THE MASS OF THE VIRGO CLUSTER*

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ABSTRACT

The lists of radial velocities now include results for thirty-two members of the Virgo Cluster, thus giving for the first time sufficient data to determine some of the physical characteristics of a cluster of nebulae.

A comparison of the velocities of fainter members of the cluster with those of brighter members shows that the line-of-sight velocity of a nebula has no dependence on its magnitude; hence, equipartition apparently does not hold in the cluster. The distribution of the velocities in right ascension and declination shows that the cluster is not in rotation and that there is no central concentration of high velocities. This result is taken to mean that the cluster is neither condensing nor breaking up, but is a fairly stable assemblage, more or less held together by its gravitational field.

From the observed distribution function for radial velocity is derived the distribution function for space velocity. For an assumed distance of 2×10^6 parsecs this function leads to 2×10^{47} g or 10^{14} ⊙ as a value of the mass of the cluster. On the basis of 500 nebulae in the cluster, the mass per nebula is 2×10^{11} ⊙.

Although far larger than Hubble's value of 10^9 ⊙ for the mass of an average nebula, other evidence lends support to the high value obtained from the Virgo Cluster. It is possible that both figures are correct and that the difference represents a great mass of internebular material within the cluster.

Masses of extra-galactic nebulae have, in a few favorable cases, been derived from a study of relative line-of-sight velocities of different parts of the nebula. Similarly, we can turn to larger-scale phenomena and, by studying the relative line-of-sight velocities of different members of a cluster of nebulae, derive the mass of the cluster. In the first instance we determine the mass within the luminous boundaries of a nebula, while in the second we derive the total mass per nebula, including any internebular material within the cluster. Our present ideas concerning the transparency of space would suggest the presence of only a negligible amount of internebular material, and thus we should expect the two results to be more or less similar. Actually a discrepancy appears. In the following paragraphs we shall find that, applied to the Virgo Cluster, the second procedure gives a mass per nebula far larger than the mass of single nebulae as determined by Hubble.

Before considering the case of the Virgo Cluster, it is perhaps well to point out that both procedures suffer from the serious limitation

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that we have no means of determining nebular accelerations. Our observations are restricted to the measurement of velocities, and hence in attempting to determine masses we must always assume that our structure is stable. As an example, suppose that a nebula consisting of a lenticular assembly of stars is set in uniform rotation. If it is observed within, say, 10^7 years, it will still appear to be in solid-body rotation. Actually, very different conditions will hold. Particles at a particular distance from the center will continue to describe circular orbits; particles within this distance and immediately outside will describe elliptical orbits; while the outermost particles will travel in either parabolic or hyperbolic orbits and will soon (say in 10^9 years) escape from the cluster. If the assembly is observed during the early part of its history, observations of rotation alone obviously will leave the mass completely indeterminate.

When we consider a cluster of nebulae, an additional uncertainty arises, namely, that the cluster may be only a statistical fluctuation in the space density of nebulae. In this case gravitational forces would play an insignificant part in the formation of the cluster, and the internal velocities would be unrelated to its mass. If, however, the cluster represents merely a statistical fluctuation, the cluster should appear to be either condensing or else evaporating into the surrounding space. In the detailed discussion of the Virgo Cluster which follows, it will be shown that, as far as can be determined, this particular cluster seems to be a stable assembly, held together by its own gravitational field. If this can be established, then the radial velocities now known should yield a satisfactory estimate for the mass of the cluster.

The material used consists of twenty-five radial velocities taken from Humason's lists and five by V. M. Slipher.[1] Nine of the fainter members were observed and measured by the writer, both to improve the sampling, since only bright members of the cluster had previously been observed, and to eliminate the possibility of a depend-

[1] *Mt. W. Contr.*, Nos. 426, 531; *Ap. J.*, **74**, 35, 1931; **83**, 10, 1936. Slipher's results are given by Strömberg in *Mt. W. Contr.*, No. 292; *Ap. J.*, **61**, 353, 1925. Humason's lists include twenty-seven velocities, but two are recent additions not used in the calculations reported here.

ence of the apparent radial velocity on magnitude.[2] The detailed discussion follows.

