THE CRAB NEBULA*

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ABSTRACT

Photographs of the Crab nebula, taken in selected regions of wave length, show that the nebulosity consists of two distinct parts, namely, an outer system of filaments and an inner mass of amorphous structure. Exposures in the *Hα* region lead to the conclusion that the line spectrum is localized in the envelope of filaments, the continuum in the inner amorphous mass. More than 80 per cent of the light of the nebula is contributed by the continuum. This high intensity can be understood only if the continuum is interpreted as an emission spectrum.

The star that excites the nebula cannot be conclusively identified at present. The north following component of the central double star is definitely ruled out. Proof that the south preceding component is the exciting star can be expected only after the proper motion of the nebula has been better determined.

The present angular rate of expansion of the nebula is discussed. The value resulting from Duncan's measures appears to be too large, and the inferred acceleration of the expansion is probably spurious.

The important new data about the nova of 1054 which Professor Duyvendak[1] has recently made available leave hardly any doubt, as Oort and Mayall[2] have shown, that this star is the parent of the Crab nebula and that at maximum it reached the absolute magnitude −16.5, which means it was a supernova. If anything could be added to Oort and Mayall's final conclusions, it is the statement, implicit in their reasoning, that the nova of 1054 was a supernova of type I.

The investigation of the remnants of former galactic supernovae[3] is of special interest for two reasons. First, it should answer the question as to the final state of a supernova. Estimates of the energy released during an outburst—from an integration of the observed light-curve—make it probable that this quantity is of the order of the total heat content of a star. It has therefore been suggested that the outburst marks the transition of a star into the collapsed state. Second, such an investigation should provide much needed information about what actually happens during the outburst. Although in recent years we have observed a number of supernovae in action, we know nothing about the physical events which take place at the time of the outburst because of our inability to interpret the spectra. Moreover, there seems to be little hope of obtaining new clues during this active stage of a supernova, unless one appears bright enough to be followed spectroscopically for at least two years. Since only a supernova within our own galaxy or the Magellanic Clouds or an exceptionally bright one in the more distant members of the local group would meet this requirement, the chances do not appear very promising.

We have, therefore, in the last two seasons undertaken an investigation of the Crab nebula. In the present article some preliminary questions are discussed; the main results thus far obtained are presented in the following paper[4] by R. Minkowski.

I. THE STRUCTURE OF THE CRAB NEBULA

A peculiar feature of the Crab nebula is the strong continuous spectrum which appears associated with a line spectrum typical of gaseous nebulae.[5] The unusual character of the

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[1] *Pub. A.S.P.,* **54,** 91, 1942. [2] *Pub. A.S.P.,* **54,** 95, 1942.

[3] In the following discussion the term "supernova" always refers to a supernova of type I. Supernovae of type II, with luminosities intermediate between those of ordinary novae and supernovae of type I, appear to be closely related to the ordinary novae. In any case during an outburst they present essentially the same phenomena as common novae.

[4] *Mt. W. Contr.,* No. 666; *Ap. J.,* **96,** 199, 1942.

[5] V. M. Slipher, *Nature,* **95,** 185, 1915; R. F. Sanford, *Pub. A.S.P.,* **31,** 108, 1919.

resulting spectrum is best reflected in E. Hubble's remark[6] that "the Crab nebula has so strong a continuous background for its bright-line images that its spectrum could be classed as continuous almost as reasonably as emission." Such continua in emission nebulae are usually due to dust particles which reflect and scatter the light of the exciting star. However, this explanation certainly does not hold in the present case. Since the exciting star of the Crab nebula is of the sixteenth magnitude, the total light of the nebula due to reflection and scattering must be of the same order. Obviously, the resulting surface brightness would be so low as to make this reflected light quite unobservable.

In an attempt to obtain further information on this continuum and its origin, I began a few years ago to photograph the Crab nebula in selected regions of wave length. Through a proper combination of plates and filters, regions varying in width from 400 to 1500 A were explored. The first of these photographs, which covered in successive steps the range between λ 3300 and λ 6500, showed nothing out of the ordinary. Although minor differences in details were noted, the plates, on the whole, repeated the familiar picture of the Crab nebula in the ordinary photographic region (see Pl. IX).

A remarkable feature, however, appeared on the first exposure to include the region around $H\alpha$ (see Pl. X). This plate, covering the range from λ 5200 to λ 6600, brought out in great strength an intricate network of filaments which, although present on the earlier plates, had escaped attention on account of its faintness. The plate also suggested that the nebula consists of two distinct parts: the network of filaments just mentioned, which forms an envelope around the nebula, and an inner S-shaped mass of amorphous character.

There could be little doubt that the greatly increased strength of the filaments on the last plate must be due to unusually strong emission in the $H\alpha$ region and experience suggested that the strong emission is either the $H\alpha$ line itself or the well-known combination of $H\alpha$ and the [*N* II] pair, λ 6548 and λ 6584.[7] In contrast to the filaments, the inner amorphous mass showed no strengthening but appeared with practically the same intensity as on the plate which covered the range from λ 5200 to λ 6500.

