LUNAR RADIATION AND TEMPERATURES[1]

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ABSTRACT

Methods of observing.—The vacuum thermocouple attached to the 100-inch telescope was used as in measuring stellar radiation. A microscope cover-glass was used to separate the reflected light from the planetary heat. A fluorite screen and water-cell were used for special purposes.

Reduction of the measurements.—Comparison stars, whose radiometric magnitudes we have already determined, were used to reduce the galvanometer deflections to absolute intensities. These are expressed in terms of the radiometric magnitude of the planetary heat in a solid angle of 1 sq. sec. of arc. The relation of radiometric magnitude to absolute temperature is given by the equation

$$\log T = 2.612 - 0.1\ (\overline{m}_r - \Delta m_r)\ .$$

An expression applicable to the screens used and to the conditions of observing is derived from this equation.

Limiting measurable temperature.—Temperatures below 100° K are measured with difficulty, since the value of m'_r for that temperature is 12.47 mag.; and it is probably not feasible to detect planetary radiation from a celestial body whose temperature is below 70° K.

Reflection of solar radiation between 8 and 14 μ.—Even if the moon reflected all the solar radiation in this region, it would add only $1\frac{1}{2}$ per cent to the planetary heat and affect the computed temperature by only 1°. With the fluorite screen the emissivity between 8 and 10 μ is shown to be the same as between 8 and 14 μ, which indicates that the silicates, if present in the lunar crust, cannot be detected by the selective emissivity between 8 and 10 μ as in laboratory tests, probably because they are in a finely divided or porous state for which the emissivity would be more like that of a black body.

Distribution of radiation over the disk.—From drift-curves it is found that the distribution of planetary heat over the disk at full moon does not follow the formula

$$E = a \cos \theta\ ,$$

derived from the Lommel-Seeliger law, but more nearly the formula

$$E = a \cos^{3/2} \theta\ .$$

This is explained by the rough surface. From the water-cell drift-curve it appears that the general trend of reflected light at full moon is uniform, but that at the limb in the E.-W. direction the intensity is 60 per cent greater than that of the neighboring *maria*.

Temperature of the subsolar point.—The measures indicate that at full moon, when the subsolar point is nearly central on the disk, its temperature is 407° K, but that at quarter-phase, when it is on the limb, they show a temperature of 358° K. The directive effect of the rough surface on the planetary heat probably accounts for this difference.

Temperatures during a lunar eclipse.—Measurements made near the south limb on June 14, 1927, show that the temperature fell from 342° to 175° K during the first partial phase, continued to drop to 156° K during totality, and rose again abruptly to nearly the original temperature during the last partial phase. From these data it is estimated that less than 0.1 cal cm^{-2} min^{-1} is conducted into the moon from the surface.

Distribution of planetary heat and reflected light about the subsolar point.—With special equipment the night-to-night values of the reflected light transmitted by the cover-

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 392.

glass and of the planetary heat eliminated by it were determined for the subsolar point as it passed over the earthward side of the moon. The mean spherical distribution of light over the hemisphere about the subsolar point is 0.85 mag. fainter and the planetary heat 0.15 mag. fainter than the corresponding intensities at full moon.

The theoretical lunar temperature.—From the water-cell measurements and the distribution-curve, the light reflected from the subsolar point is estimated to be 0.24 cal cm^{-2} min^{-1}; and combining this with the amount conducted and with the solar constant, we find the mean spherical rate of lunar emission to be 1.61 cal cm^{-2} min^{-1} and the computed black-body temperature to be 374° K. From the distribution-curve of planetary heat about the subsolar point the mean spherical rate of emission is found to be 1.93 cal cm^{-2} min^{-1} and the corresponding temperature 391° K, which is to be compared with the theoretical temperature of 374° K.

Atmospheric absorption.—A discussion of the discrepancy between the observed and theoretical rate of lunar emission gives a set of corrections to the atmospheric absorption constants, derived on the assumption that the entire difference is due to errors in these quantities.

Reflection of solar radiation from the lunar surface.—From the distribution-curve of reflected light about the subsolar point and the drift-curve made with the water-cell in the beam the formula

$$E_r = K \frac{0.46 \sec^2 i/2}{0.46 \cos \theta + \sin \theta} \tag{22}$$

was derived, where i and θ are the angles of incidence and reflection, respectively. The laws of Lambert, Lommel and Seeliger, and Euler cannot be made to fit the observational material.

Radiometric albedo.—The radiometric magnitude of the reflected light is found to be −13.3, and of the whole lunar radiation −14.8. The radiometric albedo of the reflected light is 0.093.

Temperature of the dark side.—This was found to be 120° K. The temperature is so low that considerable observing will be necessary to establish a good value.

The radiation from a planet consists of two parts: reflected sunlight which, when it reaches us, is confined practically to the limits of wave-length 0.3 to 5 μ, and the low-temperature radiation called "planetary heat," which is emitted by the warmed surface and which reaches us principally through the great water-vapor transmission band between 8 and 14 μ. The small amount of planetary heat transmitted between 0.3 and 8 μ is nearly negligible except in the case of the planet Mercury. To separate these radiations a microscope cover-glass 0.165 mm thick is superior to a water-cell, since any errors in the spectral reflection coefficients for the planet enter into the results very much less with the former screen than with the latter.

TRANSMISSION COEFFICIENTS

The transmission-curves of the microscope cover-glass,[1] water-cell,[1] fluorite screen,[2] rock-salt window,[2] and the atmosphere[3] are

[1] *Mt. Wilson Contr.*, No. 336; *Astrophysical Journal*, **66**, 43, 1927.

[2] *Smithsonian Physical Tables* (7th ed.), p. 305, 1921.

[3] *Smithsonian Miscellaneous Collections*, **68**, No. 8, 1917.

shown in Figure 1. All of these are well determined except that of the atmosphere. The only complete data on the transmission of water vapor and the atmosphere in long wave-lengths where planetary heat is radiated most abundantly are those obtained by F. E. Fowle.[1] The admirable experimental data from which these transmission coefficients were determined covered the range 0.001 to 0.082 cm of precipitable water, whereas the amount between an observer on Mount Wilson and a celestial object in the zenith is 0.7

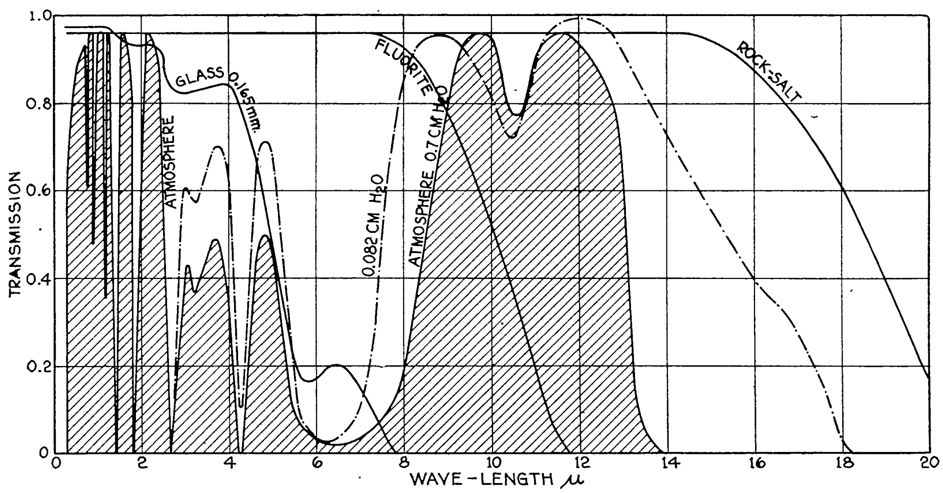


FIG. 1.—Transmission curves of (*a*) the atmosphere above Mount Wilson (the shaded curve); (*b*) water vapor 0.082 cm precipitable water; (*c*) microscope cover-glass, 0.165 mm thick; (*d*) fluorite 4 mm; and (*e*) rock salt 2 mm.

cm,[2] which necessitates an extrapolation of the observational data over a range 900 per cent greater than that observed. Probably no greater contribution to our knowledge of planetary temperatures could be made than a determination of the transmission coefficients of water vapor between 2.7 and 20 μ for amounts in the neighborhood of 1 cm of precipitable water, which is equivalent to a column of steam 16.7 m long at normal atmospheric pressure.

The shaded curve in Figure 1 is that for the atmosphere having 0.7 cm of precipitable water as recommended by Fowle,[3] with details

[1] *Ibid.*, p. 23, 1917.

[2] *Astrophysical Journal*, **35**, 149, 1912; *Annals of the Astrophysical Observatory, Smithsonian Institution*, **3**, 171, 1913.

[3] *Smithsonian Miscellaneous Collections*, **68**, No. 8, 1917.

in the near infra-red from radiation-curves which we observed for the purpose. That portion of the curve between 2 and 14 μ was made as nearly symmetrical as possible with Fowle's laboratory curve (the broken line) for 0.082 cm of precipitable water, the data for which are given in his table only in steps of 1 μ.

