THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME LXXII JULY 1930 NUMBER 1

ON THE AXIAL ROTATION OF STARS

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ABSTRACT

Broad and *shallow* absorption *lines* in stellar spectra are shown to be *due to axial rotation.* The evidence is based upon the following facts: (*a*) the *broadening depends upon wave-length,* as required by the Doppler principle; (*b*) in *spectroscopic binaries* there is a distinct *correlation* between *line width* on one side and *period and amplitude* on the other; and (*c*) as was shown by Elvey, the contours of "*dish-shaped*" *lines* agree well with the theoretical shapes of lines of rapidly rotating stars.

A survey of the spectra of stars of various types shows that *rapid rotation is peculiar to the earliest spectral classes.* The B- and A-stars display the greatest tendency toward rapid rotation. In the F's the proportion of rapidly rotating stars is smaller, and in classes G, K, and M no cases of rapid rotation have been observed in single stars.

The effect of *rotation* upon the contours of *components of spectroscopic binaries* is discussed. In α Virginis the stronger component is found to rotate with an equatorial velocity of 200 km/sec., while for the fainter component the rotational velocity is not more than about 50 km/sec. This indicates a marked *difference in the size of the components.* The ratio of the radii is roughly 4 to 1.

The contours of lines in η Ursae Majoris suggest an equatorial velocity of about 200 km/sec. This *single star* is thus *similar to* the stronger component of α *Virginis.* Both stars are found to be *stable,* but the characteristic quantity $\omega^2/2\pi\gamma\rho$ is not far from the critical value where the Maclaurin ellipsoids of rotation lose stability. It is suggested that there is a *transition between close spectroscopic binaries and rapidly rotating single stars* in the earliest spectral classes. Observations do not establish the direction in which this transition proceeds.

I. INTRODUCTION

Many theoretical discussions of the origin of spectroscopic binaries and of other cosmogonic problems have been based upon the dynamical consequences of rapid axial rotation of single stars. The fission theory originated by Sir George Darwin and developed by Sir James Jeans[1] is built upon the assumption that stars lose energy

[1] *Astronomy and Cosmogony,* Cambridge, 1928.

by radiation and are therefore forced to contract. In doing so they must preserve their angular momentum and this can be accomplished only if the angular velocity is increased. A simple computation shows that an average star must rotate at a very high speed before instability can become dangerous. Not until the equatorial velocity of a typical B-type star has reached a value of the order of several hundred kilometers per second is there any danger of breakup. Indeed, F. R. Moulton[1] has shown that the sun must contract until its equatorial radius is 11 miles and its mean density 3×10^{13} on the water standard, before the Maclaurin ellipsoids of rotation become unstable. Moulton concludes that "the oblateness of the sun can never approach that for which the Jacobian figures of equilibrium branch." The difficulty obviously lies in the fact that for the sun with its slow rate of rotation the angular momentum is small, and that an enormous contraction would be required to increase the angular velocity to the critical value.

The problem would assume an entirely different aspect if it could be shown observationally that there are in existence stars, with normal average densities, for which the rotational velocities are large, so large indeed that the characteristic value of $\omega^2/2\pi\gamma\rho$ is not far removed from the critical point where instability sets in.

Until recently such observational evidence was completely lacking. It seems to have been the opinion of many of the leading observational astronomers that stars rotate with equatorial velocities of a few kilometers per second at the most. The relative velocity at the equatorial limbs of the sun amounts to only 3.9 km/sec., and by analogy it was inferred that similar conditions must hold for the stars in general.[2]

[1] *Carnegie Institution of Washington, Publication* No. 107, 150, 1909. Also *Astrophysical Journal*, **29**, 1, 1909.

[2] J. Evershed says in *Monthly Notices of the Royal Astronomical Society*, **82**, 395, 1922: ". . . . The angular speed [of Sirius] will be over three times that of the Sun, a complete rotation taking eight days or less. This high speed of rotation seems improbable considering that Sirius is in an earlier stage of evolution, and is therefore less condensed than the Sun. . . . ." Similarly J. A. Carroll expresses the opinion (*ibid.*, **88**, 555, 1928): ". . . . We are thus able to say that no stars have been found rotating with an equatorial velocity much greater than say 50 km/sec. and that . . . . the rotational speed must have been more nearly of the order of 10 km/sec. or less." The same idea is stated by H. C. Vogel (*Astronomische Nachrichten*, **90**, 75, 1877): "Ein Aequator-

However, in the light of observations made within the past few years[1] our early conceptions of this subject must be radically changed. It appears that rapidly rotating stars are by no means rare exceptions. They are quite common in the earlier spectral classes, and their equatorial velocities occasionally exceed 200 or even 250 km/sec. Such velocities are characteristic for many single stars, as well as for spectroscopic binaries of short period and large amplitude. This naturally leads to the inference that there is a continuous transition between single stars having rapid axial rotations and close spectroscopic binaries of short period.

The observations do not indicate in which direction this transition proceeds. It remains uncertain whether a rapidly rotating single star breaks up into a binary (as would follow from the fission theory), or whether the components of a binary, by falling together, give rise to a rapidly rotating single star. Nevertheless, it is of considerable cosmogonic interest that the stars of earliest spectral classes—A, B, and probably O—reveal the greatest tendency toward rapid rotation. It is in these types, then, that we should expect the binaries to originate, if we were to accept the fission theory. The rotational velocities of many single stars are of the same order of magnitude as those shown by components of the closest-known spectroscopic binaries, and are not much inferior to those predicted theoretically for the "break-up" of a single star.

This paper is not directly concerned with the fission theory. It is well known that Professors F. R. Moulton[2] and W. D. MacMillan[3] have raised a number of important objections to it, and it is not here possible to pursue the problem of the origin of double stars farther. Neither do we touch upon the question as to how the rapid rotations in single stars have originated. It is not easy to reconcile

punkt von α Aquilae . . . . würde die immer noch ansehnliche Geschwindigkeit von 25 geogr. Meilen haben. Es erscheinen diese Geschwindigkeiten, zumal im Vergleich mit der Rotationsgeschwindigkeit unserer Sonne (Aequatorpunkt 0.27 geogr. Meilen) als im hohen Grade unwahrscheinlich." See also the article by K. Walter in *Die Sterne*, **10**, 9, 1930.

[1] Shajn and Struve, *Monthly Notices of the Royal Astronomical Society*, **89**, 222, 1929. This paper contains references to earlier investigations on the subject of stellar rotation; C. T. Elvey, *Astrophysical Journal*, **71**, 221, 1930; *ibid.*, **70**, 152, 1929.