VELOCITY AS A FUNCTION OF CLUSTER CO-ORDINATES

The data were first examined for cluster rotation. Figure 1 shows the distribution of velocities across the cluster in right ascension and declination. It is evident that, if rotation is present, it must be very

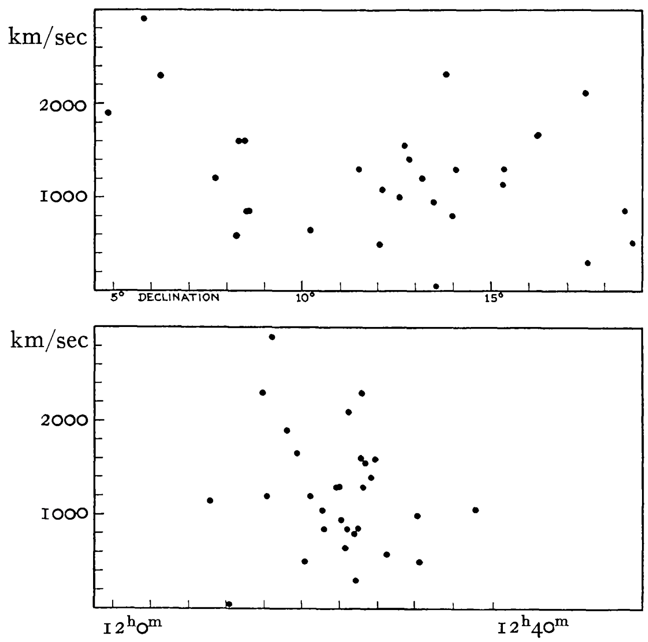

FIG. 1.—Distribution of velocities in declination and right ascension

small; and—more important for our purpose—there is no accumulation of high velocities toward the center of the cluster. A somewhat better demonstration of this point is furnished by Figure 2, in which the velocities are plotted against distance from the center of the cluster (center, $12^h\ 25^m$, $+12°30'$, 1930). Velocities of all values are more or less uniformly distributed over the range; and since this range covers the main portion of the cluster, we conclude that high line-of-sight velocities are just as likely to be found in one part of the

[2] No magnitude effect was found; and, as a corollary, there is no indication of equipartition of energy in the cluster. The mean line-of-sight velocity of the nine fainter nebulae is 1100 ± 200 km/sec, while that of the whole group is 1225 ± 330 km/sec.

cluster as another. If the cluster were either rapidly condensing or evaporating into space, a pronounced grouping of high velocities should occur near the center of the cluster. Since no such grouping

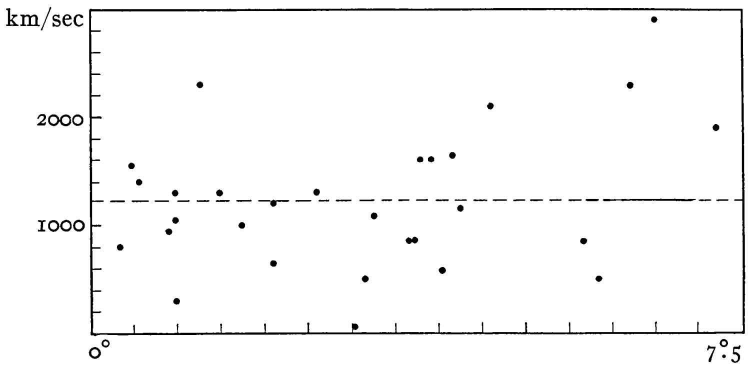

FIG. 2.—Distribution of velocities as a function of distance from center of the cluster.