Variation in the amount of atmospheric water vapor will, of course, affect the atmospheric transmission. According to the shaded curve, a variation of 50 per cent[1] in the water vapor changes the transmission of the atmosphere 11 per cent. The formula of Hann,

$$p = 1.7\, e_w \sec z, \tag{1}$$

gives the precipitable water p in the optical path for a given vapor pressure e_w. Fowle[2] has shown that in actual practice the coefficient 1.7 varies over a large range. On account of this and the uncertainty in the transmissions themselves it was thought better not to include observations when p differed largely from the mean value 0.7 cm than to apply corrections which could be of little certainty. From Fowle's extrapolations, which apply to the shaded curve only, it follows that the correction due to a variation of p, to be applied to a galvanometer deflection, B, for planetary heat from a celestial object between 300° and 500° K, is

$$\delta B = \frac{p - 0.7}{3} B, \qquad (0.3 < p < 1.5 \text{ cm}) . \tag{2}$$

OBSERVATIONAL DATA

The general method which we have used for measuring radiation from celestial objects has already been discussed.[3] The vacuum thermocouple employed for most of the lunar observations had a silver shield covering all the thermocouple and the platinum lead-wires except the receiver 0.62 mm square over one junction. The cell of the thermocouple was provided with a rock-salt window 2 mm thick.

[1] See also *Annals of the Astrophysical Observatory, Smithsonian Institution*, **4**, 284–285, 1922.

[2] *Astrophysical Journal*, **35**, 149, 1912; *Annals of the Astrophysical Observatory, Smithsonian Institution*, **3**, 171, 1913.

[3] *Mt. Wilson Contr.*, No. 369; *Astrophysical Journal*, **68**, 279, 1928.

The general scheme of observing was to make (1) a series of measurements on the moon with the microscope cover-glass, water-cell, and fluorite screen, and without any screen at all (free deflection); (2) a set of free deflections on a star whose radiometric magnitude we had previously determined;[1] and (3) a reading of the sling psychrometer. All the measurements were made at the Newtonian focus of the 100-inch telescope, where the mean diameter of the lunar image is 116.6 mm. The width of the receiver of the thermocouple is only 0.0055 of this image.

REDUCTION OF THE MEASUREMENTS

We have already pointed out[2] that the temperature of a star computed from the total radiation is given by

$$\log T = 2.638 - 0.1\ (m_r - \Delta m_r) - \tfrac{1}{2} \log d\ , \qquad (3)$$

where T is the absolute temperature, m_r the radiometric magnitude, Δm_r the loss, in magnitudes, due to the atmosphere and silvered surfaces of the telescope, and d the diameter of the star in seconds of arc. This formula assumes that the star radiates uniformly over its apparent surface—a condition which is approximately satisfied for the total radiation from the sun. From drift-curves on the sun taken in the Pasadena laboratory we have found that d from (3) should be multiplied by 1.07 for dG-type stars. In the case of a planet, d is the angular diameter of the receiver as seen from the mirror of the telescope, provided that the receiver is so small that the temperature of that part of the disk covered by it is practically constant. If the image of the planet approaches or is smaller than the diameter of the receiver, the condition of uniform distribution of radiation over the receiver no longer holds, and d is then the angular diameter of the receiver, on the one hand, or the angular diameter of the planet, on the other, corrected for distribution of radiation. Since either of these cases may occur for a planet, the last term of equation (3) has been changed to correspond to the diameter of a solid angle of 1 sq. sec. of arc of planetary heat and

[1] *Ibid.*, Table III.

[2] *Ibid.*, equation 4.

all observations have been reduced to this unit. Equation (3) then becomes

$$\log T = 2.612 - 0.1\ (\bar{m}_r - \Delta m_r)\ , \tag{4}$$

where $\bar{m}_r$ is the radiometric magnitude of a solid angle of 1 sq. sec. of arc of planetary heat. This equation gives the temperature corresponding to the mean radiation from the portion of the planet's surface, the image of which falls on the thermocouple, and assumes that the spectral distribution of energy is like that of a black body.

The values of Δm_r (Table I) have been obtained for various temperatures by combining the black-body curves with the atmospheric transmission-curves in Figure 1 and the reflecting power of silver, which for two reflections is 0.97 throughout the region of planetary heat.

The values of $\bar{m}_r$ are obtained from

$$\left.\begin{aligned} \bar{m}_r = m_r + (C-c) + 10/4 \log b - 10/4 \log B \\ -(a \sec Z - 0.16 \sec z) - P + 10/4 \log As^2\ , \end{aligned}\right\} \tag{5}$$

where m_r is the radiometric magnitude of the comparison star, $C-c$ the correction to m_r due to tarnishing of the silver, b the deflection for the star, B that part of the deflection for the planet due to planetary heat, Z and z the zenith distances of the moon and the star, respectively, a the coefficient of atmospheric extinction for planetary heat, P the correction in magnitudes from equation (2) due to the deviation of the water-vapor content of the atmosphere from the average, A the area of the thermocouple receiver in square millimeters, and s the scale of the telescope field in seconds of arc per millimeter. We will consider these quantities in the order in which they occur in equation (5).

The values of m_r for a number of stars have already been published.[1] The correction to m_r due to the differential absorption of radiation from the star and planetary heat by tarnished silver is

$$C - c = D\ (S_p - S_s)\ , \tag{6}$$

[1] *Ibid.*, Table III.

where D is the number of days since silvering, S_p the coefficient for planetary heat, and S_s the coefficient for the comparison star. S increases with spectral class[1] but becomes nearly constant at 205×10^{-6} mag. per day for the radiation from M-type stars, and, since planetary heat is of still longer wave-length, we may substitute this value for S_p in equation (6).

In obtaining B, that part of the deflection due to planetary heat, the free deflection, F, is combined as follows with Cg, the deflection observed with the cover-glass screen. Let R be that part of the light reflected from the planet, Tr the transmission of planetary heat through the cover-glass, and tr the transmission of the reflected light from the planet through the cover-glass. Then,

$$F=B+R\ , \tag{7a}$$

$$Cg=BTr+Rtr\ , \tag{7b}$$

$$B=\frac{F-\dfrac{Cg}{tr}}{1-\dfrac{Tr}{tr}}\ . \tag{7c}$$

The coefficient, a, of sec Z was determined from a series of measurements on the moon and Mars which gave a value of 0.16 mag., the same as the mean value for stellar radiation.

P is equivalent to δB in equation (2) expressed in magnitudes, and $s^2=256$ sq. sec. of arc at the Newtonian focus of the 100-inch telescope.

For planets in general we substitute B from equation (7c) into equation (5) and obtain (8):

$$\left.\begin{aligned}\bar{m}_r=m_r+D(205\times 10^{-6}-S_s)+10/4\log b-10/4\log\left(F-\frac{Cg}{tr}\right)\\ +10/4\log\left(1-\frac{Tr}{tr}\right)-0.16\,(\sec Z-\sec z)-10/4\log\frac{p+2.3}{3}\\ +10/4\log As^2\ .\end{aligned}\right\}\tag{8}$$

[1] *Ibid*, Table I.

If we substitute the value of $\bar{m}_r$ from (8) into equation (4) and collect the constants and those variables which are functions of the planetary temperature, T, we obtain

$$\left.\begin{aligned} m_r+D\,(205\times 10^{-6}-S_s)+10/4\log b-10/4\log\left(F-\frac{Cg}{tr}\right)\\ -0.16\,(\sec Z-\sec z)-10/4\log\frac{p+2.3}{3}+10/4\log As^2\\ =26.12-10\log T+\Delta m_r-10/4\log\left(1-\frac{Tr}{tr}\right)=m_r',\end{aligned}\right\}\quad(9)$$

which applies to any planet for which the spectral distribution of the reflected light is known. This equation represents the radiometric magnitude of the planetary heat eliminated by the cover-glass for a solid angle of 1 sq. sec. of arc; the second member can be calculated for any planetary temperature, T, and the first member, which will be called m_r', can be obtained from observations.

It has already been shown[1] that, in the visible spectrum at least, the energy-curve of moonlight gives a color-index on the Mount Wilson scale[2] of 0.92 mag., which corresponds approximately to that of a K0 dwarf star. The absorption of the cover-glass screen for a star of this type[3] is 0.03 mag. with sufficient approximations, and the radiation reflected by it is 0.08 mag. Hence, the value of tr for the moon is 0.90. Substituting this value of tr into equation (9), we have equation (10), which applies to observations of the moon:

$$\left.\begin{aligned} m_r+D\,(205\times 10^{-6}-S_s)+10/4\log b-10/4\log\,(F-1.11Cg)\\ -0.16\,(\sec Z-\sec z)-10/4\log\frac{p+2.3}{3}+10/4\log As^2\\ =26.12-10\log T+\Delta m_r-10/4\log\,(1-1.11Tr)=m_r'.\end{aligned}\right\}\quad(10)$$

From the atmospheric transmission-curves in Figure 1, the black-body energy-curves for various temperatures, and the reflecting power of silver, we have computed Table I for different values of T. The application of $\Delta m_r'$ will be discussed later.

In equation (10) the first and third terms of the left-hand mem-

[1] *Publications of the Astronomical Society of the Pacific*, **38**, 242, 1926.

[2] *Mt. Wilson Contr.*, No. 226; *Astrophysical Journal*, **55**, 165, 1922.

[3] *Mt. Wilson Contr.*, No. 369; *Astrophysical Journal*, **68**, 279, 1928.

ber are fixed by the conditions of observing. The correction due to tarnished silver given by the second term did not exceed 0.05 mag. The reduction for air mass given by the fifth term did not exceed 0.16 mag. The correction for atmospheric water vapor given by the sixth term would not have exceeded 0.1 mag. The last term, which depends on the area of the thermocouple receiver and the scale of the telescope, was 5.07 mag. for the thermocouple previously described when used at the Newtonian focus of the 100-inch reflector.

