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PHOTOGRAPHIC DETERMINATION OF THE DIAMETER OF MARS

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Introduction. This is the second photographic determination of the diameter of *Mars* with the 26-inch McCormick visual refractor, the first having been made by VAN DE KAMP from photographs taken in 1924, 1926 and 1927[1]. The discussion of his material indicated that small improvements in observational technique could be made in a similar investigation and that an increase in weight through a still larger range in diameter might be obtained. These aims have been largely realized, but resulting gains have been offset by the effects of adverse

TABLE 1

Plate No.	Number of Images	Date	E. S. T.	Seeing	Q	$\frac{1}{r}$	Diameter	Diff. Refr.
1	38	1934 Oct. 16	17 33	3.5	291°7	0.50	4″65	0″00
2	46	Oct. 23	17 26	3.5	292.3	0.51	4.77	.00
3	54	Oct. 23	17 48	3.5	292.3	0.51	4.77	.00
4	56	Nov. 14	17 52	3.7	293.7	0.56	5.25	.00
5	56	Nov. 14	18 15	3.6	293.7	0.56	5.25	.00
6	43	Nov. 19	17 54	3.0	293.8	0.57	5.37	.00
7	38	1935 Apr. 23	13 55	3.2	117.7	1.57	14.73	.02
8	35	Apr. 23	14 15	3.0	117.7	1.57	14.73	.02
9	32	Apr. 25	13 44	2.5	117.0	1.56	14.61	.02
10	31	Apr. 25	14 4	2.5	117.0	1.56	14.61	.02
11	9	1937 May 8	14 25	3.5	283.1	1.85	17.33	.02
12	9	May 8	14 42	3.5	283.1	1.85	17.33	.02
13	27	May 11	14 10	3.2	285.0	1.89	17.63	.02
14	27	May 11	14 22	3.2	285.0	1.89	17.63	.02
15	47	May 21	12 3	3.0	82.7	1.95	18.29	.02
16	72	May 21	12 39	2.7	82.7	1.95	18.29	.02
17	66	May 25	11 17	3.0	95.5	1.97	18.39	.02
18	66	May 25	11 48	2.8	95.5	1.97	18.39	.02
19	15	1939 Jul. 31	11 30	2.8	47.8	2.57	24.05	.04
20	28	Jul. 31	12 55	2.0	47.8	2.57	24.05	.04
21	64	Aug. 1	11 50	3.5	50.4	2.56	24.01	.04
22	81	Aug. 1	12 25	3.7	50.4	2.56	24.01	.04
23	77	Aug. 2	12 40	3.2	52.9	2.56	23.96	.04
24	61	Aug. 6	12 15	3.5	59.8	2.53	23.68	.04
25	86	Aug. 11	11 0	3.5	65.0	2.48	23.18	.04
26	92	Aug. 19	10 40	3.0	70.0	2.36	22.10	.04
27	81	Aug. 22	10 2	2.0	71.1	2.31	21.63	.04
	1337							

weather conditions. The total number of 16 observing nights turned out much smaller than was anticipated and the 25 plates with a total of 480 exposures, which were selected for measurement, of necessity include some taken under mediocre seeing. Consequently, the present result does not constitute an improved value but rather a new observation of approximately the same accuracy.

Observations and Measures. The plates were taken in the focal plane of the 26-inch visual refractor, through a Wratten yellow filter No. 12. The exposures were made by a shutter, consisting of two semi-circular cardboard discs which could be set at any desired sector opening up to 180°. This shutter revolved in a plane parallel and close to the plate by means of a small handle on the outside of the tube. The use of a star in the guiding eyepiece allowed the orderly placing of the exposures without having to manipulate the plate holder. The images were placed in rows, 2.5 mm apart, there being in the average some 8 images per row. In this way exposures could be made in rapid succession, a fact partly responsible for the total of 1337 images, of which 480 were selected for measurement.

