

CORRECTING LENSES FOR REFRACTORS

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ABSTRACT

A short history is given of the use of zero-power correcting lenses with visually corrected refracting telescopes. The object of the corrector is to change the refractor over into a photographically corrected telescope, in order that it may be better adapted for spectrum photography. An analysis and reconstruction of the optical design of the 40-inch lens of the Yerkes Observatory is presented, furnishing data useful for the proper design of the corrector lens. The design and data on the corrector lens are given, and a comparison of its performance with that of the older corrector lens, showing gain in speed. The general theory of corrector lenses of this type is presented in some detail; in particular, it is shown that there is a large outstanding optical aberration, chromatic magnification, which cannot be eliminated, but is capable of material reduction by proper design. The effect of flexure of the telescope tube is considered, and found to be negligible. It is pointed out that the most serious outstanding defect of the corrector lens is secondary axial-color aberration. The question of reducing this by suitable choice of glass is discussed, and it is shown that little can be expected in this direction.

So far as I am aware, a correcting lens for a visual refractor was first employed by H. F. Newall,¹ with the 24.8-inch refractor of the Cambridge Observatory. The object of a correcting lens of the type here considered is to change the color-curve of the objective in such a way that it can be used with a spectrograph for photography in the important blue-violet region. The correcting lens as used by Newall was placed 5 feet inside of the focus, and shortened the back focus by 18 inches. In America, correcting lenses have been in use at the Lick, Yerkes, Flagstaff, and Ottawa observatories, and have been an indispensable adjunct in carrying on the exceedingly important researches in spectrum analysis and in related problems. Considering its importance, it is surprising that so little attention has been paid to the optics of the correcting lens, with a view to increasing its efficiency. As will be seen below, the solution is by no means unique. The elementary theory of the correcting lens has been very clearly presented by J. E. Keeler,² who solved the problem of flattening the color-curve in the desired region by making use of the simple equations of conjugate foci. A very thorough investigation, from the practical side, of the correcting lens used in conjunction with the 15-inch Ottawa refractor has been made by J. S.

¹ *Monthly Notices of the Royal Astronomical Society*, 54, 373, 1894.

² *Astrophysical Journal*, 1, 101, 1895.

Plaskett.¹ The lens which he finally adopted was of 4 inches aperture, placed 59 inches inside the focus.

The correcting lens in use at the Yerkes Observatory, up to 1928, was of 57 mm aperture, and placed 100 cm inside the focus. It was thoroughly investigated by Philip Fox,² in conjunction with an investigation of the 40-inch lens. The color-curve of the combination was found to be excellent for purposes of spectroscopy. The spherical aberration was, however, quite large; the normal amount of over-correction to be expected in the violet for a visual refractor (5 mm), instead of being corrected by the lens, was apparently increased to about 6 mm.³ In July, 1928, a new correcting lens, designed by the writer and made by J. W. Fecker Company, was installed. This is of 6 inches aperture, placed 2.5 m inside the focus. Its mounting, with convenient adjustments for making the important collimation correction, was designed by G. W. Moffitt⁴ of the observatory staff. Its performance is very satisfactory, especially in the matter of speed. For example, it has been found to be twice as fast in the blue as the old corrector, with a factor increasing uniformly toward the violet end of the spectrum to a value six times at λ 3900.⁵

I. COLOR-CURVES

Of prime importance in determining the design of a correcting lens to be applied to a visual refractor is the amount of tilt of the color-curve, between the lines F and h , say, of the spectrum, which must be corrected by the lens. This depends numerically upon the kinds of glass in the refractor, and upon the correction which has been adopted in the visual region. I have collected in Table I data on the color-curves of some refractors, reduced to a common focal length of 10 m. The data for Vienna, Pulkowa, and Cambridge were taken from Newall,⁶ for Lick, from Keeler;⁷ and for Yerkes, from Fox.⁸ I have also added a computed curve for the Lick, and

¹ *Ibid.*, 25, 195, 1907, and 32, 243, 1910.

