PHYSICAL PROCESSES IN GASEOUS NEBULAE

III. THE BALMER DECREMENT

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ABSTRACT

This paper contains a numerical solution of the equations derived and formally solved in two earlier papers of the series. Various tables of general interest, including those of functions $X_n = hR/n^2kT_\epsilon$ and $-[Ei(-X_n)]$, for useful astrophysical ranges of n and T_ϵ are given. The Balmer decrement, computed under two alternative hypotheses—**A** for a nebula transparent to Lyman line radiation, and **B** for an opaque nebula—is tabulated. The latter hypothesis agrees better with the observed data. The conclusion is reached that the electron temperature, T_ϵ, of the nebular gas cannot be effectively determined from observed Balmer decrement data, because the decrement is insensitive to temperature. In view of the extreme physical conditions that exist in nebulae, the partition of atoms among the various excited levels approaches surprisingly close to the thermodynamic value.

In papers I and II[2] of this series the fundamental equations for calculating the relative intensities of lines of the Balmer series were set up and formally solved. The present paper gives the numerical results and the comparison of theory with observation.

For the benefit of those whose interest is chiefly in the final results, we shall briefly recapitulate the theory and fundamental hypotheses on which the calculations were based. The energy emitted in a given Balmer line, say in the transition from level n to level 2, is given by

$$E_{n2} = N_n A_{n2} h\nu \,, \tag{1}$$

where N_n is the number of atoms per cubic centimeter in level n, A_{n2} is the Einstein spontaneous probability coefficient, and ν is the frequency. Outside of thermodynamic equilibrium, N_n must be calculated by balancing the number of atoms leaving a given quantum level against those entering, by all possible routes. We thus have an infinite set of simultaneous equations to solve, one equation for each level. The method of solving these equations, for certain specified physical conditions, was given in II. The results are most con-

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veniently expressed in terms of a dimensionless parameter, b_n, defined by

$$N_n = b_n N_{nT} = b_n N_i N_\epsilon \frac{h^3}{(2\pi m k T_\epsilon)^{3/2}} n^2 e^{X_n} , \qquad (2)$$

(II, Eq. [4]), where

$$X_n = \frac{hRZ^2}{n^2 K T_\epsilon} . \qquad (3)$$

b_n gives the departure from thermodynamic equilibrium. For thermodynamic equilibrium, $b_n = 1$.

We have calculated the Balmer decrement on two alternative hypotheses about the nature of nebular excitation. Under **A** we assume that an electron enters quantum level n of an atom either by capture from the free state or by cascade from a higher discrete level. We neglect the radiation field of the star and assume that there is no reabsorption of Lyman line radiation. For **A** we distinguish two cases: $\mathbf{A}_1$, where an idealized hydrogen atom with the Gaunt g factor equal to unity was employed, and $\mathbf{A}_2$, where the exact g values were introduced. Comparison of the results indicates how far calculations based on the simpler assumptions may be trusted and in what directions the deviations will tend. In a nebula that is very opaque to Lyman line radiation, we may take account of the nebular radiation field, but not of the stellar, by assuming that absorptions from level 1 to level n are exactly balanced by the inverse spontaneous transitions. Zanstra has based his theoretical calculations on this assumption, which we shall designate as **B**. In II the radiation field was entirely neglected. We may allow for it, under **B**, by redefining the quantity t_n as

$$t_n = \sum_{2}^{n-1} - u_{n'n} \qquad (4)$$

instead of

$$t_n = \sum_{1}^{n-1} - u_{n'n} . \qquad (5)$$

The remaining terms of the previous equations are unaltered, and the solution is carried out as before.

