A SPECTROGRAPHIC STUDY OF THE TRIPLE SYSTEM IN 59 d SERPENTIS

Elizabeth Cornwall Tilley
University of Michigan Observatory

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ABSTRACT

In the visual double star 59 d Serpentis the primary component (5.4 mag.) is a triple system giving a composite spectrum of G0 and A2. The secondary component (7.8 mag.) is A0. Spectrograms of the triple system were taken at Yerkes Observatory in 1928–1929, at Ann Arbor in 1938–1942, and at Mount Wilson in 1941–1942. Velocities from these plates, combined with thirteen scattered observations in 1903–1909, yielded for the G-type star a period of 386.0 days. The following elements were derived: $\omega = 277°$, $K = 28$ km/sec, $\gamma = -23$ km/sec, $T = 221$ days, and $e = 0.47$.

For the orbit of the center of mass of the A-type binary system the above elements were adopted, except for $\omega = 97°$ and $K = 19$ km/sec.

A least-squares solution of the orbit of the A-type stars was made after the period of 1.8505205 days had been determined graphically from a composite plot of velocities in 1938–1942. The respective values of K were 90 and 100 km/sec, e was assumed to be zero, and γ was also assumed to be zero, after the correction obtained in the solution was absorbed by revision of K in the center-of-mass orbit.

On six plates an interstellar calcium line was measured between the components of the K line.

From a knowledge of $m \sin^3 i$, the ratio of the masses for the members of the triple system, and a dynamical parallax, approximate values were obtained for the masses and inclinations. The probable inclinations of the orbits preclude an eclipse of any components. The radii of the stars were also approximated.

Four low-dispersion spectrograms were taken of the faint visual companion. Velocities derived from these plates, relative to that of the center of mass of the triple system, suggest that this star might be a binary. A negligible change in the separation of 4″ has been observed during the past century.

INTRODUCTION

Up to the present time comparatively little work has been done on individual composite spectra which were not those of two stars having orbital motion clearly indicated by the relative shift of the lines. The present problem is an investigation of the spectrum of 59 d Serpentis ($\alpha = 18^h22^m$, $\delta = +0°9'$, 1900), one of the stars included in Hynek's list of composite stars.[1] A visual double of apparent magnitudes 5.4 and 7.8, it has been observed occasionally since 1828. It is highly probable that the visual double is a physical pair. It shows a very slowly changing position angle and an almost negligible change in its separation of 4″. The common proper motion would be over 1″ in the interval of observation; and thus, if it were an optical double, a greater change in the position angle and separation would have been observed. In the *Henry Draper Catalogue* Miss Cannon designated the spectrum as composite—the brighter component of class A; the fainter, of class G. It was not clear at the time whether the composite character applied to the bright component alone or to the visual double. The secondary component of the visual pair (7.8 mag.) has a normal spectrum of class A0; and the primary component (5.4 mag.) is alone responsible for the composite spectrum A + G, as found by Leonard[2] in 1923 and by Hynek in 1938. The latter says that the composite spectrum "is not like that artificially produced by the superposition of two stars differing in photographic magnitude by nearly three magnitudes." This has been verified with spectrograms taken in Ann Arbor by the writer.

The triplicity of the primary component was discovered by Dean B. McLaughlin[3] in 1938, when he found that the K line of the A-type spectrum was distinctly double and

[1] *Contr. Perkins Obs.*, No. 10, 1938.

[2] *Lick Obs. Bull.*, **10**, 175, 1923.

[3] *Ap. J.*, **88**, 356, 1938.

showed a periodic variation repeated about every 2 days. At the same time R. Tremblot[4] found the same characteristic and stated that the period of this variation was 1.85 days. It is this triple star which is the subject of the present investigation, and it will be referred to as "d Serpentis."

SOURCE AND REDUCTION OF MATERIAL

The majority of the spectrograms were taken at the Observatory of the University of Michigan during parts of the years 1938–1942, with the one- or two-prism spectrographs attached to the 37-inch reflector. They have a dispersion of approximately 40 A/mm at $H\gamma$. Dr. Otto Struve, of the Yerkes Observatory, kindly sent to the writer the fourteen plates taken in 1928–1929 on which he and Miss Johnson reported in 1939.[5] These plates, of dispersion approximately 26 A/mm at $H\gamma$, were remeasured at Ann Arbor in order

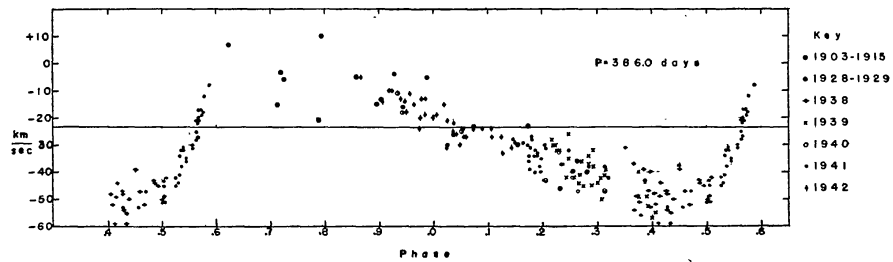

FIG. 1.—59 d Serpentis

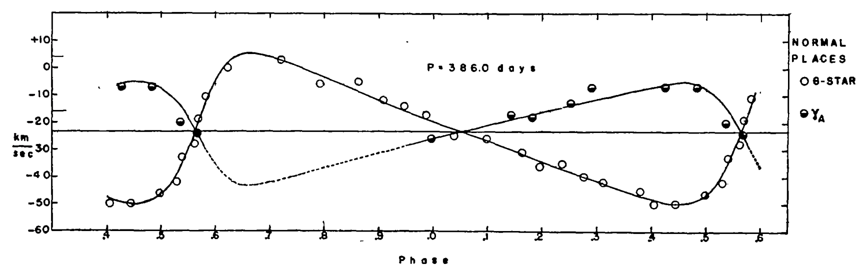

FIG. 2.—59 d Serpentis

that the same stellar and comparison lines might be used, so far as possible. Other spectrograms were taken with the 60-inch telescope at Mount Wilson, through the kindness of Dr. Walter S. Adams; and these fifteen plates supplemented those taken at Ann Arbor at the most significant phases observable in the velocity-curve of the G-type star. Their linear dispersion is approximately 36 A/mm at $H\gamma$. On many of these spectrograms which did not extend to the K line, owing to atmospheric conditions, the velocities of the A-type stars were found only from Mg II 4481, since a narrow slit on the spectrograph had been used.

