

THE SPECTRA AND ORBITS OF AR LACERTAE

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ABSTRACT

New elements of the spectroscopic binary AR Lacertae differ but slightly from Harper's. The *absorption lines* of the secondary star are wide and shallow directly preceding and following primary minimum. At other phases the profiles more nearly resemble those of the primary. The absorption lines of the primary are deeper than those of the secondary except in the phase interval 0.289–0.637 day. These variations of width and of percentage absorption preceding and following primary eclipse point to the importance of photometric observations for a considerable interval before and after primary eclipse as well as within it. Both stars have H and K in emission. These lines are not monochromatic, being 168 and 78 km/sec wide for secondary and primary, respectively. These widths give radii for the two stars that are 85 per cent of the values found photometrically with strong limb darkening. The average intensity of the emission H and K lines of the secondary, relative to those of the primary, is 3.8. Velocities from these emission lines average 6.4 km/sec less algebraically than the velocities from the absorption lines.

Photoelectric observations of the eclipsing binary AR Lacertae by F. B. Wood¹ showed that it varies from magnitude 7.31 at maximum to 8.20 when the primary star is totally eclipsed. He concluded also that the primary star was itself a variable. The spectral class of the primary is G5, that of the secondary K0, according to A. B. Wyse,² who found the H and K lines to be bright at primary minimum (total eclipse). G. E. Kron, in discussing his photoelectric observations,³ commented as follows on the variability of the G5 component:

The peculiar variations in the light curves can be explained on the hypothesis that the surface of the G5 star has upon it huge light and dark patches. . . . The patches must be of such size that a projected hemisphere at times has as much as 20 percent of its area covered, while the individual patches cover as much as 3 to 5 percent of the area. The data not only indicate that the patches are eclipsed by the K0 star but that they move, and that they form and dissolve. Furthermore, they appear and disappear around the limb, with rotation of the binary. Clearly, the phenomena are exceedingly complicated, and difficult to analyze precisely, particularly in view of the unfavorable period of rotation of the binary.

W. E. Harper⁴ has derived orbital elements from spectrograms obtained at the Dominion Astrophysical Observatory.

In 1945 I began to observe this variable with the coudé spectrograph (10 Å/mm), at first, primarily, for the bright H and K lines at total eclipse. It was soon evident, however, that these emission lines were always present outside, as well as within, eclipse; moreover, two components were observed, one belonging to the primary star, the other to the secondary. Table 1 is the journal of the 22 spectrograms obtained from 1945 to 1949 inclusive. The phases in Table 1 are derived from the formula

$$\text{Primary min.} = \text{JD } 2426624.338 + 1.98321525E \text{ (G.M.T.)}.$$

The epoch is Wood's and the period is Kron's, which he was kind enough to send me by letter.

The absorption lines of both components are present, well separated between eclipses but, of course, blended during eclipse. The lines are shallow and wide for stars of such advanced spectra, undoubtedly being widened by axial rotation of the same period as the orbital revolution. This fuzziness of the lines prevents the usual accuracy of measurement

¹ *Contr. Princeton. U. Obs.*, No. 21, 1946.³ *Pub. A.S.P.*, 59, 264, 1947.² *Lick Obs. Bull.*, 17, 37, 1934.⁴ *J.R.A.S. Canada*, 27, 146, 1933.

for stars of such late spectral type. Nevertheless, as will be seen later, velocities from the mean of several lines are quite consistent. They indicate essentially circular orbits, and therefore only γ , K1, and K2 were adjusted by the method of least squares. My corrected elements and the corresponding ones previously derived by Harper are given in Table 2. Although the lack of agreement in the values of γ , K1, and K2 is somewhat disappointing, it can be accounted for in part at least by the probable errors of these quantities. In any event, my values of $a \sin i$ and $m \sin^3 i$ and of the mass ratios differ but slightly from those found by Harper. The absorption velocities of Table 2 and the velocity-curves from my elements are shown in Figure 1.

