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PERIGALACTIC AND APOGALACTIC DISTANCES OF HIGH–VELOCITY STARS

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Introduction. In speculations about the origin of population II stars, the question arises where were the high velocity stars formed or, more particularly, did these stars originate in the central bulge of our galaxy or farther out. A clue to the answer for this question may be found in the present galactic orbits of the high-velocity stars. These orbits remain, it seems, virtually unchanged by perturbations during the lifetime of our galaxy.[1] Hence, one may assume that the place of origin of a high-velocity star lies somewhere on or very near the present orbit of that star. For this reason there are derived here the orbits or rather the perigalactic and apogalactic distances for a representative set of high-velocity stars.

For any star which passes near the sun and for which the space velocity has been determined, the galactic orbit can be computed if the gravitational potential of the galaxy is known. If the potential is assumed to be Newtonian, the orbits, particularly the orbital elements a and e, can be obtained with the help of the Bottlinger diagram.[2] A generalization of this diagram for any gravitational potential has recently been given by Trumpler and Weaver.[3] For the present purpose, however, it appears preferable to use the perigalactic and apogalactic distances, r_p and r_a, rather than the elements a and e, for characterizing the orbits. A method for obtaining r_p and r_a under any assumption regarding the gravitational potential has already been given by Bottlinger.[4]

Gravitational potential. Before one can actually compute peri- and apogalactic distances, one has to choose an approximation to the galactic potential as well as is presently possible. Near the sun the Newtonian potential should be a tolerable approximation, as shown by recent determinations of Oort's constants.[5] For distances from the galactic center larger than that of the sun, the Newtonian approximation will become even better since it is the limiting potential for large distances from the attracting mass. On the other hand, moving from the sun inwards toward the galactic center, one will find that the Newtonian potential breaks down when the main body of the galactic mass is entered. From here in, a potential proportional to the square of the distance from the center might be expected to be a tolerable approximation—an expectation which is substantiated by the measurements of the rotational velocities in the Andromeda Nebula.[6] Accordingly, the following computations are based on an assumed potential consisting of a potential proportional to R^2 from the center to a transition distance and of a Newtonian potential from there out. The transition distance has here been taken to be 60 per cent of the sun's distance from the center. The precise value of the transition distance is not critical for the present purpose; even if the Newtonian potential is taken to hold all the way to the center, the perigalactic distances most affected change only by 15 per cent of the center-to-sun distance.

In the following computations the motions perpendicular to the galactic plane will be ignored, and the potential discussed above will be applied only to the motions parallel to the galactic plane. This restriction should not cause appreciable errors for stars with small or moderate z-velocities. For stars with large z-velocities the computed peri- and apogalactic distances will be too small.

Computing scheme. With the above approximations the peri- and apogalactic distances can be derived by the following scheme.[7]

Definitions:

R	distance from center of galaxy to arbitrary point.
$R_\odot$	distance from center to sun.
R_1	transition distance from harmonic to Newtonian potential.
R_p, R_a	distances from center to star at perigalactic and apogalactic points of its orbit.
V_x, V_y	velocity components in galactic plane relative to local standard of rest (V_x positive in direction of rotation, V_y positive in anti-center direction) for stars near sun.
V_c	circular velocity at arbitrary point.
$V_\odot$	circular velocity at sun.
V_p, V_a	velocities of star at its perigalactic and apogalactic points.
F	gravitational potential at arbitrary point (per unit mass).

Nondimensional quantities:

$$r_{p,a} = R_{p,a}/R_{\odot}, \quad r_1 = R_1/R_{\odot},$$
$$v_x = (V_x + V_{\odot})/V_{\odot}, \quad v_y = V_y/V_{\odot}. \qquad (1)$$

Assumed gravitational potential:

$$F = -V_{\odot}^2 \frac{R_{\odot}}{R} \qquad \text{for } R > R_1,$$
$$F = -V_{\odot}^2 \left(\frac{3}{2}\frac{R_{\odot}}{R_1} - \frac{1}{2}\frac{R_{\odot}R^2}{R_1^3} \right) \quad \text{for } R < R_1. \qquad (2)$$

Assumed numerical value: $R_1 = 0.6\, R_{\odot}$.