In view of the unusual spectrum of the Crab nebula, it was tempting to conclude that the line spectrum originates in the filaments, the continuum in the inner amorphous mass.[8] The following test showed that this interpretation is correct. If the amorphous mass emits the continuum, whereas the line spectrum originates in the filaments, a reduction of our former range of wave lengths (λ 5200–λ 6600) to the narrower region λ 6300–λ 6700 should weaken the image of the amorphous mass to less than one-half of its former intensity without affecting the strength of the filamentary network. Plate XI, taken on an Eastman 103E plate behind a Schott RG2-filter (effective range λ 6300–λ 6700), shows that such is the case. It reveals in a striking manner the outer system of filaments with the amorphous mass faintly visible inside.

Thus far it has been shown only that the filaments emit essentially a line spectrum, the amorphous mass essentially a continuum. But the crucial spectroscopic test is now easy. Using the red photographs of the nebula as a guide, Minkowski[4] obtained a number of spectra in selected positions of the slit. These spectra, which cover the ordinary photographic region λ 5000–λ 3600, show that wherever the slit crosses a red filament the emission lines of the spectrum flare up; where it traverses the undisturbed background of the amorphous mass only the continuum shows. We thus have conclusive proof that the line spectrum of the Crab nebula originates in the outer envelope of filaments, the continuum in the inner amorphous mass. It is hardly necessary to point out that on account of these features the Crab nebula is a unique object among the galactic nebulae.

[6] *Mt. W. Contr.*, No. 241, p. 18; *Ap. J.*, **56**, 179, 1922.

[7] Later spectra have shown that the [*N* II] pair is largely responsible for the emission.

[8] The old empirical rule that filamentary structure in nebulae indicates line emission, while smooth amorphous forms indicate continuous spectra, also played some part in these considerations.

The form and structure of the filamentary system are clearly outlined on Plate XI. To obtain a picture of the inner amorphous mass, undisturbed by the filaments, it is only necessary to find a range of wave lengths free from strong lines of the outer envelope. Such a range is that from λ 7200 to λ 8400, which can be easily reached with modern plates. A photograph of the Crab nebula on an Eastman IN plate behind a Wratten No. 88 filter (effective range, λ 7200–λ 8400) shows the image of the inner mass free from interference by the surrounding filaments (see Pl. XII).

It was pointed out in an earlier paragraph that the observed continuum of the Crab nebula cannot be a reflection spectrum. With the localization of the continuum in the amorphous mass, we are now in a position to make an estimate of its integrated brightness. The integrated photographic magnitude[9] of the whole Crab nebula is $m_{\rm neb} = 9.0$. Since the photographic magnitude of the central star is $m_* = 15.9$, the corresponding intensity ratio is $I_{\rm neb}/I_* = 580$. A conservative estimate indicates that in the photographic region the amorphous mass (continuum) contributes more than 80 per cent of the total light or that $I_{\rm cont}/I_* \geqq 450$. The possibility that this large ratio might be due to obscuration of the star by the nebulosity can be ruled out at once. As N. U. Mayall has pointed out,[10] the nebula must be very transparent to light in the photographic region, because the red and the violet components of the emission lines, which come from the rear and the front parts of the shell, respectively, are on the average of equal intensities.[11] Only one explanation appears, therefore, to be plausible, namely, that the continuum is an emission spectrum, excited by a star of very high temperature. In the following *Contribution* by Minkowski it will be shown that the free-free and free-bound transitions of electrons in a highly ionized gas offer a satisfactory explanation of this emission continuum of the Crab nebula.

Plate XI shows that the filamentary system has a highly regular elliptical outline with only a minor distortion (bulge) in the south preceding quadrant. To obtain quantitative data about the form and the dimensions of the envelope, the outer contours of the filamentary system have been traced on enlarged paper prints of the best plates. The ease with which the outermost contours can be fixed in this manner is illustrated by the fact that the lengths of major and minor axes of the nebula can be determined with an accuracy of $\pm 5''$ for each plate. An ellipse drawn with the mean values, $a = 178''$, $b = 120''$, fits the observed contour perfectly (except at the bulge). For the best adjustment, the center of the ellipse coincides closely with the faint double star in the nebula, the exact co-ordinates of the center being

$$\Delta\alpha = +4''\!.7\,(\pm 3'')\,, \qquad \Delta\delta = 0''\!.0\,(\pm 3'')\,,$$

if the north following component of the double star is chosen as origin. The uncertainties of these figures are estimates indicating the amounts by which the ellipse could be shifted without destroying the fit.

The axial ratio of the ellipse is $b/a = 0.67$. The corresponding ratio for the ellipsoidal shell, if we assume it to have two equal axes, will be smaller on account of the inclination of the main plane to the line of sight. This inclination is unknown at present. However, Mayall's published spectrum[10] of the Crab nebula with the slit along the minor axis suggests that the center line of the λ 3727 emission is inclined, the north preceding end of the line being shifted toward the red. If this interpretation is correct, the main plane of the

[9] This magnitude is based on a plate taken by the writer with the schraffierkassette at the 10-inch refractor.

