

Hence from (20) and (21)

$$(G \cos \Omega + F \sin \Omega) \tan \omega = - (G \sin \Omega - F \cos \Omega) \cos i \quad (22)$$

This equation, together with (20) and (21), determines ω .

When ω has been found, the eccentricity e is obtained from (20) or (21).

The last element with which we are concerned is the semi-major axis a ; it is found by means of

$$p = a(1 - e^2),$$

in which p and e have already been evaluated.

Observatory, Cambridge :
1930 February 24.

The Use of a Floating Mirror with the Transit Circle.

By H. C. Woods, Ph.D., B.Sc.

The object of this paper is to demonstrate the value of a mirror mounted so as to float upon mercury, as a means of observing the zenith and horizontal points directly with the transit circle, the nadir being already observable by reflection at a mercury surface.

Captain Kater in 1825* made experiments with small collimating telescopes fixed on solid wooden and iron floats, 7 and 8 inches in length, floating in mercury. Observing the cross-wires of these telescopes with a fixed telescope he obtained results which showed that the float returned, after drastic disturbance, to the same inclination to the horizon with remarkable accuracy. Later† he invented a "Vertical floating collimator," which consisted of a small telescope with carefully made fiducial mark pointing downwards through the centre of an annular trough, rigidly mounted on a solid iron float, pivoted inside the trough by polished horizontal pins fitting in slots. He mounted the trough on rollers so that it could be accurately reversed, and obtained observations of the zenith with an altazimuth and a zenith telescope which he applied with success to determinations of latitude.

Since then the principle of flotation in mercury has been used by Chandler and others in the Almucantar, and by Bryan Cookson in the floating zenith telescope at present in use for observing variations of latitude at Greenwich.‡ It is estimated of the Durham Almucantar that after displacement it returns within 0".05 of its previous position. These instruments are of much larger size than that used in the present investigation.

In the recent instruments the method of eliminating the unknown inclination of the optical axis to the vertical has been to reverse the float within the trough. Captain Kater's vertical collimator was

* *Phil. Trans.*, 1825, p. 147.

† *Ibid.*, 1828, p. 257; see also *ibid.*, 1826, p. 307.

‡ *Monthly Notices R.A.S.*, 60, 572, 1900; 61, 315, 1901.

designed so that the trough and float were turned together, and in the present experiments the same method is adopted. The difficulty of designing suitable pivots for the float is greatly simplified when it is not necessary to make them capable of being thrown in and out of action, and no springs pressing on the float are required.

The vertical floating collimator in which a small telescope was used had the disadvantage of the great difficulty of obtaining optical conditions suitable for precise measurements. The use of a plane mirror of the full aperture of the object-glass of the observing telescope is, if the mirror is perfect, equivalent to the use of a floating collimator of the same size as the observing telescope, though the size of the whole instrument need not be excessive even for the largest transit instruments.

Description of Transit Circle.

The transit circle is the Draper's Company's Transit Circle at University College, London. The telescope is of $3\frac{1}{2}$ inches aperture and 44 inches focal length. The eye end is fitted with a right ascension micrometer which moves the whole set of cross-wires in the field. The mounting is on concrete piers resting on concrete foundations under the observatory. There is no apparatus for raising or reversing the instrument, and the collimation has to be obtained by reflection in mercury, the level being found with a striding-level, and applied to the reflection observations. Two lenses fixed in the north and south windows enable two meridian marks about 60 feet distant to be observed.

Accuracy in Time Determinations.

The time-signals sent out from different observatories throughout the world have shown that the determinations of time made by these observatories are subject to errors larger than were to be expected from errors in the timing of transits, and of a character which suggests that the cause might be inaccuracies of determination of instrumental constants. The two constants which can normally be determined without star observations are the level and collimation errors. It is usual to determine the collimation error with collimating telescopes, and the level error may either be determined independently by means of a striding-level, or by reflection in a bath of mercury at the nadir. The striding-level is subject to the limitations of sensitiveness and accuracy of the level bubble, and requires a correction for any possible inequality in the size of the pivots. There may also be an error of the nature of a level error, caused by flexure of the axis, of which the striding-level cannot take account.

Professor Sampson* has shown that daily observations at Edinburgh might be improved in many cases by rejecting the observations of level by reflection at the nadir and obtaining it instead from the transits themselves, the azimuth error being found from observations of the collimators, the fixity of which may be relied on from day to day.

* *Monthly Notices R.A.S.*, **82**, 193, 215, 1922 ; **85**, 560, 1925.

This method cannot completely eliminate the azimuth error, but is shown to smooth out irregularities in the determination of clock rate, and an extensive investigation by M. R. Madwar * leaves no doubt that erratic determinations of time are greatly reduced by rejection of the level error as ordinarily observed.

