Struve suggest that the amount of latitude involved in the calculations, due to uncertainty in the numerical values of these three factors, may be sufficient to account for the apparent discrepancy in the observed and calculated intensities of interstellar lines. It is hoped that the quantitative measures given above may be of some help in defining the problem more clearly, while additional physical data on the properties of the atoms of sodium and calcium should eventually make a satisfactory solution possible.

It is rather surprising to find that, within the probable errors of the determination, the intensity ratio $\frac{5889}{5895} = 1 \cdot 15$ is approximately the same for the two stars in which the absolute values of the intensities are so different. The interpretation of the variation of this ratio with absolute intensity depends to a considerable extent on the mechanism producing the true line contour; and while a first attack on the problem has been made by Unsöld, Struve and Elvey,* the nature of the mechanism is still incompletely understood. The writer hopes to extend the measures of this ratio to include a considerably larger range of absolute intensities.

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THE TEMPERATURES OF THE NUCLEI OF PLANETARY NEBULÆ.

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Introduction

The nuclei of the planetary nebulæ have such high surface temperatures that the ordinary methods of determining stellar temperatures cannot be applied to them. Zanstra † has developed two methods by which the nuclear temperature can be found from comparision of the nebular and nuclear spectra on a calibrated slitless spectrogram. In this paper a method of determining the nuclear temperature from observations of the nebular spectrum alone is developed. This method rests on the same theoretical assumptions as Zanstra's, but appears to be simpler and of wider application. This formula when applied to existing observational data yields temperatures rather lower than those at present current. Some of the assumptions on which the various formulæ are based are briefly examined, and it is found that some of the very high temperatures that have been derived for some of the planetary nuclei are not justifiable.

* *Zs. f. Astrophysik*, 1, 314, 1930.

† H. Zanstra, *Zeit. für Astrophysik*, 2, 1, 1931; or *Pub. D.A.O. Victoria*, 4, 209, 1931.

§ 1. *Derivation of the Formula from Zanstra's Model*

Zanstra's model of a planetary nebula consists of a hot central star radiating like a black body surrounded by a very extensive tenuous atmosphere composed of hydrogen with which is mingled a very small proportion of other ions, mainly those of doubly ionised oxygen. As a high degree of ionisation is observed in the actual nebulæ, it is supposed that the greater part of this hydrogen is dissociated into protons and electrons, whilst on account of the very low density in the nebula, the high dilution of the radiation flowing through it and the very short life of an excited hydrogen atom, the overwhelming majority of the remaining neutral hydrogen atoms are supposed to be in their normal state.

Three main processes are supposed to go on continuously in this nebula. The first of these processes is the recombination of free electrons and protons, resulting in the emission of the Balmer series and the formation of neutral hydrogen atoms; the second is the ionisation of neutral hydrogen atoms from their ground state by ultra-violet quanta (by which, in this paper, is meant quanta whose wave-length is less than 912 A., and which are therefore capable of ionising an hydrogen atom from its ground state); and the third process consists of inelastic collisions between free electrons and the foreign ions (*i.e.* the non-hydrogen ions) which result in the excitation of the ions, which, owing to the tenuity of the nebular material and the consequent rarity of collisions, relapse to their normal state by the emission of radiation.

The nebula is supposed to have reached a steady state in which the first process makes as many hydrogen atoms as the second destroys, whilst the kinetic energy of the photo-electrons freed by the second process is absorbed by the third. This perhaps requires some explanation. The velocity distribution of the free electrons will tend to the Maxwellian. Expelled photo-electrons get entangled in this crowd of free electrons and are quickly robbed of their initial energy, whilst electrons that have suffered a non-elastic collision and lost energy will quickly regain it at the expense of their fellows. In any interval of time as much energy is supposed fed into this electron gas by the hydrogen photo-electrons as is taken out of it by the inelastic collisions that are the direct cause of the emission of the so-called nebulium lines.

The ultra-violet quanta which cause the ionisation of the hydrogen can be divided into two classes: those which were emitted by the star (either directly or after transformation by one of the foreign ions) and those which have been emitted in the nebula as a result of captures of free electrons on the lowest level by protons.

