

ON THE SUN'S GENERAL MAGNETIC FIELD

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Summary

The time of electromagnetic decay of the Sun's general magnetic field is shown to be 10^{10} years, suggesting that the field may be a relic from a different primeval state. An alternative hypothesis, ascribing the field to thermal effects due to convection in the rotating Sun, is shown to give a field of the correct sign, but less than 10^{-7} times the observed field; and a modification of the hypothesis, supposing that the field due to thermal currents is magnified by dynamo action, is shown to demand over 10^{18} years for the field to grow to its present size. Other possible explanations of the Sun's field are suggested.

In an Appendix, the effect of the Sun's rotation on turbulence in its interior is briefly considered.

1. Our knowledge of solar magnetic fields is almost entirely derived from the work of Hale and his collaborators at Mount Wilson. By observations of polarization at the wings of spectral lines, these workers identified a general magnetic field of the Sun, similar in many respects to the Earth's magnetic field.* The field-strength is of order (say) 25 gauss; the axis of the field is inclined to the axis of rotation of the Sun at about 6° , and rotates round it in about $31\frac{1}{2}$ days. Different spectral lines indicate different intensities for the field; weak lines suggest larger fields than strong lines. Since the latter probably originate at greater heights in the Sun's atmosphere than the former, these results were interpreted as implying that the field falls off rapidly with increasing height.

The average observed separation of the Zeeman components is of the same order as the probable error of a single measurement. Because of its smallness, its reality is sometimes doubted. The present author sees no reason to share these doubts. To ensure that personal bias should not colour the results, the original observer measuring the Zeeman displacements was kept in ignorance of the hemisphere and latitude of the observations. Thus if he were prone to imagine a non-existent phenomenon, he would give it sometimes one sign, sometimes the other, and in the aggregate his estimates would cancel out. Since, in spite of this, regular results were obtained, the phenomenon being observed was clearly not imaginary. In fact, though all observers have not been able to identify real Zeeman displacements, all those who have been able to identify them have found effects with the same sign.

Moreover, no convincing explanation of the observations has been given, other than the existence of a magnetic field. The phenomenon requiring explanation is a difference in polarization between the two wings of a spectral line. Effects producing a pure displacement or pure broadening of the line cannot explain the polarization; no one has yet suggested a satisfactory alternative to the explanation in terms of a magnetic field.

We may, however, doubt whether the field does fall off rapidly with increasing height, as originally suggested. Though all observers have agreed regarding the sign

* Hale, *Ap. J.*, **38**, 93, 1913; Hale, Seares, van Maanen and Ellerman, *Ap. J.*, **47**, 206, 1918; Seares, van Maanen and Ellerman, *Proc. Nat. Acad. Sci.*, **4**, 4, 1918, and **5**, 242, 1919; Langer, *P.A.S.P.*, **48**, 208, 1936; H. D. Babcock, *P.A.S.P.*, **53**, 237, 1941.

of the magnetic field, they have not agreed well regarding its magnitude; the magnitude observed seems to have depended to a large extent on the observer and, probably, on the conditions under which he has been observing. The difference between the results for strong and weak lines certainly need not be real.

The first explanation proposed for the field was that it arises from the rotation of the solar material, supposed to be electrically charged. This was examined by Brunt *, who showed that only about 10^{-15} of the Sun's field could be explained in this way. A second explanation was proposed by Larmor †, according to which there is a circulation in meridian planes in the solar interior; electric currents induced in the moving material by reason of the Sun's field are supposed to maintain that field in the same way as the magnetism of a dynamo magnet is maintained by the currents it itself generates. In an earlier paper ‡ I have examined this theory and shown that the mechanism proposed cannot unaided maintain the field. It is not impossible, however, that it may operate to magnify the field due to some other effect. Extremely small speeds of circulation (10^{-6} cm./sec.) are adequate to produce appreciable induced currents.

In this paper, the Sun's field is considered further. An adequate theory of the field is not found; but considerations are advanced which may make clearer the conditions which an adequate theory must satisfy.