[2] *Op. cit.*, pp. 156, 160.

[3] *Science*, **62**, 69, 1925.

the facts with the contraction hypothesis, and we must at present be content with the purely observational result that rapid rotations can and do occur.

II. THE INTERPRETATION OF LINE CONTOURS

The evidence concerning rotation depends primarily upon the interpretation of the contours of stellar absorption lines. It is now known that there are three major factors[1] which influence the shape of these contours, viz., abundance of atoms, molecular Stark effect, and axial rotation.

a) The contour of an absorption line in a scattering atmosphere without collisions is known from theoretical considerations. Observations have shown that this "Unsöld contour" is closely obeyed by the majority of spectral lines in late-type stars. Small corrections due to the influence of collisions have been computed by Unsöld and tend to make the agreement between observation and theory even better. The abundance of atoms in the right state to absorb a given line depends mainly upon the temperature and pressure of the reversing layer. The actual percentage of occurrence of any one element in a stellar atmosphere is remarkably constant along the spectral series. For many lines the effect of absolute magnitude is very small and can be neglected, in which case the total amount of energy absorbed in any one line is a function of spectral type alone.

b) The hydrogen lines show anomalous contours in all spectral classes. The same is true of the helium lines in many B-type stars. These anomalies are due to Stark effect, produced by the electrical fields of neighboring ions. The resulting broadening of the lines is not the same in all stars; it varies with the pressure and is consequently related to absolute magnitude. However, as in the case of

[1] There are many physical factors which influence the contour of a line in the laboratory. Of these only the molecular Stark effect is believed to be effective in stellar atmospheres. Doppler effect due to temperature agitation is probably not appreciable (Vasnecov, *Vestnik Kral. Ces. Spol. Nauk*, **2**, 1927). Most of the other effects observed in the laboratory are due to direct interaction of the atoms. Such effects are improbable in the stars on account of the low pressures in the reversing layers. The Stark effect alone is rendered appreciable because of the high state of ionization. See also p. 5, n. 2.

abundance, the effect of absolute magnitude is comparatively small, and can be neglected if we consider only the major features of a stellar spectrum.

c) A considerable number of stars belonging to classes O, B, and A, and occasionally to class F, show broad and hazy lines for all elements, wholly different in shape from the narrow and deep contours produced by the normal absorption coefficient of Unsöld. C. T. Elvey[1] has shown that these "dish-shaped" lines agree with the contours computed by G. Shajn and the writer for rapidly rotating stars.

In section VI we shall compare the contours of certain lines of a single star (η Ursae Majoris) with those of a spectroscopic binary (α Virginis). The agreement is so satisfactory that there remains no doubt that both are produced by the same cause. Since in the binary this cause is known to be rotation[2] the single star, too, must be rapidly rotating.

III. ROTATION AND SPECTRUM

It has been customary at the Yerkes Observatory to classify all spectra according to the character of the lines. This classification was originally introduced by Professor E. B. Frost in order to provide a rough criterion as to the precision to be expected from measurements of radial velocity. Since "poor" lines are invariably dish-shaped, and since even the absence of strong lines can be interpreted as being due to excessive diffuseness, it appears that this classification is a good measure of rotation, or rather of the component of the rotational velocity in the line of sight. The distribution of all stars of types B and A observed at Yerkes[3] is shown in Tables I and II.

[1] *Astrophysical Journal*, **70**, 152, 1929.

[2] This follows from the correlation between line width and a function of period and amplitude (*Monthly Notices of the Royal Astronomical Society*, **89**, 225, 1929):

$$v = \text{const.}\ \frac{r \cdot K}{P^{\frac{2}{3}}}\ .$$

Evershed has suggested that wide and hazy lines may be due to Doppler effect caused by violent convection currents (*ibid.*, **82**, 395, 1922). This may be a contributory cause, but it does not explain the correlation just noted. The latter strongly suggests that we are dealing principally with Doppler effect due to rotation.

[3] *Astrophysical Journal*, **64**, 9, 1926; *Publications of the Yerkes Observatory*, **7**, Part I, 10, 1929.

It will be seen that the proportion of stars classified as "few, poor" is very great among the B's as well as among the A's. The number of F stars observed is too small to justify a complete tabulation, but from the existing spectrograms it appears that the proportion of dish-shaped lines is much smaller than in the B's and A's. Finally, in spectral classes G, K, and M we have never observed any stars having very wide lines. This refers mostly to the giants, since there have been only few dwarfs of late types on our program.

TABLE I

B STARS

Character of Lines	Number of Stars
Many, good	45
Few, good	59
Many, fair	17
Few, fair	101
Many, poor	7
Few, poor	139

TABLE II

A STARS

Character of Lines	Number of Stars
Many, good	118
Few, good	37
Many, fair	71
Few, fair	59
Many, poor	35
Few, poor	180

It seems safe to say that rotational speed is a function of spectral type, the fastest rotations occurring in the earliest types.

In the following sections we shall assume that all dish-shaped lines are caused by rotation alone and that, furthermore, broadening due to abundance and Stark effect is constant within any given spectral subdivision. That this assumption is permissible results from the fact that among the stars of early types variations due to dish-shaped character are by far the most important of any observed. Minor differences in abundance or pressure broadening depending upon the absolute magnitude can be neglected if we limit our discussion to stars having very flat and broad lines.[1]

[1] The assumption that Stark effect does not vary much within any one spectral subdivision is a modification of certain ideas put forward in my first article on Stark

IV. LINE WIDTH AND WAVE-LENGTH

Consider a rapidly rotating star. The lines are widened by amounts depending upon the projection of the equatorial velocity upon the line of sight. According to Doppler's principle the amount of widening is

$$\Delta\lambda=\lambda\,\frac{v\cdot\sin i}{c}\,. \tag{1}$$

But the wave-length in A.U. is related to the readings on the plate in millimeters by Hartmann's formula

$$\lambda-\lambda_0=\frac{k}{S_0-S}\,. \tag{2}$$

Differentiating this we get

$$\Delta\lambda=\frac{k}{(S_0-S)^2}\cdot\Delta S=\frac{(\lambda-\lambda_0)^2}{k}\,\Delta S\,.$$

Substituting this into (1) we find

$$\Delta S=\frac{\lambda\cdot v\cdot\sin i\cdot k}{c(\lambda-\lambda_0)^2}\,. \tag{3}$$

Consequently the width of a line broadened by Doppler effect should show the following proportionality:

$$\Delta S\propto\frac{\lambda}{(\lambda-\lambda_0)^2}\,. \tag{4}$$

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effect (*Astrophysical Journal*, **69**, 185, 1929). It was suggested there that the diffuse appearance of the hydrogen and helium lines in many stars might be due to electric fields. It now appears that this explanation is correct for the hydrogen lines only (C. T. Elvey, *ibid.*, **71**, 191, 1930; E. T. R. Williams, *Harvard College Observatory Circular*, No. 348, 1930). The variation in width of the helium lines due to Stark effect is comparatively small, though real (cf. the contours in γ Pegasi and in 67 Ophiuchi), and the dish-shaped character must be ascribed almost wholly to rotation. The existence of an effect of absolute magnitude for the hydrogen lines is largely due to changes in Stark effect, since here Stark effect is very pronounced and rotation is little effective. In the helium lines the relationship of character (*s* or *n*) to absolute magnitude must be due to a real tendency of the more luminous stars to rotate slower than the less luminous stars.