Zanstra makes three subsidiary assumptions.

The first is the assumption that there is sufficient nebular matter present to ensure that every quantum emitted, whose wave-length is less than 912 A., is absorbed and causes either directly or indirectly the ionisation of one and only one normal hydrogen atom. With this assumption, the rate of ionisation by ultra-violet quanta emitted in the nebula will equal the rate at which free electrons are captured on the first level by protons. Hence, as in a steady state the rates of recombination and of ionisation must be equal, we can assert that the rate at which protons capture free electrons on the second and higher levels is equal to the rate at which ultra-violet quanta are emitted by the star.

The second assumption is that each such recombination of a free electron with a proton, in which capture occurs on the second or higher level, results in the emission of one and only one Balmer quantum. This assumption taken with the former one enables us to say that in any second the number of ultra-violet quanta emitted by the star is equal to the number of Balmer quanta emitted by the nebula. This translated into mathematics gives equation (4).

The number of quanta with frequencies between ν and $\nu+d\nu$ emitted by the star in one second is

$$4\pi R^2 \cdot \tfrac{1}{4}c\rho_\nu d\nu \cdot \frac{1}{h\nu}, \tag{1}$$

where R denotes the radius of the star, T its temperature, and ρ_ν the energy density of full radiation corresponding to a temperature T. By Planck's law

$$\rho_\nu = \frac{8\pi h}{c^3} \cdot \frac{\nu^3}{e^{h\nu/kT}-1}. \tag{2}$$

Hence the total number of ultra-violet quanta emitted by the star in one second is

$$4\pi R^2 \int_{\nu_0}^{\infty} \tfrac{1}{4}c \cdot \frac{8\pi h}{c^3} \cdot \frac{\nu^3}{e^{h\nu/kT}-1} \cdot \frac{1}{h\nu} \cdot d\nu, \tag{3}$$

where ν_0 denotes the frequency of light which is just able to ionise a hydrogen atom.

The number of quanta emitted per second by the nebula in the light of one of the Balmer lines is $\frac{L_P}{h\nu_P}$, where L_P denotes the total energy emitted per second by the nebula in the light of that line and ν_P denotes the frequency of that light. Consequently the total number of Balmer quanta emitted per second by the nebula is

$$\sum_{\text{Hyd}} \frac{L_P}{h\nu_P},$$

where the summation includes all the lines of the Balmer series and also a term $\int_{\nu'}^{\infty} \frac{L_P'}{h\nu} d\nu$ for the continuous spectrum at the head of the Balmer series, L_P' denoting the energy in light of frequencies between ν and $\nu+d\nu$ that the nebula emits per second in this spectrum. Thus Zanstra's first two assumptions give us the equation

$$4\pi R^2 \int_{\nu_0}^{\infty} \tfrac{1}{4}c \cdot \frac{8\pi h}{c^3} \cdot \frac{\nu^3}{e^{h\nu/kT}-1} \cdot \frac{1}{h\nu} \cdot d\nu = \sum_{\text{Hyd}} \frac{L_P}{h\nu_P}. \tag{4}$$

The third assumption that Zanstra makes is that all the kinetic energy of the freed hydrogen photo-electrons is used in stimulating the nebulium spectrum. This gives us equation (7), obtained as follows :—

The kinetic energy of an electron freed by a quantum of frequency ν is given by

$$\tfrac{1}{2}mv^2 = h\nu - h\nu_0, \tag{5}$$

and the number of such quanta emitted by the star in one second is given by (1), so that the total energy made available in one second for the stimulation of the nebulium spectrum is

$$4\pi R^2 \int_{\nu_0}^{\infty} \tfrac{1}{4}c \cdot \frac{8\pi h}{c^3} \cdot \frac{\nu^3}{e^{h\nu/kT} - 1} \cdot \frac{1}{h\nu}(h\nu - h\nu_0)d\nu. \tag{6}$$