The comparison spectra on the plates from the three observatories were either iron or titanium or a combination of the two. In order to have a homogeneous system, the same comparison lines were used as standards in all spectra of the same element, and the same stellar wave lengths were used in the Hartmann solutions for spectrograms with iron as with titanium spectra. Comparison lines were chosen for freedom from blending or broadening, their wave lengths being known with greater accuracy than those in the stellar spectrum.

The 1928–1929 plates from Yerkes Observatory were measured twice, and final velocities for the G-type star were derived using statistical methods, since some plates

[4] *C.R.*, **207**, 491, 1938.

[5] *Ap. J.*, **89**, 136, 1939.

were underexposed. From the majority of the plates taken in 1938 the final velocities are also the result of two sets of measures, made approximately a year apart. The difference between the two sets of measures amounted to not more than 10 per cent of the scatter within either set, and therefore the scatter is evidently inherent in the plates.

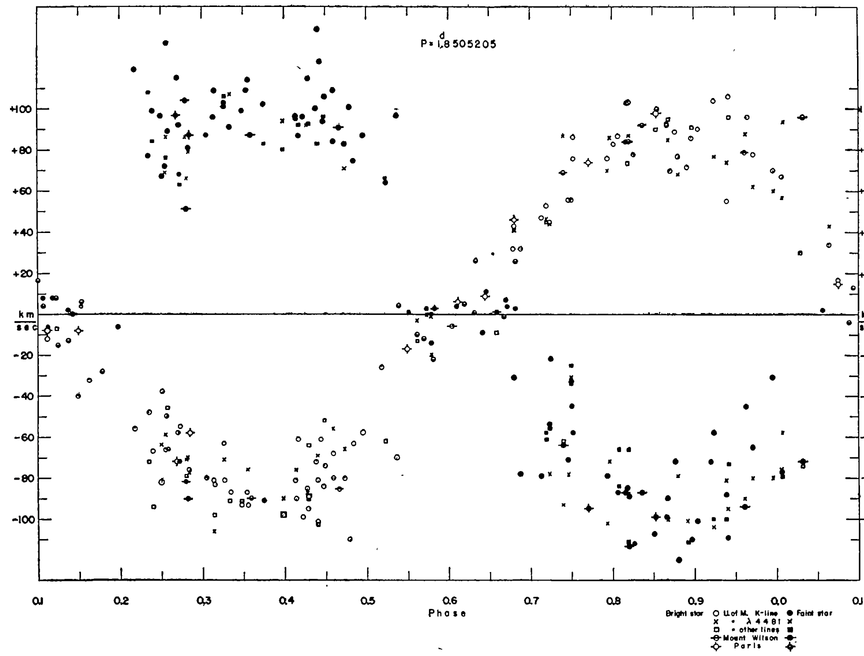

FIG. 3.—59 d Serpentis

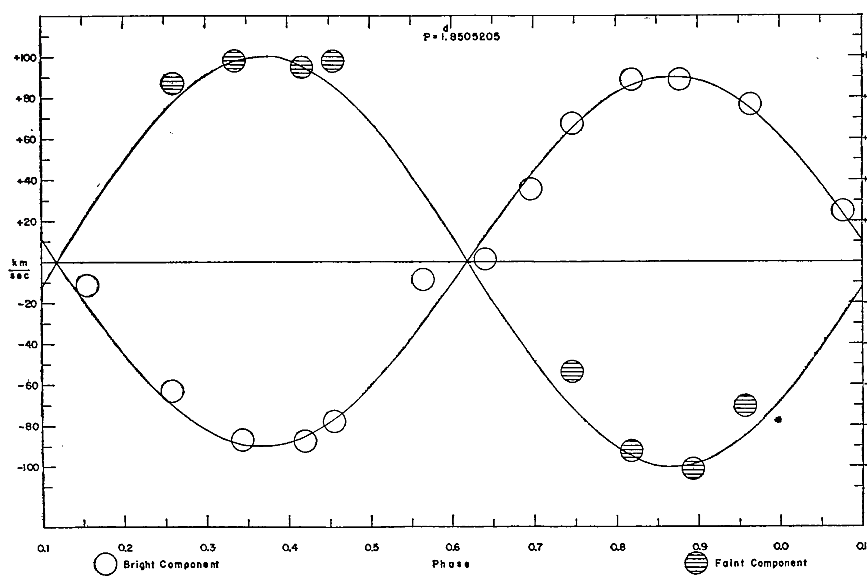

FIG. 4.—59 d Serpentis

All other spectrograms were measured once. The residuals of all individual plates are predominantly in the direction of the velocity of the stronger component of the K line. It therefore appears probable that much of the scatter is due to blending with lines of the stronger A-type spectrum. Some of the lines which are most noticeably affected are:

λ 4045 and λ 4063 of *Fe* I; λ 4215 of *Sr* II; λ 4227 of *Ca* I (asymmetry); λ 4233 of *Fe* II; and occasionally λ 4415 of *Fe* I.

Velocities of the A-type stars, as used in the computation of their orbits, depend on the K line of *Ca* II alone. However, some velocities were obtained also from the lines λ 4078 and λ 4215 of *Sr* II and λ 4481 of *Mg* II. In one or two cases a velocity has also been secured from $H\gamma$, λλ 4045, 4063, and 4187 of *Fe* I. The velocities from the K line as given by Tremblot for 1938 were not included in the computations but are shown, on Figure 3, to be satisfied by the solution.