2. I have recently considered in detail the electrical conductivity σ of an ionised gas.§ A formula roughly valid in the Sun's interior is

$$\sigma = 1.35 \times 10^{-14} T^{\frac{3}{2}} / \bar{Z}' \quad \text{E.M.U.} \quad (1)$$

In this T is the temperature, and $\bar{Z}'$ is defined as follows. Let n be the number per unit volume of Z -fold ionized atoms in any region; then the mean degree of ionization of the material is defined as

$$\bar{Z} = \sum n Z^2 / \sum n Z, \quad (2)$$

the summations being over all types of ion present. $\bar{Z}'$ is a function of $\bar{Z}$, whose values in terms of $\bar{Z}$ are tabulated in the paper quoted above.|| In what follows, we adopt the value $\bar{Z}=7$, which a rough calculation suggests to be reasonable in the Sun's interior; this gives $\bar{Z}'=2.3$.

The variation of T with the distance from the centre of the Sun is assumed to be that found in a paper by Blanch, Lowan, Marshak and Bethe.¶ Using their results, the following table is obtained, giving σ in terms of $x=r/R$, where r is the distance from the centre and R is the Sun's radius.

TABLE I

x	$10^5 \sigma$ (E.M.U.)	x	$10^5 \sigma$
0	76.5	0.5	7.1
0.05	73.5	0.55	5.3
0.1	62	0.6	3.95
0.15	47.5	0.65	2.9
0.2	36	0.7	2.05
0.25	27	0.75	1.4
0.3	20.5	0.8	0.9
0.35	15.6	0.85	0.5
0.4	12.0	0.9	0.3
0.45	9.2	0.95	0.1

* *A.N.*, 196, 169, 1913.

† *Brit. Ass. Rep.*, p. 159, 1919; *M.N.*, 94, 469, 1934.

‡ *M.N.*, 94, 39, 1934.

§ *Proc. Roy. Soc., A*, 183, 453, 1945. Equation (1) is equation (54) of that paper, with the numerical factor slightly reduced, because the encounter function $A_1(2)$ is rather smaller in the Sun's interior than near the surface.

|| *Loc. cit.* The table quoted is given at the end of section 8 of that paper.

¶ *Ap. J.*, 94, 37, 1941. The data used are taken from Table I of this paper.

These figures enable us roughly to estimate the time of decay of the Sun's magnetic field, assuming it to decay only as a result of electromagnetic effects. According to Lamb *, the maximum time of decay of a magnetic field in a sphere of uniform conductivity σ , and of radius a , is $4\sigma a^2/\pi$. Taking a as half the radius of the Sun, and σ as 10^{-4} E.M.U., this is found to be 5.10^9 years. This is so long a time that it seems desirable to investigate the matter in more detail.

3. In electromagnetic units, the electromagnetic equations for a slowly varying field are

$$\text{curl } \mathbf{H} = 4\pi\mathbf{j}, \quad \text{curl } \mathbf{E} = -\frac{\partial \mathbf{H}}{\partial t}, \quad \mathbf{j} = \sigma\mathbf{E}. \quad (3)$$

Hence

$$4\pi\sigma\frac{\partial \mathbf{E}}{\partial t} = -\text{curl curl } \mathbf{E} = \nabla^2 \mathbf{E}. \quad (4)$$

Suppose that the field decays like an exponential factor $e^{-t/\tau}$, and that it is axially symmetric. Let $\mathbf{r}$ be the (vector) displacement from the centre of the Sun, and $\mathbf{k}$ a unit vector parallel to the axis of symmetry of the field. We assume that $\mathbf{E}$ is of the form

$$\mathbf{E} = (\mathbf{k} \wedge \mathbf{r})f(r)e^{-t/\tau}; \quad (5)$$

this implies that the electric currents flow in circles round the axis of the field, and that the lines of magnetic force are in planes through the axis. Then

$$\nabla^2 \mathbf{E} = (\mathbf{k} \wedge \mathbf{r}) \left(f'' + \frac{4}{r}f' \right) e^{-t/\tau}$$

and so (4) reduces to

$$f'' + \frac{4}{r}f' = -\frac{4\pi\sigma}{\tau}f. \quad (6)$$

This equation is equivalent to the two first-order equations

$$\chi' = -\frac{4\pi\sigma}{\tau}rf, \quad \frac{d}{dr}(r^3f) = r^2\chi. \quad (7)$$