But the energy radiated per second by the nebula in the nebulium spectrum is simply $\sum_{\text{Neb}} L_P$, where L_P is defined as before, and the summation extends over all the non-hydrogen lines. Equating these two expressions for the energy emitted per second we get

$$4\pi R^2 \int_{\nu_0}^{\infty} \tfrac{1}{4}c \cdot \frac{8\pi h}{c^3} \cdot \frac{\nu^2(\nu - \nu_0)}{e^{h\nu/kT} - 1}d\nu = \sum_{\text{Neb}} L_P. \tag{7}$$

Dividing equation (7) by equation (4) we get

$$\frac{\displaystyle\int_{\nu_0}^{\infty} \frac{\nu^3 - \nu_0\nu^2}{e^{h\nu/kT} - 1}d\nu}{\displaystyle\int_{\nu_0}^{\infty} \frac{\nu^2}{e^{h\nu/kT} - 1}d\nu} = \frac{\sum_{\text{Neb}} L_P}{\sum_{\text{Hyd}} \frac{L_P}{\nu_P}}. \tag{8}$$

If we put

$$x = \frac{h\nu}{kT} \tag{9}$$

equation (8) may be written

$$\frac{J_2 - x_0 J_1}{x' J_1} = \frac{\sum_{\text{Neb}} L_P}{\sum_{\text{Hyd}} L_P\left(\frac{\nu'}{\nu_P}\right)}, \tag{10}$$

where ν' denotes some chosen frequency, say that of H_β, and

$$x_0 = \frac{h\nu_0}{kT}, \qquad x' = \frac{h\nu'}{kT},$$

while

$$J_1 = \int_{x_0}^{\infty} \frac{x^2}{e^x - 1}dx, \qquad J_2 = \int_{x_0}^{\infty} \frac{x^3}{e^x - 1}dx. \tag{11}$$

As in equation (10) only the ratios of the various L_P are involved, we may replace them by the observed relative integrated intensities I_P of the various images found from a slitless spectrogram, or, if there is no very marked variation in the sizes of the various monochromatic images, by the ordinary relative intensities measured with a slit spectroscope. We thus obtain the equation

$$\frac{\mathcal{J}_2 - x_0\mathcal{J}_1}{x'\mathcal{J}_1} = \frac{\sum_{\text{Neb}} I_P}{\sum_{\text{Hyd}} I_P \frac{\nu'}{\nu_P}}, \tag{12}$$

from which, given the observed relative intensities I_P of the various nebular lines, we may deduce the temperature of the central star.

The right-hand side of this equation is independent of T and can be determined directly from observation, whilst the left-hand side, which contains no observed quantities, is a function of T only, whose value may be tabulated for various values of T. From this table, or from a graph plotted from it, the temperature corresponding to any observed value of the right-hand side can be found immediately. Approximately values of the left-hand side of equation (12) for a series of values of T are given in Table I.

TABLE I

Values of $\mathcal{J} \equiv \frac{\mathcal{J}_2 - x_0\mathcal{J}_1}{x'\mathcal{J}_1}$ *for x corresponding to* 912 *A.*, *x′ to* 4861 *A.* (H_β)

T	$\mathcal{J}$	T	$\mathcal{J}$	T	$\mathcal{J}$
T = 15,000°	$\mathcal{J}$ = 0·59	T = 40,000°	$\mathcal{J}$ = 2·0	T = 65,000°	$\mathcal{J}$ = 3·6
20,000°	0·85	45,000°	2·3	70,000°	4·0
25,000°	1·1	50,000°	2·6	80,000°	4·8
30,000°	1·4	55,000°	2·9	90,000°	5·5
35,000°	1·7	60,000°	3·2	100,000°	6·2