The numerical calculations were laborious, and various intermediate results of astrophysical interest have been tabulated with the hope that they may prove useful to others. The tables cover a

TABLE 1

$X_n \times 10^{-p}$

n	5,000°	p	10,000°	p	20,000°	p	40,000°	p	80,000°	p	160,000°	p	320,000°	p
1	3.141	1	1.570	1	7.851	0	3.926	0	1.963	0	9.814	−1	4.907	
2	7.851	0	3.926	0	1.963		9.814	−1	4.907	−1	2.454		1.227	
3	3.490		1.745		8.724	−1	4.362		2.181		1.090		5.450	−2
4	1.963		9.814	−1	4.907		2.454		1.227		6.140	−2	3.070	
5	1.256		6.281		3.141		1.570		7.851	−2	3.930		1.970	
6	8.724	−1	4.362	−1	2.181	−1	1.091	−1	5.453	−2	2.727	−2	1.36	−2
7	6.409		3.205		1.602		8.012	−2	4.006		2.003		1.00	
8	4.907		2.454		1.227		6.134		3.067		1.533		7.67	−3
9	3.877		1.939		9.693	−2	4.847		2.423		1.212		6.06	
10	3.141		1.570		7.851		3.926		1.963		9.815	−3	4.91	
11	2.596	−1	1.298	−1	6.489	−2	3.244	−2	1.622	−2	8.11	−3	4.06	−3
12	2.181		1.091		5.452		2.726		1.363		6.82		3.41	
13	1.858		9.292	−2	4.646		2.323		1.161		5.81		2.90	
14	1.602		8.012		4.006		2.003		1.001		5.01		2.50	
15	1.396		6.979		3.490		1.745		8.725	−3	4.36		2.18	
16	1.227	−1	6.134	−2	3.067	−2	1.534	−2	7.670	−3	3.83	−3	1.92	−3
17	1.087		5.434		2.717		1.358		6.790		3.40		1.70	
18	9.693	−2	4.847		2.423		1.212		6.058		3.03		1.52	
19	8.700		4.350		2.175		1.087		5.437		2.72		1.36	
20	7.851		3.926		1.963		9.814	−3	4.907		2.45		1.23	
21	7.121	−2	3.561	−2	1.780	−2	8.902	−3	4.45	−3	2.23	−3	1.11	−3
22	6.489		3.244		1.622		8.111		4.06		2.03		1.01	
23	5.937		2.968		1.484		7.421		3.71		1.86		9.28	−4
24	5.452		2.726		1.363		6.815		3.41		1.70		8.52	
25	5.025		2.512		1.256		6.281		3.14		1.57		7.85	
26	4.646	−2	2.323	−2	1.161	−2	5.807	−3	2.90	−3	1.45	−3	7.26	−4
27	4.308		2.154		1.077		5.385		2.69		1.35		6.73	
28	4.006		2.003		1.002		5.007		2.50		1.25		6.26	
29	3.734		1.867		9.336	−3	4.668		2.33		1.17		5.83	
30	3.490		1.745		8.724		4.362		2.18		1.09		5.45	

large range of temperatures and quantum numbers. A sufficient number of figures was carried in the calculations to insure the correctness of the values as tabulated.

Table 1 contains values of X_n, equation (3), with Z set equal to unity. When Z is not unity, as for ionized helium, the tabulated temperatures must be multiplied by Z^2, to correspond with the true

TABLE 2

$g_{nn'}$

n'	n = 2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	.717	.765	.780	.786	.790	.792	.793	.794	.795	.795	.795	.796	.796	.796
2	....	.757	.822	.844	.855	.861	.865	.867	.869	.870	.871	.872	.872	.873
3	....	....	.768	.839	.865	.878	.886	.891	.895	.897	.899	.900	.901	.902
4	....	....	....	.772	.847	.874	.890	.898	.903	.908	.911	.914	.915	.916
5	....	....	....	....	.774	.851	.881	.896	.905	.912	.916	.919	.922	.924
6	....	....	....	....	....	.776	.853	.884	.899	.910	.917	.922	.926	.929
7	....	....	....	....	....	....	.777	.852	.887	.904	.915	.922	.926	.930
8	....	....	....	....	....	....	....	.779	.856	.888	.905	.916	.925	.929
9	....	....	....	....	....	....	....	....	.779	.856	.889	.906	.917	.925
10	....	....	....	....	....	....	....	....	....	.780	.858	.888	.907	.919
11	....	....	....	....	....	....	....	....	....	....	.781	.854	.890	.908
12	....	....	....	....	....	....	....	....	....	....	....	.781	.860	.891
13	....	....	....	....	....	....	....	....	....	....	....	....	.781	.861
....	....	....	....	....	....	....	....	....	....	....	....	....	....	.781