When the level error is determined by reflection at the nadir, a correction has to be applied for the collimation error. The error of such a determination of level includes the error of observation of the coincidence of wires and the error of determination of collimation. If it were possible to make an observation of the zenith with the same accuracy as the reflection observation at the nadir, then the uncertainty of a transit error at the zenith would amount only to the uncertainty of this observation. The two vertical reflection observations would determine the level and collimation together, the reading at the zenith being affected with the same collimation as at the nadir, while the displacement due to level would have altered sign. The level thus determined is the true level of the effective axis of the instrument, and the collimation is obtained in the vertical position of the telescope.

In order to compare the value of different methods of observation of level and collimation, suppose that N transits of stars have been observed at various declinations and the adopted level and collimation errors have been applied in each case. Then the clock error (Δt) and azimuth error (a) have to be found by means of the following equation, Δb and Δc being the errors of the adopted level and collimation respectively.

$$(1) \quad (\alpha - T) = \Delta t + a(\sin \phi - \cos \phi \tan \delta) + \Delta b(\cos \phi + \sin \phi \tan \delta) + \Delta c \sec \delta.$$

The equations for a least-squares solution are :

$$(2) \quad \Sigma(\alpha - T) = N(\Delta t + a \sin \phi) - a \cos \phi \cdot \Sigma \tan \delta + N\Delta b \cos \phi + \Delta b \sin \phi \Sigma \tan \delta + \Delta c \Sigma \sec \delta.$$

$$(3) \quad \Sigma(\alpha - T) \tan \delta = (\Delta t + a \sin \phi) \Sigma \tan \delta - a \cos \phi \Sigma \tan^2 \delta + \Delta b \cos \phi \Sigma \tan \delta + \Delta b \sin \phi \Sigma \tan^2 \delta + \Delta c \Sigma \tan \delta \sec \delta.$$

Eliminating a from these equations the component ($\Delta t'$) of the clock error which depends on Δb and Δc only is given by

$$(4) \quad \Delta t' = -\Delta b \sec \phi + \Delta c \cdot P,$$

where

$$P = \frac{\tan \phi (\Sigma \tan \delta \Sigma \sec \delta - N \Sigma \tan \delta \sec \delta) - (\Sigma \tan^2 \delta \Sigma \sec \delta - \Sigma \tan \delta \Sigma \tan \delta \sec \delta)}{N \Sigma \tan^2 \delta - (\Sigma \tan \delta)^2}$$

If two equal groups of stars, one near the equator and the other near the pole, are used, P becomes $-(1 + \tan \phi)$, or -2.28 in latitude 52° . A somewhat lower value will be obtained in practice, however, since when a polar star is used the residual error of collimation ($\Delta c \sec \delta$) is wholly ascribed to azimuth error (by the term $-a \cos \phi \tan \delta$) and the

* *Monthly Notices R.A.S.*, **86**, 137, 1926.

constant term in the azimuth error is of opposite sign to the correction which should be applied for the residual collimation error. If five transits are observed in declinations -20° , 0° , 20° , 40° , 60° , the value of P is -1.06 , and therefore in latitude 52°

$$\Delta t' = -1.62\Delta b - 1.06\Delta c.$$

The following three methods may now be compared:—

I. Level and collimation errors determined by reflection at zenith and nadir. Let e_z , e_n be the errors of measurement of the zenith and nadir points respectively.

Then

$$\Delta b = \frac{1}{2}(e_z - e_n), \quad \Delta c = \frac{1}{2}(e_z + e_n),$$

i.e.

$$\Delta t' = -1.34e_z + 0.28e_n.$$

II. Collimation determined by means of horizontal collimators and the level by reflection at the nadir. Let e_c = the error of measurement of collimation.

Then

$$\Delta b = e_c - e_n, \quad \Delta c = e_c,$$

i.e.

$$\Delta t' = 1.62e_n - 2.68e_c.$$

III. Level obtained from the zenith and nadir observations and collimation from the horizontal collimators.

Then

$$\Delta b = \frac{1}{2}(e_z - e_n), \quad \Delta c = e_c,$$

i.e.

$$\Delta t' = -0.81e_z + 0.81e_n - 1.06e_c.$$

The above figures indicate the relative importance of each of the three determinations, namely, zenith and nadir points, and collimation. So far as the zenith and nadir observations are concerned, method III., which gives them equal importance, is the best.