[10] *Pub. A.S.P.*, **49**, 104, 1937.

[11] Star counts within the nebula and in four adjacent areas confirm this conclusion, although for obvious reasons they have less weight. On the red plates of long exposure (limiting magnitude fainter than $20^{\rm m}$), 117 stars were counted within the area of the nebula, compared with the mean of 139 ± 1 stars for four adjacent fields of equal area. The deficit of 16 per cent in the nebula is entirely due to faint stars lost in bright filaments.

PLATE IX

PHOTOGRAPH OF THE CRAB NEBULA: λ 3600–λ 5000

PLATE X

PHOTOGRAPH OF THE CRAB NEBULA: λ 5200–λ 6600

ellipsoid would be tilted southward by perhaps 20°–30°, and the true ratio b/a for the ellipsoid would be about 0.60.

Although the amorphous mass is much less regular in outline than the outer envelope, it undoubtedly is more flattened. This difference between the two systems becomes quite obvious when one traces on Plate XII the contour of the inner mass for threshold intensities. At the ends of the major axis (along the main plane of the shell) it reaches nearly to the limits of the filaments, whereas at right angles to this direction it terminates well inside the envelope. The structure of the envelope itself reflects this fact. The filaments are best developed in the direction of the minor axis, while at the ends of the major axis they appear weakened. Using as a rough measure of the flattening the extension of the inner mass in these two directions, one obtains $b/a = 0.54$—a value decidedly smaller than that for the outer envelope, 0.67.

A final remark may conclude this discussion of the structural features of the Crab nebula. All our present data about the expansion of the nebula refer to the outer envelope. Since in a shell which has been expanding for more than 800 years the moving matter is arranged according to velocities—the fastest moving particles are farthest out —this means that the measured velocities represent the upper limit of the velocity distribution. According to Mayall's measures,[10] this limit is of the order of 1300 km/sec. It seems, therefore, that the velocities with which matter is ejected during a supernova outburst are quite moderate and of the same order as those found for ordinary novae.

II. THE CENTRAL STAR OF THE CRAB NEBULA

A. *Magnitude and color.*—It has long been assumed that one of the components of the faint double star at the center of the Crab nebula is the source exciting the nebulosity. Since the exciting star must have an unusually high temperature, it should be outstanding on account of its color. Curiously enough, all attempts to identify the star in this way have been indecisive. Neither one of the two stars near the center seems to have an abnormally low color index, and a search among the fainter stars down to magnitude 20 proved equally unsuccessful.

TABLE 1

MAGNITUDES AND COLORS OF THE TWO CENTRAL STARS

Star	Photographic			Photovisual			C.I.
	I	II	Mean	III	IV	Mean	
S. prec.......	15.85	15.94	15.90	....	15.45	15.45	+0.41
N. foll........	15.90	15.96	15.93	15.18	15.08	15.13	+0.80

Since these earlier tests were merely qualitative, the color indices of the components of the central double star were recently determined from polar comparisons at the 60-inch reflector. All exposures were made with the mirror diaphragmed down to 40 inches to reduce as much as possible the disturbing effects of the nebulosity. The resulting photographic and photovisual magnitudes of the central double star, derived from two pairs of plates of excellent quality, are given in Table 1.

Since on account of a defect the south preceding component could not be measured on one of the photovisual plates, the photovisual-magnitude difference between the two stars was determined on a number of plates taken at the 100-inch reflector. Each of these plates received two exposures, with the mirror diaphragmed down to 84 and 58 inches, respectively, the reduction in magnitude amounting to 1.10 mag. The magnitude difference of the two stars from four such plates is

$$\Delta m_{pv} = 0.40 \pm 0.02 ,$$

the south preceding component being the fainter. This value was taken into account in computing the color index of the south preceding star in Table 1.

The measured color indices confirm the earlier conclusions that neither of the two stars is distinguished by an abnormal color. The actual color indices will be somewhat smaller, since, on account of the low latitude ($b = -4.3°$) and the distance ($D \sim 1000$ parsecs) of the Crab nebula, the effects of space reddening are perceptible.

To allow for the space reddening, the color excess for the Crab nebula has been estimated in two ways:

a) By using Stebbins'[12] color excesses of faint B-type stars in the vicinity of the nebula. This leads to an average color excess of +0.15 mag. on Stebbins' scale, if stars of distances comparable to that of the Crab nebula are considered.[2] The corresponding color excess on the international scale is