ELEMENTS OF THE ORBIT OF THE G-TYPE STAR

A somewhat detailed description of this portion of the work appears to be useful, in view of the very special difficulties that arose in this case. In 1938 the data from published velocities by Frost and Lee[6] for 1903, 1904, 1905, 1906, and 1909, some single velocities from Lick Observatory in 1913 and 1915,[7] and a list of fourteen by Struve and Johnson in 1928–1929 were not sufficient for a determination of the period, though McLaughlin suggested a value of 15 years. Observations at Michigan were accordingly continued to 1941, when it was evident that no period of a number of years would satisfy these observations.

TABLE 1

Phase	Julian Day 2400000+	Velocity	Phase	Julian Day 2400000+	Velocity
0.600	32270	−36	0.800	32347	−37
.650	32289	43	0.900	32386	−31
0.700	32308	−42			

A more careful examination of the individual velocities then showed that a steady progressive change within each season had been masked by the scatter and that each year's observations (which were grouped within a few months) fell a little earlier in the star's cycle than the set for the preceding year. The true period was evidently slightly more than a year or half a year. Several trials indicated a probable period from which it was expected that the velocity-curve would rise rapidly from minimum to maximum in the late autumn of 1941. The expected rise occurred during November, and observations were continued until the star was close to conjunction with the sun. Observations were resumed as early as possible in March, 1942, and these indicated that the period was close to 1 year instead of 6 months. A period of 386 days was finally derived, and continued observations fully confirmed this result when the velocities in July and August repeated those of June and July, 1941. The A-type stars also showed a center-of-mass-curve which reflected the velocity of the G-type star. It now appears that all doubt as to the period has been removed, even though there is a large unobserved region in the velocity-curves which cannot be filled in completely until 1947. (See Table 1 for predicted values corresponding to the dotted-line portions of Figure 2, which will be observed beginning about March 25, 1947.)

Individual observations were given a weight of either ½ or 1, and twenty-three normal places (see Table 2) were formed for a preliminary solution of the orbit. This was carried out by Russell's short method,[8] since this is most suitable for a curve having no observations precisely at the maximum.

[6] *Ap. J.*, **30**, 67, 1909.

[7] *Pub. Lick Obs.*, **16**, 269, 1928.

[8] *Ap. J.*, **40**, 282, 1914.

The period of 386 days was adopted as known, and the remaining five elements were then determined by a least-squares solution for the twenty-three normal places. The resulting elements are:

Elements	Preliminary	Final
P	386.0 days	386.0 days
γ	−23.4 km/sec	−23.3±0.22 km/sec
T	218.7 days	221±2.9 days
ω	272°	277°±5°
e	0.46	0.47±0.02
K	27 km/sec	28±2 km/sec
$a \sin i$	127,000,000 km	131,000,000 km

ORBIT OF THE CENTER OF MASS OF THE A-TYPE STARS

The velocities of the K line belonging only to the bright component of the system of period 1.85 days were used for the determination of the orbit of the center of mass of this system. (We shall refer to the velocity-curve of the latter as the "γ_A-curve.") From inspection of the plot of velocities in the 1.85-day orbit it was evident that there would be no justification for assuming orbits of these stars other than circular, and therefore

TABLE 2

NORMAL PLACES FOR THE G-TYPE STAR

No.	Phase	Vel.	Wt.	O − C	No.	Phase	Vel.	Wt.	O − C
1	0.038	−25.0	1.0	−2.0	13	0.540	−33.0	0.4	+1.4
2	.099	26.0	0.5	+1.0	14	.562	−28.0	.4	−3.1
3	.164	31.0	1.0	+1.0	15	.570	−19.0	.5	+1.4
4	.196	36.0	1.0	−2.0	16	.583	−10.7	.2	+2.7
5	.237	35.0	1.2	+2.0	17	.623	0.0	.1	−2.0
6	.277	40.0	0.9	0.0	18	.720	+ 3.0	.2	−1.0
7	.311	42.0	0.7	0.0	19	.792	− 6.0	.2	−3.0
8	.380	45.0	0.9	+2.0	20	.862	− 5.0	.2	+4.0
9	.404	50.0	0.9	−2.0	21	.909	−12.0	.4	+1.0
10	.443	50.0	1.0	0.0	22	.948	−14.0	.5	+2.0
11	.499	46.3	0.9	+0.6	23	0.988	−17.0	0.5	+2.0
12	0.530	−42.0	0.4	−2.8					

each individual observation was compared to the ordinate of a sine-curve to give residuals, each of which might be regarded as a measure of the velocity of the center of mass for the date considered. The mean residuals and mean phases in the 386-day period for groups of the velocities then gave normal places for the γ_A-curve. This method was used to find one value of γ_A for 1938, one for 1939, one for 1940, three for 1941, and two for 1942.

The elements of this orbit, except for K, were taken from the orbit of the G-type giant, since the great number of observations obtained for the giant gave accurately those elements which must be the same for the two components of a system. The semiamplitude of the orbit was estimated from the ratio of the ordinate (in km/sec) of the γ_A-curve to that of the G-type star at the same phase. An ephemeris from these elements, with ω equal to $\omega_G + 180° = 97°$, $P = 386$ days, $\gamma = -23.3$ km/sec, $e = 0.47$, and an approximate semiamplitude, yielded residuals predominantly negative and indicated an immediate revision to $K = 18$ km/sec. A least-squares solution was then carried out for δK, but the correction δK was not considered significant; therefore the value $K = 18$ km/sec was accepted, with the above elements for construction of the velocity-curve of the center of mass of the A-class stars. This curve was used in the solution of the orbits of the binaries, but upon its completion the γ_A-curve required a final adjustment, which gave $K = 19$ km/sec.

SOLUTION OF THE ORBITS OF THE A-TYPE STARS FROM THE K LINE OF *Ca* II

The γ_A velocity was eliminated from the observed velocities of the individual A-type stars, giving their velocities in the 1.85-day orbit. The date of each observation was corrected for the equation of light due to the orbital motion of the earth and also for that of the center of mass of the A-type pair.