§ 2. *Application of the Formula*

Unfortunately there is at present very little observational data to which formula (12) may be applied. It may be of interest, however, to apply it to the line intensities found by Plaskett * for N.G.C. 7027, a very bright planetary with no well-defined nucleus to which Zanstra's methods cannot be applied (Berman's † application of Zanstra's formulæ is probably fallacious; finding no trace of a nuclear spectrum he used instead the faint continuous spectrum emitted by the nebulosity which may be due to free-free electron switches as predicted theoretically by Cillié ‡). Using Plaskett's values for the ordinary lines and Berman's † for the ultra-violet lines which Plaskett has not measured, formula (12) gives a temperature of about 43,500° for the stimulating star of N.G.C. 7027. If we use Plaskett's values of the line intensities for the Orion Nebula in formula (12) we get a kind of mean stimulation temperature for that nebula of 17,000°. (This estimate should be

* H. H. Plaskett, *Publ. D.A.O. Victoria*, 4, 187, 1928.
† L. Berman, *Lick Obs. Bull.*, 15, 86, 1930.
‡ G. G. Cillié, *M.N.*, 92, 820, 1932.

slightly increased, as it has been impossible to include the ultra-violet lines, which Plaskett has not measured.)

§ 3. *Comparison with Zanstra's Methods*

Zanstra's methods of determining the nuclear temperature depend on the determination from a slitless spectrogram of the ratio of the energy emitted in a monochromatic image (L_P) to the energy per unit frequency emitted in the stellar spectrum $\left(\frac{\partial L_s}{\partial \nu}\right)$, where $\left(\frac{\partial L_s}{\partial \nu}\right)$ is measured at the same frequency as L_P. His methods have the great advantage that comparisons are only made between lights of the same frequency that have traversed the same optical paths ; no subsidiary standard black body is needed. On the other hand, it is difficult to compare accurately light in the very narrow nuclear spectrum with that in the nebular image, especially when there is a very great difference in intensity between them. For very few of the bright planetaries will the nuclear spectrum be at all comparable in intensity with the nebular images, especially near N_1, N_2 and H_α, which make the most important contributions to the quantities from which the nuclear temperature is determined.

Formula (12), however, is independent of the nuclear spectrum, and can be applied equally well to the many small bright nebulæ in which the nucleus is inconspicuous as to those in which the nuclear spectrum is strong enough to be measured. It has the disadvantage of requiring a subsidiary standard black body, but when once that drawback has been overcome, the reduction of the observations is much simpler and quicker than those needed by Zanstra's methods.

§ 4. *Consideration of the Subsidiary Assumptions on which the Formula is Based*

We have assumed that each stellar quantum whose frequency is greater than ν_0 is absorbed and causes the ionisation of one and only one hydrogen atom. This cannot be true unless there is a great abundance of normal hydrogen present in the nebula. Moreover, some of the ultra-violet radiation is used in keeping the oxygen and other atoms present ionised. This radiation, however, is not completely lost to the hydrogen, for if the nebula is in a steady state, for each ionisation of such an atom there is a corresponding recombination which results in the emission of a quantum of radiation of frequency lower than that of the stellar quantum absorbed, yet still of high enough frequency to ionise a hydrogen atom. [This is true of all the ions involved in the actual nebulæ with the exception of N^+ and N^{++} which make only a partial return.] The photo-electric absorption coefficient decreases with increasing frequency, so that the foreign ions actually help the hydrogen to get its full share of the very high frequency stellar quanta. It is to be noted that if He and He^+ serve the hydrogen in this way (and presumably they do), the helium lines should be considered as belonging to the nebulium spectrum when we apply the formula, for they have drawn the energy that they emit from the same source, viz. the energy remaining after the hydrogen atoms have been ionised.

The second assumption was that each recombination of free electron and proton in which capture occurs on the second or higher level results in the emission of one, and only one, Balmer quantum. The justification of this assumption given below also requires a great abundance of normal hydrogen, for it requires that the free path of any Lyman quantum emitted be small compared with the dimensions of the nebula.

If an electron is captured on the second level a quantum in the Balmer continuous spectrum is emitted and the newly formed atom relapses to the ground state with the emission of an L_α quantum (L_α denoting the first line in the Lyman series). Owing to the scarcity of hydrogen atoms in the right state to absorb it, the Balmer quantum will presumably find its way straight out of the nebula, but the L_α quantum will be absorbed by a normal hydrogen atom. It will, however, quickly be re-emitted, for owing to the conditions prevailing in the nebula, the absorbing atom is unlikely to be disturbed during its short excited life of about 10^{-8} seconds. After many such scatterings, the L_α quantum will eventually find its way out of the nebula.