$$CE_{\text{Int}} = +0.23 \text{ mag.} \tag{1}$$

b) From counts of extragalactic nebulae in the field of the Crab nebula. A one-hour exposure of the nebula on an ordinary photographic plate at the 100-inch reflector revealed two faint extragalactic nebulae. Their reality has been confirmed on the red plates of long exposure, which reveal 16 additional faint nebulae. These data indicate that the obscuration in the field is close to the critical value at which the number of nebulae—for standard exposures of one hour on ordinary photographic plates at the 100-inch—drops to zero. Because of the close correlation between obscuration and reddening, this point should be marked by a critical value of the color excess as well. This is, indeed, what Stebbins and Whitford[13] have found in their investigation of the reddening of globular clusters, i.e., that the number of nebulae for standard exposures at the 100-inch drops to zero when the color excess (on their scale) reaches the value +0.20 mag. The quick transition from presence to absence of nebulae in the neighborhood of this value is illustrated by their finding that, when the color excess is smaller than +0.19 mag., nebulae are seen on the plates; when it is greater than +0.20 mag., no nebulae are found. We adopt therefore as an upper limit of the color excess in the field of the Crab nebula +0.20 mag. on Stebbins' scale, corresponding to the value

$$CE_{\text{Int}} = +0.32 \text{ mag.} \tag{2}$$

on the international scale.

The small difference between the two values (1) and (2) suggests that most of the reddening in the field of the Crab nebula is caused by particles within less than 1000 parsecs of the sun. For the further discussion we use the mean of the two values

$$CE_{\text{Int}} = +0.27 \text{ mag.}$$

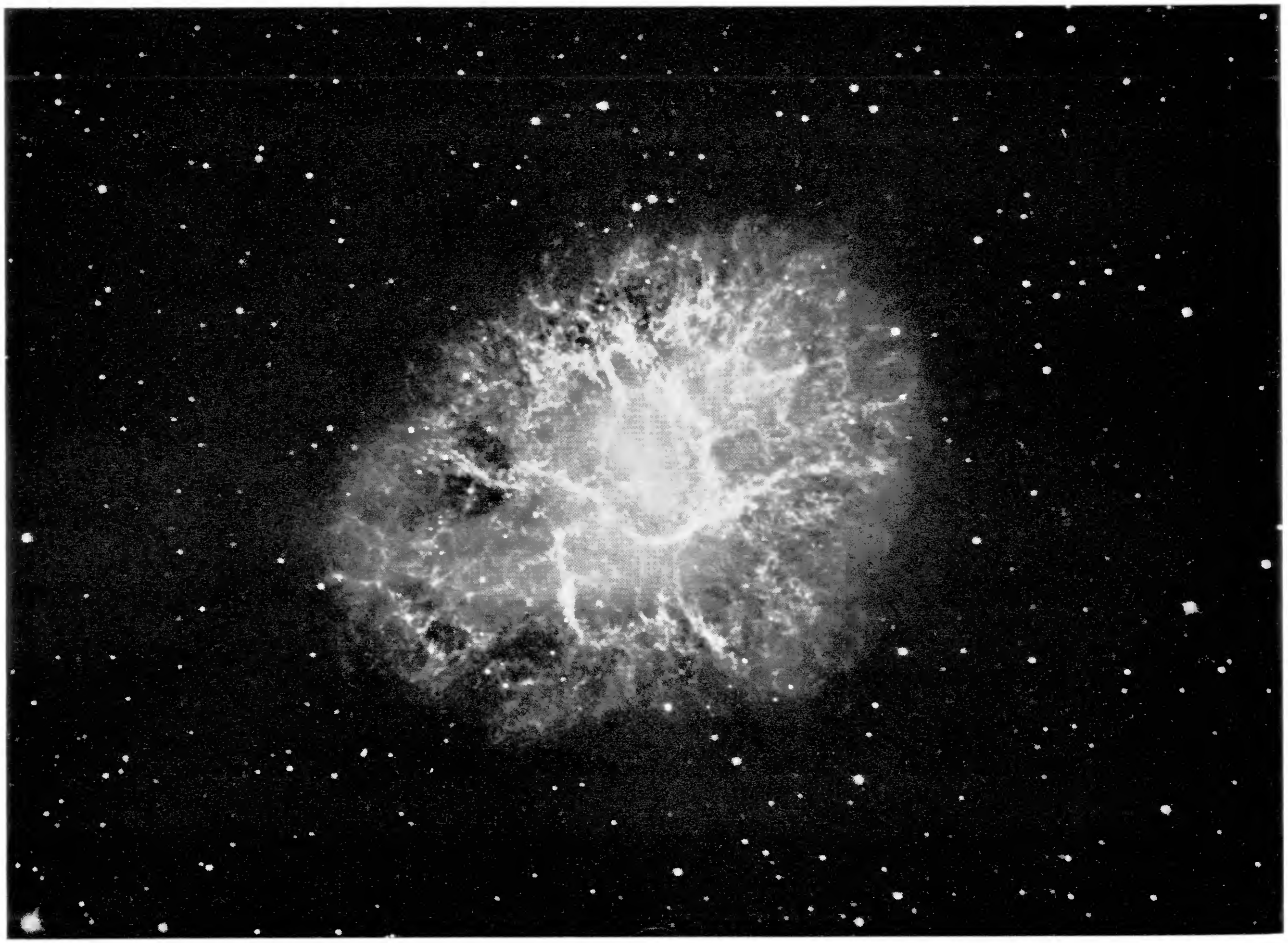
Subtracting this color excess from our measured values in Table 1, we obtain for the color indices, freed from absorption:

s. prec. star . . . . +0.14 mag.,
n. foll. star . . . . +0.53 mag.