Corresponding circular velocity:

$$V_c = V_{\odot} \left(\frac{R_{\odot}}{R} \right)^{\frac{1}{2}} \qquad \text{for } R > R_1,$$
$$V_c = V_{\odot} \left(\frac{R_{\odot}}{R_1} \right)^{\frac{1}{2}} \frac{R}{R_1} \qquad \text{for } R < R_1. \qquad (3)$$

Conservation laws:

$$\text{Energy} \quad \tfrac{1}{2}(V_x + V_{\odot})^2 + \tfrac{1}{2}V_y^2 + F_{\odot} = \tfrac{1}{2}V_{p,a}^2 + F_{p,a}, \qquad (4)$$

$$\text{Momentum} \quad (V_x + V_{\odot})\, R_{\odot} = V_{p,a}\, R_{p,a}. \qquad (5)$$

Equation for peri- and apogalactic distances (obtained from conservation laws by eliminating $V_{p,a}$):

$$\tfrac{1}{2}(V_x + V_{\odot})^2 + \tfrac{1}{2}V_y^2 + F_{\odot} = \tfrac{1}{2}(V_x + V_{\odot})^2(R_{\odot}/R_{p,a})^2 + F_{p,a}, \qquad (6)$$

or with assumed potential and non-dimensional quantities:

$$v_x^2 \left(\frac{1}{r_{p,a}^2} - 1 \right) - v_y^2 = 2 \left(\frac{1}{r_{p,a}} - 1 \right) \qquad \text{for } r_{p,a} > r_1,$$
$$v_x^2 \left(\frac{1}{r_p^2} - 1 \right) - v_y^2 = 2 \left(\frac{3}{2}\frac{1}{r_1} - \frac{1}{2}\frac{r_p^2}{r_1^3} - 1 \right) \qquad \text{for } r_p < r_1. \qquad (7)$$

With the help of (7) r_p and r_a can be derived for any star which passes near the sun and for which the velocity components V_x and V_y (or according to (1) v_x and v_y) are known.

Galactic velocity diagram. Instead of employing (7) numerically it can be applied graphically in the galactic velocity diagram with the coordinates v_x and v_y (Fig. 1). In this diagram (7) gives two families of curves. A curve of the first family (hyperbola) connects all points in the velocity diagram which represent orbits, all passing near the sun, with equal perigalactic distances. A curve of the second family (ellipse) represents orbits of equal apogalactic distances. The grid formed by the two curve families permits one to read off r_p and r_a for any point on the diagram.

To apply the above to actual high-velocity stars, Miczaika's list[8] of stars with $|V| > 63$ km/sec has been taken as representative, and his 555 stars have been plotted in Fig. 1. (Dr. Miczaika kindly transmitted the following corrections to his catalogue of high-velocity stars. For star No. 85: $W = 480$. For star No. 466: $l_a = 201°$ and $b_a = -8°$.) For plotting the high-velocity stars in Fig. 1 a definite value for the circular velocity at the sun, $V_{\odot} = 250$ km/sec, was assumed. This was necessary to connect the non-dimensional quantities v_x and v_y, in terms of which the curves were drawn, with actual stellar velocities in km/sec. An error in the assumed value of $V_{\odot}$ will only change the relative scales for the points and curves in Fig. 1. Since the error in $V_{\odot}$ is probably at most 20 per cent, the following discussion is little affected by the uncertainty in $V_{\odot}$. The accuracy of the individual points in Fig. 1 should be in many cases not high since the data are based to a large extent on spectroscopic parallaxes. Particularly in the outer fringes of the point distribution an appreciable fraction of erroneous points might be expected.

The data shown in Fig. 1 may be considered in three parts: 1) the bulk of the high-velocity stars; 2) the stars with large apogalactic distances, and 3) the stars with small perigalactic distances.