TABLE 1
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PLATE No. CE	DATE	JD 243	PHASE (DAYS)	RADIAL VELOCITY (KM/SEC)					
				Primary		Blended		Secondary	
				Abs.	Em.	Abs.	Em.	Abs.	Em.
3825.....	1945 June 16	1623.964	1.924	— 0.1				— 59.8	— 58.8
3831.....	17	1624.975	0.951			—48.4	—18.4		
3865.....	July 18	1655.906	0.151	— 86.5	— 98.7			+ 30.4	+ 21.0
3871.....	19	1656.905	1.150	+ 16.7	+ 7.7			— 88.9	— 95.0
3904.....	Aug. 19	1687.969	0.482	—151.8	—152.2			+ 66.4	+ 69.6
3970.....	Sept. 17	1716.818	1.566	+ 82.2				—142.8	—145.0
3978.....	18	1717.860	0.625	—143.0	—144.9			+ 75.3	+ 53.2
4285.....	1946 June 11	1983.903	0.917			—52.2	—22.9		
4343.....	July 20	2022.981	0.331	—134.4	—144.3			+ 69.6	+ 56.0
4351.....	Aug. 9	2042.889	0.407	—142.1	—149.3			+ 74.4	+ 64.8
4357.....	10	2043.988	1.506	+ 83.7	+ 81.8			—145.4	—142.7
4501.....	Dec. 13	2168.647	1.222	+ 38.8	+ 43.7			—110.2	—104.6
4504.....	14	2169.650	0.242	—121.7	—119.5			+ 51.2	+ 33.8
4871.....	1947 Sept. 26	2455.858	0.867			—72.6			
4876.....	27	2456.840	1.849	+ 13.6	— 10.8			— 84.7	— 80.6
4961.....	Nov. 2	2492.683	0.011			—31.5	—22.6		
4983.....	Dec. 2	2522.683	0.263	—120.6				+ 51.3	
5282.....	1948 Aug. 17	2781.882	1.644	+ 68.8	+ 67.8			—123.2	—130.6
5286.....	18	2782.858	0.636	—141.4	—141.5			+ 73.1	+ 58.4
5404.....	Oct. 24	2849.702	0.051			—29.8	—18.3		
5408.....	25	2850.715	1.064			—42.1			
5453.....	Nov. 23	2879.688	0.289	—123.4	—131.7			+ 58.7	+ 49.2

Although the absorption lines of the two stellar components of AR Lac are much broadened by axial rotation and their equivalent widths consequently difficult to measure, the mean percentage absorption at the line centers should have significance in determining the relation between the absorption lines of the two stellar components. The three $Fe\ I$ lines $\lambda\lambda$ 4045, 4063, and 4071 were chosen for this mean because they are so much stronger than near-by lines that blends would not be serious. Those spectrograms with the absorption lines of both components well separated have been traced with the microphotometer and the mean percentage absorption obtained for the three $Fe\ I$ lines. These means were then corrected to the continuous spectrum to which each belonged, by assuming that 54.3 per cent of the total light of this variable is furnished by the primary and 45.7 per cent by the secondary, as given by Wood.¹ The density of the continuous spectrum to which these measures are referred can affect the accuracy of the percentage absorption on the various spectrograms much more than it can the ratios of these percentage absorptions. Hence only these ratios are given in Table 3.

It is apparent that the ratios average less for the interval between primary and secondary minimum (0.242–0.673 day) than for the interval between secondary and primary minimum (1.151–1.849 days). Moreover, these ratios have a somewhat orderly progression, with a minimum near phase 0.5 day and a maximum near phase 1.5 days. The lines of the primary have, therefore, their relatively strongest absorption when the primary is receding fastest and their relatively weakest absorption when the primary is approaching fastest.

Since the widths of the absorption lines of AR Lac are primarily caused by axial rotation, the ratios of the widths of the lines of the primary to those of the secondary

TABLE 2
ORBITAL ELEMENTS OF AR LACERTAE

Element	Sanford	Harper
<i>P</i>	1.98321525 days	1.983244 days
<i>T</i> (Prim. min.) ..	JD 2426624.338 G.M.T.	JD 2426624.377 G.M.T.
<i>K</i> ₁	116.1 km/sec	120.0 km/sec
<i>K</i> ₂	115.6 km/sec	119.2 km/sec
<i>γ</i>	– 33.7 km/sec	– 36.25 km/sec
<i>a</i> ₁ sin <i>i</i>	3.167 × 10 ⁶ km	3.263 × 10 ⁶ km
<i>a</i> ₂ sin <i>i</i>	3.151 × 10 ⁶ km	3.247 × 10 ⁶ km
<i>m</i> ₁ sin ³ <i>i</i>	1.29 ⊙	1.39 ⊙
<i>m</i> ₂ sin ³ <i>i</i>	1.30 ⊙	1.40 ⊙
<i>m</i> ₁ / <i>m</i> ₂	0.995	0.993