[12] Stebbins, Huffer, and Whitford, *Mt. W. Contr.*, No. 621; *Ap. J.*, **91**, 20, 1940.

[13] *Mt. W. Contr.*, No. 547; *Ap. J.*, **84**, 132, 1936.

PLATE XI



Photograph of the Crab Nebula: λ 6300–λ 6700

PLATE XII

PHOTOGRAPH OF THE CRAB NEBULA: λ 7200–λ 8400

The corresponding spectral types would be F1 and G4, respectively.[14] These photometric results are best discussed in connection with the spectra of the two stars which Minkowski has obtained.

The spectral type of the north following component, as inferred from the color, is G4, in agreement with the observed spectrum for which the various estimates range from an early F- to an early G-type. The north following component of the double star is therefore definitely eliminated as an exciting source of the Crab nebula.

For the south preceding star the verdict is more difficult. There is essential agreement as to the color, since, according to Minkowski, the intensity distribution in the spectrum suggests a late B-type, if not a more advanced one. What makes the case puzzling is the fact that no lines could be seen in the spectrum. In view of the rather small dispersion, this would not be surprising if we were dealing with weak lines. But we should expect strong hydrogen lines if the star were of a late A- or early F-type as its color suggests, and it is difficult to understand how they could have escaped detection. The present data suggest, therefore, that the star is abnormal, displaying at the same time high-temperature characteristics (absence of lines) and a low color temperature. If this be true, the star probably is the exciting star of the Crab nebula. However, additional evidence that no lines are present in the spectrum is required before this interpretation can be accepted without reservation.

B. *The proper motions of central star and nebula.*—In view of these inconclusive results concerning the central star, it seemed of interest to examine the recent measures of the motions of the Crab nebula which J. C. Duncan has published.[15] Since Duncan has determined the proper motions of both the nebula and the two stars, a discussion of his data might throw light on the question whether one of the stars is associated with the nebula.

Duncan represented his measured motions in the nebula by

$$x = x_0 + t\Delta x , \qquad y = y_0 + t\Delta y ,$$

where x, y are the co-ordinates of the measured point; Δx, Δy its annual motions in the two co-ordinates. The equations are solved for the unknowns t—the time which has elapsed since the outburst—and x_0, y_0—the point at which the nebula started at the time of the outburst. The distance of this point from the present center of the nebula represents the total proper motion of the nebula since 1054. Duncan's procedure involves the assumption that the nebula has expanded at a constant rate; and his resulting value of t places the outburst in the year 1172, 118 years later than the date indicated by the Chinese annals. Since we are now certain that the Crab nebula started its expansion in 1054, the difference between observed and computed values would be an indication that the expansion has been accelerated since the outburst.

Although this circumstance should not affect Duncan's computed values of x_0, y_0, a more appropriate solution would define x_0, y_0 simply as the converging point of the measured velocity vectors. It involves the less objectionable assumption that the ejected particles have moved in straight lines since the outburst. The equations of condition in this case are, in the same notation as above,

$$y_0\Delta x - x_0\Delta y - (y\Delta x - x\Delta y) = 0 .$$

The resulting co-ordinates of the convergent point, referred to the north following component of the double star as origin, are

$$x_0 = +24''.3 \pm 5''.8\ (m.e.) , \qquad y_0 = +\ 1''.7 \pm 4''.8\ (m.e.) ,$$

[14] Based on a relation between Harvard spectral type and international color index which has been freed from the effects of interstellar reddening. I am greatly obliged to Dr. F. H. Seares for putting these data at my disposal.

[15] *Mt. W. Contr.*, No. 609; *Ap. J.*, **89**, 347, 1939.

in close agreement with Duncan's values:

$$x_0 = +20\overset{''}{.}3 \pm 3\overset{''}{.}3\,(m.e.)\,, \qquad y_0 = -\ 1\overset{''}{.}8 \pm 3\overset{''}{.}0\,(m.e.)\,.$$

That the computed errors in Duncan's solution are the smaller is to be expected since he included t as an additional unknown quantity. The real uncertainty in the position of the convergent point is clearly better represented by the errors of the new solution. For the following discussions the new values x_0, y_0 will therefore be adopted.

