

NEGATIVE IONS OF HYDROGEN AND THE OPACITY OF STELLAR ATMOSPHERES

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ABSTRACT

Compared to the current theory of the atmospheric absorption coefficient, the photospheric level is raised considerably in all spectral types later than F_5 by including the contribution to the opacity made by the negative ions of hydrogen. Likewise, the discontinuity at the Balmer limit is greatly reduced, so the values of this quantity observed in F stars can be represented without the assumption, advocated by Unsöld, of an abundance ratio of hydrogen to metals equal to 50:1. The abundance ratio adopted by Russell and Pannekoek ($H:M = 1000:1$) appears to be compatible with the observations of the Balmer discontinuity.

The current theory of the atmospheric absorption coefficient is beset with two difficulties: the lack of an adequate theory of the continuous absorption of metallic atoms, and the great uncertainty with respect to the relative abundance of hydrogen and the metals in the outer layers of the stars. Since these matters have been comprehensively reviewed by Unsöld and by Page,¹ there is no need for a more detailed introduction to this paper, which is to present the results of a computation of the contribution to the opacity of stellar atmospheres made by negative ions of hydrogen. An announcement of the astrophysical significance of these particles was made in a paper² read at a symposium held at the Yerkes Observatory in June, 1938. In a later note³ some of the results of the present paper were anticipated, utilizing the absorption coefficients of H^- given by Jen.⁴ However, Jen's data are impaired by an error, which was kindly pointed out to the writer by Professor H. S. W. Massey, of London. Then the work was repeated and amplified on the basis of new absorption coefficients of H^- derived by Professor Massey and communicated in advance of publication to the writer, who gratefully acknowledges this assistance.

Massey's absorption coefficients refer to the bound-free transi-

¹ A. Unsöld, *Physik der Sternatmosphären*, Berlin: Julius Springer, 1938; T. L. Page, *M.N.*, **99**, 385, 1939.

² *Ap. J.*, **89**, 295, 1939.

³ R. Wildt, *Pub. A.A.S.*, **9**, 236, 1939.

⁴ *Phys. Rev.*, **43**, 540, 1933.

tions (photoelectric ionization), the ground state of H^- being represented by Hylleraas⁵ wave function and the continuous state by a plane wave. Throughout the visual range and the near ultraviolet, Massey's and Jen's absorption coefficients are of the same order of magnitude, about $2 \cdot 10^{-17}$ cm² per H^- ion; but in the infrared Massey's curve drops rapidly and reaches zero at the ionization limit ($\lambda 17400$), whereas Jen's curve retains a finite value at the ionization limit. The absorption coefficients for the free-free transitions of H^- have been taken from Pannekoek's⁶ Table 13b, which was compiled from data given by Menzel and Pekeris.⁷ After adding the terms for the bound-free and the free-free transitions, the Rosseland mean of the sum was computed. Since each of these two terms is proportional to the electron pressure, only one graphical integration was needed for every temperature. Furthermore, the Rosseland mean for the sum of the absorption of H and H^- was computed. Of course, it was necessary to carry out a graphical integration for every electron pressure at a given temperature, because the relative contributions of H and H^- to the total depend on the ionization equilibrium between H^+ , H , and H^- . It should be clearly understood that the electrons involved in this equilibrium are not supposed to have originated from ionization of the hydrogen atoms alone but are thought to have been released from the metals of low ionization potential present in stellar atmospheres (cf. n. 2, above). It has recently been suggested to the writer by Professor H. N. Russell that all the former investigations of the ionization equilibrium in stellar atmospheres of medium and late spectral types should have led to an ionization degree systematically too low, by inadvertently neglecting the fact that neutral hydrogen acts as an acceptor for free electrons and, therefore, tends to increase the ionization of the metals. Most fortunately, a quantitative analysis has revealed that this effect, though doubtless operating in stellar atmospheres, is entirely negligible numerically.⁸

⁵ *Zs. f. Phys.*, **60**, 624, 1930, and **63**, 291, 1930.

⁶ *Pub. Astronomical Institute of U. of Amsterdam*, No. 4, Addendum.

⁷ *M.N.*, **96**, 77, 1935.