For single-prism spectrograms taken with the Bruce spectrograph of the Yerkes Observatory the constant λ_0 is approximately equal to 2300 A.

I have measured the line widths in a microphotometer tracing of a spectrogram of α Virginis. This is a binary showing two very unequal components, but the particular plate selected for measurement shows only one set of lines, the fainter component being superposed over the stronger.

The results of the measurements, given in column 3, clearly show a relationship to λ in rough agreement with formula (4). However,

TABLE III

RELATION BETWEEN LINE WIDTH AND WAVE-LENGTH

Wave-Length	Central Intensity	Width	Corrected Width
3926...........	0.11	7.0 mm	7.0 mm
4009...........	.06	5.5	6.2
4026...........	.13	7.0	5.3
4121...........	.09	5.0	5.0
4144...........	.10	5.0	5.0
4388...........	.09	5.0	5.0
4471...........	.13	7.0	5.0
4481...........	.04	3.5	4.6
4552...........	0.05	3.5	4.2

better results can be obtained if small corrections are introduced to take care of differences in central intensity. By a simple graphical procedure all line widths were reduced to a uniform central intensity of 0.10. The values thus obtained are given in the last column. The decrease in width with wave-length is very nearly that required by formula (4). The evidence is in favor of the rotation hypothesis, since it affects all lines.

V. APPLICATION TO THE STUDY OF SPECTROSCOPIC BINARIES

Figure 1 shows a microphotometer tracing of the spectrum of 67 α Virginis near maximum separation of the lines. Two components are visible for each of the two lines, λ 4472 and λ 4481. It is at once obvious that the shapes of the lines are not the same. The violet components of both lines are shallow and very broad, while the red components are much narrower.[1] Figure 3 shows the contours evaluated by means of the sensitometer intensities given at the right margin of Figure 1.

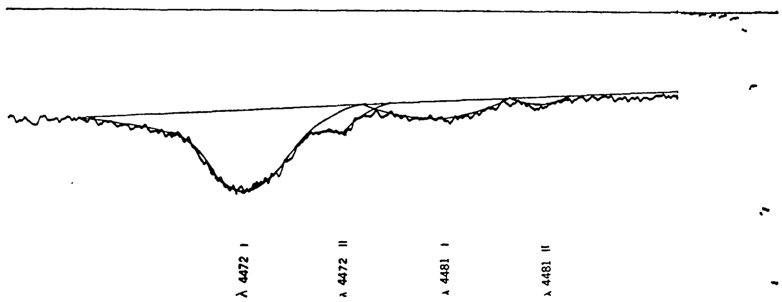


FIG. 1.—Microphotometer tracing of Plate R 1788 of α Virginis, taken on April 24, 1930, at 5^h 35^m U.T. The sensitometer intensities, of which only nine are shown near the right-hand margin, correspond to the following differences in intensity, expressed in stellar magnitudes (from the top): 0.00; 0.39; 0.70; 1.13; 1.47; 1.93; 2.26; 2.75; 3.11; 3.45; 3.86; 4.23; 4.48; 5.06.

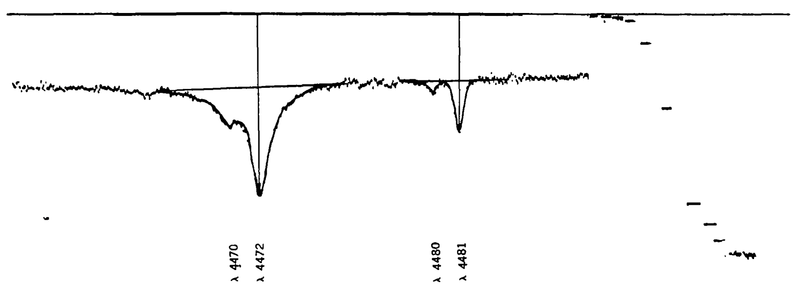


FIG. 2.—Microphotometer tracing of Plate R 1599 of γ Pegasi. Note the sharpness of the lines. The forbidden helium line at λ 4470 is visible to the right of the strong helium line λ 4472. The line at λ 4480 is due to *Al* III. The plate was taken on September 21, 1929, at 4^h38^m U.T.

The conclusion is obvious: One component of the binary is in rapid rotation while the other is not. This is a rather striking result and it leads to several interesting consequences.

The spectroscopic elements of α Virginis, as derived by R. H. Baker,[2] are:

[1] This is true of all other lines not shown in the figure. The narrow component is particularly well visible in *He* 4713. It has also been measured in *He* 4026 and *He* 4388.

[2] *Publications of the Allegheny Observatory*, 1, 65, 1909. Measures of our spectrograms agree well with Baker's orbit.

Velocity of system	+1.6 km/sec.
Period	4.01416 days
Eccentricity	0.10
Time of periastron passage	1908 Jan. 14. 846
Longitude of periastron	328°
K_1	126.1 km/sec.
K_2	207.8 km/sec.
$a_1 \sin i$	6,930,000 km
$a_2 \sin i$	11,400,000 km
$m_1 \sin^3 i$	9.6☉
$m_2 \sin^3 i$	5.8☉

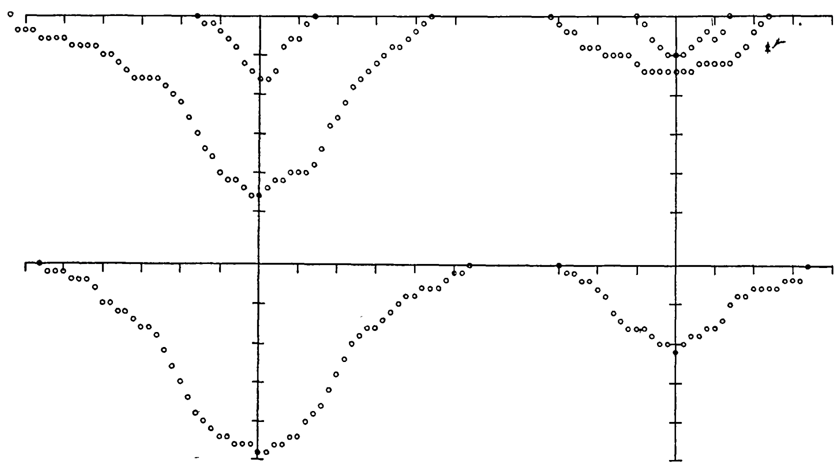


FIG. 3.—Observed contours of lines in α Virginis (top) and η Ursae Majoris (bottom). The contours of both components of α Virginis are shown. One unit in the abscissa corresponds to 1.28 A.U., and one unit in the ordinate to an absorption of 0.05 of the continuous spectrum.