The convergent point and the time that has elapsed since the outburst being known, we can derive the proper motion of the nebula as soon as we have fixed the present center of the nebulosity. Two reasons, however, make it advantageous to conduct the discussion in terms not of proper motion but of the position of the convergent point. One is the uncertainty about the present center of the nebulosity, especially its center of gravity. The other is the possibility that exciting star and nebula may not have the same proper motion. This possibility has to be seriously considered since there is good reason

TABLE 2

POSITIONS OF CENTER OF NEBULA AND OF THE TWO CENTRAL STARS FOR 1054

(Origin: Present Position of North Following Star)

Object	x_0	y_0	Authority
Nebula...........	$+24\overset{''}{.}3\pm5\overset{''}{.}8$	$+1\overset{''}{.}7\pm4\overset{''}{.}7$	Duncan; new reduction
S. prec. star......	$\{+14\overset{''}{.}6\pm3\overset{''}{.}2$	$-4\overset{''}{.}3\pm3\overset{''}{.}0$	Duncan ($\mu_\alpha=-0\overset{''}{.}019$, $\mu_\delta=0\overset{''}{.}000$)
	$\{+13\overset{''}{.}7$	$-1\overset{''}{.}7$	Poulkovo ($\mu_\alpha=-0\overset{''}{.}018$, $\mu_\delta=-0\overset{''}{.}003$)
N. foll. star.......	$0\overset{''}{.}0$	$-1\overset{''}{.}8$	Poulkovo ($\mu_\alpha=0\overset{''}{.}000$, $\mu_\delta=+0\overset{''}{.}002$)

to believe that the mass of the nebula is comparable to that of the star. If during the outburst more matter were ejected in one direction than in others, the center of mass velocity of the shell would be shifted in that direction, that of the star in the opposite direction, with the result that the proper motions of shell and star would no longer be identical. Both star and shell will, however, be at the same point—the convergent point—at the date of the outburst if their present motions are extrapolated backward.

In Table 2, therefore, are given the positions of the nebula and the two central stars for the year 1054, referred to the present position of the north following component as origin. In this system the present co-ordinates of the south preceding component are $\Delta x = -2\overset{''}{.}19$, $\Delta y = -4\overset{''}{.}35$. Included in Table 2 are the recent Poulkovo measures of the motions of the two stars by Deutsch and Lavdovsky.[16]

These figures confirm our former conclusion from color and spectrum that the north following component of the central double star is not associated with the nebula but must be a background star. On the other hand, there is fair agreement in the positions of the south preceding component and the nebula for 1054. A discordance of $9\overset{''}{.}7$ in the x-co-ordinates is of the order of the computed mean error, $\pm6\overset{''}{.}6$. Altogether, one would be inclined to accept the existing agreement between these data as an indication that the south preceding star and the nebula are associated.

Nevertheless, there remain serious doubts whether this agreement is significant. They arise from the fact that the data of Table 2 imply an improbable motion of the nebula. Converted into linear measure, the peculiar transverse velocity of the nebula becomes $\mu_{\text{pec}} = 109 \pm 47$ km/sec in position angle 275°, if we adopt $\mu_\alpha = -0\overset{''}{.}0221 \pm 0\overset{''}{.}0073$, $\mu_\delta = -0\overset{''}{.}0019 \pm 0\overset{''}{.}0063$ as the proper motion of the nebula.[17] This velocity in itself

[16] *Poulkovo Obs. Circ.*, No. 30, 1940.

[17] These values result from x_0, y_0 as given in Table 2 and the present center of the nebula $\Delta x = +4\overset{''}{.}7 \pm 3''$, $\Delta y = 0\overset{''}{.}0 \pm 3''$ mentioned on p. 190.

would be unobjectionable and merely add the interesting feature that the nebula is a high-velocity object. What makes this high velocity suspicious is the fact that it is directed toward the hemisphere which is avoided by the apices of the high-velocity stars.[18] The violation of this rule, if interpreted on the basis of galactic rotation, is so serious that a check of the proper motions presented in Table 2 appeared desirable.

Such a check is possible at present only for the proper motions of the two stars for which a series of plates, taken for parallax by van Maanen in the years 1920–25 at the 100-inch Newtonian focus, is available. Second-epoch plates were therefore taken recently by the writer to duplicate some of van Maanen's earliest exposures. The proper motions of the two stars derived from two pairs of plates, measured with the large blink comparator, are in Table 3. The results are based on eight comparison stars of mean photographic magnitude 17.5. For comparison the values of μ_α, which van Maanen derived from his parallax series,[19] have been added in the last line. The mean errors of

TABLE 3

NEW PROPER MOTIONS OF THE TWO CENTRAL STARS

Plate Pair	Δt	South Prec. Star		North Foll. Star	
		μ_α	μ_δ	μ_α	μ_δ
I	20.ʸ14	−0.″0101	+0.″0037	+0.″0014	0.″0000
II	19.18	−0.″0086	+0.″0061	+0.″0029	−0.″0003
Mean	....	−0.″0094	+0.″0049	+0.″0022	−0.″0002
van Maanen	....	−0.″007 ±.001	....	+0.″001 ±.002	....

a final μ_α or μ_δ are ±0.″0015, as determined from the residuals of the two stars, together with those of three other faint proper-motion stars measured on the same plates.

The new values for the north following component show that the proper motion of this star is barely perceptible, in agreement with the results of the earlier observers.