n'	n = 16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	.796	.796	.796	.796	.796	.797	.797	.797	.797	.797	.797	.797	.797	.797	.797
2	.873	.874	.874	.874	.874	.875	.875	.875	.875	.875	.875	.875	.875	.875	.875
3	.902	.903	.904	.904	.904	.904	.905	.905	.905	.905	.906	.906	.906	.906	.906
4	.917	.918	.919	.920	.920	.921	.921	.921	.922	.922	.922	.922	.922	.923	.923
5	.926	.927	.928	.929	.930	.930	.931	.931	.932	.932	.932	.933	.933	.933	.933
6	.931	.933	.934	.936	.937	.937	.938	.939	.939	.940	.940	.940	.941	.941	.941
7	.933	.935	.937	.939	.940	.941	.942	.943	.943	.944	.945	.945	.945	.946	.946
8	.933	.937	.939	.941	.943	.944	.945	.946	.947	.948	.948	.949	.949	.950	.950
9	.930	.934	.938	.940	.942	.944	.945	.947	.948	.948	.949	.950	.950	.951	.952
10	.927	.932	.936	.940	.942	.944	.946	.948	.949	.950	.951	.952	.953	.953	.954
11	.919	.927	.933	.937	.940	.943	.945	.947	.949	.950	.951	.952	.953	.954	.955
12	.909	.920	.928	.933	.938	.941	.944	.946	.948	.950	.951	.952	.953	.954	.955
13	.892	.910	.921	.929	.935	.939	.943	.946	.948	.950	.952	.953	.954	.955	.956
14	.862	.894	.911	.923	.931	.936	.941	.945	.947	.950	.952	.954	.955	.956	.957
15	.782	.864	.894	.912	.923	.931	.937	.941	.945	.948	.950	.953	.954	.956	.957
16	....	.782	.866	.894	.912	.923	.931	.937	.942	.946	.949	.951	.953	.955	.956
17	....	....	.782	.868	.894	.912	.924	.932	.938	.942	.946	.950	.951	.954	.955
18	....	....	....	.782	.862	.894	.912	.924	.932	.938	.942	.946	.949	.952	.954
19	....	....	....	....	.782	.862	.894	.912	.924	.932	.938	.943	.947	.950	.952
20	....	....	....	....	....	.782	.862	.895	.912	.924	.932	.938	.943	.947	.950
21	....	....	....	....	....	....	.782	.862	.895	.913	.924	.932	.939	.943	.947
22	....	....	....	....	....	....	....	.782	.862	.895	.913	.924	.933	.939	.943
23	....	....	....	....	....	....	....	....	.782	.862	.895	.913	.924	.933	.939
24	....	....	....	....	....	....	....	....	....	.782	.863	.895	.913	.924	.933
25	....	....	....	....	....	....	....	....	....	....	.782	.863	.895	.913	.925
26	....	....	....	....	....	....	....	....	....	....	....	.782	.863	.895	.913
27	....	....	....	....	....	....	....	....	....	....	....	....	.782	.863	.895
28	....	....	....	....	....	....	....	....	....	....	....	....	....	.782	.863
29	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.782

temperatures. For example, the solution for He^+ at $T_\epsilon = 40{,}000^\circ$ is equivalent to that for hydrogen at $T_\epsilon = 10{,}000^\circ$.

The Kramer-Gaunt factors for the discrete-discrete transitions are given in Table 2. The functional form of this factor, $g_{nn'}$, is