TABLE I

T	Δm_r	Tr	m'_r	$\Delta m'_r$
100° K	6.35	0.000	+12.47	−0.49
150	...	...	7.80	.38
200	2.12	.000	5.23	.28
250	...	...	3.76	.24
300	1.39	.013	2.76	.20
350	...	...	2.01	.19
400	1.28	.063	1.45	.18
450	...	...	0.99	.18
500	1.28	.147	.60	.18
550	...	...	+ .27	.17
600	1.30	.249	− .01	.17
650	...	...	.25	.17
700	1.31	.364	.46	.17
750	...	...	.67	.16
800	1.25	0.468	− 0.85	−0.16

The lower limit of planetary temperature which can be measured is fixed by the sensitiveness of the thermocouple circuit and the area of the receiver, if the receiver is smaller than the image of the planet. As has been pointed out,[1] stars of radiometric magnitude +7 can be observed with the routine equipment which we have used; and with especially constructed thermocouples a gain of two additional magnitudes can be made. By adding the last term of equation (8) for the thermocouple described above, planetary heat of radiometric magnitude +12 could be measured with an accuracy of about 10 per cent, and with effort this might be increased to +15 or +16 if a large surface were available. Table I shows that planetary heat eliminated by the cover-glass of radiometric magnitude

[1] *Ibid.*

+12 corresponds to a temperature of 100° K, and of magnitude +16 to 70° K. From this it is seen that the temperatures of celestial objects much below 100° K can be determined only with difficulty.

REFLECTION OF SOLAR RADIATION OF WAVE-LENGTH 8–14 μ

It is well known that silica and to a less extent the silicates, of which the lunar crust may contain a large percentage, have high reflecting power in the region 8–14 μ. If these substances are present, it might be supposed that the solar radiation reflected from the lunar surface would be great enough to affect the temperatures calculated on the assumption that all the radiation in these wave-lengths is planetary heat radiated by the warmed surface of the moon. A simple calculation from Planck's black-body formula shows, however, that the reflected solar radiation would be less than $1\frac{1}{2}$ per cent of the planetary heat, even if the reflecting power of the lunar surface were perfect over the whole atmospheric transmission band 8–14 μ. This would decrease the computed temperature about 1°, which is less than the uncertainty arising from other sources. It is not easy, therefore, to determine by this means whether the lunar crust has a high silicate content.

Since the emissivity of silica (and to a less extent of the silicates) is abnormally low between 8 and 10 μ,[1] some indication of its presence should be revealed by the transmission of planetary heat through a fluorite screen, which, as an examination of Figures 1 and 2 will show, isolates just this spectral region. We will now compare the ratio, H, of the observed planetary heat transmitted by the fluorite screen to that without a screen and the similar ratio computed (*a*) by assuming unit emissivity and (*b*) by assuming the emissivity to be that of silica. From the curves in Figure 1 it will be seen that this ratio is essentially that of fluorite *minus* cover-glass to free deflection *minus* cover-glass, or of the energy between 8 and 10 μ to that between 8 and 14 μ.

From drift-curves in Figure 3 we find the following data, which

[1] Wood, *Physical Optics*, p. 602, 1911; Sosman, *The Properties of Silica*, p. 738, 1927.

apply to the subsolar point where the temperature is approximately 400° K:

$$F = 145 \text{ mm},$$
$$Fl = 79 \text{ mm},$$
$$Cg = 41 \text{ mm},$$
$$H = \frac{Fl - Cg}{F - Cg} = 0.37,$$

where Fl is the deflection with the fluorite screen in place.

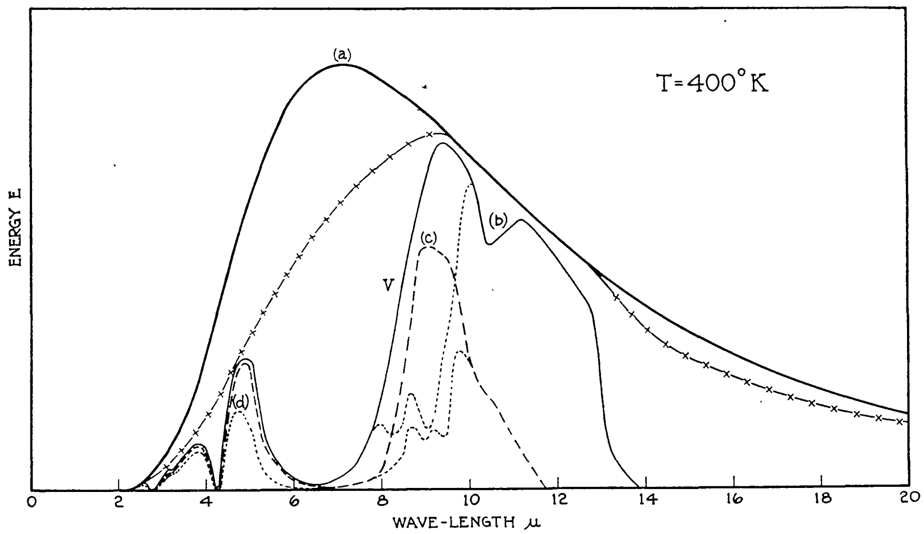


FIG. 2.—Energy-curve (*a*) of a black body at 400° K; (*b*) the same after the radiation has passed through the earth's atmosphere, the dotted line between 8 and 10 μ being the deformation due to presence of silica; (*c*) after passage through fluorite screen, the dotted line between 8 and 10 μ being the deformation due to presence of silica; (*d*) after passage through microscope cover-glass (red limit at 6 μ). The dash-and-cross line is the deformation of (*a*) necessary to explain the discrepancy between the observed and theoretical lunar temperatures.

From these data it can also be seen that the reflected light, R, which is essentially represented by Cg, is 0.39 as great as the planetary heat represented by $F - Cg$.

From the ordinates of the curves in Figure 1 multiplied into those of a black body at 400° K, we obtain Figure 2, where (*a*) is the spectral energy-curve for a black body at 400° K, (*b*) the same after transmission by the atmosphere, (*c*) after further transmission by the fluorite screen, and (*d*) after transmission by the cover-glass.

From this we find that the area which represents planetary heat in the free deflection is 2054, and that when the fluorite screen is in place it is 773. The ratio of these numbers represents H approximately, but to get a more exact value we must apply equations (7*a*) and (7*b*), which allow for the small amount of planetary light lost by the cover-glass. From these we find

$$Fl-Cg=0.05R+\ 773\ ,$$

$$F-Cg=0.10R+2054\ ,$$

whence

$$H=\frac{Fl-Cg}{F-Cg}=\frac{41+\ 773}{82+2054}=0.38\ ,$$

where R is taken at 0.39 Cg as above, and the reflecting power used for fluorite is 0.05. While the observed value $H=0.37$ is less than the 0.38 computed from the transmissions on the assumption of equal emissivities in the regions 8–10 μ and 8–14 μ, it can scarcely be interpreted as implying the presence of silicates, since with such a close agreement no surprise would have been caused had the figures been interchanged. From this, together with the nature of the lunar surface indicated by the low conductivity as shown in the discussion of eclipse observations to follow, it appears that practically all the planetary heat in the region 8–10 μ is accounted for by the assumption of unit emissivity.

We will now compute H on the assumption that the moon has a relatively smooth silica surface. The computed spectral energy-curves of planetary heat transmitted by the atmosphere and by the fluorite used above must be multiplied by the emissivity coefficients of silica before the ratio H is determined. The adopted coefficients, e, are as follows:[1]

λ	e	λ	e
7.75 μ	1.00	9.0 μ	0.20
8.0	0.76	9.25	.24
8.25	.38	9.5	.42
8.5	.30	9.75	.68
8.62	.40	10.0	0.98
8.75	0.32	10.25	1.00

[1] *Ibid.*

From these data we find

$$H=\frac{41+505}{82+1691}=0.30 .$$

This value of H is so radically different from the observed value that we conclude that the emissivity of the lunar surface in the region 8–10 μ is unity for practical purposes and unaffected by any silica content it may have. This does not necessarily mean that the silica content of the lunar crust is small, for if it were finely divided like sand or porous like pumice, which probably would be its state if present, its radiating properties would be nearly those of a black body.[1] We can only say then that if silica or silicates are present, they are probably in the finely divided or porous state. It is possible that spectroscopic examination of the violet slope V (Fig. 2) of the band at 8–14 μ might reveal a deformation due to the emissivity of small amounts of non-porous silicates which would escape detection by the foregoing methods. It will be recalled, however, that the energy-curve[2] of lunar radiation obtained by Langley and Very showed no effect of this kind.