For the south preceding component the agreement between the new values of Table 3 and those of Table 2 is poor—so poor, indeed, that one set of data must be seriously in error. It seems that the source of the discrepancy is a curious systematic error in Duncan's proper motions which affects one of the two stars but not the other. Its cause became obvious when the plates measured by Duncan were examined. Evidently the earlier of his two plates—the one taken by Ritchey in 1909—was taken in excellent seeing but with a disturbed figure of the mirror. As a consequence the star images appear elliptical with an asymmetrical intensity distribution. In measuring such images the observer will set the micrometer wire on the geometrical center of the ellipse for images of high densities, because he is unable to recognize the asymmetrical intensity distribution. For lower densities the point of "bisection" will be shifted more and more toward the center of gravity of the density distribution. The transition from one mode of setting to the other may be quite abrupt, depending on the gradation of the plate. This seems to be true for Ritchey's plate of the nebula. Of the two central stars, the north following clearly shows the asymmetrical intensity distribution, whereas the south preceding appears as an ellipse of uniform intensity. Since Duncan's comparison stars are all faint—about a magnitude fainter than the two central stars—we should therefore expect a magnitude error for the southern component, the northern component being unaf-

[18] At present the argument rests on the transversal component alone, since the radial velocity of the nebula is not accurately known. There is, however, good evidence that the latter is not very large.

[19] *Mt. W. Contr.*, No. 356, 1928.

fected. The error acts in the right direction to explain the measured proper motion of the south preceding star since the center of the ellipse lies eastward of the center of the intensity distribution; moreover, the size of the error is plausible. Altogether, we have good reason to believe that Duncan's value of the proper motion of this star is too large. How much his measures of the nebular points have been affected by the same error is difficult to judge at present. The final decision as to whether south preceding star and nebula are associated must therefore be deferred until the recent photographs in the red can be used for a determination of the proper motion of the nebula.

III. THE ANGULAR RATE OF EXPANSION OF THE NEBULA

A somewhat uncertain element in previous determinations of the distance of the Crab nebula has been the present angular rate of expansion. With the data now at hand it can be examined more closely. Since we are mainly interested in the expansion along the major axis of the nebula, we select from Duncan's measured points those closest to it: the points 6–9 on the preceding side of the nebula, the points 17–20 on the following

TABLE 4

ANGULAR RATE OF EXPANSION ALONG MAJOR AXIS OF NEBULA

Point	$\Delta x-\mu_a$	$\Delta y-\mu$	ΔS	Reduction Factor	ΔS_a	
6	−0″149	+0″143	0″2065	1.125	0″2323	Mean prec. points 0″234±0″014
7	− .186	+ .105	.2136	1.090	.2328	
8	− .098	+ .086	.1304	1.549	.2020	
9	− .194	+ .031	.1965	1.371	.2694	
17	+ .079	− .139	.1599	1.443	.2307	Mean foll. points 0″236±0″010
18	+ .111	− .110	.1563	1.333	.2083	
19	+ .142	− .066	.1566	1.594	.2496	
20	+0.184	−0.022	0.1853	1.371	0.2540	

side. To obtain from the measured motions the angular rate of expansion ΔS_a along the major axis the following corrections have to be applied.

1. The measured motions have to be freed from the proper motion of the nebula and from systematic errors with which the measures may be affected. The proper motion of the nebula derived from the convergent point and the present center of the nebula should take care of both factors, since, according to our previous discussion, it is the combined effect of both. For the following discussion we adopt the values derived on page 194:

$$\mu_a = -0''.0221 \pm 0''.0073 , \qquad \mu_\delta = -0''.0019 \pm 0''.0063 .$$

2. The measured points are at different distances from the center of the nebula. The resulting values for the outward motion therefore need reduction to a common contour line. As such, the outermost contour of the filamentary system has been chosen since it corresponds to the tips of the emission lines on both Mayall's and Minkowski's spectra.

3. Since the measured points do not lie on the major axis, a reduction to that axis is necessary.

The reductions (2) and (3) were performed graphically by means of the tracing of the outer contour mentioned on page 190. The results in Table 4 need no explanation, except that the reduction factor in the fifth column represents the combined corrections (2) and (3). The mean values in the last column of Table 4 show that all nonradial components in the measured motions have been successfully eliminated by our procedure, the rate of the expansion being the same for points both preceding and following the major axis.

The final mean of the present angular expansion along the major axis is

$$\Delta S_a = 0\overset{''}{.}235 \pm 0\overset{''}{.}008 \text{ per year.}$$

Since we found for the present length of the major axis $a = 178'' \pm 5''$, we obtain $\Delta t =$ 758 years for the time which has elapsed since the outburst,[20] if the expansion has proceeded at the present rate. On the other hand, the mean rate of the expansion since 1054 has been

$$\overline{\Delta S_a} = 0\overset{''}{.}201 \pm 0\overset{''}{.}006 \text{ per year.}$$

The difference between the two values, $0\overset{''}{.}034 \pm 0\overset{''}{.}010$, would be a measure of the acceleration which has taken place since the outburst in 1054.