² *Ibid.*, 27, 237, 1908.

³ *Ibid.*, p. 253.

⁴ *Journal of the Optical Society of America*, 20, 457, 1930.

⁵ Frost, Barrett, and Struve, *Publications of the Yerkes Observatory*, 7, Part I, 2.

⁶ *Op. cit.*, p. 374.

⁷ *Op. cit.*, p. 104.

⁸ *Op. cit.*, p. 252.

two computed curves for the Yerkes. The curve for the Lick was computed on the assumption that the glass was of the same kind as that of the Yerkes, the computation then being carried on so as to obtain the same difference between the C and F lines as that which was observed by Keeler. Separation and thicknesses, not being known, were neglected. The column YC_1 for Yerkes was obtained in the same way, adjusting for the C and F difference given by Fox. The column YC_2 was computed on the basis of adjusting for Fox's value of the D and G' difference. It appears to the writer that

TABLE I
COLOR-CURVE FOR LARGE REFRACTORS
(Reduced to a focal length of 10 m)

WAVE-LENGTH		YERKES 40-IN. $F=19.35$ m			LICK 36-IN. $F=17.63$ m		VIENNA 27-IN.	PULKOWA 30-IN.	CAM- BRIDGE 25-IN.
		Fox	YC_1	YC_2	Keeler	LC	$F=10.36$ m	$F=14.12$ m	$F=8.84$ m
		mm	mm	mm	mm	mm	mm	mm	mm
B.....	6870	9.4			6.4				
C.....	6563	4.8	3.9	3.8	3.0	3.4	2.6	2.1	3.5
D.....	5893	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E.....	5270	1.2			1.1				
F.....	4861	6.2	5.5	6.3	6.5	6.9	5.8	4.5	4.5
G'.....	4341	26.3	24.2	26.4	27.4	27.1	22.7	23.3	17.4
<i>h</i>	4102	45.9	42.8	46.2	46.3	46.8			35.3
	4047*	52.7	47.1	50.8		51.3			
<i>h</i> -F.....		39.7	39.3	41.2	39.8	41.1			30.8

* A mercury line.

the curve obtained by Fox is too steep in the violet. This is to be expected, on account of the overcorrected spherical aberration of the 40-inch lens in the violet, which lengthens the focus for the edge zone and thus acts in the same direction as the color error. It is of interest to note the differences in the color-curves of these lenses in the visual region, where they have been designed to give the best performance. In interpreting these differences, however, considerable allowance must be made for observational error in deriving them. The last row in Table I, in which the values of difference in back focus for the F and *h* lines are given, is of especial importance in our problem, for its value governs the design and efficiency of the correcting lens. A small value is desirable, as will be shown later.

The values for Lick and Yerkes are identical, while that for Cambridge is 25 per cent smaller. It would appear that this lens must be made of special or unusual glasses, which give the highly desirable reduced secondary spectrum. It is to be noted, however, that the value for the h line for Cambridge is relatively larger than the value for the G' line.

2. CONSTANTS OF THE 40-INCH LENS

In order that a correcting lens may be properly designed, it is desirable to know the optical constants of the refractor which is to be corrected. The published data on the 40-inch lens are exceedingly

TABLE II
GLASS CONSTANTS OF THE 40-INCH LENS

WAVE-LENGTH	INDICES	
	Crown	Flint
6563.....	1.513193	1.610092
5893.....	1.515797	1.614866
4861.....	1.522032	1.626824
4341.....	1.527065	1.636989
4047.....	1.530882	1.645006

meager. I have accordingly made an attempt to collect all the necessary data, in order that the information which is thus placed on record may be complete on all important and interesting points. The observatory has in its possession prisms of the glass used in the construction of the lens. Dr. Frost in 1928 forwarded these prisms to the United States Bureau of Standards, with a request that their indices be determined as accurately as possible. It appears from this determination that the glass of the positive lens is of the Schott type O203, with a dispersion factor of 58.35, and that of the negative lens of type O118, dispersion, 36.75. The indices, in detail, are shown in Table II. These wave-lengths are for the customary Fraunhofer lines, with the exception of 4047, which is a mercury line adopted by Schott as the standard point in the far violet. Schott unfortunately called this line h , which had been pre-empted by Fraunhofer as the designation of $H\delta$ at λ 4102. In order to avoid confusion, the Schott line will be referred to by its wave-length, in what follows.