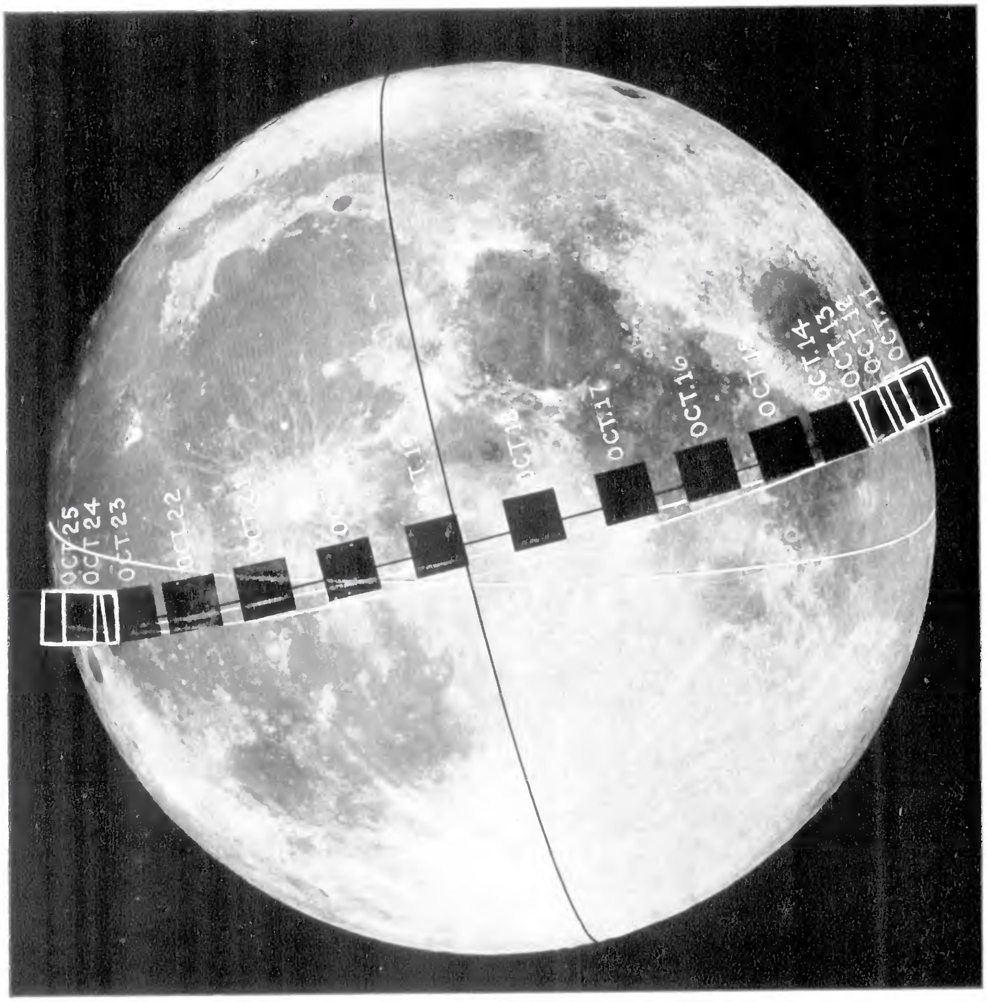
DISTRIBUTION OF RADIATION OVER THE DISK

Figure 3 shows drift-curves along a parallel of declination passing through the center of the disk of the full moon on June 16, 1924, when the phase angle was 0°. The curves were made by unclamping the telescope and allowing the image of the moon to transit the thermocouple. They represent (1) free radiation, that is, without filters, (2) radiation transmitted by 4 mm of fluorite, (3) radiation transmitted by 0.165 mm of glass, and (4) radiation transmitted by 1 cm of water. Number 1 represents all the planetary heat and all the reflected light transmitted by our atmosphere and reflected by the telescope mirrors. Number 2 represents about 35 per cent of the planetary heat and about 90 per cent of the reflected light. Number 3 represents about 11 per cent of the planetary heat and about 95 per cent of the reflected light, while No. 4 represents about 60 per

[1] *Ibid.*, p. 740.

[2] *Memoirs of the National Academy of Sciences*, **4**, Part II, 193 ff., 1889.

PLATE VIII



The white lines represent the width of path traversed by the thermocouple-receiver in recording drift-curves such as in Figure 3; the black lines are the axes of lunar co-ordinates. The black squares represent the size and position of the receiver in measuring the energy from the subsolar point between first and last quarters. These measures were used to obtain the distribution-curves in Figure 7.

cent of the reflected light. The principal features of the moon corresponding to points on the curve are indicated below.

Plate VIII is a photograph of the full moon made at the Lick Observatory, upon which are plotted the axes of co-ordinates (in

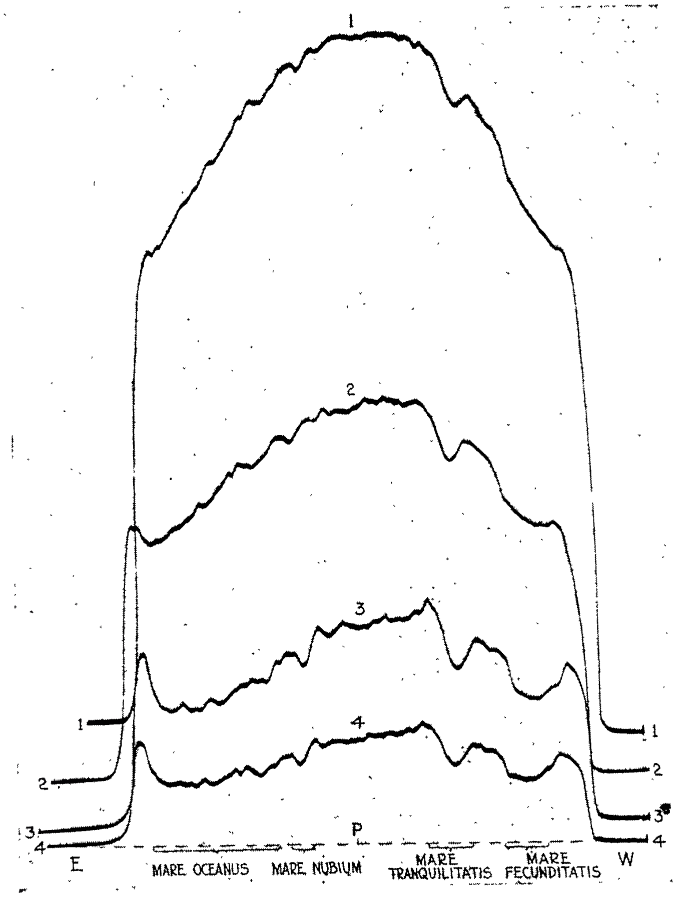


FIG. 3.—Drift-curves across full moon: (1) free radiation; (2) fluorite screen; (3) cover-glass; (4) water-cell.

black) and the paths (in white) traversed by the thermocouple junction on the three dates when drift-curves were taken. The width of the white lines is that of the thermocouple-receiver as used at the 100-inch telescope in making the drift-curves. The black squares refer to the dimensions of the thermocouple-receiver as used at the Snow telescope in the focus of a 27-inch mirror of 67 inches focal length in the study of the distribution of energy about the subsolar point to be described later. We will first discuss the distribution of planetary heat over the disk.

DISTRIBUTION OF PLANETARY HEAT OVER THE DISK

As has been known from experimental observations for more than a hundred years, the radiation incident on the thermocouple-receiver from a given solid angle presented by a uniformly heated surface is independent of the angle of projection.[1] If we consider a smooth sphere, not in rapid rotation, exposed to parallel sunlight, the amount of solar radiation which is incident on and can be absorbed by any small-unit area and converted into heat will vary with the cosine of its angular distance from the subsolar point. The amount of planetary heat radiated by the small area in any directed unit solid angle, if it is a constant fraction of that absorbed, will follow the same law. Hence, if we pass a thermocouple junction over an image of this sphere as seen from any direction, the galvanometer deflections due to planetary heat emitted by it will follow the cosine law (eq. [13])

Two circumstances modify this principle as applied to the moon. The increasing absorption, Δm_r, of the earth's atmosphere for planetary heat from the moon as the receiver moves away from the subsolar point, due to the falling temperature, causes the deflection observed toward the limb or terminator to be too low. A more serious difficulty is the roughness of the lunar surface, which makes the deflections anywhere on the disk near full moon higher, but near quarter-phase lower, than those calculated from the cosine law. The first consideration is obvious from an examination of Table I, but the second may need added explanation.

Two factors which unfortunately cannot be separated appear to explain the effect of roughness. Consider a cavity or bowl-shaped crater. Reflection of planetary heat from the interior by the sides of this depression, or re-emission of absorbed radiation, will tend to make the radiation toward the selenographic zenith greater, and toward the horizon less than it should be, thus making deflections at the center of the disk higher and at the limb lower than the cosine

[1] *Mémoires de l'Académie des Sciences, Paris*, 5, 179–213, 1826.

law predicts. Again, if we consider an almost vertical mountain side near the terminator and facing the sun, it is clear that its temperature ought to be equal to that of the subsolar point, and its radiation, measured at full moon with a thermocouple-receiver smaller than its image in the field of the telescope, will give the same deflection as that from the subsolar point. At quarter-phase, however, such a mountain face at the terminator is invisible, and the area covered by its shadow will give no deflection. Thus arises the idea of directional temperature. The thermocouple measures the mean radiation of a small area of the moon as seen from the particular direction of the earth at the time, and this is interpreted in terms of temperature. Viewed from a different direction, the mean surface involved would not be the same because of its rough character.