A period of 1.85053 days was obtained before the solution of the center-of-mass orbit was made. When the above corrections had been applied to the velocities and to the Julian days, respectively, the longest base line available for refinement of the period was the span of four years, 1938 through 1942. The method of reciprocal periods was used, and a very small correction to the above period presented a more favorable coincidence of cycles; thus the solutions of the orbits were made with the assumed period

TABLE 3

NORMAL PLACES FOR THE VELOCITY OF THE CENTER OF MASS OF THE A-TYPE STAR SYSTEM

Year	Wt.	Phase	Vel.	O − C*	Year	Wt.	Phase	Vel.	O − C*
1942......	14	0.996	−26	0	1938......	27	0.426	− 7	−1
1942......	11	.144	17	+2	1941......	6	.483	7	−1
1941......	16	.182	18	−1	1941......	5	.536	20	−6
1940......	8.5	.252	13	+1	1941......	9	0.568	−24	+1
1939......	23	0.290	− 7	+5					

* When $K = 19$ km/sec.

of 1.8505205 days. However, the use of the period carried to eight figures was merely a computational procedure required by the adopted value of the reciprocal. Only with future observations can the period be surely found to seven places. The probable error of the period is estimated as 0.000009 days from an examination of the possible shift of the velocity-curve of 1942 with respect to 1938 (not the maximum allowable shift). That the true period is the one given above, and not a value slightly more than 2 days, is shown by the satisfactory agreement of velocities from the plates taken at Mount Wilson and those measured by Tremblot in Paris, the difference in longitude between these observatories being 8 hours.

Final phases from $P = 1.8505205$ days and radial velocities (due to the motion of this binary alone) were plotted to form a composite curve for each star from which the normal places were formed (see Table 4). Weights were assigned to individual velocities on the basis of the quality and separation of the components of the K line, and the weights of the normal places depended solely on the sums of these single weights.

The value of K for the bright component was first estimated closely by superimposing a sine-curve upon the composite velocity plot of that star. At the same time it was possible to read off T', the time of superior conjunction of the brighter star.

For the ephemeris of a preliminary orbit we had the following elements: $P = 1.8505205$ days; $T' = 0.222$ day; $K = 90$ km/sec; we assume: $e = 0$, $\gamma = 0$. It was desirable next to carry out a least-squares solution for only three unknowns: δK, $\delta T'$, and $\delta\gamma$. Actually, there should be no correction to the center-of-mass velocity, since it has been accounted for in each season; but from the fitting of the sine-curve to the composite velocity plot and from the least-squares solution the correction was found to be significant. From the order of magnitude of the probable errors the corrections to the assumed T' and K were inconsequential.

The elements of the orbit of the bright star then are:

Elements	Preliminary	Corrected
P	1.8505205 days	1.8505205 days
ω	97°	97°
e	0.0	0.0
γ	0.0	0.0
K	90.0 km/sec	90±2 km/sec
T'	0.222 days	0.219± .008 days
$a \sin i$	....	2,270,000 km

For the faint component only the observations near maximum and minimum of the velocity-curve are entitled to any weight, since others are affected by blends with the stronger component and would give a much less accurate value of K. Four normal places were thus available at maximum, and four at minimum. When the same center-of-mass velocity as that of the bright component was adopted, the residuals showed an asymmetry. The best fit was obtained by trial and error, with $K = 100$ km/sec and with a center-of-mass velocity displaced by +2 km/sec relative to that of the bright star, making a total displacement of +3.8 km/sec relative to the adopted zero of the γ_A-curve.

TABLE 4

NORMAL PLACES FOR THE A-TYPE STARS

Bright Component					Faint Component				
No.	Phase	Vel.	Wt.	O − C	No.	Phase	Vel.	Wt.	O − C
1.....	0.077	+25.0	0.6	+ 0.2	1.....	....	....	......	....
2.....	.115	+ 1.9	0.6	− 1.7	2.....	....	....	......	....
3.....	.156	−11.0	0.6	+ 8.2	3.....	....	....	......	....
4.....	.258	−63.3	1.0	+ 3.3	4.....	0.261	+ 87.3	1.0	+ 7.4
5.....	.344	−86.8	1.0	− 0.6	5.....	.336	+ 98.0	1.0	− 1.8
6.....	.420	−87.5	0.8	− 4.8	6.....	.417	+ 95.1	0.8	− 2.0
7.....	.455	−78.1	0.8	− 3.7	7.....	.458	+ 98.3	0.7	+12.0
8.....	.564	− 8.5	0.6	+19.4	8.....	....	....	......	....
9.....	.640	+ 1.4	0.6	−12.4	9.....	....	....	......	....
10.....	.696	+35.7	0.8	− 7.9	10.....	....	....	......	....
11.....	.747	+67.4	1.0	+ 1.2	11.....	.746	− 53.8	0.8	+16.2
12.....	.819	+88.9	1.0	+ 2.2	12....	.818	− 92.5	0.9	+ 0.2
13.....	.877	+88.7	1.0	− 2.1	13.....	.893	−101.6	0.9	− 1.0
14.....	0.964	+76.5	1.0	+ 1.2	14.....	0.957	− 70.3	0.9	+12.8

Since the position of the center-of-mass axes for both A-type stars differed from zero, further revision of the γ_A-curve was necessary. Accordingly, residuals for each season were redetermined with the new elements for the bright star, and the value of K was changed from 18 km/sec to 19 km/sec. This only slightly altered the maximum and minimum parts of the curve, and thus only twelve of the individual observations of the A-type stars were affected by +1 km/sec. They are listed with this adjusted velocity in the "Journal of Observations," Table 6. Since the latest residuals which were used for the redetermination of K for the γ_A-curve were obtained with new elements and an assumed $\gamma = 0$, the correction to $\gamma = 0$ found in the orbit solutions of the two A-type components was then absorbed.