The orbital period is so short that the rotational periods are doubtless equal to it. Consequently the two components have identical angular velocities. Since

$$v_1 = r_1\omega_1 \sin i \,,$$
$$v_2 = r_2\omega_2 \sin i \,,$$
$$\omega_1 = \omega_2 \,,$$

the disparity in the v's must mean a disparity in the radii. We arrive at the conclusion that the stronger component of α Virginis (violet in Fig. 1) has the greater diameter. Information concerning stellar radii can thus be obtained by a purely spectroscopic method.

The binary is not resolved visually; consequently the light of both components enters the slit of the spectrograph simultaneously. Suppose the radial velocities are such that the two components are completely resolved. The continuous spectrum of one star will overlap the line of the other. Let the intensities of the continuous spectra outside the lines be i_1 and i_2 and let the real intensities within the two lines, in the absence of overlapping, be given by

$$j_1=f_1(\lambda) \qquad \text{and} \qquad j_2=f_2(\lambda) \; .$$

The spectral types of the two components are the same. Consequently, if there were no rotation, we should have

$$\frac{j_1}{i_1}=\frac{j_2}{i_2} \; .$$

It should be noted here that the contours are usually expressed in units of the intensity of the continuous spectrum. Hence the fractions in the foregoing expression.

The rotational effect may not be the same in both stars, and the foregoing equality will not, in general, be fulfilled. But rotation changes only the shape of the contour, leaving the total amount of absorbed energy unaffected. Consequently the integrals taken over the functions j_1/i_1 and j_2/i_2 should be identical:

$$A=\int_{-\infty}^{+\infty}\left(1-\frac{j_1}{i_1}\right)d\lambda=\int_{-\infty}^{+\infty}\left(1-\frac{j_2}{i_2}\right)d\lambda \; .$$

In reality we do not observe j_1 and j_2 separately. Both components are photographed simultaneously, and the plate records the combined effect of the lines and of the continuous spectra. As a result of this overlapping we observe for the two components the quantities A_1 and A_2, which are not, in general, identical:

$$A_1=\int_{-\infty}^{+\infty}\left(1-\frac{j_1+i_2}{i_1+i_2}\right)d\lambda=\frac{i_1}{i_1+i_2}A \; , \tag{5}$$

$$A_2=\frac{i_2}{i_1+i_2}A \; . \tag{6}$$

From these

$$\frac{A_1}{A_2}=\frac{i_1}{i_2}\ .$$

The observed areas of the contours of the two components are proportional to the intensities of their continuous spectra. Since the spectral types are the same, we obtain the difference in absolute magnitude:

$$M_1-M_2=-2.5\ \log\frac{i_1}{i_2}=-2.5\ \log\frac{A_1}{A_2}\ . \tag{7}$$

By means of this formula we compute the values of ΔM if A_1 and A_2 are known from the observations.

We shall now reconstruct from the observed contours the real contours j_1 and j_2, which the lines would have had if overlapping with the continuous spectrum did not occur. It follows from (5) and (6) that this is accomplished by multiplying the ordinates of the observed contours by $(1+[A_2/A_1])$ and by $(1+[A_1/A_2])$, respectively.

The numerical evaluation of A_1 and A_2 from the curves of Figure 3 gives roughly:

$$\frac{A_1}{A_2}=8.8\ .$$

Consequently[1]

$$M_2-M_1=2.4\ \text{mag.}$$

Figure 4 shows the corrected contours as they would have been without overlapping. The difference in shape is very marked, confirming that the rotational effect is not identical in the two components.

We now proceed to evaluate the rotational velocity. Let us assume that the shape of the line λ 4472, not affected by rotation, is that obtained by J. Pauwen[2] for the star γ Pegasi (Fig. 2). The

[1] It may be noted that the lack of broadening of the fainter component, its greater "compactness", makes it possible to observe it even though (M_1-M_2) is rather large. If the second component were as broad and diffuse as the primary it could not have been seen on the background of the continuous spectrum. The actual value obtained, 2.4 mag. should be considered as a rough approximation only. It is probable that since the line is very faint even on our best plates, the measured area A_2 is slightly too small. That this is so may be seen from the fact that a line can be lost completely if it is too diffuse and broad.

[2] *Astrophysical Journal*, **70**, 263, 1929.

spectral types of α Virginis and γ Pegasi are approximately the same. Measurement of the areas shows that the line in γ Pegasi is not as strong as the line in α Virginis. In order to make the two areas identical, we multiply all ordinates of the contour for γ Pegasi by 1.8. The resulting contour is shown in Figure 5. We proceed in a manner similar to that used by Shajn and the writer, which was also

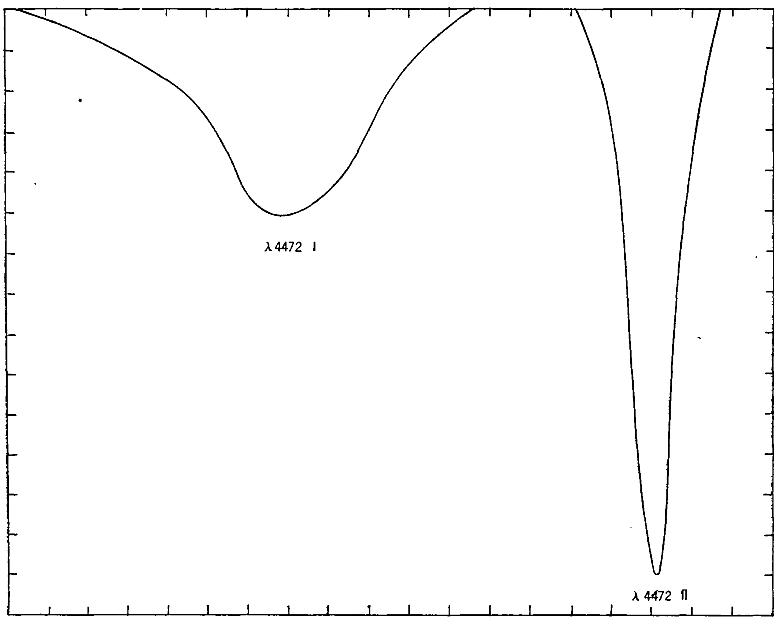


FIG. 4.—Corrected contours of the two components of α Virginis. The curves show the shapes of the lines as they would have been in the absence of overlapping with the continuous spectra. The units in the abscissa and in the ordinate are the same as in Fig. 3.