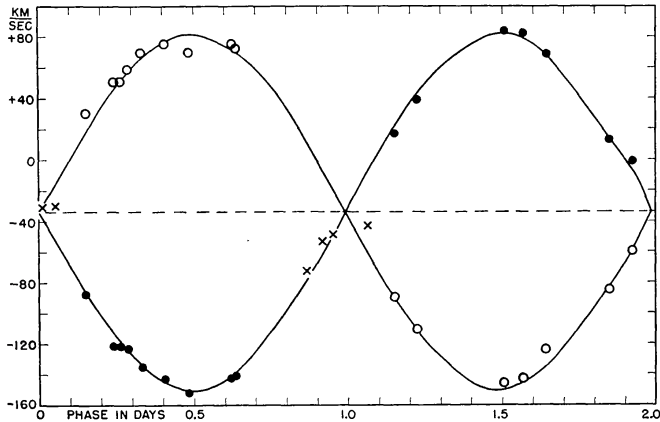


FIG. 1.—Velocity-curves of AR Lacertae. Dots represent the velocities of the primary component; circles, those of the secondary; crosses, velocities derived from blended lines. See Table 1. The velocity-curves and the γ -line are from the elements in Table 2.

should be close to the ratio of the respective radii, or, according to Wood, $1.82/2.99 = 0.61$. Therefore, the ratios of the mean equivalent widths of the three *Fe* I lines should be roughly 61 per cent of the ratios in the third column of Table 3. This equivalent-width ratio varies from 0.40 to 0.80, approximately.

Six sample tracings are reproduced in Figure 2. The spectral region contains the three *Fe* I lines $\lambda\lambda$ 4045, 4063, and 4071. The first tracing has the profiles of the secondary star only, since the primary is in total eclipse. The second and third tracings belong to the first half-period, i.e., when the primary is approaching. The fourth is a tracing of a spectrogram obtained near annular eclipse of the secondary star. The last two tracings have the profiles of the lines of the two stellar components when the primary is receding.

The profiles of the secondary's lines on Ce 4504 are in strong contrast with profiles on Ce 3904. There is a similar contrast of flat, shallow profiles against relatively deep, sharp-bottomed ones between Ce 4357 and Ce 4501. An examination of all suitable tracings shows that flat, shallow profiles characterize the two intervals 1.506–1.849 and 0.152–0.242 days, i.e., preceding and following primary minimum. Kron⁵ has called attention to "low portions of the curve just preceding and following primary minimum" in his light-curve for 1946. Kron's curve is for 1946 only; my spectrograms extend from 1945 to 1948, inclusive. Hence it would be important to obtain more observations of both light and spectrum within phases 1.5–0.25 days.

The heating of the stellar atmospheres by bombardment of interstellar or circum-stellar particles might account in part for the foregoing observations. Whichever star is approaching would be overtaking these particles and hence would be hit by more than the average number of particles and with higher-than-average velocities of impact than

TABLE 3
PHOTOMETRIC DATA OF ABSORPTION AND EMISSION LINES OF AR LAC

PLATE CE	PHASE (DAYS)	RATIO OF PER CENT ABSORPTION PRIM.: SEC. (MEAN FOR $\lambda\lambda$ 4045, 4063, 4071)	E.W. OF EMIS. (H+K)		
			Prim. (A)	Sec. (A)	Ratio Sec.: Prim.
4504.....	0.242	1.00			
4983.....	0.263	1.00			
5453.....	0.289	0.83	0.40	0.98	2.45
4343.....	0.331	0.71			
4351.....	0.407	0.85	.33	1.12	3.40
3904.....	0.483	0.82	.28	1.20	4.30
3978.....	0.625	0.67			
5286.....	0.637	0.75	.29	1.20	4.10
3871.....	1.151	0.90			
4501.....	1.223	0.90	.16	0.86	5.40
4357.....	1.506	1.38	.22	0.88	4.00
3970.....	1.566	1.21			
5282.....	1.644	1.37	0.29	0.92	3.20
4876.....	1.849	0.86			
Mean.....					3.84

for recession. When a component is approaching, therefore, the temperature of its atmosphere on the visible side would be higher than average; when it is receding, lower than average. Higher temperature would mean an earlier-than-average spectral type and consequently weaker-than-average *Fe I* lines.

Each component of AR Lac has emission lines H and K of *Ca II*. Their equivalent widths were measured upon microphotometer tracings of seven spectrograms of AR Lac which have complete separation of the lines of the two component stars. The fourth and fifth columns of Table 3 give the sums of the equivalent widths of H and K for primary and secondary, respectively, and the sixth column gives their ratio. The mean of these ratios is 3.8. The ratios increase progressively for the four spectrograms in the first half-period and decrease for the three in the second half-period. Four tracings of the H and K region (top of Fig. 2) illustrate the profiles of these emission lines. The first and third typify primary and secondary minimum, respectively; the second tracing shows the profiles of these lines when the primary is approaching, the fourth when it is receding.

These emission lines are not monochromatic, the average width being 168 and 78 km/sec for secondary and primary, respectively. If it is assumed that emission H and K

⁵ *Op. cit.*, p. 262.

lines come from all parts of the stellar surfaces or at least from complete equatorial zones, then the half-widths of these emission lines should be a simple function of the respective rotational velocities. The radii of primary and secondary stars are 1.54 and 2.52 solar radii, upon the assumption that the periods of axial and orbital rotation are the same. These radii are about 85 per cent of those obtained by Wood, who assumed heavy limb darkening.