TABLE 3a

$[-E_i(-X_n)]$

n	T_ϵ						
	5,000°	10,000°	20,000°	40,000°	80,000°	160,000°	320,000°
2	0.0000	0.0041	0.0525	0.2264	0.5712	1.0590	1.6399
3	0.0071	0.0700	0.2727	0.6453	1.1524	1.7449	2.3857
4	0.0525	0.2264	0.5712	1.0590	1.6399	2.2746	2.9377
5	0.1451	0.4296	0.8720	1.4252	2.0443	2.6993	3.3731
6	0.2727	0.6453	1.1524	1.7449	2.3857	3.0521	3.7318
7	0.4188	0.8574	1.4080	2.0256	2.6799	3.3533	4.0365
8	0.5712	1.0590	1.6400	2.2746	2.9377	3.6167	4.3012
9	0.7235	1.2483	1.8512	2.4976	3.1669	3.8481	4.5352
10	0.8720	1.4252	2.0443	2.6993	3.3731	4.0565	4.7448
11	1.0252	1.5905	2.2217	2.8832	3.5703	4.2454	4.9345
12	1.1524	1.7449	2.3857	3.0521	3.7318	4.4182	5.1079
13	1.2832	1.8896	2.5379	3.2082	3.8899	4.5772	5.2675
14	1.4080	2.0256	2.6799	3.3533	4.0365	4.7246	5.4153
15	1.5268	2.1537	2.8128	3.4888	4.1732	4.8620	5.5530
16	1.6400	2.2745	2.9377	3.6157	4.3012	4.9906	5.6818
17	1.7480	2.3890	3.0555	3.7352	4.4216	5.1114	5.8028
18	1.8512	2.4976	3.1669	3.8481	4.5352	5.2253	5.9170
19	1.9498	2.6009	3.2726	3.9550	4.6427	5.3331	6.0249
20	2.0443	2.6993	3.3731	4.0565	4.7448	5.4355	6.1274
21	2.1348	2.7933	3.4689	4.1532	4.8419	5.5328	6.2249
22	2.2217	2.8832	3.5703	4.2454	4.9345	5.6257	6.3178
23	2.3053	2.9694	3.6529	4.3337	5.0231	5.7144	6.4066
24	2.3857	3.0521	3.7318	4.4182	5.1079	5.7994	6.4917
25	2.4632	3.1318	3.8124	4.4993	5.1893	5.8809	6.5732
26	2.5379	3.2082	3.8899	4.5772	5.2675	5.9592	6.6516
27	2.6101	3.2820	3.9645	4.6523	5.3428	6.0346	6.7270
28	2.6799	3.3533	4.0365	4.7247	5.4153	6.1072	6.7997
29	2.7474	3.4221	4.1061	4.7945	5.4853	6.1773	6.8699
30	2.8128	3.4887	4.1732	4.8620	5.5530	6.2450	6.9376

given by Menzel and Pekeris.[3] For n' from 1 to 6, and for n from 2 to 30, the accurate expressions were employed; for all other values, the asymptotic expression

$$g_{nn'} = 1 - \frac{0.1728\left(1 + \frac{n'^2}{n^2}\right)}{\left(1 - \frac{n'^2}{n^2}\right)^{2/3}(n^2)^{2/3}} \tag{6}$$

[3] *M.N.*, **96**, 77, 1935.

was used. This expression differs from the exact one for n' equal to 6 by a small number in the third decimal place. Consequently, an adjustment was made to the asymptotic values in order to remove the discrepancy. It is evident that this factor converges only very slowly to unity as n becomes infinite.

TABLE 3*b*

$\bar{g}[-E_i(-X_n)]$

n	T_ϵ						
	5,000°	10,000°	20,000°	40,000°	80,000°	160,000°	320,000°
2	0.000	0.004	0.053	0.229	0.578	1.374	1.884
3	0.007	0.071	0.279	0.662	1.185	1.808	2.483
4	0.053	0.231	0.588	1.093	1.699	2.374	3.081
5	0.148	0.440	0.900	1.479	2.130	2.832	3.556
6	0.278	0.663	1.193	1.816	2.496	3.213	3.946
7	0.427	0.882	1.459	2.112	2.811	3.535	4.276
8	0.583	1.091	1.702	2.377	3.088	3.822	4.564
9	0.739	1.287	1.922	2.611	3.331	4.071	4.816
10	0.891	1.470	2.124	2.824	3.550	4.295	5.043
11	1.048	1.641	2.309	3.018	3.760	4.498	5.249
12	1.178	1.800	2.480	3.196	3.933	4.684	5.437
13	1.312	1.950	2.640	3.362	4.102	4.856	5.611
14	1.440	2.091	2.788	3.515	4.259	5.016	5.773
15	1.562	2.224	2.928	3.659	4.406	5.166	5.924
16	1.678	2.349	3.059	3.794	4.544	5.306	6.065
17	1.789	2.468	3.182	3.921	4.674	5.437	6.199
18	1.895	2.581	3.299	4.041	4.797	5.562	6.325
19	1.996	2.688	3.410	4.155	4.913	5.680	6.445
20	2.093	2.790	3.516	4.263	5.023	5.792	6.558
21	2.186	2.888	3.617	4.366	5.128	5.899	6.667
22	2.275	2.981	3.723	4.465	5.229	6.001	6.770
23	2.361	3.071	3.810	4.559	5.325	6.099	6.869
24	2.443	3.157	3.894	4.649	5.417	6.192	6.964
25	2.523	3.240	3.979	4.736	5.505	6.282	7.055
26	2.600	3.319	4.060	4.820	5.590	6.368	7.142
27	2.675	3.396	4.139	4.900	5.672	6.451	7.226
28	2.746	3.470	4.215	4.977	5.751	6.531	7.307
29	2.815	3.542	4.288	5.052	5.827	6.608	7.385
30	2.882	3.611	4.359	5.125	5.900	6.682	7.461