The plate brands used are Wratten M for plates 1-10, Eastman Special III-G for plates 11-18 and Eastman Spectroscopic 103-G for plates 19-27. It will be shown later that no appreciable systematic errors were introduced by the different characteristics of these types of plates. The developers used were Carbonal for plates 1-12 and 19, Eastman Super-Fine-Grain for plates 13-18 and Eastman DK20 for plates 20-27, also without systematic effects upon the measures.

Table 1 contains the detailed information concerning the plates. In the fourth column the estimated average value of the seeing (on a scale of 5) for each plate is given. Columns 5, 6 and 7 give the values Q, $1/r$ and Diameter, taken from the American Ephemeris.

In the selection of images for measurement, care was taken to include only those with well defined edges, thus eliminating images affected by vibrations of the instrument during the exposure. Moreover, only those were chosen which exhibit clearly the typical surface markings of the planet, thus excluding all taken under poor seeing as well as the over-exposed images.

The diameter of each image was measured in the

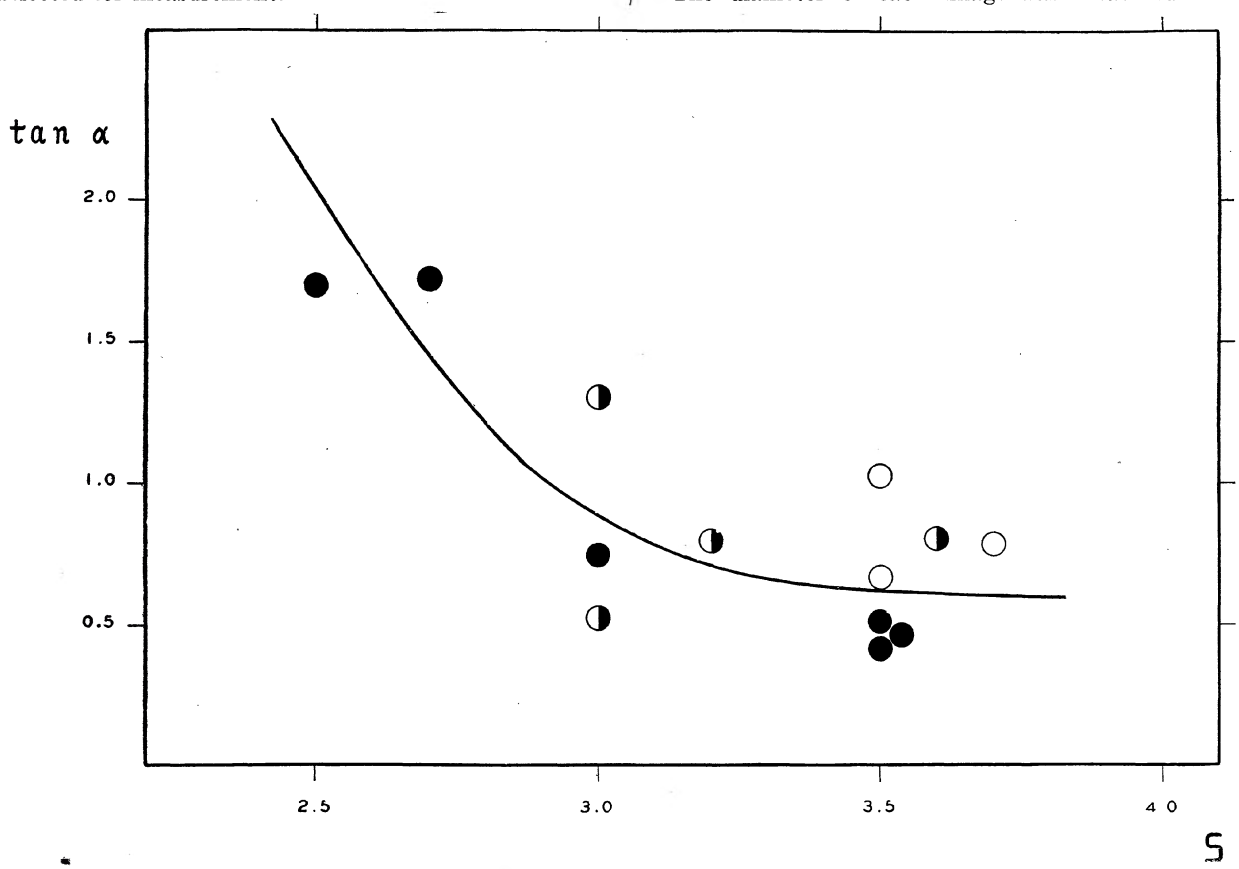

Figure. The rate of increase of the measured diameter with the density of the *Mars* image as a function of the seeing. Filled, half-filled and open circles designate weights of 1.0, 0.7 and 0.5, respectively.

direction perpendicular to that of the greatest phase effect, the plate having been oriented by means of a star trail together with the value of Q, listed in Table 1. Measurements were made in two positions of the plate, differing 180°, in each of which two settings were made on each limb. The 25 plates thus constitute 50 sets of measurements which were performed in random order. All measures were made on a Gaertner long screw engine under a magnifying power of 30 x.

The densities of all images measured for diameter were determined by means of the Schilt microphotometer, using a diaphragm of 1 mm aperture. The effective area on the plate, as determined by Mr. C. A. Wirtanen, amounts to 0.2 mm, or 4″. On the larger images of *Mars* the readings obviously vary with the position and an average value was adopted. If S is the galvanometer deflection on the surface of the planet and F that on clear film, the quantity $T=S/F$ is closely related to the transmission, and we may represent the densities of the images by values of $\mathfrak{D} = \log\frac{1}{T}$.