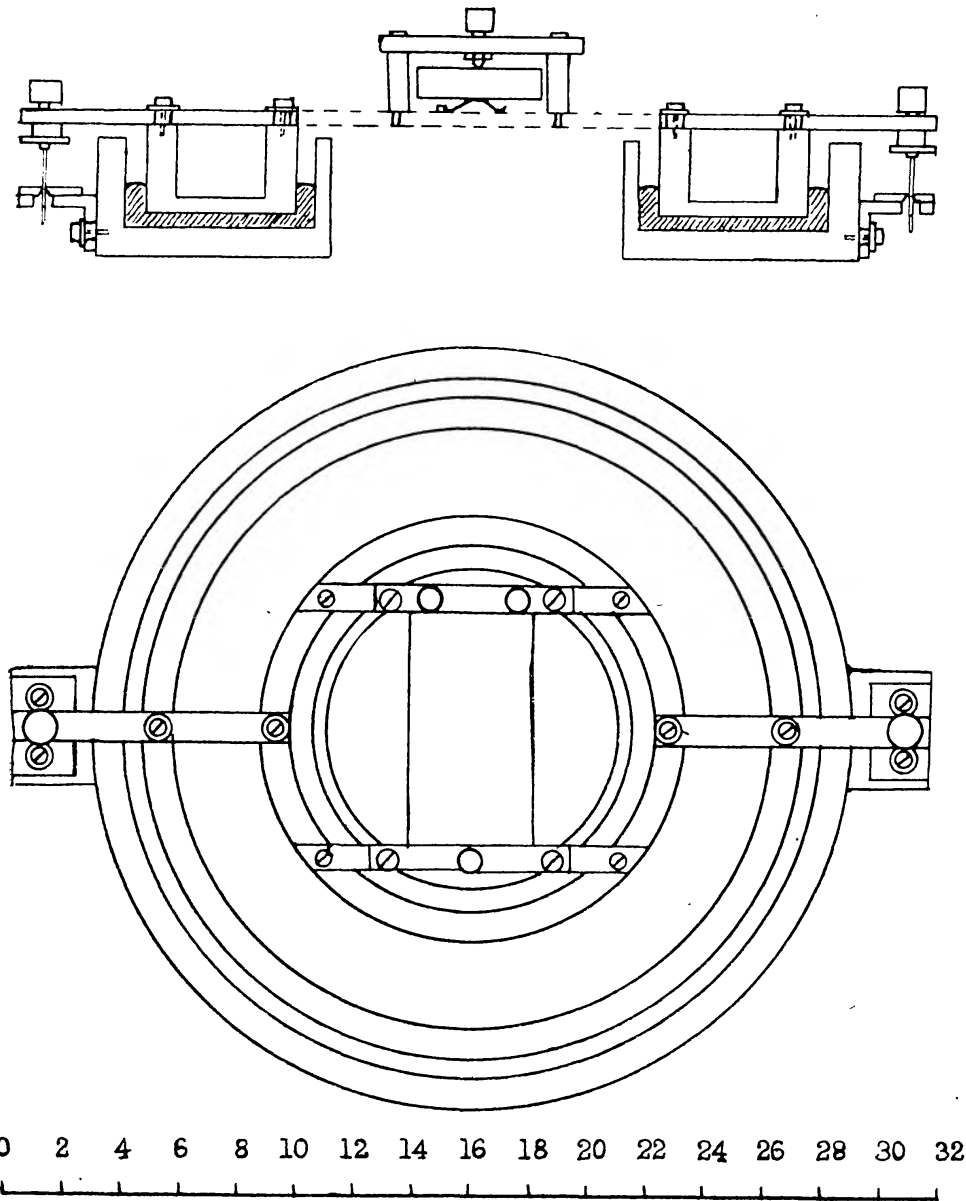
Description of Floating Mirror.

Preliminary experiments were made with a light wooden trough and ring, the arrangement being similar to that of the iron float and trough described below. This gave encouraging results and showed good accordance between mirror and mercury surface observations.

The mirror used is an optically plane parallel plate, made by Adam Hilger, Ltd., 1 cm. thick and having an available rectangular aperture $4\frac{1}{4}$ cms. by 8 cms. The wire images obtained with the mirror in collimation observations are good, having regard to the restriction of aperture, which affects the image seriously only for the wires at right angles to those being observed, the longer axis of the mirror being placed at right angles to the pivot axis of the float.

The wires are illuminated by means of a Bohnenberger eyepiece, and the setting is made with the R.A. micrometer screw, the value of one revolution of which is 4.477 secs. ($67''\cdot2$). The vertical wires

consist of six transit wires and a close pair about 20" apart in the centre. The settings in the observations referred to below were made so that the centre pair of wires were superimposed on the reflected pair.



Scale of cms.
Elevation and Plan of Floating Mirror.

The iron float and trough are made of cast iron turned to the dimensions shown in the diagram. The sides of the float and the outer side of the trough are 1 cm. thick, and the bottom of the trough the same. The bottom of the float and the inner side of the trough are $\frac{1}{2}$ cm. thick. The float is turned and polished on the sides and bottom. There is a free surface of mercury both inside and outside

the float of $\frac{3}{4}$ cm., and a clearance of $\frac{1}{2}$ cm. at the bottom. It required 10 lbs. of mercury to fill the trough to these dimensions.