The familiar exponential integral function

$$[-E_i(-X_n)] = \int_{X_n}^{\infty} \frac{e^{-\zeta}}{\zeta}\, d\zeta \qquad (7)$$

is given in Table 3*a* as a function of n and T_ϵ. In the computations we made use of formulae (12) and (13) of paper II, and of Jahnke and Emden's tables. The ordinary tables, with uniformly spaced argument, are somewhat difficult to use because of the involved interpolation.

The expression

$$\bar{g}[-Ei(-X_n)] = \int_{X_n}^{\infty} \frac{e^{-\zeta}}{\zeta} g_{\kappa n} d\zeta \tag{8}$$

is given in Table 3*b*. The Kramer-Gaunt factor for the bound-free transitions, $g_{\kappa n}$, was obtained from the formulae of Menzel and Pekeris. The integration was carried out by numerical integration formulae. Tables 3*a* and 3*b*, by multiplication with e^{X_n}, are convertible immediately into the functions S_n and $\bar{g}S_n$, which are given in paper II, equations (11) and (12).

Table 4*a* gives the expression

$$t_n = \sum_{1}^{n-1} \frac{2n^2}{n'(n^2 - n'^2)} \tag{9}$$

and Table 4*b*,

$$t_n = \sum_{1}^{n-1} \frac{2n^2 g_{nn'}}{n'(n^2 - n'^2)} \, . \tag{10}$$

The first four tables of this paper give directly and indirectly all the functions of n and T_ϵ necessary in the evaluation of b_n by equation (17) of paper II, both for **A** and for **B**. Since the function V_n is only very slowly convergent, the calculations proved to be lengthy. An expression of the form

$$\sum_{i=31}^{\infty} \frac{S_i u_{in}}{t_i} = a + bn + cn^2 \tag{11}$$

was developed to complete the first summation accurately. This summation is the kernel of the succeeding double summation, which reduces, therefore, to a single summation. The first three terms of V_n were computed directly. The contribution of the remaining terms was obtained graphically by careful asymptotic extrapolation based on the plot of S_n and of the first three terms of V_n against position in the sequence. The b_n's for the two cases $\mathbf{A}_1$ and $\mathbf{A}_2$, which were smoothed in the last figure to take out slight numerical inaccuracies, are tabulated in Tables 5*a* and 5*b*.

In Table 6, under the column headed b_n, are given the values, computed on hypothesis **B**, for an atom with an infinite number of

TABLE 4*a*

t_n (CASE **A**₁)

2	2.6667	11	8.1614	21	10.1363
3	4.0500	12	8.4285	22	10.2775
4	4.9905	13	8.6738	23	10.4124
5	5.7044	14	8.9005	24	10.5415
6	6.2801	15	9.1112	25	10.6653
7	6.7627	16	9.3082	26	10.7841
8	7.1782	17	9.4929	27	10.8984
9	7.5430	18	9.6670	28	11.0086
10	7.8682	19	9.8321	29	11.1148
		20	9.9881	30	11.2174

TABLE 4*b*

t_n (CASE **A**₂)

