THE PREDICTABILITY OF THE PROBABLE FEATURES OF THE SUNSPOT CYCLES

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ABSTRACT

This paper contains the description of a procedure by which predictions of solar activity with a given probability are obtained. By application of this procedure it is shown that the author's method of predicting the probable features of the sunspot cycles, which has led to successful predictions for the present sunspot cycle, would have yielded satisfactory results also for the last two cycles. This confirms the reliability of this method. It is pointed out that the "most probable" values used by H. T. Stetson for his criticism of the author's method do not yield any argument against this method.

In the September, 1942, issue of this *Journal*[1] I published probability laws of sunspot variations which permit us to predict the probable features of a sunspot cycle from the characteristics of the preceding cycles. The application of this method to the then next sunspot cycle, the eighteenth of the Zürich statistics, led to the prediction that, with a very high degree of probability, a steep ascent of solar activity to a maximum of great intensity was to be expected.[1, 2, 3, 4, 5, 6] In the meantime the maximum of cycle 18 has passed, and we know that my prediction has come true; in fact, we experienced in 1947 the second highest maximum of sunspot history.

There can be no doubt that *one* successful prediction is not sufficient to prove that the method which has led to this prediction is a reliable one. In this respect it would be of interest to learn whether satisfactory results would also have been obtained by this method for past cycles. I therefore suppose here that my method was known before the beginning of cycle 16 and that it would have been applied, first, to cycle 16 and later on to cycle 17. The predictions which, by this supposition, are obtained for cycles 16 and 17 can be compared with the actually observed features of these cycles; thus we shall be able to get a better insight into the reliability of my method of predicting solar activity.

The prediction of the probable features of a sunspot cycle requires the knowledge of the characteristics of the four preceding cycles. For our purpose, therefore, we are in need of the values of the characteristics of cycles 12–15 and 13–16, respectively. These data can be taken from the tables of the characteristics of the sunspot cycles which I have published in some previous papers,[1, 7, 8, 9]

By the same reasoning that led to my predictions for cycle 18 and which has been explained in detail in two earlier papers[2, 10] the following equations can easily be deduced

[1] *Ap. J.*, **96**, 234, 1942.

[2] *Pub. Istanbul U. Obs.*, No. 15, 1941.

[3] *Terr. Mag. and Atm. Elec.*, **48**, 243, 1943.

[4] *Nature*, **156**, 539, 1945.

[5] *Bull. Amer. Math. Soc.*, **26**, 351, 1945.

[6] *Pub. Instanbul U. Obs.*, No. 29, 1945.

[7] *Observatory*, **63**, 215, 1940.

[8] *Ibid.*, **65**, 24, 1943.

[9] *Zs. f. Ap.*, **22**, 376, 1944.

[10] *Pub. Istanbul U. Obs.*, No. 22, 1943.

for cycle 16 from the probability laws of sunspot variations:[1]

$$\text{prob}\ [t_r < m] = 1 - \text{erf}\,(1.88 - 0.04m)\,, \tag{1}$$

$$\text{prob}\ [R_M > n] = \frac{1}{2} + \frac{1}{2}\sum_{d=0}^{\infty} P(d)\,\text{erf}\,(0.36d + 1.66 - 0.018n)\,, \tag{2}$$

$$\text{prob}\ [t_l < p] = \frac{1}{2} + \frac{1}{2}\sum_{d=0}^{\infty} P(d)\,\text{erf}\,(0.36d - 2.12 + 0.036p)\,, \tag{3}$$

$$\text{prob}\ [t_f > q] = \frac{1}{2} + \frac{1}{2}\sum_{d=0}^{\infty} P(d)\,\text{erf}\,(0.36d + 2.92 - 0.072q)\,. \tag{4}$$

Here m, n, p, and q denote positive numbers which can be chosen arbitrarily, the symbol "prob" means the probability that the inequality which follows this symbol in brackets will be fulfilled, and the other notations are the same as those used in some of my previous papers.[1, 2, 10]

By means of a table of the error function and using the table of $P(d)$ which I have published,[1] we can compute the probabilities on the left-hand side of equations (1), (2), (3), and (4) for any given value of m, n, p, and q, respectively. This is the method which I used for cycle 18 in my previous papers. It has the disadvantage of yielding predictions with different degrees of probability. Instead of choosing m, n, p, and q and computing the corresponding probabilities, it is better to change the procedure so that we can determine the values of m, n, p, and q in such a way that the probabilities on the left-hand side of equations (1)–(4) take prescribed values. In this case it is possible to make predictions with a given degree of probability.

The question of how to choose a suitable degree of probability for our predictions presents itself. If the probability that we choose is too small, the percentage of failing predictions will be too high; but if we choose a too high probability, only trivial and non-interesting predictions can be obtained. I believe the choice of a probability of 0.90 will be suitable for our purpose; in this case only one out of ten predictions, on the average, will fail, and such a small percentage of failures seems to me admissible.