On the outside of the trough at opposite ends of a diameter are two brackets of brass, each of which carries a brass plate drilled with a conical hole which lies over a larger hole in the bracket. These plates are held in position by bolts passing through holes in the plates drilled wide to allow of adjustment both for distance from the trough and laterally. The brackets themselves are also slotted to allow of vertical adjustment so that the narrow ends of the conical holes are in the level of the mercury surface.

Two brass arms are screwed to the top edges of the float to carry steel pins, 2 mm. thick, over the holes in the plates. These pins are held by passing through bolts screwed into the horizontal arms and secured with lock nuts. The conical holes are just large enough to allow the pins to shake slightly when in position. The arms carrying these pins are also slotted to adjust them so that the float is symmetrically placed in the trough.

Two pieces of flat brass rod are secured over the top of the float parallel to the axis of the pivots and with 8 cms. clearance between them. In the middle of each a similar short piece of brass rod is held on upright posts 2 cms. from the lower one. Through these pass the pointed screws against which the mirror is held by springs fixed to the lower rods. There is one screw on one of the frames and two on the other to afford an adjustable three-point suspension of the mirror. The adjusting screws are provided with lock nuts to ensure their complete rigidity. The pressure of the springs was made as light as possible in order to avoid distortion of the mirror, and the arrangement was found to give good definition of the reflected images with quite sufficient stability of adjustment.

Adjustments and Accuracy of Floating Mirror.

In estimating the accuracy of observations with the float, attention must be paid to the following points :—

(a) The absence of bubbles under the float or irregular surface tension of the mercury where it makes contact with the float, the complete freedom of the float to rotate about a horizontal axis, and the absence of all forces, apart from the effects of its own buoyancy and weight, tending to produce such a rotation.

(b) The rigidity of the mirror mounting.

(c) The steadiness of the float while observations are being made.

(d) The position of the axis of the float with respect to the plane in which the tilt of the mirror is to be measured, taking into account the tilt of the mirror in a vertical plane parallel to this axis.

(e) Optical conditions and accuracy of setting.

With regard to (a), the method of using the float does not require it to be removed from the mercury, and if the mercury is kept clean by wiping the surface from time to time, no difficulty should be experienced from particles of dirt on the sides of the float.

The resultant surface tension of the mercury as it affects the float has two components, one horizontal in the plane of the circle of contact, and the other vertical and displaced from the axis of the float to a distance depending on the irregularities of the tension at different points of the float. The existence of a horizontal component may be demonstrated by allowing the float to drift unconstrained in the trough, when it tends to be gradually drawn to the side. When the float is centred this component is very feeble and may be made harmless by placing the pivoting points as near as possible to the effective surface of the mercury. If the float beneath the surface makes clean contact with the mercury, the only disturbance of flotation is that due to the adhesion of the mercury round the float, causing a difference in the angle of contact from place to place. This adhesion will not be changed by small oscillations of the float, and in rotating the whole instrument the mercury flows under the surface film without causing any considerable change in the arrangement of the irregularities. It may be expected therefore that if the float after drastic disturbances of its position in the mercury returns with high accuracy to its original position, still greater precision should be attainable by careful handling in making the necessary reversals, preferably by placing the whole instrument on a ball-bearing.

The surface of the mirror being accurately plane and resting against three firmly fixed points, there should be no danger of movement altering the inclination to the horizontal. Temperature effects in the mounting may cause a slow movement. In this case the use of brass on an iron float would increase the effect, but it was found that holding the brass frame in the hand for a short time did not produce a noticeable displacement, so that in the interval between reversals no serious shift is likely, and a uniform rate of change produces no effect in a symmetrical set of observations.

The oscillations of the float are of two kinds, those due to the surging of the mercury round the trough, and superimposed on them the metacentric oscillations of the float itself. The iron float has a metacentric period of $\frac{1}{4}$ second approximately, and the longer oscillation is of 2-second period. After a disturbance of the float, if it is not subjected to further vibration, the metacentric oscillations die away in a few seconds, while the surging of the mercury may continue to produce evident displacements for some time. If the float has been only slightly disturbed, as when the trough has been rotated, the reflected images become sufficiently steady to make an observation in about half a minute. However, this observatory is situated within a few yards of a street (Gower Street) in which the traffic is constant and heavy, and it is very rarely that the oscillations are completely absent. The iron float does not vibrate with such liveliness as the wooden one, and it is not difficult to take advantage of quiet moments. It is, of course, absolutely essential to protect the instrument from draughts, as these might cause a steady tilt in the same direction and thus affect the determination of the zenith point.