2	1.911	11	6.943	21	8.886
3	3.084	12	7.205	22	9.026
4	3.929	13	7.442	23	9.160
5	4.589	14	7.668	24	9.287
6	5.132	15	7.876	25	9.410
7	5.590	16	8.070	26	9.528
8	5.989	17	8.253	27	9.641
9	6.339	18	8.426	28	9.750
10	6.657	19	8.589	29	9.859
		20	8.740	30	9.956

levels. For purposes of comparison, we also give values of b_n derived from the work of Cillié.[4] Cillié based his calculations on assumption **B**, but he made two approximations to facilitate the numerical work. A model atom consisting of only fourteen levels was assumed, and the higher discrete states were neglected. For the fourteen levels, accurate values of the Einstein *A*'s were taken; but for the electron captures, the *g* factor was set equal to unity. For purposes of comparison, we have also given the b_n for a fourteen-state atom at $T_\epsilon =$

[4] *Ibid.*, **96,** 771, 1936.

TABLE 5*a*

b_n (CASE A₁)

n	T_ϵ						
	5,000°	10,000°	20,000°	40,000°	80,000°	160,000°	320,000°
2.......	0.000034	0.002893	0.03365	0.1377	0.3306	0.5873	0.8809
3.......	.003273	.02945	.1097	.2447	.4251	.6237	.8303
4.......	.01824	.07359	.1781	.3182	.4843	.6558	.8268
5.......	.04280	.1180	.2330	.3705	.5264	.6801	.8293
6.......	.07199	.1588	.2764	.4115	.5538	.6994	.8335
7.......	.09926	.1937	.3116	.4431	.5780	.7141	.8472
8.......	.1259	.2237	.3387	.4663	.5978	.7260	.8532
9.......	.1499	.2486	.3629	.4888	.6161	.7358	.8555
10.......	.1710	.2714	.3836	.5062	.6281	.7439	.8604
11.......	.1912	.2903	.4002	.5190	.6381	.7509	.8632
12.......	.2074	.3072	.4147	.5302	.6449	.7568	.8657
13.......	.2233	.3223	.4275	.5399	.6525	.7622	.8697
14.......	.2376	.3358	.4388	.5485	.6595	.7667	.8699
15.......	.2507	.3478	.4489	.5561	.6657	.7708	.8718
20.......	.3016	.3931	.4891	.5899	.6889	.7852	.8780
25.......	.3359	.4269	.5203	.6121	.7033	.7912	.8801
30.......	0.3581	0.4487	0.5400	0.6237	0.7085	0.7936	0.8818

TABLE 5*b*

b_n (CASE A₂)

n	T_ϵ						
	5,000°	10,000°	20,000°	40,000°	80,000°	160,000°	320,000°
3.......	0.00434	0.0393	0.146	0.296	0.576	0.843	1.136
4.......	.0234	.0961	.233	.420	.639	.861	1.103
5.......	.0540	.150	.296	.478	.685	.877	1.089
6.......	.0905	.202	.350	.528	.714	.888	1.079
7.......	.123	.242	.391	.561	.736	.898	1.068
8......	.154	.277	.422	.586	.753	.905	1.061
9.......	.182	.311	.448	.609	.770	.910	1.058
10.......	.206	.340	.470	.626	.780	.918	1.056
11.......	.228	.359	.488	.641	.790	.921	1.054
12.......	.248	.376	.503	.651	.800	.925	1.052
13.......	.266	.392	.517	.663	.808	.928	1.050
14.......	.281	.406	.528	.673	.813	.930	1.048
15.......	.296	.419	.539	.680	.818	.933	1.046
20.......	.350	.467	.583	.712	.842	.943	1.041
25.......	.385	.498	.611	.732	.858	.952	1.037
30.......	0.408	0.519	0.630	0.750	0.871	0.962	1.031

20,000°, with the capture g factor taken into account. The effect of the neglected higher levels is clearly exhibited by the actual crossing of the true b_n's by Cillié's b_n's at quantum number 6.