Solution. Mean values of the diameters in the "direct" and "reversed" positions of the plate were formed and converted into arc by means of the scale value, 1 mm being equivalent to 20″.75. After applying the small correction for differential refraction, the resulting values of the diameters were compared with those in the American Ephemeris, and the differences, in the sense "measured—A.E." were plotted against the corresponding values of $\mathfrak{D}$. Thus each plate furnishes a *diameter-density* relation, the measured diameter increasing with increasing density, as could be expected. In all cases a straight line represents the plotted points satisfactorily. The angle α with the $\mathfrak{D}$-axis appears to depend upon the seeing, decreasing with increasing quality of S. This relation is presented graphically in Fig. 1, and numerical values representing the smooth curve to be used in the reduction are given in Table 2. The value $\mathfrak{D}=0.70$ was found to represent a well-exposed image and accordingly was adopted as the standard value of the density to which all images have been reduced. Forming for each plate the internal probable error of a single image, it is found that the value of this error is reduced in the average by a factor 1.44 through application of the density correction. After this correction is applied the mean value becomes ±0″.16.

The mean diameter D_m of all the measured images of a plate after reduction to standard density, is represented by the equation

$$\frac{D_o}{r} + c = D_m,$$

where D_o is the diameter of the planet at unit distance, r the distance, and c a constant common to all measures. In order to facilitate the computations, the following equation has been used

Table 2

S	Tan α
3.7	0.60
3.6	0.61
3.5	0.62
3.4	0.63
3.3	0.66
3.2	0.71
3.1	0.78
3.0	0.88
2.9	1.02
2.8	1.21
2.7	1.45
2.6	1.73
2.5	2.04

$$\frac{\Delta D_o}{r} + c = D_m - D,$$

where ΔD_o is the correction to the diameter at unit distance used in the American Ephemeris, namely 9″.36, and D the value given there for the time of observation.