If the accuracy of flotation were perfect, no matter how the axis

of the pivots might be oriented, the mean taken in R.A. or Declination would give the true vertical, provided the rotation of the trough between the two measures were exactly through 180° . But if the pivot axis, which for the first measurement was very nearly at right angles to the plane in which the tilt is measured, were on rotating inclined at a small angle α to its original direction, then any tilt which the mirror might have in the direction of the axis would have a component in the direction of measurement, while the tilt about the line of the pivots would still give approximately the same component. Hence an error $x \sin \alpha$ is introduced into the second measurement of tilt, where x is the tilt from the horizontal in the line of the pivots. If it is required to make observations with a maximum error of $0''.1$, the tilt in the line of the pivots must be less than $6''$ if the possible inaccuracy of rotation amounts to 1° . In practice this adjustment is easily made with a much smaller margin. The axis of the pivots in both positions should be as nearly as possible perpendicular to the direction of measurement, since though no error can be introduced if the flotation is perfect, the component of the tilt which is affected by the sliding of the pins is subject to error. This error was found to be very small with the iron float, and the adjustment therefore does not require much precision.

The shape and size of the mirror which has been used in this investigation introduces limitations to the optical perfection of the images obtainable by reflection. The longer axis of the mirror is always set so as to define best the wires being used, and since the clear aperture in this direction is 8 cms. (3.15 inches) no serious loss of definition results. The mirror must, however, be set centrally with regard to the telescope axis. The illumination of the wires is by means of a Bohnenberger eyepiece, a compound microscope of low power, before the object-lens of which is a diagonal plain glass reflector, reflecting part of the light of an electric lamp on to the cross-wires. This lamp must be placed so that the illumination completely fills the aperture in use; if this is not so, shifts of the wire images can be seen on moving it to and fro. With this eyepiece there are present in the field, both with the mirror and the mercury surface, a group of easily distinguishable interference fringes, six or seven in number, with a separation of about $20''$. They are in focus when the eyepiece is adjusted to show the cross-wires, and become indistinct on either side of this plane. The bands extend in a direction nearly perpendicular to that from which the illumination comes. Their centre is two or three bands away from that point in the field which is coincident with its reflection, and therefore in making a setting with the illumination from the side the central transit wire and its image are brought to coincide in the part of the field which is affected with the bands. A slight change in distinctness of the reflected images is observable as they pass over the bands, which must be considered a possible source of error. The location of the bands is, however, constant with regard to the wire which is being observed in coincidence with its own reflection. Interference fringes are theoretically present with a parallel plate reflector in the eyepiece (see *Journ. de Physique*, 1892, p. 414, M. A. Hurion, "Sur les franges visible dans un oculaire

nadiral"), but should vary in separation at different distances of the horizontal mirror from the object-glass of the telescope, the separation being infinite when the distance is equal to the focal length of the object-glass. The fringes observed with this instrument, however, do not fulfil this condition, but are nearly the same width in all positions of the mirror. In the case of a parallel glass reflector, each ray from the source of illumination is divided into two by the partial reflections from the two faces of the reflector. When this double ray is returned to the reflector each component is again divided, and of the four rays resulting two have described nearly the same length of path and are therefore in a condition to interfere. The fact that in general the angle of incidence on the reflector is not the same as the angle at which the rays left it causes the ray which undergoes double reflection the second time to interfere with that which was doubly reflected the first time, the path difference depending on the change of the angle of incidence. For the reflector used (2 mm. thick) these fringes should be too wide apart, even with the mirror in a favourable position, to be easily observable under this power, but a similar effect can be produced by an inclination to one another of the two faces of the reflector, so that the rays at the commencement of their path and at the end have different thicknesses of mirror to traverse. The linear displacement on the reflector of the points of departure and incidence is but little affected by the position of the mirror, and the cause of this optical defect in the present case must therefore be the prismatic form of the reflector, the direction of the fringes in the field being by chance nearly that which they would have in the other case.

With this eyepiece a small difference in focus is very appreciable, and the fact that the vertical and horizontal wires lie in slightly different planes makes it impossible to obtain sharp focus of the wires direct and reflected for both sets. The focus of the telescope is permanent, and a compromise must be made when making settings by reflection.

These optical defects of the instrument, besides producing apparent displacements of the wires, particularly the reflected images, may also cause a difference of aspect of the reflected and direct images. This difference may vary according to the mirror or mercury surface used as reflector, and it therefore requires to be shown that the floating mirror is capable of giving results identical with those which the mercury surface affords. Normally there is considerable vibration at this observatory, and this is especially difficult to remove when the mirror is mounted on the overhead beams to observe the zenith. It can be reduced to a fairly small amplitude, and settings are made so that the reflected wire is seen as much to one side as the other of the wire on which it is being set. This differs from the method in the case of the mercury surface, in which case the image is made faint by the vibrations but remains steady, the reflected wire at the moment of coincidence being just completely hidden.