The apparent distribution of planetary heat over the lunar surface can be obtained from the ordinates of curves Nos. 1 and 3 in Figure 3 by applying equation (7*c*) where $tr=0.90$. The distribution of temperature can be obtained from equation (10). The real distribution of planetary heat over the lunar surface can be obtained from the distribution of temperature. This has been done for plates taken on April 19 and June 16, 1924, and June 14, 1927. The measurements were made for points located at intervals of 0.025 diameter of the apparent disk, and were plotted after reducing the positions of the measured points to the positions they would have for equatorial phase angle 0° when the subsolar point is on the hour circle passing through the center. This may be done by the usual formulae for the measurement of positions on a sphere; but, since the paths in every case pass very close to the subsolar point, the simpler formula of projection in one plane applies:

$$Y=\sin\,(\sin^{-1} Y_0-i_0)\ , \tag{11}$$

where Y is the distance in lunar radii from the subsolar meridian, Y_0 the observed distance from the central meridian in the same units, and i_0 the component of the phase angle in right ascension given by $i_0=180°-(\alpha\odot-\alpha☾)$. Y and Y_0 are positive west of the central meridian.

The results of such measurements are plotted in Figure 4. The scale of abscissae gives fractional parts of a radius of the lunar disk, the center of which is on the subsolar meridian indicated in the middle of the figure. The scales of ordinates show the galvanometer deflection B in millimeters due to apparent planetary heat, the radiometric magnitude m_r' of a square second of arc of planetary heat eliminated by the cover-glass, the absolute temperature T, and the energy E emitted by the lunar surface in cal cm^{-2} min^{-1}. The temperatures are computed from the data in Table I.

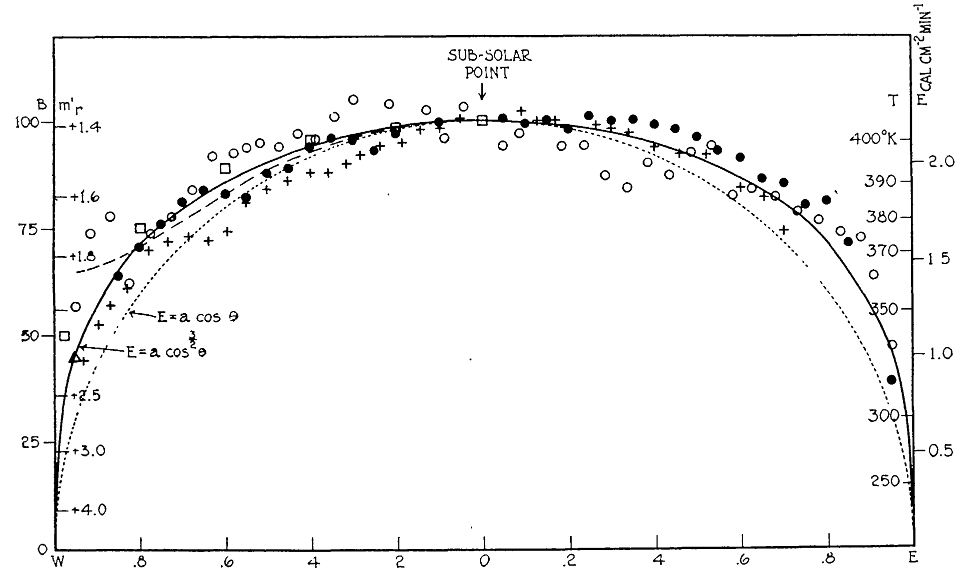


FIG. 4.—Distribution of planetary heat over the disk at full moon

The heavy full line in Figure 4 which represents the plotted observations is also the locus of the equation

$$E=a \cos^{3/2} \theta \,, \tag{12}$$

and, as the observed points show, represents the apparent distribution of planetary heat over the lunar disk. An inspection of Figure 4 shows that from the center to 0.95 radius from the center the temperature varies only between 400° and 320° K, and since Δm_r is nearly constant over this range (Table I), the apparent distribution of radiation over most of the lunar surface at full moon is nearly the real one and is practically represented by equation (12).

The semicircle (dotted line) is that which represents the ideal distribution of planetary heat emitted by a smooth, slowly rotating sphere without atmosphere exposed to sunlight according to the simple formula

$$E = a \cos \theta \, , \tag{13}$$

where θ is the angular distance of a point from the subsolar point and a is a constant, made equal to the radius of the sphere in this plot.

That roughness of the lunar surface will explain qualitatively, in part at least, the deviation of the drift-curve of planetary heat from the cosine-curve is illustrated by the following simple consideration. If the roughness on the moon is due to non-conducting spheres uniformly distributed over its surface, each sphere will present at full moon a distribution of energy approximating the cosine law, and its radiation in the direction of the earth at that phase will be two-thirds that which the surface it occults would have radiated. It is easily seen then that the energy E emitted in an earthward beam of unit cross-section, small compared to the radius of the moon, will be given by

$$E = E_0 \left[\cos \theta + \frac{na}{k} \left(\frac{2}{3 \cos \theta} - 1 \right) \right] \, , \tag{14}$$

where E_0 is the energy from the area k in the beam at the subsolar point, θ the angular distance from the subsolar point, n the number of spheres of sectional area a in the area k (at the subsolar point). This equation will hold to a value $\theta = \theta'$, where the spheres would begin to occult each other. The constant na/k was determined from Figure 4 by inserting the observed value of E for $\cos \theta = 0.6$ in equation (14) and was found to be 0.316. From this it appears that the spheres would be separated 3.39 radii, which gives $\theta' = 53° \, 50'$. The locus of this equation appears in Figure 4 as a broken line. An additional point was found for the case where for one orientation of the field the spheres appear mutually tangent, while for another orientation their maximum occultation is 0.78 radius.

In an extended series of measurements during the lunar eclipse of June 24, 1927, a point 0.05 radius from the south limb of the moon was selected for continuous observation. This gave the point indicated in Figure 4 by a triangle, which agrees fairly well with similar points obtained from the drift-curves above.

DISTRIBUTION OF REFLECTED SOLAR RADIATION OVER THE DISK

Several conclusions concerning the reflected light from the moon follow directly from an examination of curve No. 4 of Figure 3 taken through the water-cell: (*a*) the limb of the moon is 60 per cent brighter than the neighboring surface, which in this case consists of *maria;* (*b*) the center of the moon is 10 to 15 per cent brighter than the limb; (*c*) the brightest point on the E.–W. diameter is on the eastern boundary of Mare Tranquilitatis near longitude 17° W.; (*d*) the *maria* are only three-fourths as bright as the mountainous regions; (*e*) the general trend of the curve is toward uniform brightness, which would be expected from the Lommel-Seeliger or Euler laws of diffuse reflectivity,[1] but not from Lambert's law, which would give a distribution more like that obtained above for planetary heat. A discussion of the laws of diffuse reflection which apply to the moon will be given in another section.

TEMPERATURE OF THE SUBSOLAR POINT

The temperature of the subsolar point has been determined at full moon from the drift-curves described above, and at quarter-phase by observing deflections on the middle of the limb. When the phase angle in the latter case was not exactly 90°, the deflections due to planetary heat were reduced to quarter-phase by equation (13). The results are given in Table II, where m_r° is the radiometric magnitude of the moonlight transmitted by the water-cell and the other symbols have the meanings previously defined.

The solar constant has been reduced to the heliocentric distance of the moon at the time of observation, and the average temperatures to 1.93 cal cm^{-2} min^{-1} by the fourth-power law. That the temperature of the subsolar point measured at first quarter is 49° C. lower than that measured at full moon may be explained by the rough surface. The work of Rosse[2] and of Langley[3] showed that the

[1] Müller, *Photometrie der Gestirne*, p. 68, 1897.

[2] *Philosophical Transactions of the Royal Society*, **163**, 587, 1873.

[3] *Memoirs of the National Academy of Sciences*, **4**, Part II, 107, 1889.

lunar temperature was probably as high as boiling water, and the value of 407° K obtained by us for the subsolar point at full moon is in substantial agreement with that found by Menzel[1] from the water-cell observations of Coblentz,[2] the known visual albedo of the moon being used to interpret the eliminated planetary heat in terms of temperature.

TABLE II

TEMPERATURE OF THE MOON'S SUBSOLAR POINT

Date P.S.T.	Solar Const.	Wc.	m_r°	m_r'	T	E
	Full Moon					
1924 Apr. 19	1.92	2.42	+3.44	+1.37	408°K	....
Apr. 20	1.92	2.50	3.55	1.35	410	....
June 16	....	2.01	....	....	....	....
1927 June 14	1.87	2.00	+3.05	1.44	401	....
1929 Oct. 17*	1.93	....	....	+1.40	405	....
Corrected average	1.93	2.23	+3.35	+1.39	407	2.24
	Quarter-Phase					
1923 Mar. 24†	1.93	2.49	+4.18	+1.69	377	....
July 19†	1.85	2.35	3.99	1.96	354	....
Oct. 14†	1.95	2.30	4.28	2.02	349	....
Nov. 14†	1.98	2.89	3.80	1.80	367	....
1924 June 9†	1.90	2.36	4.20	1.81	366	....
July 24‡	1.88	1.90	+4.05	+2.21	335	....
Corrected average	1.93	2.38	+4.08	+1.92	358	1.36

* Corresponds to Greenwich Civil date, Oct. 18, on Plate VIII.
† First quarter.
‡ Last quarter.

TEMPERATURES DURING A LUNAR ECLIPSE

A continuous series of measurements of lunar radiation was made during the eclipse of June 14, 1927. A point 48″ (0.05 radius) from the south limb was chosen, since this limb passed nearest to the center of the earth's shadow, the distance at mid-totality being 11′ north. The duration of total phase for this point was 2^h40^m,

[1] *Astrophysical Journal*, **58**, 65, 1923.