It is to be noted that b_n can be greater than unity. Referring to paper II, equation (17), we see that S_n and V_n must increase with T_ϵ. For any T_ϵ, as we have shown in II, $b_n \rightarrow 1$ as $n \rightarrow \infty$. There must,

TABLE 6

b_n (CASE B)

n	T_ϵ: 5,000° b_n	5,000° b_n (Cillié)	10,000° b_n	10,000° b_n (Cillié)	20,000° b_n	20,000° b_n (Cillié)	20,000° 14-State b_n	40,000° b_n	80,000° b_n
3......	0.0098	0.0120	0.089	0.111	0.330	0.409	0.420	0.670	1.304
4......	.0406	.0448	.166	.188	.404	.453	.466	.729	1.108
5......	.0840	.0878	.233	.251	.460	.494	.510	.743	1.065
6......	.132	.131	.296	.298	.512	.520	.538	.773	1.045
7......	.173	.166	.341	.333	.550	.538	.558	.789	1.036
8......	.211	.195	.379	.357	.577	.547	.568	.802	1.032
9......	.244	.218	.417	.373	.600	.549	.570	.816	1.031
10......	.271	0.236	.448	0.383	.620	0.545	0.565	.825	1.030
15......	.371		.526		.676			..853	1.027
20......	.428		.571		.713			.871	1.025
25......	.464		.600		.736			.882	1.024
30......	0.486		0.618		0.750			0.893	1.024

then, be a temperature for which $b_n \sim 1$, when the conditions of thermodynamic equilibrium are closely approximated by the nebula. For assumption $\mathbf{A}_2$, the temperature at which the pseudo-thermodynamic equilibrium occurs is about 235,000°. For **B** it occurs near 71,000°.

The relative intensities I_n of the Balmer lines, referred, as is customary, to $H\beta$ as unity, are listed in Tables 7a, 7b, and 8 for the assumptions $\mathbf{A}_1$, $\mathbf{A}_2$, and **B**, respectively. In the last table are given, also, the values of I_n as derived by Cillié. The intensities are calculated from the formula

$$I_n = \left(\frac{b_n}{b_4}\right) e^{X_n - X_4} \frac{(4)^3}{n^3} \frac{g_{n2}}{g_{42}} = \left(\frac{b_n}{b_4}\right) I_{nT} , \qquad (12)$$

TABLE 7*a*

I_n (CASE A₁)

n	T_ϵ						
	5,000°	10,000°	20,000°	40,000°	80,000°	160,000°	320,000°
3	1.953	2.035	2.118	2.205	2.289	2.364	2.438
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5	0.592	0.576	0.561	0.546	0.533	0.519	0.508
6	0.389	0.368	0.350	0.334	0.317	0.305	0.294
7	0.271	0.252	0.235	0.220	0.205	0.195	0.187
8	0.198	0.180	0.165	0.152	0.141	0.132	0.126
9	0.149	0.135	0.121	0.111	0.101	0.094	0.089
10	0.115	0.104	0.091	0.083	0.075	0.069	0.066
11	0.092	0.081	0.071	0.063	0.057	0.052	0.049
12	0.074	0.065	0.056	0.050	0.044	0.040	0.038
13	0.060	0.052	0.045	0.040	0.035	0.032	0.030
14	0.050	0.043	0.037	0.032	0.028	0.026	0.024
15	0.042	0.036	0.030	0.026	0.023	0.021	0.019
20	0.020	0.017	0.014	0.012	0.010	0.009	0.008
25	0.011	0.009	0.007	0.006	0.005	0.005	0.004
30	0.007	0.005	0.004	0.004	0.003	0.003	0.002

TABLE 7*b*

I_n (CASE A₂)