If an electron is captured on the third level, a quantum in the continuous Paschen spectrum is emitted and the newly formed atom may reach the ground state either by emitting H_α followed by L_α or by emitting L_β. The H_α and L_α quanta would find their way out of the nebula unchanged, but before the L_β quantum could escape it would be absorbed by a normal hydrogen atom, which it would raise to the third level. In about eight cases out of nine this atom would re-emit L_β, but in the ninth it would emit H_α followed by L_α. If the free path of a Lyman quantum is small compared with the dimensions of the nebula, the probability that any L_β quantum emitted is split into H_α and L_α before it leaves the nebula is very large, and the chance of its survival as L_β correspondingly small. Thus after thirty free paths, the chance of survival of an L_β quantum is less than 2 per cent., so that no great error will be made in supposing that all the L_β quanta emitted get split into H_α and L_α quanta.

The last of the series of transitions made by an electron captured on one of the other levels must be one to the ground state, as a result of which a quantum in the Lyman series is emitted. If this quantum is not L_α—if it were the electron must have come from the second level and so already have caused the emission of a Balmer quantum—it will be reabsorbed, and by the process described for L_β will be split into a number of quanta, one of which is L_α and another a Balmer quantum.

It appears, then, that the assumptions, from which equation (4) is derived, are likely to hold if, and only if, there is a great abundance of hydrogen in the nebula.

The third assumption on which equation (12) is based is that all the kinetic energy of the freed hydrogen photo-electrons is used in stimulating the nebulium spectrum. This cannot be absolutely true, as the electrons captured by the protons on the second and higher levels have some kinetic energy which will be converted into radiation in the Balmer and other continuous hydrogen spectra; but as the chance of capture varies like $1/v^4$, only slow electrons are likely to be caught, so that the total energy of the electrons dissipated in this way is likely to be small. It is possible to make some correction for this by including in the nebulium spectrum the energy emitted in the continuous Balmer and Paschen spectra, while, at the same time, including in the hydrogen spectrum the number of quanta emitted in the Balmer continuous spectrum.

§ 5. *Criticisms of some of the Applications of Zanstra's Formulæ*

Equation (4) is essentially the equation from which Zanstra calculates the temperature of the central star from observations of a recombination spectrum. He applies it separately to the observed spectra of H, He and He^+. We have seen that the application of equation (4) to the hydrogen spectrum is probably justifiable if there is a large abundance of hydrogen in the nebula, but the above considerations do not justify its application to the spectrum of ionised helium. We cannot assert that each suitable stellar quantum ionises an He^+ ion, since the He^+ and O^{++} ions have to share such quanta, nor can we find the number of quanta emitted in the Balmer series of ionised helium; we can only guess it. We cannot observe the Balmer series which is in the ultra-violet, nor can we observe the whole of the Paschen series of ionised helium, and even if we could, it would only tell us the number of quanta emitted in the first line of the Balmer series and not the total relevant number of recombinations that occur in one second.

The realisation that Zanstra's recombination formula cannot be applied satisfactorily to the observed spectrum of He^+ (of which 4686 is the most important line) is of some importance, since it is from its application to 4686 that Zanstra * and Berman † get temperatures of the order of 70,000° to 80,000° for some of the planetary nuclei, and Beals ‡ gets equally high temperatures for Wolf-Rayet stars and novæ. The breakdowns in the assumptions upon which the formula depends, which have already been pointed out, offer us no escape from these high temperatures; on the contrary, they rather suggest that the application of the formula would lead to temperatures that were too low.

It appears probable, however, that there might be a breakdown in the basic assumption that the *He* II spectrum is emitted as the result of the recombination of free electrons and alpha particles, *the balancing ionisations being accomplished by stellar quanta.* The stellar quanta may not be the only ionising agency at work.