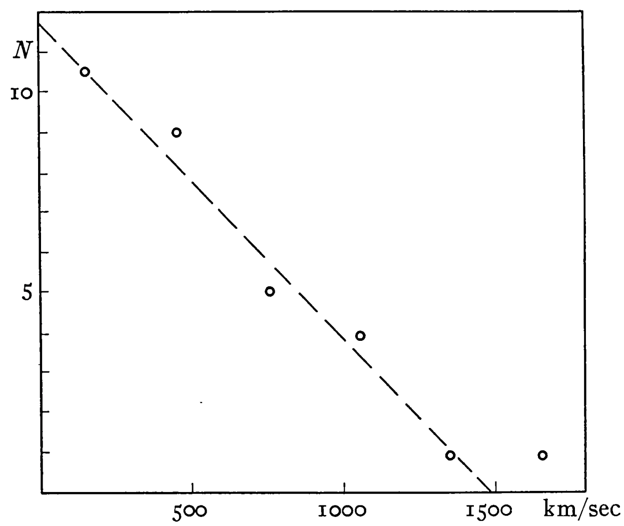

FIG. 3.—Distribution function of cluster velocities. Ordinates are numbers of nebulae for a velocity interval of 300 km/sec.

appears, it seems reasonable to assume that we are dealing with a more or less stable structure. If the orbits of the individual nebulae were largely circular, and random distribution of the orbital planes is assumed, we should expect the line-of-sight velocities to decrease systematically as we approach the center of the cluster. On the other hand, if the orbits were in the main highly elliptical, we should expect just the reverse radial distribution. The observed distribution agrees well enough with the hypothesis that we are dealing with a mixture of both types of orbits, and that orbits of all eccentricities are to be found in the cluster.

THE LINE-OF-SIGHT VELOCITY DISTRIBUTION FUNCTION

The line-of-sight velocity distribution for the whole cluster is plotted in Figure 3. In this case the mean velocity (1225 km/sec) has been subtracted from each observed value, and the difference (without regard to sign) has been taken to be the peculiar velocity of the individual. These velocities are grouped in intervals of 300 km/sec. The dotted line is drawn in as a possible representation of the distribution function.

If we assume random distribution for the orbits, Figure 3 tells us that the actual space velocities in the cluster probably cover the entire range from zero to a maximum of about 1500 km/sec, for, if the space velocities were more or less the same for all nebulae, we should expect a very different distribution function. For example, in the case of space velocities having the same magnitude but random distribution, the number dN_v showing line-of-sight velocities between v and $v+dv$ (considering only a hemisphere), would be

$$dN_v = N_u \frac{2\pi U dv}{2\pi U^2} = \frac{N_u}{U} dv \; .$$

Hence, all line-of-sight velocities between 0 and V would have the same frequency. In this case the velocity distribution-curve would be a horizontal straight line between zero and the maximum velocity, 1500 km/sec.

If we assume that the number of observed line-of-sight velocities varies linearly with the observed velocity as shown in Figure 3, i.e.,

$$dN \approx c\left(1 - \frac{v}{U}\right) dv \; , \tag{1}$$

the actual space velocity distribution function can be derived as follows:

Let $\phi\,(u)du$ be the number of nebulae having a velocity between u and $u+du$ and randomly distributed in direction over the hemisphere $\theta=0$ to $\pi/2$, $\phi=0$ to 2π. For the number of nebulae with velocities between u and $u+du$ which lie in the range v to $v+dv$, we can write

$$dN=\frac{\phi(u)}{u}\,dudv\ .$$

For the total number in range dv we then get

$$dN=dv\int_v^U\frac{\phi(u)}{u}\,du\ .$$

But observationally we have equation (1) as an expression for dN, and by comparison obtain

$$\int_v^U\frac{\phi(u)}{u}\,du=c\left(1-\frac{v}{U}\right)\ .$$

By differentiating with respect to the lower limit,

$$-\frac{\phi(u)}{u}=-\frac{c}{U}\ ,$$

or

$$\phi(u)=\frac{c}{U}\,u\ , \tag{2}$$

between $u=0$ and $u=U$.

Since $\phi\ (u)$ increases with u to a sharp cut-off at $u=U$, it is reasonable to regard U as the velocity of escape from the cluster.