In a letter from one of the designers, Mr. R. Lundin, to Dr. Frost, of June 28, 1907, are given the radii of the four spherical surfaces of the 40-inch lens. The thicknesses and separation of the two components are not stated. These have been given, however, in round numbers, by Dr. Hale.¹ The data furnished by Lundin are:

Crown, equiconvex, $R_{1,2} = \pm 22$ ft. 9.6 in. (6950 mm) ,
 Flint, int. surf., $R_3 = -22$ ft. 1.92 in. (6754 mm) ,
 Flint, ext. surf., $R_4 = +4000$ ft. (1219 m) concave .

Preliminary computations showed that the foregoing value for the interior flint surface must be wrong, and should have read:

$$R_3 = 20 \text{ ft. } 1.92 \text{ in. (6145 mm) .}$$

The central or axial thicknesses of the lenses and the separation of the adjacent surfaces were measured by the writer, with the assistance of F. R. Sullivan. They are as tabulated, with Hale's values given for comparison. The error in our measurement of the thickness of the positive is not greater than 0.5 mm; of the negative, 1.0 mm; and of the separation, 0.2 mm.

	R. and S.	H.
Central thickness, pos.,	58 mm.....	2.5 in. (63 mm)
Central thickness, neg.,	28	1.5 (38)
Central sep., adj. surf.,	215.2	8.37 (213)

Using all the foregoing data, with the corrected value of R_3 as noted, I have computed the following values of the focal length, and the back focal difference, D-G'. For comparison are tabulated the observed values of these quantities, using E. E. Barnard's² value for the focal length (visual) and Fox's value of the color difference (Table I). It is seen that the color difference is well represented, but

	Computed	Observed	Diff.
Focal length at 50° F, for D.....	19,550 mm	19,354 mm	196 mm
Color difference, D-G'.....	52.7	51.0	1.7

that the computed focal length is nearly 8 inches long. I assumed, therefore, that errors were made by the makers in measuring the

¹ *Astrophysical Journal*, 6, 43, 1897.

² *Astronomical Journal*, 31, 24, 1917.

curvature of the surfaces, and computed what they should have been, in order to satisfy the observed data. In making this computation it is necessary to assume that the positive is equiconvex, and that the radius given for the flat exterior surface is nearly correct. On these assumptions I have obtained the following new radii:

$$R_{1,2} = \pm 6886.7 \text{ mm} ,$$

$$R_3 = -6090.0 \quad ,$$

$$R_4 = +1209290 \quad .$$

It is interesting to note that Lundin's values for these radii are larger by 0.90, 0.89, and 0.83 per cent, respectively, suggesting an error of his spherometer. Using these new radii, I have computed a table of the optical constants of the 40-inch lens (Table III). To get the data for the H (*Ca*) line given in the table, the necessary indices of refraction were obtained by extrapolation, using the Cauchy formula. The last column, "Gauss Point," is the distance from the lens of the second Gauss point (the point from which focal length is measured), measured outward from the front surface of the lens. This comparatively large distance is due to the unusual amount of separation between the two components of the lens. The remaining computed data are:

TABLE III
OPTICAL CONSTANTS OF THE YERKES 40-INCH OBJECTIVE

WAVE-LENGTH	FOCAL LENGTH	BACK FOCAL LENGTH	Fox		
			Color-Curve	Obs. Color-Curve	Gauss Point
A	mm	mm	mm	mm	mm
6563.....	19359.3	18666.4	+ 7.4	+ 9.3	391.7
5893.....	19355.0	18659.0	0.0	0.0	394.8
4861.....	19375.9	18671.3	+ 12.3	+ 12.0	403.4
4341.....	19422.9	18710.3	51.3	51.0	411.4
4047.....	19477.0	18757.6	98.6	102.3	418.2
3969.....	19493.0	18771.6	112.6	119.3	420.2