The floating mirror with the iron float was compared at the nadir with the mercury surface. The trough was placed on a board under the telescope, and marks were made on the trough and on the board

by means of which the necessary reversals could be made with an error not exceeding 1° . Four observations were made in each position of the mirror, the float being depressed between each consecutive pair and set into oscillation. The measurements were made with the R.A. micrometer. Observations of the mercury surface consisted of at least five settings. The results were as follows:—

Mirror: $59^r.162$	Mercury: before $59^r.167$
	after $59^r.161$
„ $59^r.182$	„ after $59^r.182$
„ $59^r.180$	„ before $59^r.180$
	after $59^r.179$
„ $59^r.182$	„ before $59^r.187$
	after $59^r.181$.

These observations are affected by changes taking place in the constants of the transit circle, and possibly by changes in the observer's personality in the setting, but there is no reason to think that any systematic difference exists between the two methods.

Method of Mounting Float at the Zenith.

Two oak planks, 1 inch thick and 11 inches wide, were mounted on their edges across the observatory on either side of the telescope. The ends rest on the walls of the observatory and the top edge is planed smooth. Across these beams is placed a wooden frame consisting of two boards fastened together just inside the beams by laths. The distance between the boards is sufficient to leave a clear aperture for the mirror. The trough is placed on a smaller smooth board, with a circular hole in it, mounted on levelling screws and placed across the opening in the wooden frame. The reversal of the trough was made by sliding it round on this board and setting marks on the board against marks on the trough. The distance between the beams is a little over 2 feet, almost exactly the same as the distance between the piers of the transit circle, thus leaving plenty of room for the observer to make the adjustments of the trough from a ladder. The frame is pushed to the end of the beams when not in use.

The vibrations to which the float is subjected in this position are severe. They are principally due to lateral impulses on account of the shape of the beams. If the beams are fixed by struts those impulses which cause slow and wide swinging of the float can be very largely removed, but the rapid oscillations which at times even blur the definition are increased. However, the amplitude of the vibrations was greatly reduced by placing on the beams under the four corners of the wooden frame small pads of soft rubber which easily yield to the lateral strains. When the impulses are momentarily absent the swing of the float takes place evenly from side to side and a setting can be made at the central point of the swing of the reflected wires.

The following observations were made with the floating mirror at the zenith. Seven observations were made consecutively in one position

(A) of the trough, each consisting of two micrometer settings, the float being disturbed after each observation, followed by a similar set in the other position (B), and repeated in the same order :

I.	A	58 ^r .683	.685	III.	A	58 ^r .667	.661
		.680	.689			.670	.670
		.688	.682			.671	.669
	Mean	.687	.680		Mean	.675	.672
	58 ^r .681	.676	.676		58 ^r .670	.671	.670
		.679	.679			.670	.670
		.680	.677			.669	.670
II.	B	59 ^r .147	.147	IV.	B	59 ^r .131	.140
		.149	.144			.130	.132
		.147	.144			.142	.137
	Mean	.148	.144		Mean	.137	.132
	59 ^r .145	.150	.150		59 ^r .135	.138	.138
		.140	.140			.134	.137
		.144	.138			.130	.132

The change in reading for both A and B positions is in the same direction and might therefore be the effect of a change in the instrumental constants such as affects also the mercury surface readings at the nadir, or it might be due to a slight steady draught in the observatory, though the roof was kept shut throughout. The differences in the two readings made between each pair of disturbances indicate how much of the variations should be ascribed to errors of setting, though there is distinct evidence of bias in the frequency with which the same reading is repeated. The average difference between these pairs of settings is $\pm 0^r.003$, and the average deviation of each observation from the mean of the group of seven is $\pm 0^r.0028$. This suggests that errors of flotation cannot be of very great importance in determining the zenith point, but it is clearly necessary to eliminate errors arising from vibrations if high accuracy is to be attained. The settings made at the zenith involve more risk of bias on the part of the observer on account of the uncertain nature of the observations, but where vibration is not present and a full aperture mirror is used, the results should be quite as reliable as those with the mercury surface, with which two sets of ten readings showed an average deviation from their means of $\pm 0^r.0018$.

Observations of Level, etc., with Floating Mirror.

The following observations were made with the iron float at the zenith. The level and collimation shown by the striding-level with the nadir observation and by the zenith and nadir method are compared, and the last column of the table shows the correction to the striding-level results on the supposition that the reflection observations of level are correct. Each observation of the zenith given below consisted of two settings of the float in position B interposed between two in position A. The float was disturbed between the B observations. In each of the four cases three readings of coincidence were made. The large value of the

level error on the first two days introduced an uncertainty of the scale value of the striding-level, this being nearly the extreme value observable. Between the two observations on June 3 the level of the transit circle was adjusted. Both methods indicated a change of collimation after this adjustment. Throughout this series of observations the tilt of the mirror to the horizontal in the line of the pivots was very small (not exceeding 2").