[2] *Scientific Papers, Bureau of Standards*, **18**, 535 (No. 438), 1922.

and the partial phases were 1^h each. For the point selected the time of mid-totality was 12^h16^m A.M., P.S.T., which made it a favorable eclipse for continuous measurement. Figure 5 shows the circumstances of the eclipse, *P* being the position of the thermocouple junction.

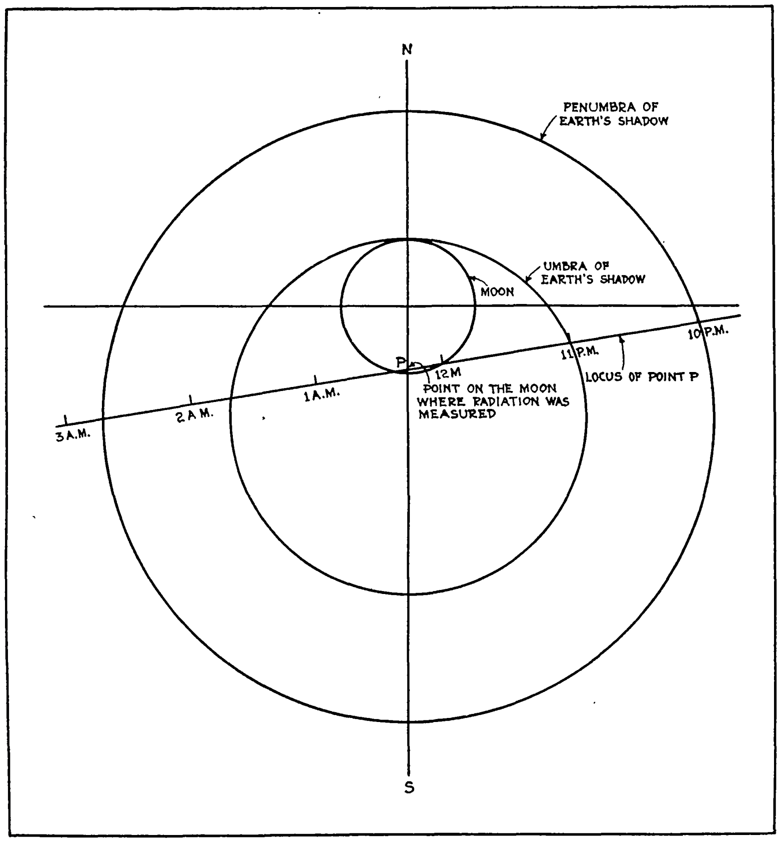


FIG. 5.—Circumstances of the total lunar eclipse of June 14, 1927. The radiation measurements were made at the point *P*.

The program of observation included several drift-curves before and after eclipse, and deflections on and off the south limb for free radiation, and with the cover-glass and the water-cell in the beam, all taken in the usual manner. Arcturus and Vega were used as comparison stars.

The deflections have been reduced to the zenith as previously described. Since the radiation varied greatly during the eclipse, the sensitivity used in the electrical circuit was changed in four steps, so that during totality it was 4.2 times as great as before and after the eclipse. The time required to obtain a single set of deflections for free radiation or for that through water-cell or cover-glass was 3 minutes. To simplify the record in Table III the deflections have been read for the same instant from a plot of the observed deflections against the time.

TABLE III

RADIATION DURING LUNAR ECLIPSE

TIME P.S.T.	OBSERVED DEFLECTIONS			m'_r	T	E	E_R
	Free	Cg	Wc				
	mm	mm	mm			cal cm^{-2}	min^{-1}
9^h22^m*	617	289	191	+2.09	342°	1.13	1.85
10 04	617	289	182	2.09	342	1.13	1.85
10 30	347	166	113	2.74	300	0.67	1.00
10 52	79.5	33.9	20	4.2	231	.24	0.19
11 13	6.46	1.00	0.80	6.4	174	.08	.09
11 34	4.47	0.07	.64	6.7	170	.07	.00
11 47	5.13	.12	.07	6.5	173	.07	.00
12 02†	2.57	.09	.07	7.3	160	.05	.00
12 28	4.08	.01	.14	6.8	169	.07	.00
12 58	1.48	.20	.16	8.0	149	.04	.00
1 22	2.89	0.42	0.20	7.3	160	.05	.00
1 46	8.92	....	6.31	7.6	156	.05	.09
1 54	55.0	31.6	21.9	5.0	205	.15	.28
1 59	100	63.1	39.8	4.6	219	.19	0.48
2 28	513	263	174	2.41	320	0.87	1.63
2 41	617	302	200	2.15	339	1.09	1.85
3 04	603	309	204	+2.23	333	1.02	1.85

* P.M. † A.M.

The values of m'_r were computed from equation (10), and the absolute temperatures, T, were read from Table I. E, the radiation emitted by the moon in cal cm^{-2} min^{-1}, was obtained from the black-body formula by using the temperature T. The energy, E_R, received by the moon from the sun, was calculated from the conditions of the eclipse and the distribution of energy on the solar disk.

The total radiation along a radius of the sun is expressed by the formula given by Jeans:[1]

$$\frac{J}{J_0}=0.465+0.535\sqrt{1-r^2}\,, \tag{15}$$

[1] *Monthly Notices of the Royal Astronomical Society*, **78**, 35, 1917.

where J_0 is the total radiation at the center of the disk and r the distance in radii from the center to the point where the total radiation is J; the constants have been derived from Abbot's data.[1] The total radiation which reaches the moon at any instant as the sun is eclipsed by the earth is the summation over all the uneclipsed area, each element of which is to be weighted according to equation (15). Table IV gives the resulting intensity of solar radiation on the moon as a function of the fraction of a solar diameter occulted by the earth.

TABLE IV

Fraction of Solar Diameter Occulted by Earth	Fraction of Total Solar Radiation Received by Moon
0.1	0.96
.2	.89
.3	.79
.4	.67
.5	.54
.6	.41
.7	.27
.8	.15
0.9	0.05

Figure 6 is a plot of the data in the last three columns of Table III. The full line indicates the absolute temperature, T, of the point P in Figure 5 obtained from the data in Table I. The dotted line represents the solar radiation E_R received, and the broken line the energy E radiated by the moon in cal cm^{-2} min^{-1}.

In general, the fall and rise in temperature lags behind E_R, the energy received by the moon, as might be expected.

The temperature falls from 342° to a value between 200° and 175° K during the first partial phase, and during totality continues to fall slowly, reaching 156° K at the end. For a few minutes after totality the temperature does not change materially, then it rises abruptly during the next 20 minutes. From this it follows that the surface materials of the moon have low heat conductivity.

DISTRIBUTION OF PLANETARY HEAT AND REFLECTED LIGHT ABOUT THE SUBSOLAR POINT

It will now be our purpose to compare the radiation and temperature of the moon as determined from direct observations with

[1] Abbot, *The Sun*, 2d ed., p. 107, 1929.

the thermocouple (Table II) with the values derived theoretically from the solar constant by making suitable allowances for reflected light and conducted heat. Before this can be done the distribution of both reflected light and planetary heat over the hemisphere about the subsolar point must be investigated in much the same manner as we study the distribution of radiation about an electric light. This was accomplished by a continuous series of measurements on the subsolar point made throughout the lunation between first and last quarters in October, 1929.

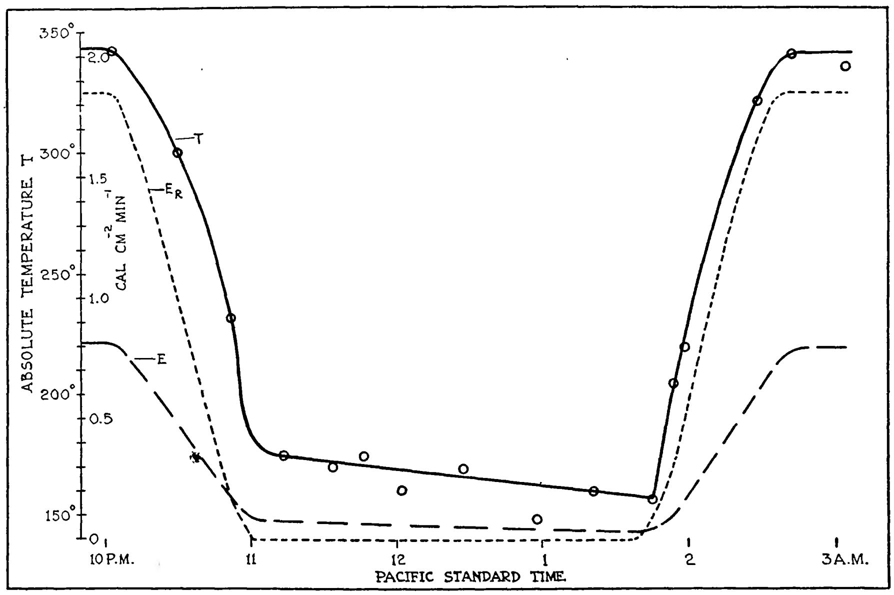


FIG. 6.—March of absolute temperature, T, of energy received from the sun by the moon, E_R, and of energy radiated, E, during the total lunar eclipse of June 14, 1927.

The apparatus employed was a reflecting telescope of 27 inches aperture and 67 inches focal length fed by the coelostat of the Snow telescope.[1] The image of the moon was approximately 15.5 mm in diameter. The deflections, which were of the order of 200 mm for free radiation and 50 mm with the cover-glass, were read visually at intervals during the night. The planetary heat eliminated by the cover-glass and the light transmitted by it were reduced to the zenith by Bouguer's formula in the usual manner.