⁸ The effect described is an instructive analogue, of opposite sign, to King's well-known experiment performed in the electric furnace, where the decrease of the ionization of a metallic vapor on addition of another metal of lower ionization potential can be

Space does not permit the publication of all the monochromatic absorption coefficients. The values of the Rosseland mean of the mass-absorption coefficient are separately given in Table 1 for atomic hydrogen alone (reproduced from Pannekoek's work,⁶ Table 8a of Addendum), for negative ions of hydrogen alone, and finally for the sum of both, i.e., the total absorption produced by hydrogen. In the upper left corner and the lower right corner of Table 1 the contributions of the atoms and the negative ions, respectively, are predominant. At temperatures lower than 4000° the free-free transitions of H^- become more and more important, compared with the bound-free transitions, on account of the shift of the maximum flux into the infrared. Menzel and Pekeris did not extend their calculations into the infrared because of the lack of convergence of the approximations they used. Their data had to be slightly extrapolated in computing the Rosseland mean at 4852° and 4200°. At still lower temperatures the continuous absorption by molecular hydrogen, too, would gain prominence. It is of interest to note that molecular hydrogen also forms a negative ion. Eyring, Hirschfelder, and Taylor⁹ have shown that H_2^- has stable electronic states. Although these cannot be reached by the collision of an electron with H_2 , they might be formed through unusually favorable three-body collisions between H^- , H , and a third atom or molecule. The probability of such a process appears to be so low that it is not surprising that the ion H_2^- has not yet been observed in the mass spectrograph.¹⁰

observed directly. It is not difficult to verify the following formula, pertaining to the hydrogen effect:

$$\frac{x^2}{1-x} - \frac{x_0^2}{1-x_0} = \frac{AK_1}{p_e + K_3} - \frac{A}{B} \frac{K_1 K_2}{p_e (K_2 + p_e)},$$

where x_0 is the ionization degree of a certain metal (ionization constant K_1) at the electron pressure p_e , resulting from Saha's formula $x_0/(1-x_0) = K_1/p_e$. If now there are added another metal (ionization constant K_2) and atomic hydrogen (K_3 ionization constant of the negative ions of hydrogen), the ionization degree of the first metal assumes the value x at the electron pressure p_e . A and B denote the abundance of hydrogen in units of the first and second metal. The first term on the right-hand side of the equation represents the small increase of ionization produced by the electron affinity of hydrogen, which, however, may be masked by the presence of another metal, as indicated by the negative sign of the second term.

⁹ *J. Chem. Phys.*, **4**, 479, 1936.

¹⁰ J. W. Hiby, *Ann. d. Phys.*, **34**, 473, 1939.

In a preceding note³ it had been pointed out how well the correction of Pannekoek's monochromatic absorption coefficients, by in-

TABLE 1

ROSSELAND MEAN OF THE ABSORPTION COEFFICIENT OF HYDROGEN

$\log_{10} \bar{k}(H)$
 $\log_{10} \bar{k}(H^-)$
 $\log_{10} \bar{k}(H+H^-)$

LOG ₁₀ P_e DYNES/CM ²	5040/T AND T° K							
	0.5 10,080°	0.6 8400°	0.7 7200°	0.8 6300°	0.9 5600°	1.0 5040°	1.1 4852°	1.2 4200°
-3.0	{ 6.03	6.42 0.29	6.71 1.99	6.94 3.58	6.73 4.73	5.67 4.97 5.92	5.01	4.94
-2.0	{ 7.03 0.43	7.42 2.29	7.70 3.98	7.75 5.39	6.89 5.89 6.96	5.67 5.98 6.56	6.01	5.94
-1.0	{ 8.03 2.43	8.42 4.29	8.63 5.91	8.10 6.74 8.18	6.91 6.91 7.45	5.67 6.98 7.25	7.01	6.94
0.0	{ 9.03 4.43	9.39 6.26 9.26	9.22 7.50 8.50	8.15 7.79 8.09	6.91 7.91 8.09	5.67 7.98	8.01	7.94
+1.0	{ 0.02 6.42	0.21 8.08 9.57	0.37 8.65 9.57	0.16 8.80 9.07	6.91 8.91 8.93	5.67 8.98	9.01	8.94
+2.0	{ 0.96 8.36	0.57 9.44 0.68	0.39 9.67 0.04	0.16 9.80 9.86	6.91 9.91	5.67 9.98	0.01	9.94
+3.0	{ 1.59 9.99 1.62	0.62 0.49 1.01	0.39 0.67 0.77	0.16 0.80 0.81	6.91 0.91	5.67 0.98	1.01	0.94
+4.0	{ 1.77 1.17 2.22	0.63 1.50 1.63	0.39 1.67 1.71	0.16 1.80	6.91 1.91	5.67 1.98	2.01	1.94