TABLE 5*

JOURNAL OF OBSERVATIONS FOR THE G-TYPE STAR
($P = 386.0$ Days)

Year	Julian Day 2400000+	Phase	Vel. in Km/Sec
1903........	16272.80	0.157	−30
	278.82	.173	−27
1904........	16677.78	.207	−43
1905........	17104.58	.312	−47
1906........	17459.64	.232	−46
	468.65	.256	−40
	471.58	.263	−36
	478.60	.281	−40
1909........	18448.84	.795	+10
1911........	19188.72	.712	−15†
1913........	19963.82	.720	− 3
	65.70	.724	− 6
1915........	20762.66	.789	−21
1928........	25421.70	.859	− 5
	435.75	.896	−15
	439.70	.906	−14
	444.76	.919	− 4
	450.63	.934	−11
	453.64	.942	−18
	454.68	.945	−16
	486.63	.027	−30
	490.60	.038	−26
	495.60	.051	−25
	496.60	.053	−24
	505.59	.076	−23
1929........	25716.40	.623	+ 7†
	857.76	.989	− 5†
1938........	29085.67	.351	−31†
	091.65	.367	−37
	092.65	.369	−54
	093.69	.372	−49
	094.71	.375	−51
	096.65	.380	−56†
	098.64	.385	−30
	099.63	.388	−43
	101.66	.393	−47†
	103.63	.398	−53
	103.74	.398	−40
	105.63	.403	−48
	107.65	.408	−52
	108.67	.411	−59†
	109.65	.413	−49
	110.64	.416	−44
	114.64	.426	−47
	114.74	.427	−48
	115.62	.429	−54
	116.65	.432	−59†
	117.62	.434	−55
	119.64	.439	−50
	123.66	.450	−39†
	127.63	.460	−47
	130.64	.468	−52
	137.61	0.486	−43
1938........	29138.62	0.488	−44
	140.61	.494	−45
	143.57	.501	−50
	144.61	.504	−49
	146.59	.509	−42
1939........	29419.80	.217	−28
	420.72	.219	−30
	420.81	.220	−30
	423.71	.227	−32
	423.88	.228	−32†
	427.80	.238	−37
	429.81	.243	−42†
	431.70	.248	−35
	431.85	.248	−26
	435.77	.258	−42
	438.77	.266	−44
	439.82	.269	−41†
	440.72	.271	−36
	441.67	.274	−45†
	445.65	.284	−37
	445.73	.284	−34
	447.80	.290	−45
	448.67	.292	−32†
	448.80	.292	−38†
	451.69	.300	−44
	453.65	.305	−42
	454.80	.308	−50†
	456.63	.312	−41
	456.72	.313	−39
	480.63	.374	−49
	481.71	.377	−40
	485 67	.388	−43
	486 66	.390	−49
	487.58	.392	−53
	490.63	.400	−57
	492.66	.406	−55
1940........	29785.80	.165	−29
	806.76	.204	−30
	810.69	.230	−32
	812.69	.235	−37
	817.65	.248	−32
	820.67	.256	−42
	823.80	.264	−47
	831.63	.284	−38
	842.68	.312	−38
	845.70	.320	−42
1941........	30164.74	.147	−28
	170.67	.162	−30
	171 68	.165	−36
	171.76	.165	−34
	172.68	.167	−39
	172.75	.168	−31
	177.63	0.180	−27

* Note that the phases have no unit, being a fraction of the period.

† Half-weight.

TABLE 5—*Continued*

Year	Julian Day 2400000+	Phase	Vel. in Km/Sec	Year	Julian Day 2400000+	Phase	Vel. in Km/Sec
1941........	30177.70	0.180	−28	1941........	30327.55	0.569	−21†
	179.63	.185	−34		328.46	.571	−17
	180.66	.188	−32		329.55	.574	−19
	180.72	.188	−40		330.55	.576	−18
	181.63	.191	−33		331.55	.579	−12
	181.79	.191	−34		335.48	.589	− 8
	183.70	.196	−38	1942........	30442.93	.868	− 5
	184.62	.198	−40		457.90	.906	−14†
	184.68	.198	−35		462.89	.919	−10†
	185.62	.201	−43		463.88	.922	−10†
	187.66	.206	−31		470.88	.940	−13†
	273.55	.429	−54		473.84	.948	−14
	275.54	.434	−48		474.84	.950	−18
	284.53	.457	−53		477.85	.958	−11
	289.54	.470	−47		479.98	.964	−15
	301.48	.501	−51		483.89	.974	−24†
	301.50	.501	−51		484.90	.976	−20†
	302.47	.504	−43		486.00	.979	−13
	302.48	.504	−45		487.80	.984	−19
	311.46	.527	−42		488.84	.986	−13
	311.47	.527	−45		493.87	.000	−20
	312.48	.530	−44		496.86	.007	−19
	313.47	.532	−41†		501.79	.020	−15
	313.48	.532	−34†		502.79	.023	−31
	315.46	.537	−32		503.91	.026	−21
	315.48	.537	−38		507.87	.036	−25
	316.47	.540	−31		508.76	.038	−22
	316.49	.540	−32		509.85	.041	−26
	318.46	.545	−35		512.93	.049	−30†
	318.48	.545	−36†		515.79	.056	−27
	323.46	.558	−30		515.95	.057	−27
	323.48	.558	−31†		522.78	.074	−24
	325.46	.563	−28		528.80	.090	−24
	325.48	.563	−25		534.84	.106	−24
	325.55	.563	−21		535.80	.108	−27
	326.47	.566	−22†		541.68	.123	−27†
	326.56	.566	−27†		543.63	.128	−33
	327.46	.568	−17		549.66	.144	−31
	327.49	0.568	−20		551.76	0.150	−29

The effect of reflection on the velocity-curve of a close spectroscopic binary, as reported upon by Kopal, does not affect the velocity-curve of these A-type stars, since the largest correction to any velocity is 0.5 km/sec.

THE VISUAL COMPANION TO THE TRIPLE SYSTEM

The spectrum of the faint visual companion (7.8 mag.) exhibited very strong hydrogen lines and a strong K line and is classified as A0. $H\epsilon$ appears unusually strong and broad. Some faint metallic lines in the spectrum were, in all probability, due to the bright component from which some light may have been spread over onto the slit during moments of poorer seeing. Since the position angle is approximately 318°, the difficulties of guiding upon the faint component alone increase with advancing hour angle when atmospheric refraction causes rapid apparent drift in declination.