employed by Elvey. Consider the disk of the star, and imagine that the x-axis lies in the equatorial plane. If the equatorial velocity is $v \sin i$ then any point on the disk has a projected velocity

$$\frac{x}{r} v \cdot \sin i ,$$

where r is the radius. Imagine that the star is subdivided into an infinite number of sections, parallel to the axis of rotation. Each section gives an intensity $j = f(\lambda - \lambda_0)$, where

$$\lambda_0 = \lambda_0' + \lambda_0' \frac{v \cdot x}{r} \sin i .$$

The area of each section is $2\sqrt{r^2-x^2}.dx$. Multiplying j by this area and integrating over x we obtain the intensity of the light from the whole disk. To express this, as is usual, in units of the continuous spectrum, we divide the result of the integration by the integrated intensity of the continuous spectrum, which is equal to $\pi r^2 i_1$.

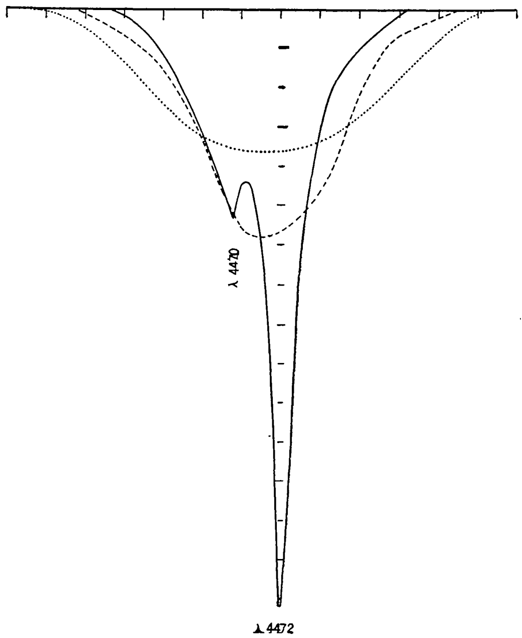


FIG. 5.—Effect of rotation upon the contour of the line λ 4472 The full line represents a non-rotating star. The two other curves correspond to equatorial velocities of 170 and 340 km/sec. The units in the abscissa and in the ordinate are the same as in Fig. 3.

Since the shape of the function f is not algebraically known we perform the integration graphically. The disk is divided into finite sections (forty in this case), each having an area of

$$a=2\int_{x_1}^{x_2}\sqrt{r^2-x^2}\,dx=\left\{x\sqrt{r^2-x^2}+r^2\text{ arc sin }\frac{x}{r}\right\}_{x_1}^{x_2}.$$

Each section contributes a certain intensity $j=f(\lambda-\lambda_0)$ proportional to its area a. Rotation shifts each of these contributory contours by an amount equal to

$$\frac{x_1+x_2}{2r}\,v\cdot\sin i\,.$$

We thus have a number of contours,

$$j_n = f\left[\lambda - \left(\lambda_0' + \lambda_0' \frac{x_1 + x_2}{2r} v \cdot \sin i\right)\right] a_n .$$

The final curve is obtained by taking:

$$I = \frac{1}{\pi r^2 i_1} \sum j_n a_n .$$

This summation has been made for two arbitrary values of v sin i, viz., 340 and 170 km/sec. (Fig. 5). It will be seen by superposition of the contour of the violet component of Figure 3 with the curves of Figure 5 that v sin $i=200$ km/sec. gives a good representation.

We have neglected here the effect of darkening at the limb. From the computations of Shajn and the writer it would seem that this is not important. It might perhaps tend to increase slightly the value of v sin i.

The second component has almost no rotation. Since it is doubtful whether it would be possible to measure by this method a rotational velocity of less than 50 km/sec. we can only state that the rotation of the second component does not much exceed this value. If we tentatively assume v_2 sin $i=50$ km/sec., we obtain

$$\frac{v_1}{v_2} = \frac{r_1}{r_2} = 4 .$$

This is in good agreement with expectation. We have for the two components[1]

$$\log r_1 = \frac{5900}{T_1} - 0.2 M_1 - 0.02 , \tag{8}$$

$$\log r_2 = \frac{5900}{T_2} - 0.2 M_2 - 0.02 , \tag{9}$$

where T_1 and T_2 are the temperatures and M_1 and M_2 the visual absolute magnitudes. In this case $T_1 = T_2$ and $M_1 - M_2 = -2.4$. By subtraction:

$$\log \frac{r_1}{r_2} = 0.2(M_2 - M_1) .$$

[1] Russell, Dugan, and Stewart, *Astronomy*, **2**, 738, 1927.

Consequently $r_1/r_2=3$, in fair agreement with the value obtained before.

While too much stress should not be placed upon the numerical values of $v \sin i$, it is clear that interesting information is contained in the contours of the components of spectroscopic binaries. For example, if the two corrected contours, j_1 and j_2, are similar, we can safely say that the diameters of the two stars are also about the same. Of course, this applies only to double stars with short periods, since it is essential that the periods of revolution and of rotation are the same. We do not know exactly at which stage this equality begins to break down. However, there is very little doubt that it is fulfilled in all binaries with periods of the order of a few days.

VI. ROTATION IN η URSAE MAJORIS

Figures 6 and 3 contain the original tracing and the contours of λ 4472 and λ 4481 for the star η Ursae Majoris. This is not a spectroscopic binary of short period and large amplitude[1] like α Virginis. The value of $v \sin i$ is again close to 200 km/sec. We are thus dealing here with a single star of the same type and presumably the same luminosity as the stronger component of α Virginis. Now this latter star is what Jeans calls a very young binary. Using equations (8) and (9) and substituting $T_1=T_2=20,000°$ and $M_1=-2.0$, we find[2]

$$r_1=3\times 10^6 \text{ km} ,$$

$$r_2=1\times 10^6 \text{ km} .$$

From the orbit we have

$$a_1 \sin i=7\times 10^6 \text{ km} ,$$

$$a_2 \sin i=11\times 10^6 \text{ km} .$$

[1] It was announced as a spectroscopic binary by L. L. Mellor (*Publications of the Observatory of the University of Michigan*, **3**, 72, 1923) but the range is small and the period is probably long. None of our plates shows any indication of duplicity. Consequently the observed spectrum refers to a single star.