Spherical aberration, D line = 49.8 mm (edge rays long) ,

Spherical aberration, G' line = 54.7 ,

Spherical aberration, λ 4047 = 55.4 ,

The calculated coma, taken as the difference between the sine condition and spherical aberration, is, for all wave-lengths, -14 mm. The spherical aberration for the D line, reduced to angular value, is $27''.4$, in good agreement with the value $24''.9$ obtained from third-order algebraic theory. This value is unusually large, being more than three times that of a single lens of best form ($R_2 = 6R_1 \pm$), of the same aperture and focal length, which is but $7''.7$. The necessary labor of reducing this to zero by hand refiguring must have been excessive, unless the curves originally ground and polished departed strongly from spheres. The value, -14 mm, for the coma means that the comatic flare is inward, and that at a distance of 100 mm from the axis the outer zone of the lens images to a circle 0.14 mm in diameter, whose center is displaced from the image formed by the small central zone by 0.14 mm or $1''.4$.

3. THE CORRECTING LENS

Before taking up the general theory of the corrector, the data for the new correcting lens computed for the 40-inch objective will be given. This consists of a positive lens of flint, followed by a negative of crown, of the same power. The glasses available were of the types Schott F₃ and K₃. The indices are given in the accompanying tabu-

WAVE-LENGTH	INDICES	
	Flint	Crown
6563.....	1.61312	1.51042
5893.....	1.61789	1.51283
4681.....	1.62996	1.51900
4341.....	1.64022	1.52394
4047.....	1.64831	1.52768

lation. The values for the F and G' lines were determined by the United States Bureau of Standards. Those for the remaining wave-lengths were obtained by utilizing the partial dispersions given in Schott's *Catalogue*, for these types of glass. The elements of the correcting lens, which were computed from these data, are as follows:

$$\begin{aligned}
 R_1 &= +1125.5 \text{ mm} , \\
 R_2 &= -1125.5 \quad , \\
 R_3 &= -1125.5 \quad , \\
 R_4 &= +773.5 \quad .
 \end{aligned}$$

Central thickness for positive lens is 12 mm; for negative lens, 10 mm; separation of components, 0.3 mm. The lens is placed inside the tube of the telescope with its rear surface at a distance of 2492 mm from the focus for the F line.

Table IV contains the values of the focal length and back focal length for the combination of 40-inch lens with corrector. The back focal length is the distance of the focus from the rear surface of the corrector, and determines the color-curve. The last column in the table, "Gauss Point," gives the position of the point from which focal length is measured, with reference to the front surface of the 40-inch lens, corresponding to the last column in Table III.

TABLE IV
DATA FOR 40-INCH WITH CORRECTOR LENS

Wave-Length	Focal Length	Back Focus	Color-Curve	Gauss Point
A	mm	mm	mm	mm
6563.....	20110.4	2569.5	+79.1	+1060
5893.....	19935.0	2542.8	52.4	912
4681.....	19544.3	2499.5	9.1	564
4341.....	19222.5	2490.4	0.0	+ 252
4047.....	18969.6	2496.4	+ 6.0	- 7

Distance between outer surfaces of 40-inch lens and corrector = 16480.5 mm

Distance between inner surfaces = 16157.0 mm

It will be seen from Table IV that achromatization has been effected at wave-lengths 4047 and 4760, the minimum falling at about 4340. This correction is farther to the violet than that of the old corrector, which probably accounts for a portion of the greater speed of the new lens in the ultra-violet (p. 185). The table discloses the enormous range of focal length with color, when a correcting lens of this type is employed. This characteristic of correcting lenses was not brought out in Keeler's paper on the subject;¹ in fact, his method of treatment precluded its recognition. Considering only the spectral range from 4047 to 4681, which is less than the range of the average spectrogram, it is noted in the last column of Table IV that the effective difference in focal length is 57 cm. This means that for a star which is only 1 mm, or 10'', from the axis, its image on the slit of the spectrograph is spread out into a spectrum whose length between the lines in question is 0.03 mm, which is the order of the slit

¹ *Op. cit.*

width. With the old corrector, this dispersion amounts to 0.07 mm (p. 195). It is for this reason that the collimation of the correcting lens is of such importance, as previously noted (p. 185).