If we had assumed a Gaussian law for the line-of-sight distribution function, the transformation to space velocities would have had the usual form; but in this case both distribution functions would extend to infinity, and hence would not be suitable for our purpose. Probably the actual space velocity distribution function in the cluster lies between equation (2) and a Gaussian distribution-curve.

In principle, a further extension of the analysis is possible, since the velocity distribution function for any small region in the cluster will depend both on the distance of the region from the center and on the density distribution in the cluster. Practically, such an extension is not justified at present, since the observations are not now sufficient to establish the form of the distribution function even for the cluster as a whole. One can only say that the observations are compatible with a linear distribution function for radial velocities which, by equation (2), makes the space velocity distribution function more or less linear.

THE MASS OF THE CLUSTER

In spite of the fact that the foregoing linear distribution function can be regarded as only a first approximation, it does show with considerable certainty that cluster velocities up to 1500 km/sec must be fairly common. In accordance with the preceding discussion, we should adopt this value as the velocity of escape; but since Figure 2 shows some grouping of high velocities at large distances from the center, it is also reasonable to assume that the outermost particles move in circular orbits with a speed of 1500 km/sec. Hence we can write either

$$m=\frac{v^2r}{2G} \qquad \text{or} \qquad \frac{v^2r}{G}\,,$$

the difference being of small importance.

Taking the circular orbit form, and assuming for radius of the cluster 2×10^5 parsecs (i.e., 0.1 times its distance), we find for the mass

$$2\times 10^{47}\,\text{grams} \qquad \text{or} \qquad 10^{14}\,\odot\;.$$

Assuming 500 nebulae in the cluster and no internebular material, we find for the mean mass of a single nebula

$$4\times 10^{44}\,\text{grams} \qquad \text{or} \qquad 2\times 10^{11}\,\odot\;.$$

This value is some two hundred times Hubble's[3] estimate of $10^9\odot$ for the mass of an average nebula. The cause of the discrepancy is not clear. In the determination of the mass of the cluster, the only source of a large error in the result is a possibility already mentioned, namely, that the cluster is simply a statistical fluctuation in space density. The extremely small probability of such an occurrence, to-

[3] *Mt. W. Contr.*, No. 485; *Ap. J.*, **79**, 8, 1934.

gether with the evidence already presented, seems to rule out this possibility.

On the other hand, the view that the cluster possesses a powerful gravitational field is strongly supported by the fact that the mean peculiar velocity of cluster nebulae is about four times the 150 km/sec found by Hubble[4] for isolated nebulae.[5] We can hardly interpret this fact in any other way than that nebulae leaving a cluster lose energy and nebulae captured by a cluster gain energy.

A consideration of other groups offers additional support. The Coma Cluster, which resembles the Virgo Cluster, shows a similar range in velocity,[6] while the small group to which our galaxy belongs shows a very small range. We therefore conclude from the available evidence that our procedure is justified and that the mass derived for the Virgo Cluster should be approximately correct.

The difference between this result and Hubble's value for the average mass of a nebula apparently must remain unexplained until further information becomes available. A statistical study of the relative velocities of close pairs of nebulae may possibly furnish the required data. It is also possible that both values are essentially correct, the difference representing internebular material, either uniformly distributed or in the form of great clouds of low luminosity surrounding the nebulae, as suggested by the recent great extension of the boundary of M 31.[7] Whatever the correct answer, it cannot be given with certainty at this time.

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[4] *Mt. W. Comm.*, No. 105; *Proc. Nat. Acad.*, **15**, 168, 1929.

[5] I am indebted to Dr. Hubble for suggesting this point.

[6] F. Zwicky has pointed out (*Helv. Phys. Acta*, **6**, No. 2, p. 110, 1933) that the velocity range in the Coma Cluster indicates non-luminous matter which is some four hundred times the amount of the observed luminous material.

[7] Stebbins, *Mt. W. Comm.*, No. 113; *Nat. Acad. Proc.*, **20**, 93, 1934.