Outside the boundary ($r=R$) of the Sun, $\mathbf{H}$ reduces to a dipole field

$$\mathbf{H} = -\text{grad } (M\mathbf{k} \cdot \mathbf{r}/r^3) = \text{curl } (M\mathbf{k} \wedge \mathbf{r}/r^3).$$

Hence $\mathbf{E}$ is of the form

$$\mathbf{E} = -\frac{M}{\tau r^3} \mathbf{k} \wedge \mathbf{r},$$

i.e. $f(r)$ is proportional to $1/r^3$ outside $r=R$. Thus the boundary condition to be satisfied at $r=R$ is

$$rf' + 3f = 0 \quad (8)$$

or, by the second of equations (7),

$$\chi = 0. \quad (8')$$

It is possible to find a solution of (6), finite at $r=0$, and satisfying (8) at $r=R$, only if τ has one of an infinite set of values. Equation (6) is, in fact, similar in form to the differential equations giving the periods of vibration of a continuous system. We require the least "eigen-value" τ^{-1} of equation (6); this corresponds to the part of the field which decays slowest.

* *Phil. Trans.*, **174**, 519, 1883.

In practice we study (6) and (7) in the non-dimensional forms

$$f'' + \frac{4}{x}f' = -\frac{4\pi\sigma R^2}{\tau}f \equiv -\beta\sigma f, \quad (6')$$

$$\chi' = -\beta\sigma x f, \quad \frac{d}{dx}(x^3 f) = x^2 \chi, \quad (7')$$

where, as before, x denotes the non-dimensional variable r/R , and dashes now denote differentiations with respect to x . The boundary condition is that $\chi=0$ when $x=1$.

4. In solving (6') and (7'), we may use any of the known methods of obtaining the periods of vibration of a continuous system. One of these is based on Rayleigh's principle.* In the present connection, this principle states that the least value of β for which the equation

$$\frac{d}{dx}\left(\frac{1}{\sigma}\chi'x^2\right) = -\beta x^2 \chi \quad (9)$$

(equivalent to (7')) possesses a solution, finite at $x=0$ and vanishing at $x=1$, is the least value of the ratio

$$\int_0^1 \sigma^{-1} \chi'^2 x^2 dx \bigg/ \int_0^1 \chi^2 x^2 dx \quad (10)$$

when χ is a regular function of x in the range $0 \leq x \leq 1$, and vanishes at $x=1$.

Near $x=1$, $\sigma \propto (1-x)^{\frac{3}{2}}$; hence, from (9), $\chi' \propto (1-x)^{\frac{3}{2}}$, $\chi \propto (1-x)^{\frac{5}{2}}$. We therefore assume in turn

$$\begin{aligned} \chi &= A(1-x)^{\frac{5}{2}}, \\ \chi &= A(1-x)^{\frac{5}{2}} + B(1-x)^{\frac{7}{2}}, \\ \chi &= A(1-x)^{\frac{5}{2}} + B(1-x)^{\frac{7}{2}} + C(1-x)^{\frac{9}{2}}, \\ &\dots \end{aligned}$$

adjusting the ratios $A:B:C:\dots$ to make (10) a minimum. This gives reasonably accurate values for β ; the value of χ is, however, not well determined, and even in determining β the numerical work is surprisingly laborious.

A second method, which gives reasonably good values for f and χ as well as β , is that of iteration. Preliminary work with Rayleigh's method suggested that $\chi_1 \equiv (1-x)^4$ is a better first approximation to χ than $(1-x)^{\frac{5}{2}}$. Successive approximations to χ and f are then determined by the equations

$$\begin{aligned} x^3 f_1 &= \int_0^x \chi_1 x^2 dx, & \chi_2 &= \int_x^1 \beta_1 \sigma x f_1 dx, \\ x^3 f_2 &= \int_0^x \chi_2 x^2 dx, & \chi_3 &= \int_x^1 \beta_2 \sigma x f_2 dx, \end{aligned}$$

etc. The quantities $\beta_1, \beta_2, \dots$, which give successive approximations to β , are found by imposing the condition $\chi_r=1$ when $x=0$ for all values of r .