The collimation error seems to have altered between June 10 and 11, which is probably due to exposure of the telescope to strong sunlight before the observations on June 11. Apart from this the reflection observations give consistent results for the collimation, which is shown to have remained nearly constant from June 3 to 10 and 11 to 24. The striding-level shows much wider fluctuations. Since the level error has varied considerably over this period the changes of the nadir observation afford confirmation of the zenith results.

Date.	Zenith.	Nadir.	Collimation.		Level.		Correc- tion to S. Level.
			Ref.	S. Level.	Ref.	S. Level.	
1925.	r	r	r	r	s	s	s
May 29	58.819	59.188	59.003	59.028	-0.824	-0.714	-0.110
June 2	.789	.235	.008	.037	-1.010	-0.880	-0.130
	.778	.232					
" 3	.789	.241	.015	.060	-1.009	-0.810	-0.199
" 3	.999	.012	.006	.022	-0.031	+0.047	-0.078
" 4	59.019	58.985	.002	.013	+0.045	+0.127	-0.082
	.017						
" 5	58.972	59.026	.001	.023	-0.116	-0.013	-0.103
	.979						
" 8	.986	.022	.004	.019	-0.074	-0.013	-0.061
	.984						
" 9	.980	.039	.007	.017	-0.143	-0.100	-0.043
	.971						
" 10	.959	.057	.007	.015	-0.224	-0.187	-0.037
	.954						
" 11	.972	.071	.021	..	-0.223
" 12	.952	.089	.022	.039	-0.300	-0.226	-0.070
	.957						
" 22	.900	.140	.020	.039	-0.538	-0.453	-0.085
" 23	.917	.119	.017	.019	-0.457	-0.446	-0.011
	.913						
" 24	.930	.112	.021	.026	-0.398	-0.385	-0.013

Observations of a millimetre scale placed in the focus of the lens in the north window were made to test the micrometer, which had previously undergone repair for a large periodic error due to faulty bearings. It was found that a small error of the same kind was present,

but the maximum correction required was $\pm 0^{\text{s}}\cdot 005$. A correction has been applied for this error to the zenith observations and the accompanying nadir observations.

The striding-level is obviously not capable of much precision, since its readings vary greatly on the same day when the nadir remains constant. It was tested with a level tester, readings being made at intervals of from 5" to 2", as shown by the tester. The readings obtained as the tilt was varied in one direction differed systematically from those in which the tilt was altered in the opposite direction, the difference varying according to the interval between successive settings. Experiments were made placing the level on the transit circle in such a way that the bubble approached from one side or the other, as desired. It was found that in the ordinary use of the level it might be expected to approach from the East. Four sets of four readings each way were made with the following result:—

Bubble from E	.	Level Error =	$- 0^{\text{s}}\cdot 434$
" " W	.	" " =	$- 0^{\text{s}}\cdot 505$
" " E	.	" " =	$- 0^{\text{s}}\cdot 393$
" " W	.	" " =	$- 0^{\text{s}}\cdot 534$

The difference E-W is $+ 0^{\text{s}}\cdot 106$, and therefore the correction to the usual striding-level observations is $- 0^{\text{s}}\cdot 053$, which is in quite good agreement with that indicated by the reflection observations.

The above reflection observations indicate a mean square error of a collimation determination by the zenith and nadir method of not more than $\pm 0^{\text{s}}\cdot 002$. Taking the same value for the mean square error of the nadir observation, then the mean square error of the zenith observation (σ_z) is given by

$$\frac{1}{4}(\sigma_z)^2 + \frac{1}{4}(0^{\text{s}}\cdot 002)^2 = (0^{\text{s}}\cdot 002)^2,$$

i.e.

$$\sigma_z = \pm 0^{\text{s}}\cdot 0035.$$

If the mean square errors of zenith and nadir observations are taken to be:

$$\begin{aligned}\sigma_z &= \pm 0^{\text{s}}\cdot 0035, \\ \sigma_n &= \pm 0^{\text{s}}\cdot 0020,\end{aligned}$$

and the mean square error (σ_c) of collimation determinations by means of horizontal collimators be taken as $\pm 0^{\text{s}}\cdot 002$, then the effect on determinations of clock error of these errors of measurement in the three cases previously mentioned is as follows:—

I. $(\sigma_{\Delta t})^2 = (1\cdot 34\sigma_z)^2 + (0\cdot 28\sigma_n)^2,$

i.e.

$$\sigma_{\Delta t} = \pm 0^{\text{s}}\cdot 0047 \quad \text{or} \quad \pm 0^{\text{s}}\cdot 021.$$

II. $(\sigma_{\Delta t})^2 = (1\cdot 62\sigma_n)^2 + (2\cdot 68\sigma_c)^2,$

i.e.