n	T_ϵ						
	5,000°	10,000°	20,000°	40,000°	80,000°	160,000°	320,000°
3	1.859	1.915	1.984	2.073	2.165	2.241	2.302
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5	0.598	0.576	0.560	0.548	0.539	0.524	0.513
6	0.396	0.374	0.353	0.338	0.321	0.307	0.296
7	0.274	0.255	0.236	0.221	0.207	0.196	0.185
8	0.199	0.182	0.165	0.153	0.141	0.132	0.124
9	0.149	0.136	0.120	0.111	0.101	0.093	0.086
10	0.114	0.105	0.091	0.082	0.075	0.069	0.063
11	0.090	0.081	0.070	0.062	0.056	0.052	0.048
12	0.073	0.065	0.055	0.050	0.045	0.040	0.036
13	0.059	0.052	0.043	0.039	0.035	0.032	0.029
14	0.049	0.042	0.036	0.032	0.029	0.025	0.023
15	0.041	0.035	0.030	0.025	0.023	0.020	0.019
20	0.019	0.016	0.014	0.012	0.010	0.009	0.007
25	0.011	0.009	0.007	0.006	0.005	0.004	0.004
30	0.006	0.005	0.004	0.003	0.003	0.003	0.003

where I_{nT} is the value of the relative intensity for thermodynamic equilibrium. Because of the approximations, the decrements calculated by Cillié for a given temperature actually correspond to those for a temperature much higher.

In Table 8, under the heading I_n (Obs.), are given the means of the observed data for 17 nebulae, as determined by Berman.[5] The

TABLE 8

I_n (CASE **B**)

n	T_ϵ									
	5,000°		10,000°		20,000°		40,000°		80,000°	160,000°
	I_n	I_n (Cillié)	I_n	I_n (Cillié)	I_n	I_n (Cillié)	I_n	I_n (Obs.)	I_n	I_n
3	2.43	2.70	2.50	2.78	2.59	2.88	2.71	2.77	2.83	2.93
4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5	0.53	0.51	0.51	0.50	0.50	0.48	0.49	0.50	0.48	0.47
6	0.33	0.30	0.31	0.29	0.30	0.27	0.29	0.26	0.27	0.26
7	0.223	0.193	0.206	0.180	0.192	0.168	0.179	0.18	0.169	0.159
8	0.157	0.134	0.143	0.120	0.130	0.110	0.120	0.12	0.112	0.104
9	0.115	0.094	0.105	0.084	0.093	0.076	0.085	0.09	0.078	0.072
10	0.087	0.069	0.079	0.061	0.069	0.054	0.062		0.057	0.052
15	0.030		0.025		0.021		0.018		0.017	0.015
20	0.014		0.012		0.010		0.009		0.006	0.006
25	0.007		0.006		0.005		0.004		0.003	0.003
30	0.004		0.003		0.003		0.002		0.002	0.001

means are set between T_ϵ equal to 40,000° and to 80,000°, because the correspondence is closest for these temperatures. We draw attention to the fact that the observations seem to agree best with theory in the same region where thermodynamic equilibrium nearly prevails, although we attach no especial significance to this result. The Balmer decrement shows only a very slow change with temperature. Observations of extreme accuracy will be required if electron temperatures are to be determined from line intensities. Selective interstellar absorption, as discussed by Page[6] and by Berman,[5] only adds to the difficulties. Furthermore, as a comparison of Tables

[5] *Ibid.*, p. 890. [6] *Ibid.*, p. 604.

7*b* and 8 shows, the decrement is much more sensitive to the mode of excitation than it is to electron temperature.

We conclude, therefore, that measures of the Balmer decrement are unsuited to determinations of the temperatures of the electron gas. The result, disappointing at first sight, proves to be fortunate, after all. The observed Balmer decrement can be used to indicate the physical nature of the excitation. For example, the general agreement of the observational data with the entries of Table 8 probably signifies that assumption **B** lies nearer to the truth than **A**.

Attention is also drawn to the close numerical agreement of Tables 7*a* and 7*b* for quantum numbers greater than 4. For the higher series members, the introduction of the Gaunt factors does little to change the relative intensities, although the absolute intensities change by as much as 20 per cent.

One advantage of the present type of solution, aside from its numerical accuracy, is the fact that the absolute intensities of both low and high series members, and even of the continuum beyond the Balmer limit, are determinate. The emission intensity, depending secondarily upon the temperatures, can be used to evaluate nebular densities to a high degree of astrophysical accuracy. Further discussion of the point is deferred until a later paper of the series. We may conclude the present contribution with the remark that perhaps the most surprising result of our investigation is that the individual b_n's fall as close to unity as they do, under conditions apparently far from those of thermodynamic equilibrium.

HARVARD OBSERVATORY
December 9, 1937