[2] Since $r_1=3\times 10^6$ km and $P=4.0$ days, the theoretical equatorial velocity of rotation should be $v_1=2\pi r_1/P=55$ km/sec. This seems to indicate that we have used too faint a magnitude for M_1 in (8).

The inclination[1] is probably close to 90°, so that

$$a_1+a_2=18\times 10^6 \text{ km} .$$

The distance between the surfaces of the two components is only about 3.5 times the sum of their radii. Consequently it is probable that the angular rotational velocity of the stronger component of α Virginis is not far from the critical value at which a double star merges into a rapidly rotating single body. For η Ursae Majoris the

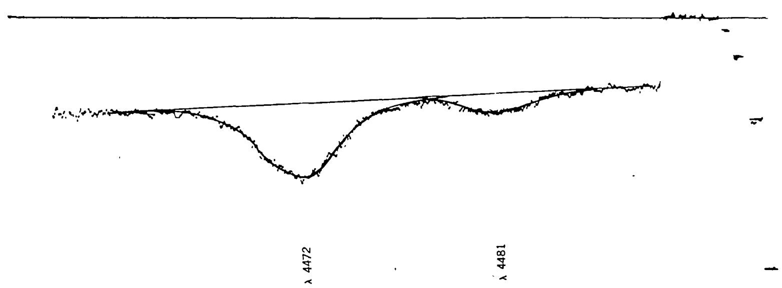


FIG. 6.—Microphotometer tracing of Plate R 1774 of η Ursae Majoris, taken on March 3, 1930, at 6^h 35^m U.T.

angular velocity is of the same order of magnitude. We conclude therefore that this star is also rather close to the critical stage where break-up may occur.[2]

VII. STABILITY

It may be of interest to compute the value of the quantity $\omega^2/2\pi\gamma\rho$, which is characteristic for the state of stability of a rotating star. Following Tissérand's method,[3] we have for the star

$$\frac{\omega_1^2}{2\pi\gamma\rho_1}=\frac{2\pi}{\gamma\rho_1 P_1^2} ,$$

[1] α Virginis was tentatively announced as an eclipsing variable by J. Stebbins (*Astrophysical Journal*, **39**, 475, 1914). However, in a later publication Stebbins did not include this star among those which are definitely known to be eclipsing variables (*Publications of the Washburn Observatory*, **15**, 56, 1928). Professor Stebbins has informed me that the question is not definitely settled, since there are few suitable comparison stars of similar spectral type in the vicinity of α Virginis.

[2] The angular velocity $\omega=2\pi/P$ may be considered as a measure of the distance between the two components of a double star. Consequently, whatever the origin of the binary, there is a critical value of ω such that the components are just in contact. At this stage the double star ceases to exist as such and should be considered as a single body.

[3] *Traité de Mécanique Céleste*, **2**, 92, 1891.

and for the earth,

$$\frac{\omega_2^2}{2\pi\gamma\rho_2}=\frac{2\pi}{\gamma\rho_2 P_2^2}=0.00230 \; .$$

Consequently

$$\frac{\omega_1^2}{2\pi\gamma\rho_1}=0.00230\left(\frac{P_2}{P_1}\right)^2\left(\frac{\rho_2}{\rho_1}\right) \; .$$

For the earth $P_2=1$ day, while for α Virginis $P_2=4$ days. The density of the earth is 5.5 gr/cm³. Consequently

$$\frac{\omega_1^2}{2\pi\gamma\rho_1}=\frac{0.00079}{\rho_1} \; .$$

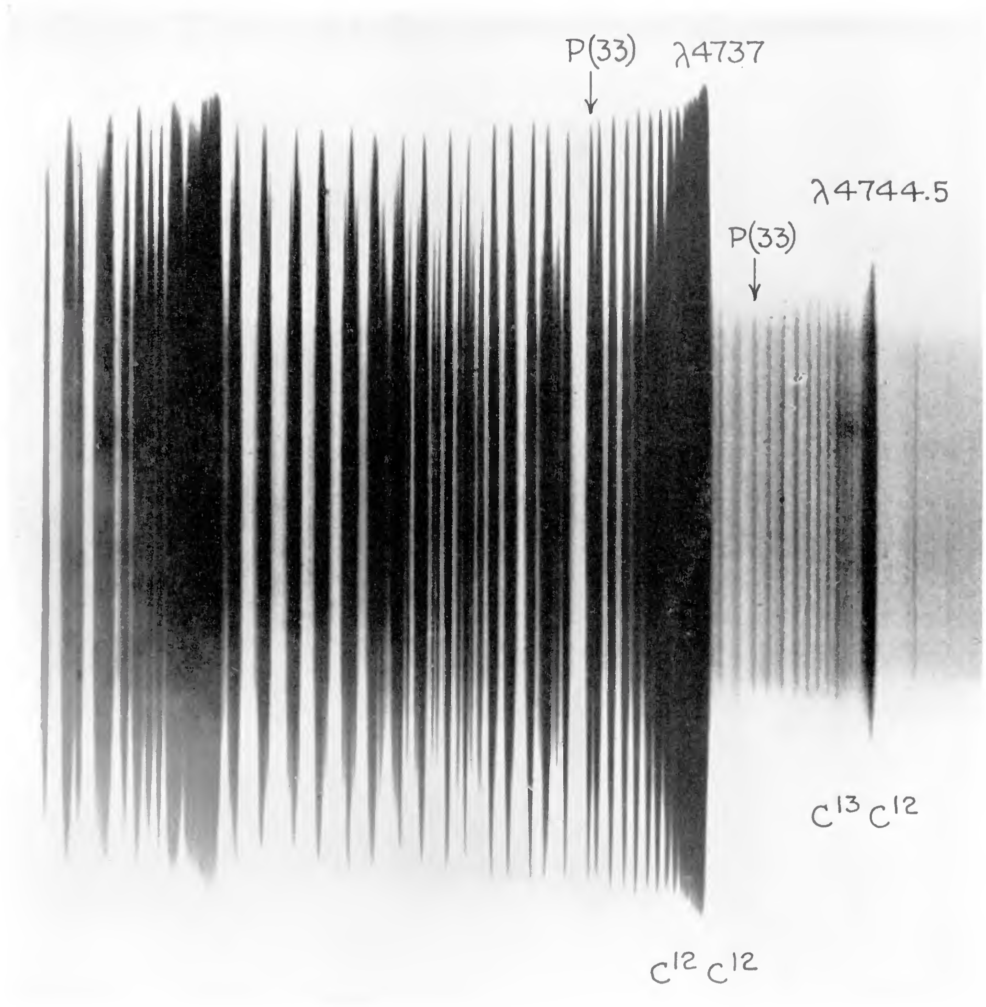
The mean density of the B-type stars ranges from about 0.01 to 0.1 gr/cm³. Using the two limits, we find