cluding the H^- absorption according to Jen's data, serves to represent theoretically the approximately gray character of the matter forming the solar atmosphere. The replacement of Jen's absorption coefficients by the improved ones of Massey has somewhat marred

the striking agreement of theory with observation, by reason of the fact that Massey's absorption-curve falls off rapidly throughout the near infrared and reaches zero at the ionization limit of H^- , whereas Jen's curve is only slightly dependent on the frequency throughout the same range. However, there is still a remarkable improvement, as will be seen on inspection of Table 2. Pannekoek's absorption coefficients k_P already contain the contribution from the free-free transitions of H^- ; so C denotes the additive correction for the bound-free

TABLE 2
MONOCHROMATIC ABSORPTION COEFFICIENTS PERTAINING TO THE
SOLAR ATMOSPHERE

λ	$\log k_P$	$\log C$	$\log k^*$	$\log k_P$	$\log C$	$\log k^*$	
7000	8.38	9.33	9.38	9.11	0.33	0.35	
6000	8.46	9.39	9.44	9.17	.39	.41	
5000	8.63	9.49	9.55	9.33	.49	.51	
4000	8.94	9.53	9.63	9.64	.53	.58	
3500	9.20	9.52	9.69	9.89	.52	.61	
3000	9.58	9.47	9.83	0.27	0.47	0.68	
7000	8.34	9.14	9.21	9.13	0.14	0.18	
6000	8.39	9.20	9.26	9.18	.20	.24	
5000	8.50	9.31	9.37	9.29	.31	.35	
4000	8.76	9.34	9.44	9.54	.34	.40	
3500	9.02	9.33	9.50	9.76	.33	.43	
3000	9.31	9.29	9.60	0.07	0.29	0.49	
	$p_e = 10$ dynes/cm ²		$p_e = 100$ dynes/cm ²				

transitions, the corrected value k^* being equal to $C + k_P$. The remaining discrepancy between theory and observation is far too great to be dismissed readily, but its origin cannot be ascertained at present.

While the energy distribution in the solar spectrum has been determined with fair accuracy, there is no consensus as to the absolute energy-curves of stellar spectra. Even differential spectrophotometric comparisons are systematically influenced, to a degree still unknown quantitatively, by the crowding of blended lines in the later spectral types. There is only one quantity which may be expected to be reasonably free from such purely observational errors, namely, the discontinuity at the Balmer limit, which in recent years

has become the subject of extensive studies by a group of French astrophysicists.¹¹ Their observations of the Balmer discontinuity, expressed as magnitude differences, have been plotted as a function of the spectral type in the right half of Figure 1; and Kuiper's temperature scale¹² has been added for sake of comparison with the left half of Figure 1, the abscissae of which are $5040/T$. The ordinates of the left half are the theoretical magnitude differences computed for the