4. GENERAL THEORY OF CORRECTING LENSES

There are but three aberrations which are of importance in the design of correcting lenses of the type here considered: (1) color difference of the back focal length, designated by K_a ; (2) color difference of the focal length, K_f ; (3) spherical aberration, S_a . There is a fourth aberration, spherical aberration depending on color, which is unimportant, and accordingly will be neglected. The well-known formulae for these aberrations, up to the fifth order, are:

$$\left. \begin{aligned} K_a &= d \sum \frac{h^2 \phi}{\nu}, \\ K_f &= -\tan \beta \sum y h \frac{\phi}{\nu}, \\ S_a &= \frac{1}{8} d^3 \sum h^4 \phi^3 A, \end{aligned} \right\} \quad (1)$$

the summation extending over all the lenses of the system, in this case four, assuming a correcting lens of two elements. In these formulae d is the aperture of the refractor, ϕ is the power of each component, or the reciprocal of the focal length, ν is the dispersion factor for which achromatization is desired, given by the formula

$$\nu = \frac{1}{2} \frac{n_\lambda + n_{\lambda'} - 2}{n_\lambda - n_{\lambda'}}, \quad (2)$$

where n_λ is the index of refraction for wave-length λ ; β is the angular distance from the axis of a star or object; y and h are, in Schwarzschild's notation, the distance from the axis, at each lens, of the chief ray at 45° , and the ray limiting the axial pencil, respectively.

Axial color K_a .—The condition for freedom from axial color, assuming as a first approximation that the corrector is composed of two thin lenses in contact, is:

$$\frac{\phi_1}{\nu_1} + h_2^2 \frac{\phi_2}{\nu_2} + h_3^2 \left(\frac{\phi_3}{\nu_3} + \frac{\phi_4}{\nu_4} \right) = 0, \quad (3)$$

the subscripts referring to each lens of the system in turn. For zero-power in the corrector, $\phi_4 = -\phi_3$, and we have for the determination of F_3 :

$$F_3 = -\frac{h^2}{\kappa} \left(\frac{1}{\nu_3} - \frac{1}{\nu_4} \right), \quad (4)$$

where we have put

$$\left. \begin{aligned} h &= h_3, \\ \kappa &= \frac{\phi_1}{\nu_1} + h_2^2 \frac{\phi_2}{\nu_2}. \end{aligned} \right\} \quad (5)$$

Equation (4) gives the approximate solution of the problem, excepting the determination of the spherical aberration. Appropriate thicknesses and separation of the two components are now to be inserted, and final adjustment of the system made by the customary ray-tracing method, after the approximate shapes have been determined from the solution for the elimination of spherical aberration (p. 195).

Equation (4) shows that if the preceding lens F_3 is of positive power, it must be of the flint type of glass, for ν_3 must be less than ν_4 , it being noted that κ is negative, as will be shown. It is indifferent whether positive or negative precedes. In any case the formula shows that the positive must be of flint, and the negative of crown, using these terms in the older sense.

By definition, h_1 is unity. In the case of the 40-inch lens, h_2 is 0.97. From the known constants of the 40-inch lens, the value of κ can be computed from equation (5). It is better, however, to obtain it from the known color-curve of the lens, from the formula

$$\kappa = \frac{k}{F^2}, \quad (6)$$

where k is the difference in back focus for the two wave-lengths for which correction is desired, and F is the focal length of the refractor. Thus, if correction is desired in the case of the 40-inch lens for λ 4047 and λ 4861, we obtain from the computed color-curve (Table III):

$$k = -86.3 \text{ mm},$$

giving

$$\kappa = -2.28 \times 10^{-7}. \quad (7)$$

If correction for F and G' is desired, we get

$$\kappa = -1.04 \times 10^{-7} . \quad (7a)$$

Application of formula (5) to the first case gives

$$\kappa = -2.24 \times 10^{-7} .$$

This value, however, is not so precise, on account of uncertainty in the value of h_2 .