Rayleigh's principle gave successively

$$\beta = 3.2 \times 10^5, \quad 2.2 \times 10^5, \quad 2 \times 10^5, \quad \dots \quad \text{cm.}^2/\text{sec.}$$

The poorness of the first approximation to β is due to the poor representation of the actual χ by $\chi = A(1-x)^{\frac{5}{2}}$. Iteration gave successively

$$\beta = 2.2 \times 10^5, \quad 1.96 \times 10^5, \quad 1.98 \times 10^5, \quad \dots, \quad 2 \times 10^5 \text{ cm.}^2/\text{sec.}$$

* For a recent astrophysical application of this principle, see Pekeris and Ledoux, *Ap. J.*, **94**, 124, 1941.

Taking $\beta = 2 \times 10^5$ cm.²/sec., we find

$$\tau = 4\pi R^2 \beta^{-1} = 3.1 \times 10^{17} \text{ sec.} = 10^{10} \text{ years.}$$

Thus the time of decay of the field is of the order of the time-scale of the expanding universe. That is, if the universe were in a different state some 10^{10} years back, the magnetic field might be a relic of that state.

I do not feel altogether happy in making this suggestion, since the assumption that the magnetic field is explained by some unassigned cause operative a long time ago is, essentially, a confession of ignorance. Moreover, in the above the field has been assumed to decay solely as a result of electromagnetic effects. If the central part of the Sun is in a state of turbulent motion, this must affect the field. Convected material

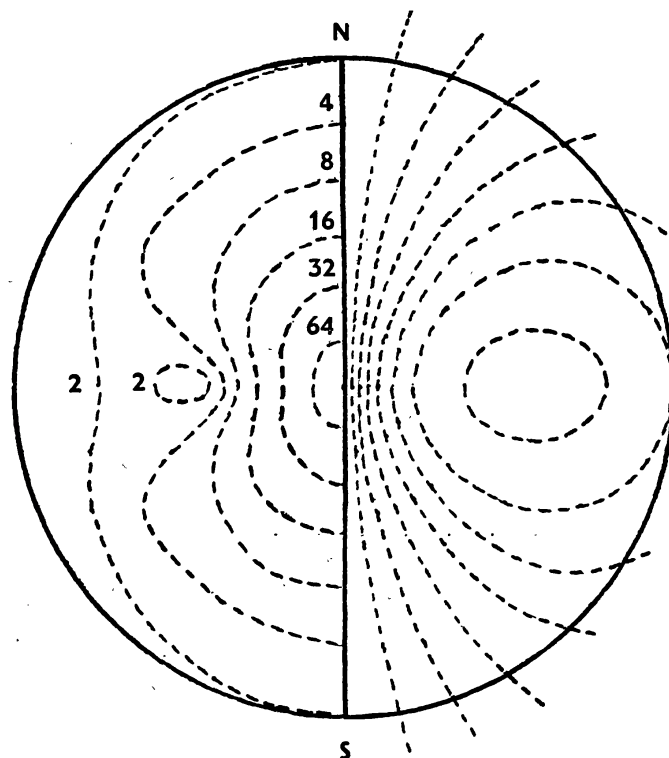


FIG. 1.—Lines of force (right half) and constant magnetic intensity (left half) of the decaying field in a plane through the magnetic axis. The magnetic intensity is given as a multiple of the equatorial intensity at the surface.

tends to carry its magnetic field with it. Thus turbulent motion may split up the field into a number of lesser elements, which may be destroyed in detail at a much faster rate than is indicated by the above calculation, much as the kinetic energy of mass motion is more rapidly dissipated by turbulence than by ordinary viscosity. The argument is not altogether certain: electromagnetic induction always acts to oppose the motion to which it is due, and in consequence effects expected from it often fail to materialize; moreover, some motions may actually reduce the rate of decay. We conclude, therefore, that it is not impossible that the Sun's magnetic field may be a relic from a previous era; but no more positive assertion is justified.

The lines of force, and lines of constant magnetic intensity, of the decaying field in a plane through the axis of symmetry, are shown in the accompanying figure. The intensity at the centre of the Sun is about 80 times that at the equator at the surface. Taking the latter as 25 gauss, the central field is 2000 gauss; save in the central core, the field is at most one or two hundred gauss. On this showing, the magnetic field of

sunspots can hardly be due solely to the convection of the general field with rising material from deep layers.