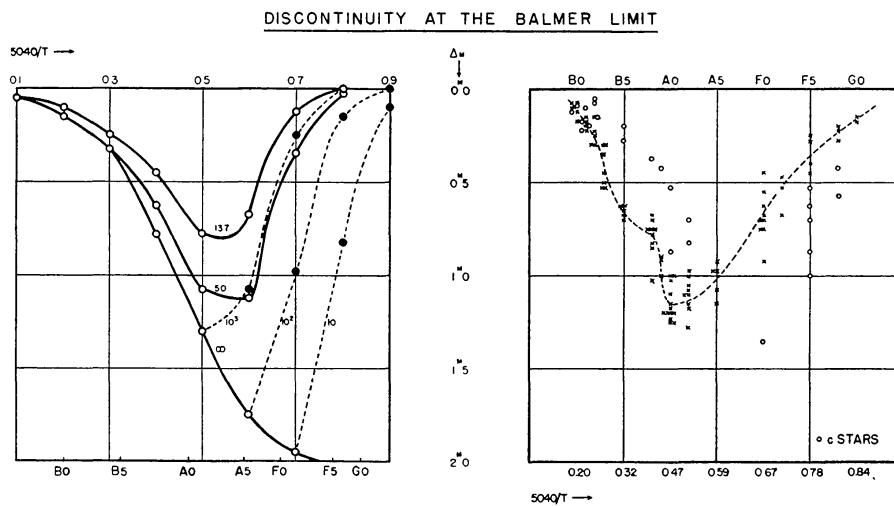


FIG. 1

Balmer discontinuity by means of Burkhardt's table,¹³ which gives the monochromatic radiation flux as function of $h\nu/kT$ and \bar{k}/k_v , on the assumption that the latter quantity does not vary with the optical depth. Three heavy lines are practically coincident with those of Figure 53a of Unsöld's book.¹ They refer to an average atmospheric electron pressure of 100 dynes/cm², assumed to be constant along the spectral sequence, and to three different compositions of the atmospheric matter (ratio of number of hydrogen atoms to number of metallic atoms equals 13.7:1, 50:1, ∞:1, respectively). These values have been taken from Burkhardt's paper, so they do not in-

¹¹ A. Arnulf, D. Barbier, D. Chalonge, *et al.*, *Annales d'astrophysique*, 1, 293 and 402, 1938.

¹² *Ap. J.*, 88, 429, 1938.

¹³ *Zs. f. Ap.*, 13, 56, 1936.

clude the opacity resulting from the negative ions of hydrogen. Under these circumstances the Balmer discontinuity increases steadily with falling temperature, in an atmosphere of vanishing content of metals; and only by adding a sufficient amount of metals can the theoretical curve be made to represent the observed relation between Balmer discontinuity and spectral type. From the theoretical curve fitting best the observational data then available, Unsöld deduced, with certain reservations, the most probable abundance ratio of hydrogen to metals to be about 50:1. In fact, this appears to be Unsöld's main argument against the abundance ratio $H:M = 1000:1$ found by Russell from his analysis of the solar atmosphere and also adopted by Pannekoek.

Furthermore, Figure 1 contains three broken curves corresponding to the absorption exerted by atoms and negative ions of hydrogen at electron pressures of 10, 100, and 1000 dynes/cm², respectively, all metallic absorption being excluded. All the three broken curves end on the heavy curve representing pure atomic hydrogen, as it should be, since the negative ions will finally be destroyed by raising the temperature. Now it becomes evident that the correction for the presence of H^- produces the same effect as the addition to the atomic hydrogen of a considerable amount of metals. Indeed, the broken curve marked $p_e = 100$ dynes/cm² runs so closely below Unsöld's curve 50:1 that it would need the addition of only an extremely small amount of metals to make the former coincide with the latter. Consequently, the existence of the negative ions of hydrogen in stellar atmospheres appears to refute Unsöld's contention that the atmospheric abundance of the metals must be as high as 1/50 of that of hydrogen. This view has always loomed, in the writer's mind, disturbingly contradictory to Russell's¹⁴ conclusion from the great strength of the hydride spectra both in the sun and in the stars, that the atmospheric abundance ratio $H:M$ is of the order of 1000:1. With this ratio being upheld, as well representing the average stellar atmosphere, it is of interest to correct Pannekoek's table of the Rosseland mean as function of temperature and electron pressure, for the presence of negative ions of hydrogen in the Russell mixture

¹⁴ *Ap. J.*, 79, 317, 1934.

of elements. This has been done in Table 3 in a way that may suffice for purposes of illustration. Strictly, the monochromatic absorption

TABLE 3
OPACITY OF STELLAR ATMOSPHERES

$$\begin{aligned} & \log_{10} \bar{k}(H+M) \\ & \log_{10} \bar{k}(H^-) \\ & \log_{10} \bar{k}(H+H^-+M) \end{aligned}$$