The factor κ in equation (4) is therefore determined by the refractor. If it has a small value, which it will if the lens has a flat color-curve, as in the Cambridge lens (Table I), the components of the correcting lens will have weak powers (equation [4]) and therefore flat curves. The formula also shows that it is desirable to choose two kinds of glass for the corrector whose dispersion values ν are as different as possible, as this also leads to weaker powers for the corrector. The most important factor, however, is h , since this enters as the square. h is the ratio of the distance s that the lens is placed inside the focus, to the focal length, F , of the refractor. As the corrector is placed nearer the focus, which means a diminishing h , the powers of the component lenses must increase. For example, with h equal to 2500 mm, the value for the present lens, computation from equation (4) gives

$$F_3 = 883 \text{ mm} ;$$

with s equal to 1000 mm, as in the superseded Yerkes corrector,

$$F_3 = 140 \text{ mm} .$$

The small values of s therefore require strong lenses, with steep curves. There is partial compensation, on the other hand, in the smaller aperture necessary.

Chromatic magnification, or color off the axis.—This is given by K_f in equation (1). To a first approximation,

$$\left. \begin{aligned} y_1 &= y_2 = 0, \\ y_3 &= y_4 = -y = s - F . \end{aligned} \right\} \quad (8)$$

Substituting these and the known values of h in equation (1), we get

$$K_f = y h \phi_3 \left(\frac{1}{\nu_3} - \frac{1}{\nu_4} \right) \tan \beta, \quad (9)$$

which gives, finally, when combined with equation (4), or the condition for axial achromatization,

$$K_f = -y \frac{\kappa}{h} \tan \beta; \quad (10)$$

eliminating y by means of the relation

$$\frac{y}{h} = \left(\frac{F}{s} - 1 \right) F,$$

the final equation becomes

$$K_f = \kappa \left(1 - \frac{F}{s} \right) F \tan \beta. \quad (11)$$

From (11) it is seen that chromatic magnification, aside from the geometric factor $\tan \beta$, varies as (1) the amount of the original axial color, κ , to be corrected; (2) as the square, nearly, of the focal length of the objective; (3) inversely as the distance, s , of the corrector inside the focal plane. Excepting choice of glass, s is the only factor at the disposal of the designer. The importance of choosing it as large as practicable has already been shown (p. 194).

Correction for spherical aberration.—The equation for spherical aberration is (p. 192),

$$S_a = \frac{o^3}{4} \sum h^4 \phi^3 A, \quad (12)$$

where A is given by

$$A = \frac{n+2}{n(n-1)^2} \sigma^2 + \frac{4(n+1)}{n(n-1)} \sigma \pi + \frac{3n+2}{n} \pi^2 + \frac{n^2}{(n-1)^2}; \quad (12a)$$

o is the semi-aperture of the objective, n the index of refraction, and σ the shape factor of each component of the correcting lens, defined, in terms of the radii of curvature, by

$$\sigma = \frac{R' + R}{R' - R}; \quad (13)$$

π is the vergency factor, defined by

$$\pi = -1 - \frac{2}{\phi s}, \quad (14)$$

where ϕ is the reciprocal of the focal length of each component and s is the distance of the focus of an axial pencil from each lens of the system. Assuming as a first approximation that the refractor itself is free from spherical aberration, we have

$$S_a = \frac{o^3}{4} h_3^4 \phi_3^3 (A_3 - A_4), \quad (15)$$

where subscripts 3 and 4 refer to the first and second component, respectively, of the corrector, and where we have put $h_4 = h_3$, and $\phi_4 = -\phi_3$. This means that the corrector is composed of two thin lenses of equal power in contact, sufficing for a first approximation. Freedom from spherical aberration then requires that

$$A_3 - A_4 = 0, \quad (16)$$

Substituting known values of the parameters, this condition finally reduces to

$$\sigma_3^2 - 1.55 \sigma_4^2 - 3.14 \sigma_3 - 3.94 \sigma_4 - 0.40 = 0. \quad (17)$$