Bulk of high-velocity stars. The bulk of the points in Fig. 1, indeed 85 per cent of them, lie between the curves for $r_p = 0.3$ and $r_a = 2.0$. Thus if our sun is assumed to be 8000 psc from the center, it appears that the majority of the high-velocity stars near the sun, as represented by Miczaika's list, move entirely within a ring zone with an inner radius of 2400 psc and an outer radius of 16000 psc.[9]

For comparison, the central bulge of our galaxy is most likely not more than 1500 psc in radius, as may be surmised by analogy with the Andromeda Nebula.[10] Hence it seems safe to con-

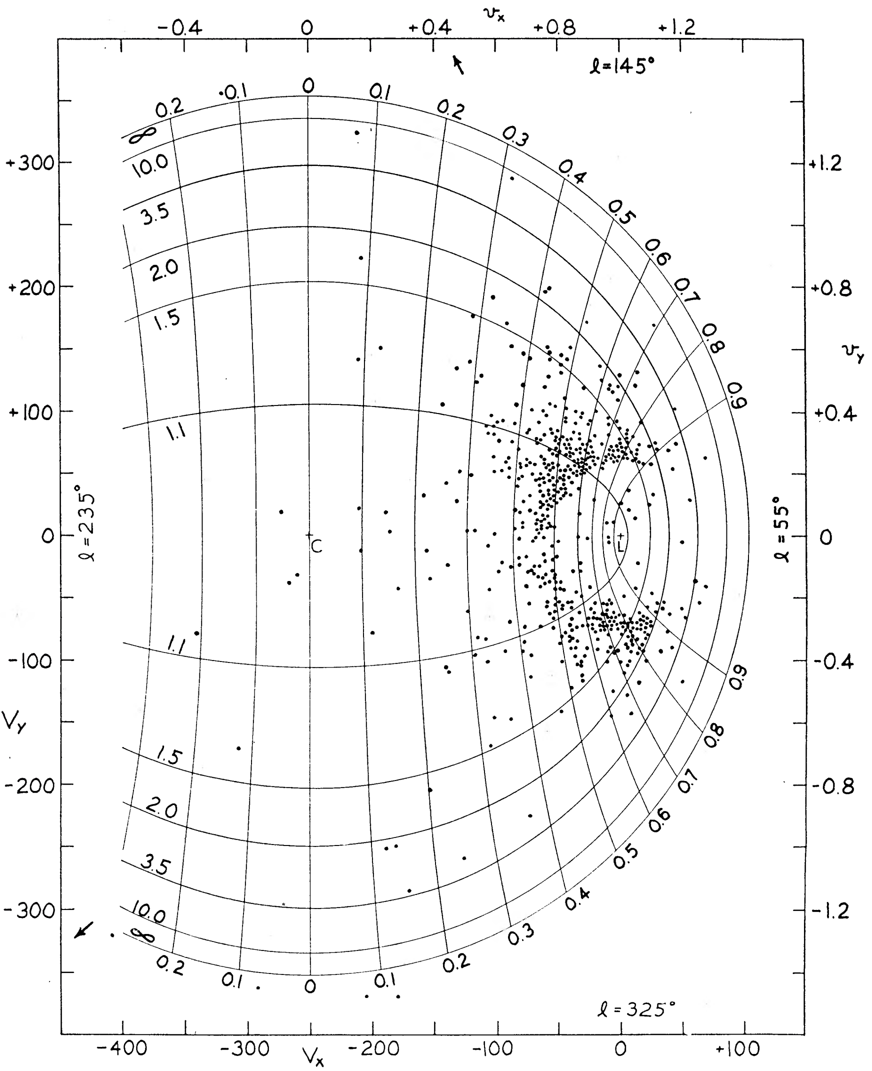

Figure 1. Galactic velocity diagram. Coordinates: velocity components in galactic plane (upwards: away from center, to right: in direction of rotation). Point L: rest in local reference system. Point C: rest relative to center of galaxy. Line CL: circular velocity at sun. Hyperbolas: curves of constant perigalactic distance. Ellipses: curves of constant apogalactic distance. Large circle: escape velocity. Numbers on curves: distances in units of center-to-sun distance. Individual points (including two arrows): 555 high-velocity stars of Miczaika's list (scarcity of points near L is caused by restriction to total velocities larger than 63 km/sec, hence usually component in galactic plane also large). Points to left of vertical line through C: retrograde orbits.

clude that the great majority of the high-velocity stars have orbits which do not penetrate the central bulge of the galaxy. Their origin, therefor, has to be sought most likely outside the central bulge.

