.[13] These are:

1. Transformations between different color or magnitude systems are, in general, nonlinear.

2. Transformations between different color or magnitude systems, one or both of which include appreciable radiation to the violet of 3800 A, are, in general, multivalued as well as nonlinear. The type of transformation depends upon the spectral types and luminosity classes of the stars, as well as the observed magnitudes and colors. Minor changes in the ultraviolet response of the photometer can produce large changes in the observed results.

3. The North Polar Sequence is clearly inadequate as a standard region. The numbers of stars of the necessary kinds are entirely too small to permit accurate transformations to be made. This is especially true if ultraviolet beyond 3800 A is included in the measurements, as in *IPg*, since very blue stars, supergiants, reddened B stars, and red dwarfs are necessary but are not available in the NPS.

4. If only radiation to the red of approximately 3800 A is included in the measurements, fairly simple (but usually nonlinear) transformations between quite dissimilar color systems are possible.

[13] W. Becker (*Veröff. Göttingen*, Nos. 79–82, 1946) has independently arrived at some of these same conclusions.