This is the necessary relation between the shape factors of the components of the correcting lens. The solution is not unique, so that another condition might be imposed—for example, the system could be cleared of coma. This, however, would be a useless refinement,

since it would only affect the field, which is already restricted by the unavoidable chromatic magnification error. It is expedient, therefore, to put $\sigma_3 = 0$, which means that the preceding or flint lens is equi-convex. Equation (17) gives, then,

$$\sigma_4 = -0.09 \quad \text{or} \quad -2.43 \quad (18)$$

for the solution of the quadratic. The smaller value will be adopted, as it leads to the best shape. These values of σ , with the focal length of the components determined by equation (4), to a first approximation, determine the lens. The final elements must be obtained by the usual ray-tracing method, in which the residual errors are eliminated by successive approximation. These errors are: adjustment of tilt of the color-curve; position of the focus; and spherical aberration. With regard to the latter, it is shown on page 185 that in the case of the Yerkes refractor there is an outstanding overcorrection of 5 mm. This is taken care of very easily by a corresponding undercorrection of the corrector, so that the system can be freed from spherical aberration, for the G' line, or the middle of the range of spectrum used. It will be noticed that the two interior radii of curvature of the corrector (p. 190) are the same. This is accidental, and without significance. It would allow a cementing of the components thus reducing reflection losses. A hard cement, such as the Canada balsam usually employed, would be objectionable, on account of strains set up. There are heavy oils, however, which might be used.

Effect of choice of glass on resultant color-curve.—The outstanding defect of a corrector lens is secondary axial color aberration, so that only a limited portion of the spectrum of a star, ideally considered as condensed to a point, can be focused on the slit. Is it possible so to choose the glass of the corrector lens that this defect can be eliminated or materially reduced? This question will now be considered. Solution of the problem requires that the back focal lengths for three colors, say F , G' , and 4047 , are all equal, or at any rate that if adjustment be made for F and 4047 , the value of the back-focus difference, $F-G'$, shall be a minimum. In the following discussion, subscripts 1 and 2 will refer to a back-focus correction for F and G' ; and

F and 4047, respectively. The corresponding values of the important κ factor are given numerically in equations (7) and (7a). It will be convenient to introduce two new variables, x and z , defined as the reciprocal of the dispersion factor (equation [2], p. 192) for flint and crown, respectively. I have calculated the numerical values of x and z for each major type of glass given in Schott's *Catalogue*. The results

TABLE V
GLASS DISPERSION CONSTANTS

FLINTS			CROWNS		
Type	100x ₁	100x ₂	Type	100z ₁	100z ₂
LLF1.....	1.207	2.222	FK2.....	0.742	1.345
BaF3.....	1.193	2.194	PK1.....	0.781	1.418
LF3.....	1.331	2.454	BK1.....	0.836	1.520
F3.....	1.537	2.840	BaLK1.....	0.882	1.607
BaSF1.....	1.446	2.670	K3.....	0.909	1.658
SF2.....	1.690	3.135	ZK1.....	0.926	1.688
K2F2.....	1.043	1.908	BaK1.....	0.936	1.708
			SK1.....	0.953	1.739
			KF1.....	1.074	1.971
			BaLF4.....	1.008	1.842
			SSK1.....	1.007	1.837

are contained in Table V. The common focal length, F (for the F line), of each component of the corrector lens is given by equation (4), or, in our changed terminology,

$$F=\frac{h^2}{\kappa} (x-z) . \tag{19}$$

The value of F will in general be different according to whether subscripts 1 or 2 are used for κ , x , and z . For example, the types of glass used for the present lens are F3 and K3. For these equation (19) gives,

$$F_1=1052 \text{ mm (cor. for F and G') ,}$$
$$F_2= 903 \text{ mm (cor. for F and 4047) .}$$