TABLE 6

JOURNAL OF OBSERVATIONS FOR THE A-TYPE STARS

Year	Julian Day 2400000+	Bright Star		Final Phase	Faint Star		Other Final Velocities			
							λ 4481		λ 4078	
		Obs. Vel.	Binary Vel.*		Obs. Vel.	Binary Vel.	Strong	Weak	Strong	Weak
1938.......	29085.671	− 19	− 10	0.562	− 19	− 10	− 3	− 3	....	....
	091.653	+ 68	+ 76	.794	− 87	− 79	+ 70	−102	....	....
	092.652	− 95	− 87	.334	+ 83	+ 91	− 91	+107	....	....
	093.691	+ 78	+ 86	.896	−118	−110	....	....	+ 91	†
	094.712	− 82	− 74	.448	+ 98	+106	....	....	− 52	+ 96
	098.647	− 5	+ 3	.574	− 5	+ 3	00	00	....	....
	099.631	− 4	+ 4	.106	− 4	+ 4	....	....	....	....
	103.635	− 65	− 58	.270	+108	+115	....	....	....	....
	103.741	− 88	− 81	.327	+ 96	+103	....	....	− 71	+106
	105.631	−100	− 93	.348	+ 92	+ 99	....	....	− 91	....
	107.652	−108	−101	.440	....	....	....	....	−101	+ 83
	109.656	− 33	− 26	.523	+ 57	+ 64	....	....	− 62	+ 66
	110.644	− 5	+ 2	.057	− 5	+ 2	....	....	....	....
	114.644	− 62	− 56	.218	+113	+119	....	....	....	....
	114.745	− 61	− 55	.273	+ 62	+ 68	....	....	†	+ 63
	115.626	+ 50	+ 56	.750	− 39	− 33	....	− 31	....	....
	116.654	− 86	− 80	.305	+ 81	+ 87	....	....	....	....
	117.619	+ 72	+ 78	.826	−118	−112	....	....	....	....
	119.643	+ 64	+ 70	.920	− 78	− 72	....	....	....	....
	123.667	+ 8	+ 13	.095	+ 8	+ 13	....	....	....	....
	127.636	− 72	− 67	.240	+ 94	+ 99	....	....	− 95	+ 81
	130.647	+ 86	+ 92	.867	− 96	− 90	+ 85	−100	+ 95	†
	137.617	− 5	+ 1	.633	− 5	+ 1	....	....	....	....
	138.625	− 34	− 28	.178	....	....	....	....	....	....
	143.571	+ 75	+ 83	.851	−115	−107	....	....	+ 90	†
	144.613	− 98	− 90	.414	+ 87	+ 95	....	....	....	....
	146.593	− 72	− 63	.484	+ 66	+ 75	....	....	....	....
1939.......	29419.803	− 7	+ 8	.123	− 7	+ 8	....	....	....	....
	420.723	− 10	+ 5	.620	− 10	+ 5	....	....	....	....
	420.815	− 8	+ 7	.670	− 8	+ 7	....	....	....	....
	423.710	− 63	− 48	.235	+ 62	+ 77	....	....	− 72	+108
	423.880	− 78	− 63	.326	+ 86	+101	....	....	....	....
	427.797	− 75	− 61	.443	+109	+123	....	....	....	....
	431.700	− 13	+ 1	.552	− 13	+ 1	....	....	....	....
	431.852	+ 12	+ 26	.634	+ 12	+ 26	....	....	....	....
	435.770	+ 73	+ 86	.751	− 58	− 45	....	....	†	− 25
	438.776	−104	− 91	.375	+ 89	+102	....	....	†	+ 83
	439.822	+ 93	+106	.941	−122	−109	†	− 95	+ 96	− 73
	440.725	−108	− 95	.429	+102	+115	....	....	− 64	+ 93
	441.669	+ 42	+ 55	.939	−101	− 88	+ 74	− 81	†	−100
	445.650	− 16	− 4	.090	− 16	− 4	....	....	....	....
	445.737	− 10	+ 2	.137	− 10	+ 2	....	....	....	....
	447.806	− 62	− 50	.256	+120	+132	†	+ 86	− 46	+ 76
	448.675	....	....	.725	− 34	− 22	....	....	....	....
	451.691	−104	− 93	.355	+103	+114	− 76	....	....	....
	453.656	− 72	− 61	.417	+ 76	+ 87	....	....	†	+ 92
	456.726	+ 6	+ 17	.076	....	....	....	....	....	....
	480.636	+ 62	+ 70	.996	− 39	− 31	+ 60	− 80	....	....
	481.715	− 22	− 14	0.579	− 22	− 14	− 20	....	....	....

* Velocity of the A-type star indicating motion in the line of sight due only to its motion about the other component.

† Velocity was measured but was too small numerically, because of blending with the G spectrum.