Simultaneous observations made with the 100-inch telescope at

[1] *Mt. Wilson Contr.*, No. 4; *Astrophysical Journal*, **23**, 6, 1906.

the time of full moon, with suitable comparison stars, furnished the scale of absolute units. The settings of the thermocouple upon the subsolar point were made from the photograph in Plate VIII, upon which were plotted the night-to-night positions of the subsolar

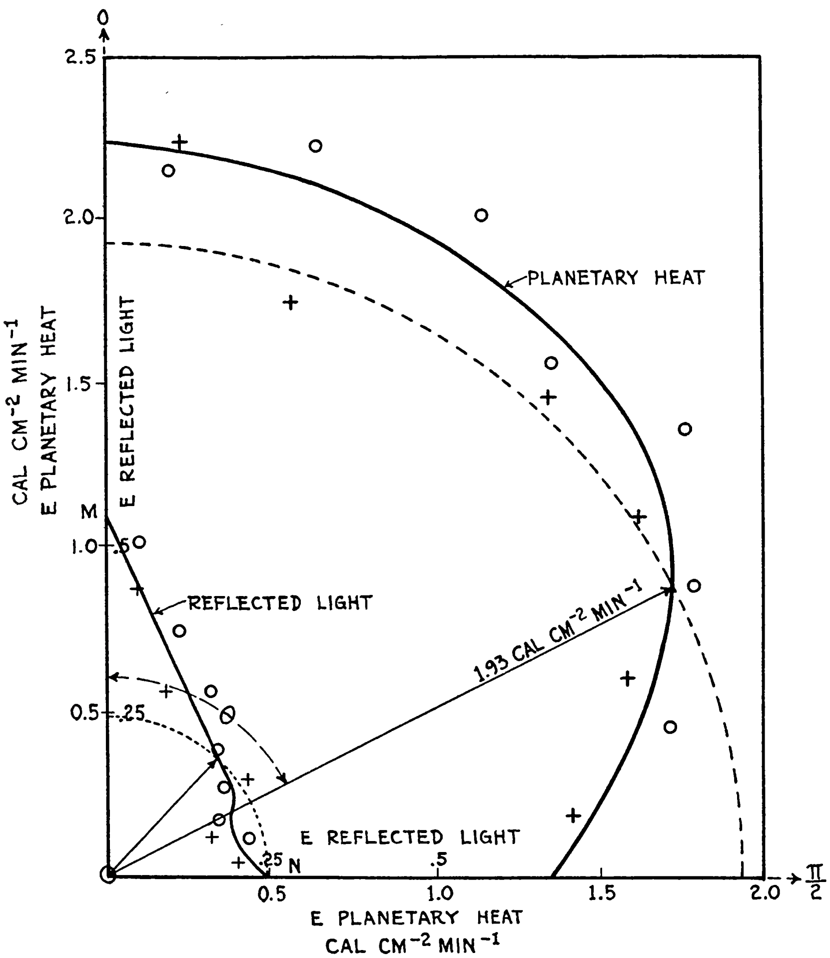


FIG. 7.—Distribution of planetary heat and reflected solar radiation about the subsolar point on the moon.

point. These are shown by the black squares which give the size and position of the thermocouple-receiver for each Greenwich Civil date. The angle between the sun and earth as seen from the subsolar point (which is equivalent to the zenith distance of the earth) was computed by the usual formula. The resulting distribution-curves of planetary heat and reflected light about the subsolar point are plotted in polar co-ordinates in Figure 7. The intensity scales E on the outside of the figure are for planetary heat and on the inside for reflected light outside the atmosphere and were calibrated by the 100-inch measurements in Table II.

Curiously, the intensities of both planetary heat and reflected light from the subsolar point after full moon (indicated by circles) are somewhat greater than the corresponding intensities before full moon (indicated by crosses). As these observations are all on the subsolar point, this difference can have no relation to accumulation of heat by the moon, which might tend to make the maximum temperature of a fixed point on the surface occur in the lunar afternoon.

It will be noted that the values of the planetary heat E in absolute units at full moon and first quarter in Table II agree with the corresponding values read from the distribution-curve of planetary heat for $\theta=0$ and $\theta=\pi/2$ given in Figure 7. The distribution of reflected light shows a sharp drop from full moon ($\theta=0$) to $\theta=75^\circ$, and a definite increase again near quarter-phase ($\theta=\pi/2$).

Simple considerations show that the mean value of the intensity of both planetary heat and reflected light about the subsolar point is the radius of the hemisphere having the volume generated by revolution of the curves in Figure 7 about the axis $\theta=0$. The arc quadrants of these hemispheres and their radii, which represent the mean spherical intensities, are also shown in Figure 7.

THEORETICAL LUNAR TEMPERATURE

The temperature of the subsolar point on the moon may be computed from the solar radiation received, E_R, if that part of it reflected from the lunar surface, E_r, and that part conducted into it, E_c, are known. The radiated energy E can then be found from the equation

$$E_R-E_r-E_c=E=\eta\sigma T^4 . \qquad (16)$$

Here η is the emissivity, which will be taken as unity in the calculation following. The value of E_R is the solar constant for the times of observation, reduced from the mean distance of the sun to the heliocentric distance of the moon by the inverse-square law. The error in the measurement of this quantity arising from uncertainties in the atmospheric transmission is small, since it depends on atmospheric transmissions principally to the violet of λ 3 μ, beyond which there is only about 1 per cent of the solar radiation. These transmissions are easily determined, while those in the region 8–14 μ are determined only with great difficulty.

E_r may be obtained from the average value of m_r° in Table II, which is +3.35 mag. This quantity is the radiometric magnitude of a solid angle of 1 sq. sec. of arc of moonlight from the subsolar point transmitted by the water-cell. Since the energy-curve of moonlight is approximately that of a Ko dwarf star, as has already been pointed out, it would be 0.41 mag. brighter outside the atmosphere; and since the water-cell absorption for a Ko dwarf star is 0.54[1] mag., the radiometric magnitude of 1 sq. sec. of reflected moonlight outside the atmosphere would be +2.40. A similar reduction for the subsolar point at quarter-phase gives +3.13 mag. for a point outside the atmosphere.

From the distribution-curve of reflected light in Figure 7 we find that the mean intensity is 0.85 mag. greater (fainter) than the intensity at full moon, which makes the radiometric magnitude of a square second of reflected moonlight outside the atmosphere +3.25 (mean spherical intensity). This is nearly the observed value at quarter-phase given above, and Figure 7 shows that the actual observed distribution-curve crosses the mean spherical distribution-curve near quarter-phase ($\theta = \pi/2$).

Since the ratio of the whole hemisphere to 1 sq. sec. is −28.57 mag., the magnitude of the total reflected light is −25.32. This, reduced to terms of energy[2] by the equation

$$E_r = 17.3 \times 10^{-12} \times 2.51^{25.32} \text{ cal cm}^{-2} \text{ min}^{-1} , \qquad (17)$$

gives

$$E_r = 0.24 \text{ cal cm}^{-2} \text{ min}^{-1} .$$

The data in Table III may be used to estimate the amount of heat, E_c, conducted into the interior of the moon. Let us assume that before a stable condition, such as existed previous to the eclipse, can be restored, approximately the same amount of heat must be

[1] *Ibid.*, p. 297. [2] *Ibid.*, p. 300.

returned to the moon as was given up by it during the eclipse. Hence, if we interpolate E and E_R from Table III for uniform time-intervals and compute E_c from equation (16), using the temperatures to determine E and various arbitrary values for E_r, until the sum of the values of E_c for the period of the eclipse is zero, we find that to fulfil this condition 52 per cent of the solar energy incident on the area observed must have been reflected, and of the remainder less than 0.1 cal cm^{-2} min^{-1} could have been conducted into the interior of the moon before and after the eclipse. At the subsolar point, where the surface temperature is higher, the conducted heat would be somewhat greater, but even there probably does not exceed 0.1 cal cm^{-2} min^{-1}.

The large value of the reflected light assumed here is borne out by the large fraction of lunar radiation transmitted by the water-cell (0.33) and cover-glass (0.50) shown in Table III, before and after the eclipse.

From the data in Figure 6, Epstein[1] has shown that $(\kappa\rho c)^{-1/2}$, in which κ is the conductivity, ρ the density, and c the specific heat of the lunar crust, is in the neighborhood of 120. This value corresponds to that of pumice or volcanic ash for which the conductivity is about 0.001. This would indicate that at a temperature of 373° K on the moon, the temperature 1 cm below the surface is 100° C. lower, that is, about at the freezing-point of water. It is therefore probably safe to assume that E_c does not exceed 0.1 cal cm^{-2} min^{-1}.

Taking the solar constant on the moon to be 1.95 cal cm^{-2} min^{-1} and applying equation (16), we find

$$\begin{aligned} E &= 1.61 \text{ cal cm}^{-2} \text{ min}^{-1} , \\ T &= 374^\circ \text{ K} . \end{aligned} \tag{18}$$

If E_c is 0.05 cal cm^{-2} min^{-1}, $T=377^\circ$ K, while if E_c is negligible, $T=379^\circ$ K.

Since the solar constant probably varies from 1.91 to 1.965 cal cm^{-2} min^{-1}, and the reciprocal square of the heliocentric distance of the moon ranges from 0.962 to 1.039, E_R may vary from 1.84 to

[1] *Physical Review*, **33**, 269, 1929.