5. We consider next the possibility that the Sun's general field may be due to thermal effects in a rotating mass.* Ordinary thermo-electric effects, due to the relative inclination of isothermal and isobaric surfaces, can be ignored; any electric currents they might produce would be in planes through the axis of rotation, and would result in a field directed in circles round that axis. We consider instead a field due to convection in an unstable central region. Here hot gases rise, cool gases sink; during their vertical motion they are subject to a Coriolis force $-2\Omega \wedge \mathbf{v}$ per unit mass, where Ω is the Sun's angular velocity and $\mathbf{v}$ the vertical velocity. This tends to deflect their motion to the east or west.

Suppose now that a mass of rising gas finds itself to the west of a mass of falling gas. The rising gas is deflected to the west, the falling to the east, by the Coriolis forces; hence they tend to move apart, and to balance the Coriolis forces a region of low pressure tends to be set up between them. Similarly when rising gas is to the east of falling gas, there tends to be a region of high pressure between them. In either case, pressure in the rising gas tends to be higher on its west side than on its east side; that in the falling gas to be higher on the east side than on the west.

A pressure gradient causes relative diffusion of electrons and ions, and so produces electric currents. A horizontal electron-pressure gradient $\partial p_e / \partial x$ is equivalent, for this purpose, to an electric force

$$-\frac{1}{en_e} \frac{\partial p_e}{\partial x}, \quad (11)$$

where $-e$ is the electronic charge, and n_e is the electron number-density. Suppose that the rising and falling masses of gas have equal dimensions in the x -direction; then $\partial p_e / \partial x$ has the same magnitude in the two. However, because the rising gas is the hotter, and the pressures in rising and falling gases are the same at a given level, n_e is smaller in the rising gas than in the falling. Hence the effective electric force (11) is rather greater in the rising gas, and, on the whole, currents tend to flow round the axis of the Sun in the direction of the electric force in the rising gas, *i.e.* from east to west. This produces a magnetic field which, at the Sun's surface, is from south to north; this is the observed sign.

This result does not actually depend on the assumption that the rising and falling gas-masses have equal dimensions in the x -direction. The effective E.M.F. driving currents round a circle round the Sun's axis is

$$-\int \frac{1}{en_e} \frac{\partial p_e}{\partial x} dx = -\int \frac{1}{en_e} dp_e,$$

which is independent of the dimensions in the x -direction, and depends only on the fluctuations in n_e and p_e on the circle. It is, however, convenient in calculations to continue to make the assumption. Then the mean effective electric force, averaged over one rising and one falling mass, is

$$\begin{aligned} -\frac{1}{2e} \left\{ \left(\frac{1}{n_e} \frac{\partial p_e}{\partial x} \right)_{\text{rising}} + \left(\frac{1}{n_e} \frac{\partial p_e}{\partial x} \right)_{\text{falling}} \right\} &= \left(\frac{k}{2ep_e} \frac{\partial p_e}{\partial x} \right)_{\text{rising}} \{ -T_{\text{rising}} + T_{\text{falling}} \} \\ &= -\frac{k\Delta T}{2ep_e} \frac{\partial p_e}{\partial x}, \quad \text{say.} \end{aligned}$$

* The ideas set forward here are similar to those advanced, in a less developed form, by W. M. Elsasser in *Phys. Rev.*, 55, 489, 1939, to explain the Earth's magnetic field.

In this, $\frac{1}{p_e} \frac{\partial p_e}{\partial x}$ is approximately $\frac{1}{p} \frac{\partial p}{\partial x}$, and $\frac{\partial p}{\partial x}$ balances the Coriolis force $2\rho\Omega v$; also, if m is the mean molecular mass, $p = k\rho T/m$. Hence the mean electric force is

$$\frac{m\Delta T}{eT}\Omega v.$$

We put $\Delta T/T = 10^{-7}$, $v = 2 \times 10^3$ cm./sec., figures whose order of magnitude is suggested by results given by the author in an earlier paper*; these figures may separately be somewhat uncertain, but their product, which is proportional to the total transport of energy by convection, is fairly accurately determinate. Then, taking $m = 3 \times 10^{-24}$ gr., $\Omega = 3 \times 10^{-6}$ radians/sec., the mean electric force is found to be 10^{-13} E.M.U. With a conductivity of order 10^{-4} E.M.U., such an electric force leads to a magnetic field which, at the Sun's surface, is less than 10^{-6} gauss.