TABLE 6—*Continued*

Year	Julian Day 2400000+	Bright Star Obs. Vel.	Bright Star Binary Vel.*	Final Phase	Faint Star Obs. Vel.	Faint Star Binary Vel.	Other Final Velocities λ 4481 Strong	λ 4481 Weak	λ 4078 Strong	λ 4078 Weak
1939.......	29485.675	+ 45	+ 53	0.719	− 64	− 56	+ 46	......	......	− 58
	486.661	− 46	− 38	.252	+ 59	+ 67	......	......	......	......
	487.587	+ 68	+ 76	.752	− 66	− 58	......	......	......	......
	490.631	−105	− 98	.398	......	......	− 90	+ 94	− 98	+ 80
1940.......	29785.800	+ 72	+ 90	.904	−129	−111	......	......	......	......
	810.691	−102	− 87	.354	+ 94	+109	......	......	......	......
	812.696	− 86	− 72	.438	+ 86	+100	......	......	......	......
	817.656	− 6	+ 8	.118	− 6	+ 8	......	......	......	......
	820.669	+ 42	+ 56	.746	− 85	− 71	†	− 78	......	......
	823.804	− 94	− 81	.440	+116	+129	− 69	†	......	......
	824.591	+ 79	+ 92	.866	−112	− 99	......	......	......	......
	842.682	− 20	− 9	.642	− 20	− 9	......	......	......	......
	845.699	− 82	− 72	.272	+ 82	+ 92	......	......	......	......
1941.......	30164.744	+ 24	+ 43	.680	− 50	− 31	+ 41	†	......	......
	171.678	−103	− 85	.427	......	......	− 87	+ 91	......	......
	171.763	− 98	− 80	.473	+ 65	+ 83	− 66	+ 71	......	......
	172.686	+ 60	+ 78	.971	− 83	− 65	+ 62	− 80	......	......
	172.752	+ 49	+ 67	.007	− 95	− 77	+ 57	− 77	†	− 79
	177.635	− 6	+ 11	.646	− 6	+ 11	......	......	......	......
	177.702	− 14	+ 3	.682	− 14	+ 3	......	......	......	......
	179.630	+ 28	+ 45	.724	− 71	− 54	+ 44	− 78	......	......
	180.665	− 93	− 76	.284	+ 64	+ 81	− 77	+ 79	......	......
	180.721	− 98	− 81	.314	+ 79	+ 96	−106	+121	− 98	†
	181.633	+ 70	+ 87	.807	−104	− 87	......	......	†	− 84
	181.790	+ 65	+ 72	.891	......	......	†	−101	†	−111
	183.700	+ 88	+104	.924	− 74	− 58	+ 77	−104	†	−100
	184.623	−115	− 99	.422	+ 80	+ 96	......	......	......	......
	184.691	− 84	− 68	.459	+ 93	+109	− 56	......	......	......
	187.662	+ 18	+ 34	.065	......	......	+ 43	......	......	......
	273.552	−116	−110	.479	+ 95	+101	......	......	......	......
	301.477	− 20	− 12	.570	− 20	− 12	......	......	......	......
	301.499	− 30	− 22	.582	− 30	− 22	......	......	......	......
	302.470	00	+ 8	.106	00	+ 8	......	......	......	......
	302.482	− 14	− 6	.113	− 14	− 6	......	......	......	......
	311.459	+ 83	+ 96	.964	− 58	− 45	......	......	......	......
	315.458	− 30	− 15	.125	− 30	− 15	......	......	......	......
	315.481	− 28	− 13	.137	− 28	− 13	......	......	......	......
	316.464	− 17	− 1	.668	− 17	− 1	......	......	......	......
	316.487	+ 10	+ 26	.681	+ 10	+ 26	......	......	......	......
	323.457	−103	− 84	.447	+ 75	+ 94	......	......	......	......
	323.478	− 99	− 80	.459	+ 65	+ 84	......	......	......	......
	325.551	− 20	+ 3	.578	− 20	+ 3	− 1	− 1	......	......
	327.460	− 21	+ 4	.610	− 21	+ 4	......	......	......	......
	327.551	− 24	+ 1	.659	− 24	+ 1	......	......	......	......
	328.457	− 66	− 40	.149	......	......	......	......	......	......
	329.551	+ 42	+ 69	.740	− 91	− 64	+ 87	− 93	......	......
	330.551	−109	− 82	.280	+ 77	+104	− 71	+ 86	− 79	......
	331.549	+ 75	+103	.820	−141	−113	+ 87	......	......	− 66
1942.......	30473.841	+ 18	+ 47	.713	−108	− 79	......	......	......	......
	474.635	−111	− 82	.250	+ 68	+ 97	− 64	......	......	......
	477.850	+ 49	+ 77	0.880	−148	−120	+ 68	− 79	......	......

TABLE 6—*Continued*

Year	Julian Day 2400000+	Bright Star Obs. Vel.	Bright Star Binary Vel.*	Final Phase	Faint Star Obs. Vel.	Faint Star Binary Vel.	Other Final Velocities λ 4481 Strong	λ 4481 Weak	λ 4078 Strong	λ 4078 Weak
1942.......	30479.984	+ 68	+ 96	0.033	−100	− 72	......	......	†	− 74
	483.889	− 27	00	.143	− 27	00	......	......	......	......
	484.897	+ 5	+ 32	.688	−105	− 78	......	......	......	......
	485.996	−117	− 90	.282	+ 24	+ 51	− 70	+ 66	......	......
	487.797	− 93	− 66	.255	+ 45	+ 72	− 59	+ 69	......	......
	488.843	+ 57	+ 84	.820	−116	− 89	......	......	......	......
	493.873	− 22	+ 4	.539	− 22	+ 4	......	......	......	......
	496.860	− 20	+ 6	.153	− 20	+ 6	......	......	......	......
	501.788	+ 59	+ 84	.816	−112	− 87	......	......	......	......
	502.793	−115	− 90	.359	+ 62	+ 87	+ 88	− 90	......	......
	503.907	+ 54	+ 79	.961	−119	− 94	......	......	......	......
	506.949	− 30	− 6	.604	......	......	......	......	......	......
	507.867	− 7	+ 17	.100	− 7	+ 17	......	......	......	......
	512.930	+ 68	+ 92	.837	−111	− 87	......	......	......	......
	515.946	−108	− 85	.466	+ 68	+ 91	......	......	......	......
	522.785	− 54	− 32	.162	......	......	− 76	†	......	......
	528.801	−103	− 81	.413	+ 74	+ 96	......	......	......	......
	534.846	+ 11	+ 32	.679	......	......	......	......	......	......
	535.805	− 27	− 6	.197	− 27	− 6	......	......	− 89	......
	543.635	−109	− 89	.429	......	......	+ 74	−111	......	......
	551.758	+ 84	+103	.818	−104	− 85	......	......	......	......
	576.631	− 82	− 66	.259	+ 73	+ 89	......	......	......	......
	576.734	− 99	− 83	.315	+ 93	+109	......	......	......	......
	577.734	+ 85	+100	.855	......	......	......	......	......	......
	577.772	+ 74	+ 89	0.876	− 87	− 72	......	......	......	......