$\log_{10} P_e$ DYNES/CM ²	5040/T AND T° K							
	0.5 10,080°	0.6 8400°	0.7 7200°	0.8 6300°	0.9 5600°	1.0 5040°	1.1 4852°	1.2 4200°
-3.0	6.46	6.75	6.94	6.47	5.51	5.07	5.17
	0.29	1.99	3.58	4.73	4.97	5.01	4.94
	5.34	5.34
-2.0	7.46	7.73	7.59	6.62	5.98	6.00	6.14
	2.29	3.98	5.39	5.89	5.98	6.01	5.94
	6.28	6.31	6.33
-1.0	8.07 2.43	8.45 4.29	8.56 5.91	7.80 6.74	6.96 6.91	6.83 6.98	6.97 7.01	7.01 6.94
	7.23	7.20	7.29	7.26

0.0	9.07 4.43	9.40 6.26	8.96 7.50	7.98 7.79	7.65 7.91	7.78 7.98	7.80 8.01	7.75 7.94
	8.17	8.06	8.17	8.19	8.14

+1.0	0.05 6.42	0.05 8.08	9.09 8.65	8.50 8.80	8.55 8.91	8.60 8.98	8.53 9.01	8.58 8.94
	9.12	9.01	8.99	9.05	9.02

+2.0	0.91 8.36	0.27 9.44	9.44 9.67	9.31 9.80	9.35 9.91	9.34 9.98	9.38 0.01	9.43 9.94
	9.84

+3.0	1.36 9.99	0.49 0.49	0.13 0.67	0.12 0.80	0.09 0.91	0.12 0.98	0.13 1.01	0.03 0.94
	0.79

+4.0	1.53 1.17 1.61	1.02 1.50 1.67	0.89 1.80	0.87 1.91	0.87 1.98	0.81 2.01	0.74 1.94	0.68 1.94

coefficients, as given by Pannekoek and Massey, should be added, and then a graphical integration of their reciprocal values should be made, which would involve an enormous amount of labor. There-

fore, an approximate procedure suggested by Unsöld¹⁵ was applied, which leads to the formula

$$\overline{(k_1 + k_2)} = \frac{4\bar{k}_1\bar{k}_2}{(\bar{k}_1 + \bar{k}_2)},$$

where \bar{k}_1 is the Rosseland mean for the Russell mixture of elements (Pannekoek, *op. cit.*, Addendum, Table 20a) and \bar{k}_2 is the Rosseland mean for the bound-free transitions of H^- . For the latter quantity there were substituted the values of $k(H^-)$ given in Table 1, though these contain the free-free transitions of H^- , already included in Pannekoek's \bar{k}_1 . The numerical difference is practically negligible, except, perhaps, at the lowest temperatures. In Table 3 the final values representative of each temperature and electron pressure have been italicized. They reveal a marked lifting of the photospheric level by taking into account the contribution of H^- to the general opacity. This is quite conspicuous at temperatures lower than about 7000° K (F stars); but at the highest electron pressures, of the order that may be realized in the atmospheres of white dwarfs, the same effect may even persist in the range of temperatures the A stars are credited with. Therefore, it is not improbable that the obliteration of the Balmer continuum in the white dwarfs, which now is regarded as a manifestation of the Stark effect, could partly be the result of the raising of the photospheric level by H^- .

To sum up, it is now to be regarded as fairly certain, on the basis of Massey's absorption coefficients, that the existence of negative ions of hydrogen constitutes a major factor ruling the structure of late-type stellar atmospheres. Hence, the question is in point whether the electron affinity of any other element might not cause a further important contribution to the atmospheric opacity. The abundant halogen atoms *F* and *Cl* have electron affinities of the order of 4 volts, and so have some astrophysically abundant diatomic molecules like *OH* and *CN*.¹⁶ However, these atoms and radicals enter

¹⁵ *Zs. f. Ap.*, **8**, 225, 1934.

¹⁶ E. Lederle, *Zs. f. Phys. Chemie*, B, **17**, 351, 1932; J. Weiss, *Trans. Faraday Society*, **31**, 966, 1935.

numerous compounds with energies of dissociation of the same order of magnitude. For this reason it is safe to assume that their electron affinity plays a role only in that range of lower temperatures and higher pressures where the present theory of atomic and ionic opacity will fail in any case because of the overwhelming influence of the molecular constituents. A case of entirely different character is He , singly charged negative ions of which Hiby¹⁰ claims to have observed in the mass spectrograph. If the stability of He^- could be established beyond doubt, both experimentally and theoretically, another amendment of the theory of atmospheric opacity may become necessary.

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