$$0.0079\leq\frac{\omega_1^2}{2\pi\gamma\rho_1}\leq 0.079 \; .$$

If the laws deduced by Jeans and others for rotating homogeneous liquid bodies are applicable to the stars, the foregoing result would indicate that the star is stable and that the figure of equilibrium is probably a Maclaurin spheroid, the meridional cross-section of which has an eccentricity of not more than about 0.5. Since the density of α Virginis is not known, the quantity $\omega^2/2\pi\gamma\rho$ cannot be determined any closer. However, a shortening of the period to about 2.8 days, with constant dimensions and density, would bring the angular velocity rather dangerously close to the point where the Maclaurin ellipsoids lose stability. It seems quite possible that such stars exist. In fact, α Virginis is only one of many examples of rapid rotations, and it is not at all improbable that such rapidly rotating binaries as V Puppis and μ^1 Scorpii approach the critical point even closer than α Virginis.

YERKES OBSERVATORY
May 22, 1930

PLATE I

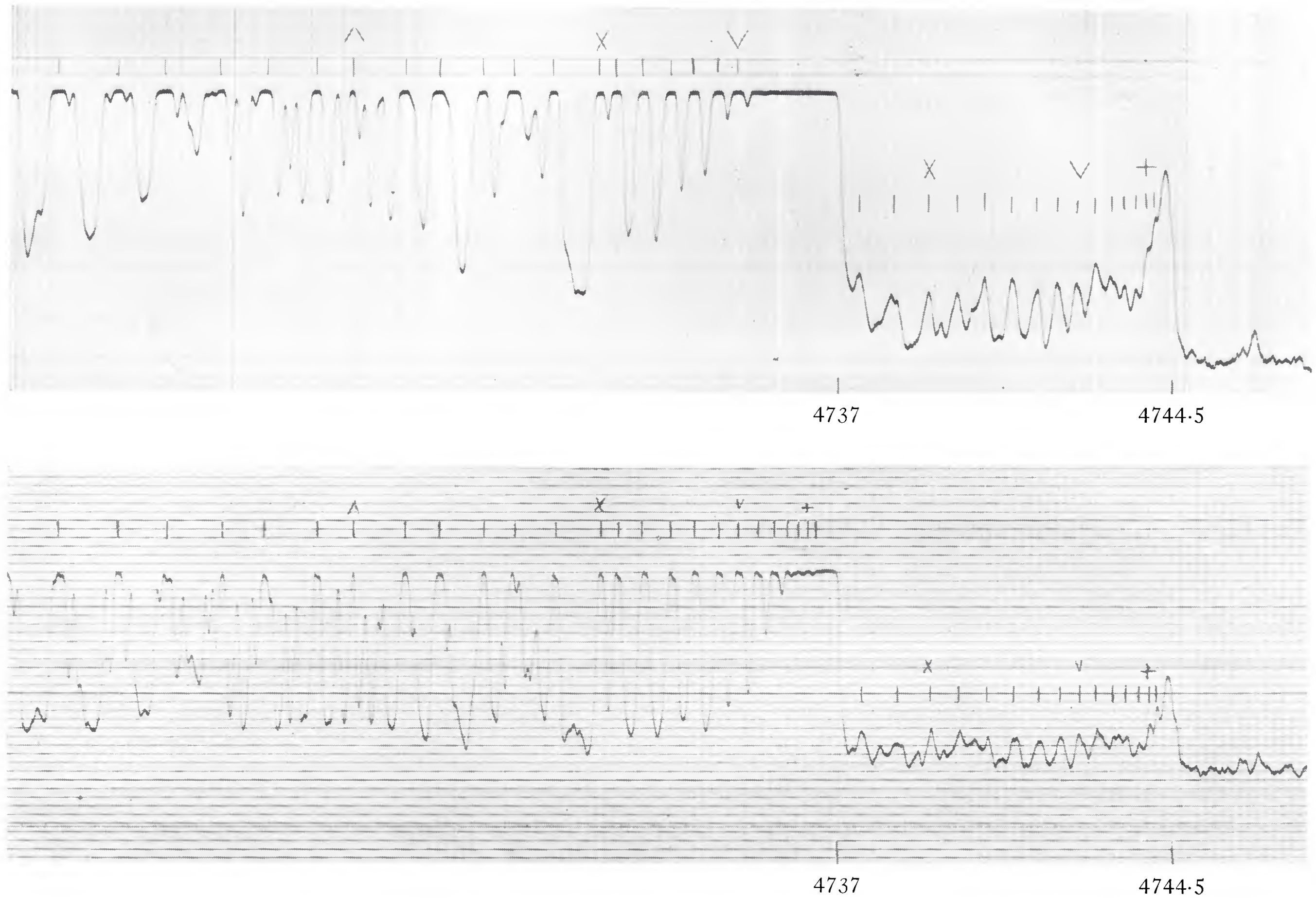


THE CARBON BAND λ 4737 AND ISOTOPE BAND λ 4744.5

PLATE II

a

b



Microphotometer Curves of the Carbon Band λ 4737 and its Isotope λ 4744.5

a) Curve from first-order spectrogram

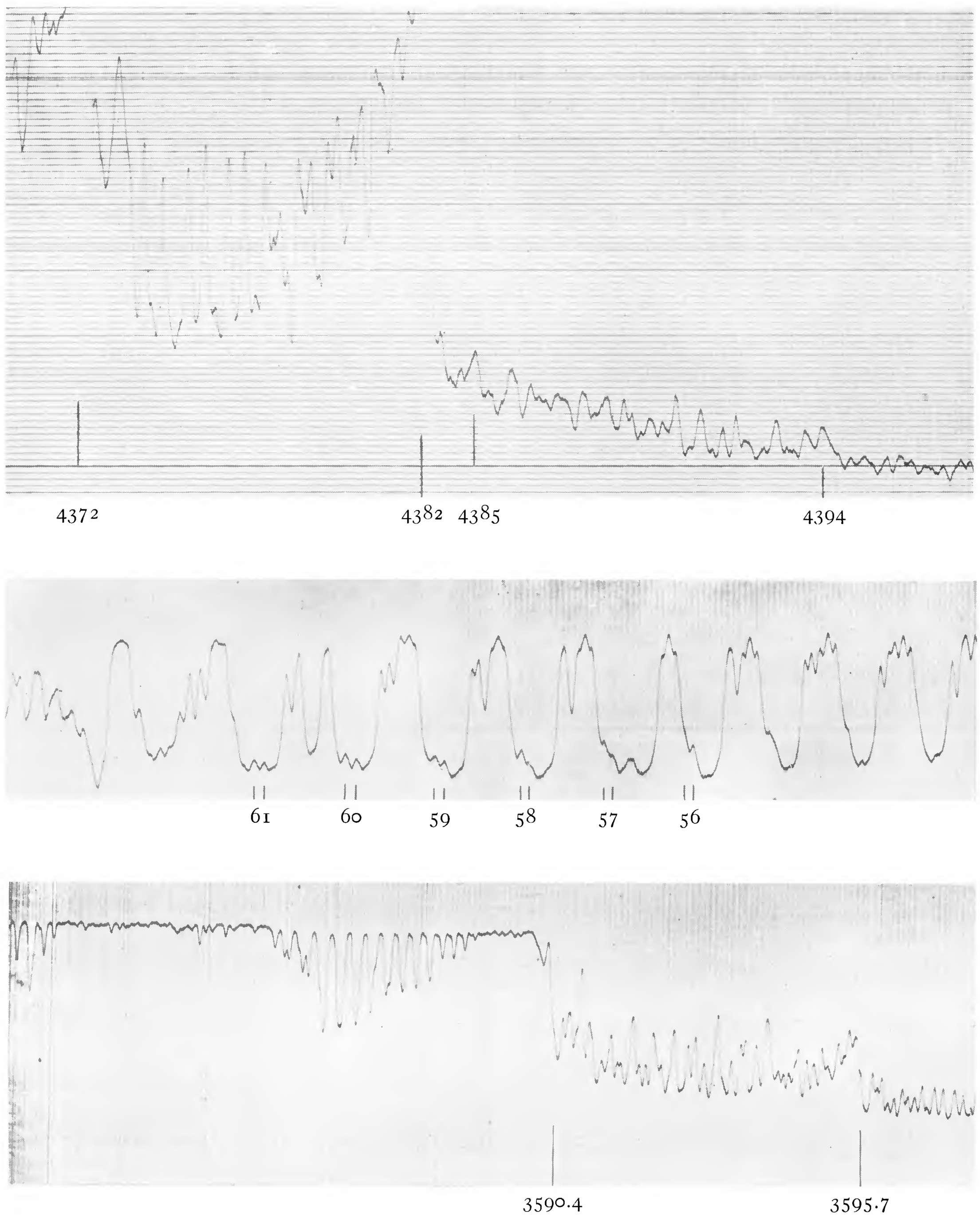
b) Curve from second-order spectrogram

PLATE III

a

b

c

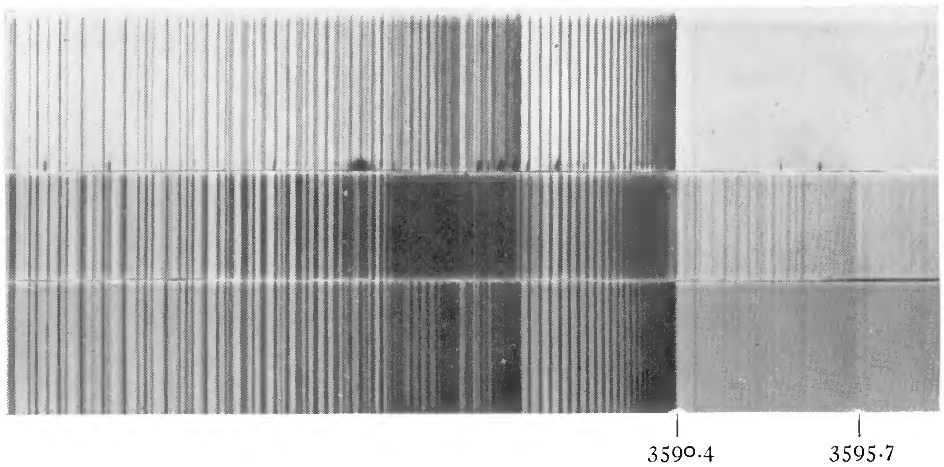


MICROPHOTOMETER CURVES OF CARBON AND CYANOGEN BANDS AND THEIR ISOTOPES

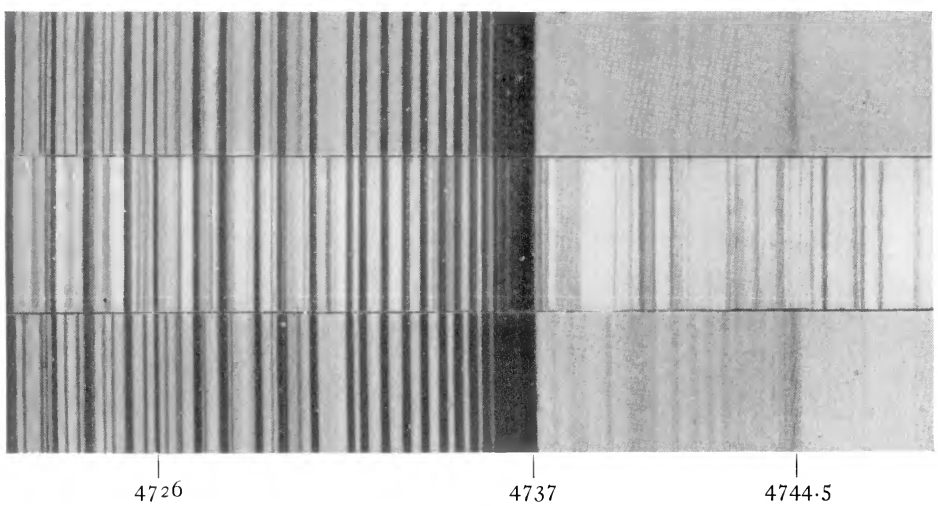
a) λ 4382 and isotope λ 4394
b) Structure in cyanogen band λ 3883
Marked lines belong to isotope band
c) λ 3590.4 and isotope λ 3595.7

PLATE IV

a



b



BAND STRUCTURES WITH ISOTOPES

a) Cyanogen band λ 3590 given by nitrogen in furnace (center), compared with vacuum furnace.

b) Furnace spectra of λ 4737 and isotope (above and below), compared with arc spectrum (center).