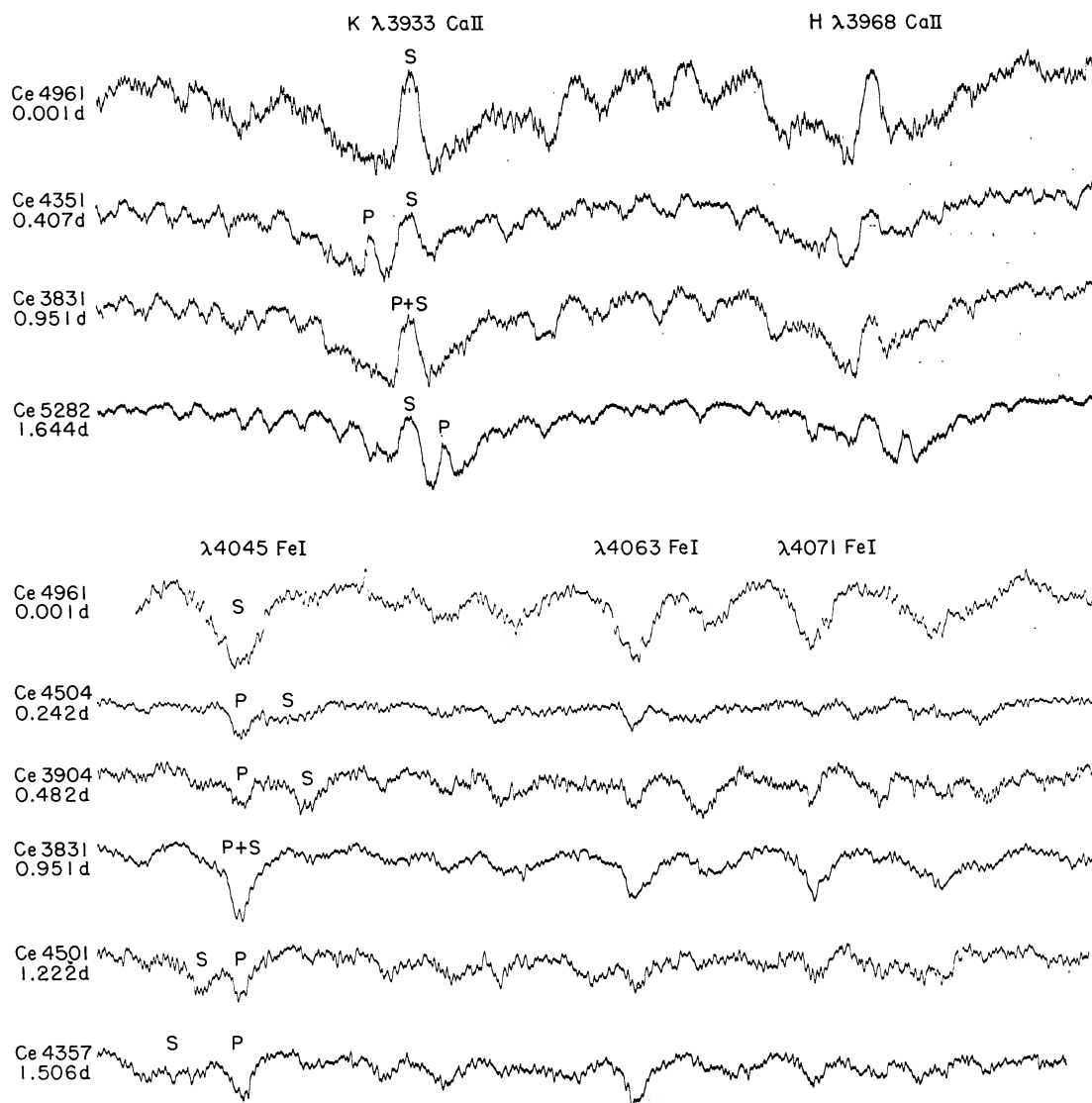


FIG. 2.—Microphotometer tracings of four spectrograms of AR Lacertae in the H and K region are at the top. The K emissions of the primary and secondary stars are identified by *P* and *S*; the H emissions have the same pattern. The six tracings in the lower group cover the region of the strong absorption Fe I lines $\lambda\lambda$ 4045, 4063, and 4071.

The velocities derived from the emission lines of H and K are, in the mean (28 comparisons), algebraically less by 6.4 km/sec than the velocities from all measured absorption lines of the corresponding components. This velocity difference may, of course, result from an outward flow of the source of the emission H and K lines.