Large apogalactic distances. Fig. 1 contains nine points outside the escape circle ($r_a = \infty$). In all nine cases the proper motions contribute greatly to the space velocity and correspondingly the parallaxes enter critically into the results. In none of these cases is the accuracy of the parallax determination sufficient to make the superescape velocity certain. Just inside the escape circle one finds in Fig. 1 nine further points with a $r_a > 3.5$. Here again it seems possible that these stars appear to have so large apogalactic distances only in consequence of inaccuracies in the data. Going to still smaller apogalactic distances, however, one finds in Fig. 1 so many points around $r_a = 2.0$ that it seems hard to escape the conclusion that our galaxy has a fringe of population II stars extending to a distance as much as twice the center-to-sun distance.[11] A detailed investigation of this fringe should, however, not be based mainly on the high-velocity stars here considered, but rather on stars with intermediate velocities since $r_a = 2.0$ can be reached with a velocity as low as 39 km/sec, as shown by Fig. 1.

Small perigalactic distances. Finally, the stars with small perigalactic distances are to be considered, mainly to derive an upper limit to the number of stars which may pass through the central bulge of the galaxy. Table 1 gives the frequency distribution of the perigalactic distances for Miczaika's stars, obtained directly from Fig. 1. This tabulation shows explicitly the small percentage of high-velocity stars near the sun which pass really close to the galactic center.

TABLE 1. FREQUENCY OF PERIGALACTIC DISTANCES FOR MICZAIKA'S HIGH-VELOCITY STARS

Range of r_p	No. of Stars
Direct Orbit	
1.0 to 0.9	27*
0.9 to 0.8	53*
0.8 to 0.7	69*
0.7 to 0.6	58*
0.6 to 0.5	111*
0.5 to 0.4	149
0.4 to 0.3	44
0.3 to 0.2	17
0.2 to 0.1	9
0.1 to 0.0	7
Retrograde Orbit	
0.0 to 0.1	4
0.1 to 0.2	2
0.2 to 0.3	3
0.3 to 0.4	0
0.4 to 0.5	2

* Incomplete because of limitation to $|V| > 63$ km/sec.

By analogy with the Andromeda Nebula it appears unlikely that the central bulge of our galaxy exceeds 1500 psc in radius. Hence $r_p = 0.2$ should be an ample upper limit for stars that might penetrate into the central bulge. There are 22 stars in Table 1 with $r_p < 0.2$. In addition, there are 5 stars with larger r_p values, but apparently retrograde orbits. It seems far from certain that these five unusual orbits are real, and one may expect that they appear only in consequence of errors in the data which put their points too far to the left in Fig. 1. Adding these 5 stars to the 22 stars with $r_p < 0.2$, one has 27 stars, or 5 per cent of 555, which may possibly pass through the central bulge. Among these stars will be several which have actually larger r_p values than here derived, owing to their appreciable z-velocities. Furthermore, errors in the data will more often reduce than increase r_p values because of the steep drop in frequency with decreasing r_p. Altogether, therefore, one may accept as a safe upper limit that at most 5 per cent of the high-velocity stars near the sun (with $|V| > 63$ km/sec) can possibly penetrate the central bulge of the galaxy.

The last result may be amplified by considering the types of the above 27 stars. Among them only one red giant is found, the rest are dwarfs or subdwarfs. The scarcity of giants among the stars of extreme velocities has been pointed out previously.[12] The distinction between dwarfs and subdwarfs among these stars has been very uncertain thus far. On the one hand, trigonometric parallaxes are of sufficient accuracy for this distinction only in a very few cases here considered. On the other hand, the spectroscopic parallaxes from the Mt. Wilson catalogue [13] were derived before the distinction between dwarfs and subdwarfs, except for A stars, was established. However, recently Dr. Joy has kindly reviewed the available spectra of the above 27 stars at the Mt. Wilson Observatory and has found that about two-thirds of them are probably subdwarfs. This change in classification has two consequences. First, the spectroscopic parallaxes in question will be increased and the space velocities, as far as they are based on spectroscopic parallaxes, will be decreased. Second, the classification change indicates that most of the exceptional orbits which may—possibly, though not certainly—penetrate the central bulge, belong to subdwarfs.[14]

Conclusion. In summarizing, one may conclude that by far the majority of the high-velocity stars near the sun, as represented by Miczaika's list, move in orbits which do not pass through the central bulge of the galaxy. Consequently, it appears that these stars must have originated not in the central bulge, but farther out in the galaxy.

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REFERENCES

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2. K. F. Bottlinger, *Veröff. Berlin-Babelsberg*, **10**, No. 2 (particularly Fig. 5), 1933.