Of the ten planetaries in which Campbell and Moore § have observed the splitting and broadening of the spectral lines, nine of them show the line 4686, while the tenth (N.G.C. 6543) has 4686 concentrated in the nucleus as

* H. Zanstra, *loc. cit.*
† L. Berman, *loc. cit.*
‡ C. S. Beals, *M.N.*, **92**, 677, 1932.
§ Campbell and Moore, *Publ. Lick Obs.*, **13**, 178, 1918.

a strong emission band. The only other nebula which shows 4686 and which has been tested with high dispersion is N.G.C. 2440, which, to quote Campbell and Moore, "is faint and not well situated for the required exposure." (To the list given by Campbell and Moore on p. 178, N.G.C. 1535 and N.G.C. 6572 might be added.)

The most natural explanation of these broadened and split spectral lines is that they are due to Doppler effect, the nebular matter moving outwards from the star with velocities of the order of 30–100 km./sec. A similar explanation has been given of the broadening of the bands in Wolf-Rayet spectra, only here the outward velocities required are much greater, while for novæ there is very strong evidence of the outward motion of matter with large velocities.

For an ionised helium atom to ionise itself by collision with another atom, it need have a velocity of about 50 km./sec. only, so that it appears possible that the helium in these expanding shells becomes ionised mainly by collision. Such ionising collisions may be supposed as resulting rather from the encounter of molecules of one cloud of gas with those of another cloud, through which the first cloud is passing, than from the random velocities of molecules within one cloud. In this case the results obtained by applying Zanstra's formula would be entirely misleading. While some of the hydrogen atoms would also be ionised by collision, it is likely that this number would be less in proportion to those ionised by stellar quanta than for helium, so that the application of Zanstra's formula to the hydrogen spectrum would be less misleading, especially as for any given temperature, the temperature given by this formula is less sensitive to change in the observed quantity when applied to hydrogen than it is when applied to ionised helium.

Equation (7) is essentially the equation from which Zanstra calculates the temperature of the central star from observations of the nebulium spectrum, and the one which forms the base of his method for deriving an approximate nuclear temperature from the difference in magnitude between the central star and the nebula, a method by which he has derived temperatures as high as 140,000° for some of the planetary nuclei. To obtain his formula, Zanstra assumes that the photographic brightness of the nebula bears a constant ratio to the intensity of N_1 and N_2. Equation (12), however, suggests that this ratio will not be constant, but will vary almost linearly with T. Moreover, Zanstra assumes that the photographic magnitude of the central star is proportional to the spectral energy at a definite wave-length, "the isophotic wave-length," which he assumes to be the same for all the nuclei. As B. Vorontsov-Velyaminov * has pointed out, this is most unlikely to hold true for nuclei which display strong emission bands superposed on a faint continuous spectrum. Further, the application of a formula of this nature assumes that the planetary nebulæ form a homogeneous class, an assumption whose truth is by no means self-evident. No great weight, then, can be attached to temperature estimates obtained from this formula, even if they are based on homogeneous and reliable observational data, which Zanstra's estimates certainly were not.

* B. Vorontsov-Velyaminov, *Russian Astronomical Journal*, 8, 122, 1931.

§ 6. *Conclusion*

Table II contains all the direct estimates of nuclear temperatures that have yet been given. The temperatures given in columns 2 to 6 are due to Berman,* with the exception of those marked with a (Z) which are due to Zanstra,† and those enclosed in brackets which are due to V. Ambarzumian,‡ but which are based on Berman's observations. The temperatures in columns 7 and 8 were obtained by applying formula (12) to the line intensities observed by Plaskett § or Berman,* together with certain assumptions about the unobserved lines. Unfortunately none of these estimates can be regarded as anything more than intelligent guesses based on the application of a theoretical formula to insufficient observational data.