$$\sigma_{\Delta t} = \pm 0^{\text{s}}\cdot 0062 \quad \text{or} \quad \pm 0^{\text{s}}\cdot 028.$$

$$\text{III. } (\sigma_{\Delta t})^2 = (0.81\sigma_z)^2 + (0.81\sigma_n)^2 + (1.06\sigma_c)^2,$$

i.e.

$$\sigma_{\Delta t} = \pm 0^r.0039 \quad \text{or} \quad \pm 0^s.017.$$

In practice, with better observing conditions, it may be expected that σ_z will be nearly equal to σ_n , which would give method I. an advantage over method II., but method III., in which the zenith and nadir observations are used together to determine the level only, seems the best all round where any uncertainty exists in the errors of the zenith observations, and provided σ_c is sufficiently small. Now the optical conditions in reflection observations have been shown to suffer from imperfections and therefore an error common to zenith and nadir observations is not improbable. In method III. no effect is produced by a systematic error in reflection observations, and errors in the collimation do not affect the level error as is the case when reflection is used only at the nadir. The choice between the methods I. and III. will depend upon the ratio of the errors of the two reflection observations, and on the relative magnitudes of the systematic errors to be expected in each of the two methods of determining collimation, as well as on the random errors of observations with the horizontal collimators. Method I. perhaps places too much reliance on the zenith reading, whereas method III. gives approximately equal importance to three independent observations, the errors common to zenith and nadir readings having no effect. It is probable that the above figure for the error of zenith observations should in fact be smaller, even for this observatory, and under better conditions all the errors σ_z , σ_n , σ_c might have a value of, say, $\pm 0^r.002$. In this case the effects on the clock error are :

$$\begin{array}{ll} \text{Method I. } \sigma_{\Delta t} = \pm 0^r.0027 = \pm 0^s.012, \\ \text{,, II. } \text{,,} = \pm 0^r.0062 = \pm 0^s.028, \\ \text{,, III. } \text{,,} = \pm 0^r.0031 = \pm 0^s.014. \end{array}$$

The mean square error due to errors of observation of instrumental constants is thus reduced by methods I. and III. to a value of the same order as that of the mean error in the timing of transits, and the only outstanding source of error is the systematic error unavoidably present in any determination of collimation, which is given much less prominence than in method II.

Other Uses of the Floating Mirror.

There are other uses to which the floating mirror may be put in connection with the transit circle. The horizontally placed mirror may also be used to obtain circle readings on the zenith, and if vertically mounted mirrors were placed north and south of the instrument, direct readings on the horizontal points might be made, giving in connection with zenith and nadir readings a means of determining possible differences of flexure in the north and south positions. If the mirror in this case were silvered on both sides the inclination of the two faces would have to be observed independently and would be a constant of the

instrument. The trough would be mounted so that it could be set accurately in azimuth and smoothly rotated.

If the mirror were inclined to the vertical the same method could be used to determine the horizontal point from observations equidistant above and below it, thus giving a value of the flexure at various zenith distances and affording a test of the law governing the R-D discordance. The mounting would be more complex since the instrument would have to be raised and lowered as well as reversed, but it should be possible to effect this without serious disturbance, as in the case of the mercury bath used for reflection observations of stars. Unless its use is restricted to a small range the trough and float must be made so that the mirror could be seen on one side from below at all inclinations. Two rectangular troughs joined by a channel at one end should give the required accuracy. In these positions of the mirror the pointing of the float and mirror must be as exact as possible, but the shake of the pivots will impose a limit on this. However, the error in azimuth setting is indicated by the reflection of the vertical transit wires, and the small corrections which might be required can easily be applied.

Conclusion.

It is hoped that this paper has indicated a means of improving substantially the accuracy of time determinations with the transit circle by including a reflection observation of the zenith, thereby making the determination of level more symmetrical and independent of the collimation. The floating mirror used at University College has shown encouraging results as regards accuracy of flotation, and a permanent instrument adequately protected from vibration should give results equivalent to those of the mercury surface. It is shown that this method diminishes the importance of errors of coincidence settings in the final result for clock error, especially in the case of the collimation error. The author wishes to acknowledge his indebtedness to Professor Filon and Mr. Gregory for their advice and assistance throughout. Some criticisms by Dr. W. M. Smart have also been of great assistance.

Discussion of Observations of Three Long-period Variables, made by A. N. Brown, M.A. No. 2, V Cas., 1909-1929 January. By Mary A. Blagg.

(These observations, down to 1926 May, have been published in detail in *M.N.*, in four papers: 1910 June, 1916 May, 1921 May, and 1926 June.)

The discussion of these careful observations of V Cas., the second of the three stars undertaken, has again proved of great interest, an interest increased by its close resemblance in many respects to the first of the set, RT Cygni. As with the latter star, no actual maximum or minimum date can be fixed precisely by observation in any single period, for the