If we put prob $[t_r < m] = 0.90$, equation (1) yields

$$\text{erf}\,(1.88 - 0.04m) = 0.10\,.$$

Since erf $(0.09) = 0.10$, we have

$$1.88 - 0.04m = 0.09\,,$$

or

$$m = 45\,.$$

Hence, for cycle 16,

$$\text{prob}\ [t_r < 45] = 0.90\,. \tag{5}$$

Let us put

$$\frac{1}{2} + \frac{1}{2}\sum_{d=0}^{\infty} P(d)\,\text{erf}\,(0.36d + x) = W(x)\,.$$

Then equations (2), (3), and (4) reduce to

$$\text{prob}\ [R_M > n] = W(1.66 - 0.018n)\,, \tag{6}$$

$$\text{prob}\ [t_l < p] = W(0.036p - 2.12)\,, \tag{7}$$

$$\text{prob}\ [t_f > q] = W(2.92 - 0.072q)\,. \tag{8}$$

I have computed the function $W(x)$ for some values of x. The results are given in Table 1. This table can always be used whenever predictions with a given degree of probability are to be derived from the probability laws of sunspot variations. From Table 1

TABLE 1

TABLE FOR PREDICTING SOLAR ACTIVITY WITH GIVEN PROBABILITY

x	$W(x)$	x	$W(x)$	x	$W(x)$	x	$W(x)$
+0.90	0.98	−0.36	0.77	−1.62	0.35	−2.88	0.10
+ .72	.97	−0.54	.71	−1.80	.30	−3.06	.08
+ .54	.95	−0.72	.65	−1.98	.26	−3.24	.06
+ .36	.93	−0.90	.59	−2.16	.22	−3.42	.05
+ .18	.90	−1.08	.53	−2.34	.18	−3.60	.04
.00	.86	−1.26	.46	−2.52	.15	−3.78	.03
−0.18	0.82	−1.44	0.40	−2.70	0.12	−3.96	0.02

we have $W(0.18) = 0.90$. Thus equations (6), (7), and (8) yield

$$1.66 - 0.018n = 0.18 \; ; \quad \text{hence} \quad n = 82 \; ,$$

$$0.036p - 2.12 = 0.18 \; ; \quad \text{hence} \quad p = 64 \; ,$$

$$2.92 - 0.072q = 0.18 \; ; \quad \text{hence} \quad q = 38 \; .$$

Thus, for cycle 16,

$$\text{prob}\,[R_M > 82] = 0.90 \; , \tag{9}$$

$$\text{prob}\,[t_l < 64] = 0.90 \; , \tag{10}$$

$$\text{prob}\,[t_f > 38] = 0.90 \; . \tag{11}$$

It is easy, of course, to obtain predictions having another probability. In this case we have to take from Table 1, if necessary by interpolation, that value of x which corresponds to the given probability $W(x)$, and then we have to put the arguments of the W-function on the right-hand side of equations (6), (7), and (8) equal to that value of x. For the prediction of t_r with any given probability, equation (1) can be used immediately.

Before we compare the values predicted for cycle 16 with the observed ones, let us first establish the predictions for cycle 17. Afterward the comparison will be carried out for both cycles together. Using the values of the characteristics of cycles 13–16 as published in previous papers,[1, 7, 8, 9] we obtain, for cycle 17, the following equations:

$$\text{prob}\;[t_r < m] = 1 - \text{erf}\,(1.44 - 0.04m)\,, \tag{1*}$$

$$\text{prob}\;[R_M > n] = \frac{1}{2} + \frac{1}{2}\sum_{d=0}^{\infty} P(d)\,\text{erf}\,(0.36d + 2.20 - 0.018n)\,, \tag{2*}$$

$$\text{prob}\;[t_l < p] = \frac{1}{2} + \frac{1}{2}\sum_{d=0}^{\infty} P(d)\,\text{erf}\,(0.36d - 2.20 + 0.036p)\,, \tag{3*}$$

$$\text{prob}\;[t_f > q] = \frac{1}{2} + \frac{1}{2}\sum_{d=0}^{\infty} P(d)\,\text{erf}\,(0.36d + 4.50 - 0.072q)\,. \tag{4*}$$

Here the notations are the same as those in the corresponding equations (1)–(4) for cycle 16. If we again choose a probability of 0.90, it follows from equation (1*) that

$$\operatorname{erf}(1.44 - 0.04m) = 0.10 ;$$

hence

$$1.44 - 0.04m = 0.09 ,$$

which gives

$$m = 34 .$$

Hence, for cycle 17,

$$\text{prob}\,[t_r < 34] = 0.90 . \tag{5*}$$

Introducing $W(x)$ as above, we can write equations (2*), (3*), and (4*) in the following form:

$$\text{prob}\,[R_M > n] = W(2.20 - 0.018n) , \tag{6*}$$

$$\text{prob}\,(t_l < p] = W(0.036p - 2.20) , \tag{7*}$$

$$\text{prob}\,[t_f > q] = W(4.50 - 0.072q) . \tag{8*}$$

From Table 1 we again have $W(0.18) = 0.90$. Equations (6*), (7*), and (8*), therefore, yield