This estimate is, of course, correct only as regards order of magnitude. In particular, the introduction of Coriolis forces tacitly implies the assumption that the Sun's angular velocity is uniform throughout the interior of the Sun, and this assumption is almost certain to be invalid. Nevertheless, errors introduced by this and similar assumptions cannot affect the order of magnitude, and we may therefore assert that currents of thermal origin cannot give rise to the observed field. The reason is simply that appeal has to be made to second-order effects, depending not only on small thermal currents but also on the small Coriolis forces.

6. One may ask whether the small field due to thermal currents may act as the seed whence larger fields may grow by dynamo action, due to the circulation of material in meridian planes (section 1). I have not examined this suggestion in detail; but it is easy to show that dynamo action, if able to produce the observed field, must take an abnormally large time to do so.

We again consider a field possessing symmetry about an axis, with the lines of force in planes through the axis. On a special circle C in the equatorial plane of the field, the magnetic field vanishes; if N is the total magnetic induction from north to south through a closed curve, N has its maximum value N_c when the closed curve coincides with C . Near C , lines of force thread round C ; the magnetic field is from north to south inside C and from south to north outside C . This implies that currents must flow round the axis of the field from east to west near C .

The E.M.F. tending to drive currents round C is the sum of the effective E.M.F. V of thermal origin, and the back E.M.F. $-dN_c/dt$ due to induction. In this, dN_c/dt is at first sight not the rate of change of the magnetic induction through C itself, but the rate of change of that through a fixed curve instantaneously coinciding with C (C may alter in radius with the time); but these two are actually identical, because N is maximum on C and so unaltered in passing to a neighbouring circle. Because there is a current from east to west round C

$$V - \frac{dN_c}{dt} > 0$$

and so

$$\frac{dN_c}{dt} < V \sim 10^{-2} \text{ E.M.U.}$$

Again, N_c must exceed the total magnetic induction across one hemisphere of the Sun's surface, which may be estimated as (say) 5×10^{23} E.M.U. Hence the time needed to produce the present field must exceed 5×10^{25} sec. $= 1.6 \times 10^{18}$ years. This is so much greater than accepted estimates of the Sun's lifetime as to make the hypothesis unacceptable.

* *M.N.*, 96, 42, 1935. The modifications of these results arising from the rotation of the Sun are discussed below in the Appendix to this paper.

Precisely the same argument shows that no effect too small to explain the Sun's field by itself can be magnified by dynamo currents to give the observed field in a time of order 10^{10} years. For 10^{10} years is not only the time of decay of the Sun's field when left to decay; it is also the time which the field needs to grow into something like its steady-state value when field-producing currents flow. Dynamo currents may be able to lift the "ceiling" that can be reached by the field; but, by the argument used above, they cannot increase the rate of growth of the field. If the "ceiling" is lifted, the time of reaching it must be correspondingly increased.

7. There seems, therefore, to be no possibility of progress along the lines which we have been considering. Thermo-electric effects produce too small a current. Dynamo effects do not help the field to grow near the locus where the magnetic field vanishes, and new lines of force have to be generated on this locus. Theories using the Hall or Nernst effects can be ruled out for the same reason.

There remain the following possibilities. In the above, the field was assumed to be in general shape similar to that of a uniformly magnetized sphere. It was supposed to possess an axis of symmetry, the lines of force lying in planes through that axis. Such a field can grow only by the production of new lines of force, which have to be generated on the locus on which the magnetic field vanishes.

If a more general type of magnetic field is considered, it is possible for the field to grow, not by the production of new lines of force, but by more efficient use of lines of force already existing. Because of the high conductivity of solar material, lines of force are effectively "frozen" into the material; when convection occurs, the lines of force are convected with the material. If the lines of force are pulled out into longer curves as a consequence of convection, they produce a larger total magnetic field. This fact was used recently by Alfvén* in developing a theory of the magnetic field of sunspots; he supposed that a strong magnetic field is produced locally by the extension in a given direction of lines of force of the general magnetic field, owing to convection in that direction, the strong magnetic field being then transported to the Sun's surface by a wave motion along the lines of force. We do not desire to consider here the validity of Alfvén's theory; but there is no doubt that strong local fields can be produced by extending already existing lines of force by convection.