The ratio of F_1 to F_2 (1.165) is a measure of the lack of achromatization for the three colors considered. Putting

$$\frac{\kappa_2}{\kappa_1} = \rho \quad (= 2.192) ,$$

it is seen that the required condition for equality of F_1 and F_2 and therefore for achromatization is, from equation (19),

$$\frac{x_2 - z_2}{x_1 - z_1} = 2.192 .$$

The left-hand member of this equation, R , is easily calculated, for all possible combinations of glasses, from Table V. For the glasses actually used, $R = 1.882$. The nearer the value of R approaches 2.192, the better the achromatization. The closest approach is obtained from LLF₁ and SSK₁, which give $R = 1.925$. Using these glasses, the focal lengths are, from equation (19),

$$F_1 = 335 \text{ mm} ,$$

$$F_2 = 294 \text{ mm} ,$$

giving ratio, (F_2/F_1) 1.140, an improvement of 15 per cent over the glasses used. This gain, while sensible, is hardly significant, and has been secured at the expense of much greater powers for the components. It must be concluded that it is impossible materially to reduce the outstanding secondary color by selection of available glasses.

5. EFFECT OF FLEXURE

J. S. Plaskett has found that for the Ottawa telescope, the effect of flexure of the tube is quite appreciable in its decentering effect on the various colors at the slit of the spectrograph,¹ necessitating a change in the position of the correcting lens for every change in zenith distance. This effect will now be examined in some detail for our own case. From a theoretical viewpoint, it will simplify matters to consider the line of collimation to be fixed, and determined by the

¹ *Astrophysical Journal*, 32, 245, 1910.

line joining the center of the slit to the center of the correcting lens. These points are so close together as a rule that flexure between them can be ignored. Owing to flexure of the tube, the objective will change its position with reference to this line of collimation, as the telescope is pointed to different portions of the sky. For an easy analysis of the effect of this shift of the objective, the center of the slit will be taken as a source of light of various colors, and two rays, the F and 4047, traced backward through the system. Since the corrector lens is assumed to be collimated, any two colors traced axially will emerge from it, on their way to the objective, coincident in direction. Assuming a symmetrical flexure of the tube of the telescope, the line of collimation, which is the direction traversed by the two rays considered, will cut the axis of the objective midway the length of the tube. It is then necessary to know where these light-rays cut the objective, or, in other words, we must know the flexure of the tube. For this information, in the case of the 40-inch lens, I am indebted to Dr. G. W. Moffitt for a series of measures made in 1927. He determined the foregoing point by a collimation device, for each 10° of zenith distance, from the north to the south point. These deflections fit a sine-curve, of the form

$$\Delta(\text{mm}) = 18 \sin z. \quad (20)$$

Using this value for the distance between the center of the objective and the line of collimation, the two rays were carried through the objective itself, by ray-tracing. The result is that for each centimeter of deflection, the rays are split after passage through the objective, the violet ray being nearer the zenith by $0''.45$. Putting this in terms of zenith distance by means of equation (20),

$$L \text{ (flexure effect, violet-blue)} = 0''.81 \sin z.$$

It will be noted that this effect is in the same direction as that caused by atmospheric dispersion, which displaces the violet light (λ 4047) toward the zenith, relatively to the blue (λ 4681) by $0''.62 \tan z$. At $z = 40^\circ$ the two effects will be equal, and therefore neutralize each other, the dispersion elevating the violet image just the right amount

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to be neutralized by the flexure effect. The outstanding resultant for other zenith distances is contained in the accompanying table. It is seen that in no case is flexure of consequence.

z	Atm. Disp.	Flex.	Diff.
20°	0".22	0".28	0".06
30°	0.35	.40	.05
40°	0.52	.52	.00
50°	0.74	.62	.12
60°	1.07	0.70	0.37

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