Judged by its mean error, the acceleration seems to be fairly well established. Difficulties arise, however, when we try to understand it in terms of known forces. Obviously, we can invoke only radiation pressure to explain an acceleration of the nebula. The following two cases serve to illustrate the difficulties.

a) Assume that the expansion has proceeded at a constant acceleration, so that the radius s of the nebula is given by

$$S = at + bt^2 ,$$

t being the time since the outburst. Since for the epoch 1938 we know both $S_a = 178'' \pm 5''$ and $dS_a/dt = 0\overset{''}{.}235 \pm 0\overset{''}{.}008$, we have for the determination of a and b the two equations

$$178 = 884 (a + 884\ b) , \qquad 0.235 = a + 2 \cdot 884\ b ;$$

hence

$$a = +0\overset{''}{.}168 \pm 0\overset{''}{.}015 , \qquad b = +0\overset{''}{.}000038 \pm 0\overset{''}{.}000014 .$$

To convert these values into linear measure, we adopt a Doppler component $V_r =$ 1116 km/sec, derived from Mayall's measures,[21] for the expansion of the nebula. The resulting equation for the velocity is

$$V_r = 798 + 0.3609t \text{ km/sec} ,$$

where the acceleration $A = 0.3609$ is given in kilometers per second per year. The corresponding value in cm/sec^2 is

$$A = 0.0011 \text{ cm/sec}^2 .$$

This value should be compared with the acceleration of the nebula due to the radiation pressure of the star in its present state. The radiation pressure of a star of radius r and temperature T is

$$F = 4\pi r^2 u_0 ,$$

where

$$u_0 = \frac{7.67}{3} \cdot 10^{-15} T^4 \text{ erg/cm}^3 .$$

Adopting, from Minkowski's discussion,

$$r = 0.02 \odot = 1.4 \cdot 10^9 \text{ cm} , \qquad T = 5 \cdot 10^5 \text{ degrees} ,$$

[20] The agreement of this value with those derived by Duncan, and by Oort and Mayall (n. 2) in an entirely different way shows that our value of ΔS_a is consistent with the other data for the expansion.

[21] This value is the mean of 9 velocities, measured by Mayall on his two spectrograms within 30″ of the center of the nebula. Mayall's adopted value, $V_r = 1300$ km/sec, is probably too large, because the highest velocities measured seem to be associated with the irregularity (bulge) mentioned on p. 190.

we obtain

$$F = mA = 3.8 \cdot 10^{27} \text{ dynes},$$

which leads to accelerations, A, of the order of $2 \cdot 10^{-6}$–$2 \cdot 10^{-7}$ cm/sec^2 for masses of the nebula between 1 and 10 solar masses ($2 \cdot 10^{33}$–$2 \cdot 10^{34}$ gr.).

The radiation pressure of the present star is therefore quite insufficient to account for an acceleration of the nebula of the order $1 \cdot 10^{-3}$ cm/sec^2, which disposes of our assumption that the expansion of the nebula has proceeded at a constant rate of acceleration.

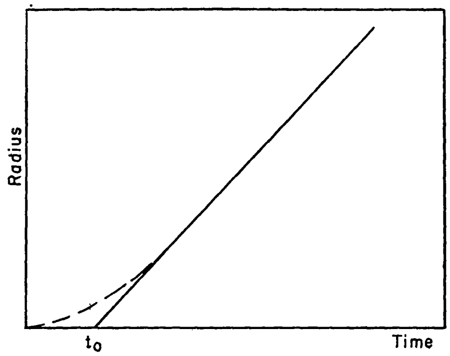


FIG. 1.—Radius-time diagram of expanding nebula with initial acceleration.

b) There remains the possibility that the acceleration took place during the months or perhaps years immediately following the outburst and that it had ceased by the time the star reached its present state. The growth of the radius of the nebula would then be of the type represented in Figure 1, with t_0 the date of the outburst as inferred from the final rate of expansion. The difficulty in this case arises from the fact that, according to Duncan's measures, the time difference between t_0 and the true date of the outburst amounts to about 100 years. Since it is obvious from Figure 1 that the acceleration is still near its maximum value at the time t_0, the bolometric intensity of the star 100 years after the outburst must have been at least 10^3–10^4 times as high as at present to explain the observed acceleration. This is highly improbable because it implies that the Crab nebula, after its decline below the sixth magnitude in 1056, was again a naked-eye object of apparent magnitude 0 some 100 years later. There seems to be only one way out of these difficulties, namely, to assume that the measured rate of the angular expansion is too large and that the resulting acceleration of the expansion is spurious. To settle this question, as well as that of the proper motion of the nebula, we shall have to wait until the recent red photographs can be used for a redetermination of the motions.

In the meantime it will be best to use the mean rate of angular expansion $\overline{\Delta S_a} = 0\overset{''}{.}201$ for determining the distance of the nebula. This value, together with a radial component of $V_r = 1116$ km/sec, leads to a distance modulus $m - M = 10.35$.

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MOUNT WILSON OBSERVATORY
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