The first solution was made from 25 observational equations, the quantities $D_m - D$ being listed in Table 3a. They were given equal weight, with the following result:

$$D_o = 9''.42 \pm 0''.06$$
$$c = -0''.24 \pm 0''.10$$

The probable error of a single plate equals ±0″.20. It is easily seen that the size of this error, with an image error amounting to ±0″.16, does not warrant the assignment of weights depending upon the number of images per plate. A comparison of the 18 plates which were taken in pairs on the same nights has been made, giving ±0″.16 as the average difference of a pair. From this value the probable error of a single plate in this set of pairs is derived to be ±0″.10. The fact that this value is small compared with the plate error derived from the solution of all 25 plates shows that each pair of plates on the same night is affected by a common "night" error. Undoubtedly, this is due to a large extent to the positions of the surface markings near the points of measurement on the limbs. In view of this it appears advisable to combine the entire material into nightly means of equal weight, thus reducing the number of observational equations to 16. The data for this solution are given in Table 3b and the results are as follows:

$$D_o = 9''.44 \pm 0''.07$$
$$c = -0''.26 \pm 0''.13$$

The probable error of a single night is $\pm 0''.21$.

TABLE 3a

Plate No.	Number of Images	$D_m - D$	v
1	16	−0″.57	−0″.36
2	24	− .32	− .11
3	15	− .31	− .10
4	23	− .17	+ .04
5	23	+ .21	+ .42
6	18	+ .14	+ .35
7	28	− .68	− .53
8	22	− .42	− .27
9	16	+ .02	+ .17
10	8	+ .03	+ .18
11	5	− .16	− .03
12	6	− .11	+ .02
13	7	− .63	− .50
14	9	− .51	− .38
15	28	− .19	− .06
16	48	+ .23	+ .36
17	21	+ .08	+ .21
18	13	+ .09	+ .22
19	5	− .06	+ .03
21	18	− .15	− .06
22	23	− .33	− .24
23	4	− .25	− .16
24	30	− .29	− .20
25	57	+ .39	+ .48
26	13	+ .41	+ .51
25	480		

Errors. The double settings made on each limb afford a means of evaluating the probable error of setting. This error is found to amount to $\pm 0''.049$ and is independent of the seeing. From this value it follows that the probable error of setting of the mean of two ("direct" and "reversed") measurements of the diameter of an image is $\pm 0''.035$. From the differences "direct-reversed" it is found that the probable error of the mean of two measurements of an image amounts to $\pm 0''.078$. The vectorial difference of $\pm 0''.069$ is the personal error introduced into the measurements by the process of reversing, i.e., the error due to the varying perception of the image border by the measurer. Errors of the screw are also incorporated in this value, but are known to be negligible.

The image error has been considerably reduced through the application of the density correction, as is shown by the factor 1.44 for the material as a whole. The image error after correction is found to be $\pm 0''.164$.

TABLE 3b

Night No.	Plate No.	Number of Images	$D_m - D$	v
1	1	16	−0″.57	−0″.35
2	2– 3	39	− .32	− .10
3	4– 5	46	+ .02	+ .24
4	6	18	+ .14	+ .36
5	7– 8	50	− .57	− .44
6	9–10	24	+ .02	+ .15
7	11–12	11	− .13	− .02
8	13–14	16	− .56	− .46
9	15–16	76	+ .08	+ .18
10	17–18	34	+ .08	+ .18
11	19	5	− .06	− .01
12	21–22	41	− .25	− .20
13	23	4	− .25	− .20
14	24	30	− .29	− .24
15	25	57	+ .39	+ .44
16	26	13	+ .41	+ .48
	25	480		

The plate error is conveniently evaluated by means of the expression derived by VAN DE KAMP, and from the solution of 25 plates is found to be $\pm 0''.238$. However, as has been stated before, the solution from 16 nightly means is preferred to that from 25 individual plates. We thereby determine the value of the night error to be $\pm 0''.196$. Therefore the combined effect of instrumental, personal, image and night errors upon the measurement of the diameter of a *Mars* image is represented by the probable error

$$\pm \sqrt{(0''.078)^2 + (0''.164)^2 + (0''.196)^2} = \pm 0''.267.$$

TABLE 4

Seeing	Number of Nights	Mean Residual	Probable Error
3.5	7	−0″.03	±″.08
3.1	5	− .05	.09
2.8	4	+ .12	.10

In order to determine if any systematic trend depending upon the seeing is present in the residuals, mean values of the residuals have been formed for 3 different groups of seeing. The results, given in Table 4, show that such a dependence, if it exists, is negligible.