Table II

Estimates of the Nuclear Temperature

Nebula	(Recombination)			Nebulium	Colour Temperature	Formula (12)	
	H	*He* I	*He* II			$H_\alpha : H_\beta$ =4·0 : 1	$H_\alpha : H_\beta$ =5·8 : 1
6572	43,500 41,000 (Z)	43,500 34–41,000 (Z)	...	41,500 38,000 (Z)	...	34,000	27,000
6543	36,100 37,500 (Z)	36,400	...	32,000 35,000 (Z)	30,000	24,000	18,500
II 4593	25,000	...	...	24,000	25,000	20,000	15,500
6826	26,500	29,500	...	27,000	25,000	27,000	22,000
7009	40,000	39,000	70,000 (115,000)	40,000	40,000(?)	51,000	41,000
7662	43,000	...	81,000	51,000	...	35,000	29,000
7027	> 52,000	> 40,000	> 86,000 (165,000)	> 53,000	...	...	43,500

It has already been shown that the application of the recombination method to *He* II is unjustifiable and perhaps misleading; the application to *He* I rests on no better theoretical grounds. The intensity of H_α, the biggest contributor to the number of quanta emitted in the Balmer series, was not measured either by Zanstra or by Berman. In order to obtain a value for the nuclear temperature, Berman assumed an intensity of H_α found by extrapolation of the observed-intensity curve of the Balmer series from H_β to H_η. This makes the ratio of the intensity of $H\alpha$ to that of H_β about 2 : 1, which is probably too low. The only measurements of this ratio are

* L. Berman, *loc. cit.*
† H. Zanstra, *loc. cit.*
‡ V. Ambarzumian, *Pulkovo Obs. Circ.*, No. 4, 1932.
§ H. H. Plaskett, *loc. cit.*

due to Plaskett, who found a value of 4·0 : 1 for the Orion Nebula and 5·8 : 1 for N.G.C. 7027. It seems reasonable to expect that the values for other planetaries will lie between these limits. Similarly when applying the nebulium method, Berman, having no measurements of lines in the red, had to ignore them, though it would appear from Plaskett's measurements for N.G.C. 7027 that 6548 and 6584 (the N II equivalents of the O III N_1 and N_2 lines) radiate quite an appreciable amount of energy. Fortunately, however, the temperatures derived from Zanstra's methods are very insensitive to errors in the observed quantities for temperatures above 30,000° [the quantity to be observed for the hydrogen method varies approximately as T^3, and for the nebulium method approximately as T^4], so that we may expect the temperatures shown in columns 2 and 5 to be of the right order, with the possible exception of those for N.G.C. 7027 (see § 2).

The temperatures given in columns 7 and 8 are open to similar objections due to lack of observational data. These temperatures are not so reliable, as for this method the quantity to be observed varies approximately linearly with T. Column 7 gives the value of the temperature obtained from the observed-line intensities when a value of 4·0 : 1 is adopted for the ratio of the intensity of H_α to that of H_β, while column 8 gives the temperature found by adopting a value of 5·8 : 1 for this ratio. The temperatures given in column 7 are probably more accurate than those given in column 8 for the lower temperatures, and *vice versa* for the higher ones. Where the intensity of N_1 has not been measured, it has been taken as three times that of N_2, which is the theoretical value according to Stevenson.* It has been necessary to ignore all other unmeasured lines and also the Balmer continuous spectrum, which for some of the nebulæ might make an appreciable contribution to the number of quanta emitted in the hydrogen spectrum.

Formula (12) appears to give values for the temperature which are rather smaller than those derived by Zanstra's methods. The observational data is, however, far too scanty for us to attempt to draw any conclusions from this, such as the breakdown of the initial assumptions through lack of abundance of hydrogen, or through our ignoring a possible accumulation of hydrogen atoms in the 2S metastable state, etc. When more extensive observational data becomes available, it may be possible to draw some such conclusion from differences in the temperatures obtained by these three alternative methods which, though they rest on the same theoretical assumptions, are observationally independent.

In conclusion I should like to express my deep-felt thanks to Sir Arthur Eddington, not only for his interest and advice in connection with this paper, but also for his constant encouragement and for the many stimulating discussions on planetary nebulæ that I have been privileged to share.

The Observatory, Cambridge :
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* A. F. Stevenson, *Proc. Roy. Soc.*, A, **137**, 298, 1932.