In our problem, in order to multiply the Sun's general magnetic field, it is necessary to use the same lines of force over and over again. For example, consider a line of force round a circle with the Sun's axis as axis of symmetry. Suppose that, as a result of convection, this is twisted into a spiral wound on an anchor ring with the original circle as its central line. Then a field will have been produced in meridian planes, whose magnitude depends only on how tightly the spiral is wound. It is easy to imagine distortions of this type, which are able locally to produce strong fields. It is more difficult to imagine a distortion which will reinforce the field as a whole, and still more difficult to imagine a convection which will produce the required distortion. Nevertheless, because of the smallness of the convection currents needed, the possibility must be considered.

A second possibility is that material in the far interior of the Sun is capable of permanent magnetisation. This demands that very hot and ionized material is capable of acquiring a regular, crystal-like structure, a possibility normally disregarded. In view of the difficulties of other hypotheses, the possibility may, however, be worthy of further study.

APPENDIX

Turbulence and the Sun's Rotation.—In section 5, turbulent convection was assumed not to be materially impeded by the Sun's rotation. It has often been stated that

* *Arkiv. f. Mat., Ast., och Fys.*, **29A**, nos. 11 and 12, 1943; *M.N.*, **105**, 3, 1945.

rotation in a star has a stabilising influence; G. Randers has recently * suggested that in consequence turbulence will not appear near the equatorial plane of the Sun. His argument appears to be unsatisfactory, for two reasons. First, he ignores the possible effect of pressure gradients of the type envisaged in section 5. Second, he uses my estimate of the excess of the temperature gradient above the adiabatic, made ignoring the rotation. This estimate was based on two equations †, one of which balanced the kinetic energy dissipated by viscosity against that created by the instability of the gas. If rotation does exert a stabilising influence, this equation will have to be modified; its modified form must indicate a radial temperature gradient rather larger than that required to overcome the stabilising influence, though still only insignificantly greater than the adiabatic gradient. Randers's argument is invalidated by this modification.

A similar argument has been used by C. Walén ‡ in connection with Alfvén's theory of sunspots. He supposes that the sunspot cycle has its origin in alternate periods of quiescence and turbulence in the Sun's interior. During a quiet period, turbulence is prevented by the stabilising influence of rotation; the excess heat conveyed away by radiation accumulates in the Sun's core, until at last the temperature gradient becomes large enough to overcome the effect of rotation, and there is an outburst of turbulence. This outburst removes the excess heat, and leaves the Sun's core in its original stable state.

The argument implies a sort of oscillation about equilibrium. The equilibrium state is, in this case, one in which turbulence is just sufficiently violent to remove excess energy generated in the Sun's core, and the unstable temperature gradient is just great enough to overcome the stabilising effect of rotation and generate enough kinetic energy to balance that destroyed by viscosity. One may ask why this equilibrium state should ever be overshoot. Walén assumes that an outburst of turbulence reduces the temperature gradient to the adiabatic value and then dies away. In fact, however, turbulence begins to be destroyed by viscosity faster than it is created, rather before the temperature gradient is reduced to such a value that the effect of rotation balances the instability due to the temperature gradient. When the turbulence begins to diminish, the convective transport of energy is reduced, and the reduction in the temperature gradient is checked. In consequence, the turbulent velocities and the temperature gradient steadily approach their equilibrium values, and cannot overshoot them. Walén's argument resembles terrestrial arguments criticised by Jeffreys §, which suggest oscillations about equilibrium due to thermal conduction, whereas thermal conduction cannot of itself produce oscillations, but only cause steady approach to an equilibrium state.

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* *Ap. J.*, **95**, 454, 1942.

† *M.N.*, **96**, 42, 1935, equations 31 and 34. The equation referred to above is the second of these.

‡ *Arkiv. f. Mat., Ast., och Fys.*, **30A**, no. 15, 1944.

§ *The Earth*, p. 323, Cambridge, 1929.