TABLE 7

	G	*A* (Primary Plus Secondary)	*A* Primary	*A* Secondary
K........................	28	19 km/sec	90 km/sec	100 km/sec
$m^3 \sin^3 i/(m_1+m_2)^2$........	0.189	0.606	0.192	0.140
$m \sin^3 i$..................	1.16	1.70	0.694	0.624
m_2/m_1..................	1.47		0.90	

The spectrum was photographed twice on July 16 with a camera lens 3.25 inches in focal length and twice on August 25, 1942, with a camera lens 6 inches in focal length attached to the two-prism spectrograph of the 37-inch reflector. The dispersions of these plates were approximately 145 A/mm and 80 A/mm, respectively. In all the spectra the $H\epsilon$ line was very strong and wide, and therefore was omitted from the measures. Below are recorded the velocities found on the four plates from the three lines: *Ca* II K, $H\delta$, and $H\gamma$. Those obtained with the shorter camera must be given low weight. No deductions can definitely be made from these velocities, but it will be interesting to ascertain from spectrograms obtained in the future whether or not this star is a binary. There is a large probability that this is so from consideration of the order of size of these negative velocities, even allowing a large error. It should be remembered that the velocity of the center of mass of the triple system of the bright component is -23.3 km/sec.

Date	Julian Day	Camera	Velocity	Total Weight
July 16.....	2430557.706	3-inch	−54 km/sec	2.5
16.....	557.729	3-inch	44	2.5
Aug. 25.....	597.611	6-inch	32*	2.0
25....	597.671	6-inch	−44	2.5

* This velocity is affected by the measures of overexposed comparison lines showing a coma effect.

INTERSTELLAR CALCIUM

On several plates where the components of the K line were widely separated, there seemed to be an extremely faint line between them. On the possibility of this being an interstellar line, and not just grain effect, measures were made of the line on six plates which showed it most distinctly. The six velocities measured are listed here:

Julian Day	Velocity	Julian Day	Velocity
2429116.654............	−10 km/sec	2430180.721............	−19 km/sec
130.647............	17	184.623............	15
146.593...........	− 7	503.907...........	−25

The mean of the above velocities is -16 km/sec. The line-of-sight component of the solar motion in the direction of d Serpentis is -17.5 km/sec.

DISCUSSION

The mass functions and the values of $m \sin^3 i$ for each star of the triple system are given in Table 7, with the mass ratio for each binary. If the G-type star is to be a giant, as indicated by the strength of its spectrum compared with that of the two A-type stars, its mass must be much more than $1.16\odot$, and therefore i cannot be close to 90°. Since the values of $m \sin^3 i$ for the two A-type stars are small, compared with their total $m \sin^3 i$ as determined from their 386-day orbit, and since the latter orbit has an inclination far from 90°, it is evident that no eclipse can occur in the A-type binary system.

With the foregoing knowledge, the appearance of the spectra, and the dynamical parallax of the system,[9] we can estimate the masses themselves. From a comparison of the spectra of d Serpentis and α Aurigae (Sp. G0, $M_v = M_b = +0.1$) and an examination of the "filling-up" of the G-type absorption lines in d Serpentis, it was estimated that the continuous spectra due to the G-type star and to the A-type pair were of about equal intensity at $\lambda = 4200$. The difference in absolute magnitude (ΔM) between the giant and one of the A-type stars was found from the use of Planck's formula for the region λ 4200A with $T_G = 5200°$ K and $T_{A2} = 9700°$ K. Then $E_G = 3.98 \times 10^{14}$ ergs/cm²sec and $E_A = 8.63 \times 10^{15}$ ergs/cm²sec. For simplicity, it may be assumed that we have two A-type stars of equal luminosity and mass. Hence, $L_G = 2L_A$ at $\lambda = 4200$, and it follows that $R_G/R_A = \sqrt{2E_A/E_G} = 6.58$. For visual light of effective wave length 5290 A, $E'_G = 4.88 \times 10^{14}$ ergs/cm²sec and $E'_A = 5.82 \times 10^{15}$ ergs/cm²sec. These

[9] Russell and Moore, *The Masses of the Stars*, University of Chicago Press, 1940.

quantities and the ratio of the radii give $L'_G/L'_A = 3.64$. Thus the difference between the visual absolute magnitudes of the G-type and one A-type star is $\Delta M = 1.40$. When the corrections for stars of types G and A_2 have been applied,[10] the difference in bolometric absolute magnitude for use with the mass-luminosity relation is $\Delta M_0 = 1.13$. The empirical formula for the slope of the mass-luminosity-curve[11] is $\Delta \log \overline{\text{mass}^{1/3}} = 0.04$ (ΔM_b) and yields for the two A-type stars, relative to the G-type star, a mass ratio of 1.46. The observed value given in Table 7 is 1.47.

It is interesting to note that, if we assume $M_v = 0$ for the star with gG0 spectrum and use the bolometric corrections and formula for the mass-luminosity relation mentioned above, the masses of the stars are approximately 3.7$\odot$, 2.9$\odot$, and 2.7$\odot$ for the G-type, A-type primary, and secondary, respectively. The inclinations which follow are approximately 43° for the orbit of the G-type star and center of mass of the binary system, and approximately 38° for that of the binary system itself. The values of the semimajor axes then would be 193,000,000 km for the G-type star and 3,700,000 km and 4,100,000 km for the primary and secondary A-type stars. The apparent visual magnitudes corresponding to the above masses and absolute magnitudes are 5.8, 7.0, and 7.5, respectively. These individual magnitudes yield a composite apparent visual magnitude of 5.3, which agrees satisfactorily with the photometrically determined value 5.33 mag. given in the *Henry Draper Catalogue*. From Russell's approximate formula, $M_v = (29{,}500/T) - 5 \log R - 0.08$, using $T_G = 5200°$ K and $T_A = 9700°$ K, the radii of the individual stars of d Serpentis are about 13$\odot$, 2.2$\odot$, and 1.8$\odot$, or 9,000,000 km, 1,500,000 km, and 1,300,000 km.

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[10] *Ap. J.*, **88**, 446, 1938.

[11] Russell and Moore, *op. cit.*