Similarly, a possible dependence upon the plate brands and the developers used has been investigated. It has been stated before that no systematic effect is present in either case. The numerical results for the different plate brands are given in Table 5, those for the developers in Table 6. Regarding the presumably somewhat higher sensitivity in the red of the Wratten M plate it is found that this has not caused an increase in the diameters.

TABLE 5

Year	Plate Brand	Number of Nights	Mean Residual	Probable Error
1934	M	4	+0″04	±0″10
1935	M	2	− .14	.15
1937	III-G	4	− .03	.10
1939	103-G	6	+ .04	.09

TABLE 6

Developer	Number of Nights	Mean Residual	Probable Error
Carbonal	8	−0″02	±0″07
Super-fine-grain	3	− .03	.12
DK20	5	+ .05	.09

The probable explanation is that, in spite of higher plate sensitivity, the out-of-focus red light was too weak to register, even near the upper limit of exposure. One may recall that this has been kept low by selecting only those images with clearly visible surface details. As to the developers used, the results agree with the expectation of practically identical action upon the images. This, of course, refers only to the position of the boundary and not to its sharpness. The latter depends upon the combination of plate brand, developer and quality of the seeing. In fact, the seeing ordinarily constitutes the dominating factor, except in cases of perfect definition.

Summary. The planet *Mars* has been photographed with the 26-inch McCormick refractor in the years 1934, 1935, 1937 and 1939, on 25 plates with a total of 480 measured images, taken on 16 different nights. The measured diameters have been corrected for systematic effects depending upon the density and the seeing. The probable values of the measurement error, the image error and the error inherent in all image diameters photographed on the same night, are ±0″08, ±0″16 and ±0″20, respectively. The photovisual diameter is found to be 9″44±0″07, the value of the constant term being −0″26±0″13. This value agrees closely with the previous determination of 9″48±0″06 by VAN DE KAMP. It is identical with the value 9″44±0″03 given by RABE[2)] as the weighted mean of 17 values, of which 15 are the result of visual and 2 those of photographic determinations.

It has been shown that in problems of this kind it is necessary to establish accurately the dependence of the measured diameter upon the density. This requires a sufficiently large number of images per plate and a sufficient range in density. If the entire material is obtained under uniform, good seeing the dependence upon the seeing may become negligible. The most important requirement is the spreading of the material over as many different nights as possible, as is shown by the size of the night error. It is obvious that uniformity in the methods of observing will exclude possible errors depending upon plate brand and developer. However, it is evident that the night error must largely depend upon the position of the surface markings near the points of measurement on the borders.

REFERENCES

1) P. VAN DE KAMP, A Determination of the Diameter of *Mars*, *Astronomical Journal*, **38**, 61, 1928.
2) W. RABE, Untersuchungen über die Durchmesser der grossen Planeten, *Astronomische Nachrichten*, **234**, 197, 1928.

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THE TRIGONOMETRIC PARALLAXES OF THIRTY-SEVEN STARS

DETERMINED BY PHOTOGRAPHY WITH THE 26-INCH MCCORMICK REFRACTOR

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This is a continuation of the McCormick parallaxes of which the most recent list was given in *A. J.* 1118. Additional McCormick parallaxes appear in this same issue below. The details for most of the stars in the two lists now appearing will be found in McCormick *Publications*, Volume VIII. The proper motions in right ascension included for comparison with the values derived from the photographs are taken from the *General Catalogue* where available or from *Cincinnati*.

For several of the stars in this list, the value given is from a second McCormick determination, and for one star, γ *Leonis*, from a third determination; in each case