$$2.20 - 0.018n = 0.18 ; \quad \text{hence} \quad n = 112 ,$$

$$0.036p - 2.20 = 0.18 ; \quad \text{hence} \quad p = 66 ,$$

$$4.50 - 0.072q = 0.18 ; \quad \text{hence} \quad q = 60 .$$

Thus, for cycle 17,

$$\text{prob}\,[R_M > 112] = 0.90 , \tag{9*}$$

$$\text{prob}\,[t_l < 66] = 0.90 , \tag{10*}$$

$$\text{prob}\,[t_f > 60] = 0.90 . \tag{11*}$$

We now proceed to compare the predictions which we have obtained for cycles 16 and 17 with the observed characteristics of these cycles. For convenience I have summarized the predictions as expressed by equations (5), (9), (10), and (11) for cycle 16 and by equations (5*), (9*), (10*), and (11*) for cycle 17 in Table 2, together with the observed values of the characteristics of these cycles, which have been taken from previous publications.[1, 7, 8, 9] The comparison between prediction and observation shows that all predictions would have been true, with one exception: the maximum of cycle 16 was a little lower than expected. This means that one out of the eight predictions which could have been made for cycles 16 and 17 would have failed. This result is not very different from the percentage of failures expected theoretically; for, as stated above, in a sufficiently long series of predictions computed with a probability of 0.90, one out of ten predictions on the average, should fail.

We can therefore draw the following conclusion from the above calculations: The reliability of my method of predicting solar activity, which had been indicated by its success for cycle 18, is strongly confirmed by the results of its application to cycles 16 and 17

In his recent discussion of my method of predicting solar activity, H. T. Stetson[11] lays stress upon the fact that, if my method had been used for a prediction of the height of the last two maxima, the "most probable" values of the highest smoothed relative numbers would have been far above their actual values. This discrepancy, however, is not, as Stetson's statement might lead one to think, a sign of the unreliability of my method. The apparent contradiction between Stetson's statement and the calculations given above for the last two cycles arises merely from the misleading term, "most probable value." By this term is really meant the value for which the probability of being (or of not being) exceeded by the actually observed value is 0.5. This means only that, in a sufficiently long series of most probable values, half will be above, and the other half below, the actual values. But it does not mean, of course, that in such a series the most probable values are alternately larger and smaller than the observed ones. Thus, from the fact that in two or three consecutive sunspot cycles the observed height of the maximum lies below its most probable value, not the slightest objection can be raised against my method of predicting solar activity.

TABLE 2 See p. 555.

PREDICTED AND OBSERVED VALUES OF THE CHARACTERISTICS OF CYCLES 16 AND 17

Characteristic	Cycle 16		Cycle 17		Characteristic	Cycle 16		Cycle 17	
	Predicted	Observed	Predicted	Observed		Predicted	Observed	Predicted	Observed
t_r	45	43	34	23	t_l	64	39	66	45
R_M	82	78.1	112	119.2	t_f	38	40	60	63

To clear up this point completely, I have computed the most probable values of all the characteristics of cycles 16 and 17. For this calculation one needs those values of x for which erf $x = 0.50$ and $W(x) = 0.50$, respectively. Since erf $(0.48) = 0.50$ and, according to Table 1, $W(-1.16) = 0.50$, we have, from equations (1), (6), (7), and (8) for cycle 16,

$$1.88 - 0.04m = 0.48\,,$$

$$1.66 - 0.018n = -1.16\,,$$

$$0.036p - 2.12 = -1.16\,,$$

$$2.92 - 0.072q = -1.16\,;$$

ikewise we have, from equations (1*), (6*), (7*), and (8*) for cycle 17,

$$1.44 - 0.04m = 0.48\,,$$

$$2.20 - 0.018n = -1.16\,,$$

$$0.036p - 2.20 = -1.16\,,$$

$$4.50 - 0.072q = -1.16\,.$$

[11] *Sunspots in Action* (New York: Ronald Press Co., 1947), pp. 154 ff. and 216.

These eight equations yield the following most probable values (in parentheses the observed values as given in Table 2 are repeated):

for cycle 16:

$$t_r = 35 \quad (43), \qquad R_M = 157 \quad (78.1),$$

$$t_l = 27 \quad (39), \qquad t_f = 57 \quad (40);$$

for cycle 17:

$$t_r = 24 \quad (23), \qquad R_M = 187 \quad (119.2),$$

$$t_l = 29 \quad (45), \qquad t_f = 79 \quad (63).$$

These data show that in five cases the most probable values are higher than the observed ones and in three cases the observed values exceed the most probable ones. As this result differs very little from what we should expect, the most probable values of the characteristics of the last two sunspot cycles do not at all contradict the conclusion drawn above, viz., that the reliability of my method of predicting solar activity is confirmed by the results obtained by its application to sunspot cycles 16 and 17.

I am indebted to Mr. Herbert Luft, of New York, for having prepared excerpts from chapters xi and xv of Dr. Stetson's book, of which no copy has hitherto been available in this country.