2.04 cal cm^{-2} min^{-1}. If we assume that E_r is proportional to E_R and that E_c is practically constant, E varies from 1.52 to 1.69 cal cm^{-2} min^{-1}, and T for the subsolar point would vary from 368° to 378° K.

From the distribution-curve of planetary heat in Figure 7 we find that the mean spherical intensity which is to be compared with the theoretical value in equation (18) is

$$\begin{aligned} E &= 1.93 \text{ cal cm}^{-2} \text{ min}^{-1} , \\ T &= 391^\circ \text{ K} , \\ m_r' &= +1.54 \text{ mag.} \end{aligned} \tag{19}$$

The planetary heat emitted by the lunar surface is therefore 0.32 cal cm^{-2} min^{-1}, or 20 per cent in excess of that which can be accounted for by the theory, and corresponds to a temperature 17° higher than the solar constant permits after reflection of solar radiation and conducted surface heat on the moon have been deducted.

This result can be explained by any one or by a combination of the following conditions: (1) The solar constant is greater than 1.93 cal cm^{-2} min^{-1}; (2) the emissivity η is greater than unity in the region 8–14 μ; (3) the atmosphere is more transparent in the region 8–14 μ than Fowle's extrapolated coefficients show.

In view of the careful work on the solar constant in spectral regions and under climatic conditions where the atmospheric transmissions which affect the problem are relatively easy to determine, it is hardly possible to allow the increase of 20 per cent necessary to account for the foregoing discrepancy between the theoretical and the observed temperatures of the moon.

As to the second postulate, we have already found, in the section on "Reflection of Solar Radiation of Wave-Length 8–14 μ," evidence that in this region of the spectrum the moon radiates like a black body. The deformation of the energy-curve of the moon necessary to satisfy the observed planetary heat is shown by the dash-and-cross line in Figure 2. We do not regard this deformation as very probable.

This leaves us with the third postulate, which requires that the atmospheric transmissions in the region 8–14 μ be increased about

19 per cent. This can be done easily without seriously encroaching on the transmission-curve for the maximum water-vapor content used in the laboratory determinations and, in view of the large extrapolation from the observed data, would not be regarded as presuming an unreasonable systematic error. Formally then, we compute the magnitude $\Delta m_r'$, which must be added to m_r' in Table I, or to Δm_r, from which m_r' is computed, to make the observed value of T agree with that deduced from the solar constant. To do this it is necessary to assign to the theoretical temperature $T=374°$ K in equation (18) the value $m_r'=1.54$ mag. found from direct measurement in equation (19). The atmospheric transmission-curve in Figure 1 was accordingly symmetrically adjusted between 8 and 14 μ to give this result for a black body at 374° K, and from these data a new set of values of m_r' was computed which appear as corrections, $\Delta m_r'$, to be added to the original values of m_r' in order to make the theoretical and observed temperatures agree. In computing the temperatures of planets with equation (9), $\Delta m_r'$ should be added to m_r' in Table I unless circumstances are found which modify this conclusion.

The effect of $\Delta m_r'$ is to lower the temperature computed from the radiation measurements of Mercury[1] about 45° C., Venus[2] 9°, Mars[3] 10°, and the moon 17°.

REFLECTION OF SOLAR RADIATION FROM THE LUNAR SURFACE

In general, three different theoretical laws of diffuse reflection have been used in discussions of the photometric properties of planets. As applied to apparent solid angles of radiation they are[4]

$$E_1=K_1 \cos i \qquad \text{(Lambert)}\,, \tag{20a}$$

$$E_2=K_2 \frac{\cos i}{\cos i+\cos \theta} \qquad \text{(Lommel-Seeliger)}\,, \tag{20b}$$

$$E_3=K_3 \frac{\cos i}{\cos \theta} \qquad \text{(Euler)}\,, \tag{20c}$$

[1] *Popular Astronomy*, **33**, 299, 1925.

[2] *Ibid.*, **32**, 614, 1924.

[3] *Ibid.*, p. 601, 1924.

[4] Müller, *Photometrie der Gestirne*, p. 68, 1897.

where i is the angle of incidence, θ the angle of reflection, and the K's are constants. We will compare these equations with the observed distribution of reflected light about the subsolar point in Figure 7, and the drift-curve (No. 4 in Figure 3) taken through the water-cell across the full moon.

We may consider the curve of distribution MN about the subsolar point O in Figure 7 to be a straight line. Its intercept N on the axis of $\pi/2$ is 0.46 of the intercept M on the axis of 0°, and since for this curve the angle of incidence i is always zero, the equation which represents it is

$$E'=K_4 \frac{0.46}{0.46 \cos \theta+\sin \theta} . \tag{21}$$

This equation gives us no knowledge as to the effect of varying the angle of incidence i, but we can determine this from drift-curve No. 4 in Figure 3.

The distribution of reflected solar radiation across the disk of the full moon shown in this curve has a general uniform trend, but is high in the middle near the subsolar point, decreases toward the limb, and then rises again at the limb itself. For this curve the values of i and θ are always equal, and their sines are proportional to the abscissae measured from the subsolar point P (Fig. 3). By multiplying E_r' in equation (21) by a power of the secant of a function of i, it may be made to fit both the drift-curve in Figure 3 and the distribution-curve in Figure 7, since i is zero for the latter case. By trial we find that

$$E_r=K \frac{0.46 \sec^2 i/2}{0.46 \cos \theta+\sin \theta} \tag{22}$$

fits both the drift-curve and the distribution-curve fairly well and therefore represents the law of diffuse reflectivity for the moon as derived from these radiometric observations. More complicated functions of i can be found which give better agreement with the observed drift-curve; but as the surface features of the moon probably affect the drift-curve in a manner not entirely dependent on i, such a procedure seems unjustified until more extended observations are obtained.

For $i=0$, equation (22) reduces to the straight line (21) and therefore deviates from the mean curve MN of Figure 7 only in the curved portion of this line. Equation (20*a*) would give a quadrant of a circle of radius OM centered about the origin O. Equation (20*b*) would be a curve intercepting the 0° axis halfway between the origin O and M and the $\pi/2$ axis at a distance from the origin equal to OM. Equation (20*c*) is the locus of a straight line parallel to the axis of $\pi/2$ and passing through M. It is therefore easy to see that equations (20*a*), (20*b*), and (20*c*) do not fit the observed curve at all, while equation (22) agrees fairly well. Table V gives a comparison

TABLE V

INTENSITY OF REFLECTED SUNLIGHT

$i=\theta$	Obs.	Eq. (22)	Eq. (20*a*)	Eq. (20*b*)	Eq. (20*c*)
0°	51	51	50	50	50
15	46	34	48	50	50
30	41	28	43	50	50
45	32	26	35	50	50
60	32	28	25	50	50
75	41	34	13	50	50
90	44	47	0	50	50

of the intensities on an arbitrary scale for various values of $i=\theta$ read from the drift-curve in Figure 3 with those computed from formulae (22), (20*a*), (20*b*), and (20*c*). Here again formula (22) perhaps represents the observations better than formulae (20*a*), (20*b*), and (20*c*.)

RADIOMETRIC MAGNITUDE AND ALBEDO OF THE MOON

Since the radiometric magnitude of a solid angle of 1 sq. sec. of moonlight from the subsolar point at full moon transmitted by the water-cell is +3.35 and the water-cell absorption 0.54 mag., the radiometric magnitude of 1 sq. sec. of reflected full moonlight transmitted by the atmosphere is +2.81. This gives −13.3 for the radiometric magnitude of the reflected light from the whole full moon at mean distance. From the areas under the water-cell and free curves in Figure 3 we find the radiometric magnitude of the whole radiation from the full moon to be −14.8.

The ratio of the solar energy reflected from the subsolar point to that received by it is 0.124. This may be considered as the albedo of the subsolar point but should not be confused with the albedo[1] of the moon, 0.073, which is the ratio of light reflected by the whole hemisphere to that received by it. The albedo, A, may be expressed as

$$A = pq \ ,$$

where p is the ratio of actual brightness of the moon at full phase to that of a self-luminous body of the same size and position, which radiates as much light from each unit of its surface as the moon receives from the sun under normal illumination, and q is a factor depending on the law of diffuse reflection from the moon. For visual light[2] the value of q is 0.694 and $p=0.105$. The ratio p for light transmitted by the earth's atmosphere may be obtained from the equation

$$\log p = 0.4[m_r \text{ (sun)} - m_r'' \text{ (moon)}] + \log \pi - 2 \log \sin 1'' \ .$$

The radiometric magnitude, m_r'', of a solid angle of 1 sq. sec. of the sunlight reflected from the full moon is practically constant over the lunar disk, as may be seen from Figure 3. The mean ordinate of curve No. 4 is 15 per cent lower than at the subsolar point, but since the *maria,* which seem to be responsible for the depressions, cover only one-third of the apparent disk of the full moon, the error in this assumption is probably only 5 per cent, i.e., 0.05 mag. Substituting for m_r'' the value $+2.8$ found above, and for m_r the value -27.18 mag. found for the radiometric magnitude of the sun,[3] we find

$$p = 0.135 \ ;$$

and if q is the same for all wave-lengths the radiometric albedo of the moon is

$$A = 0.093 \ .$$

[1] Russell, *Astrophysical Journal,* **43,** 179, 1916.

[2] *Ibid.*

[3] *Mt. Wilson Contr.,* No. 369; *Astrophysical Journal,* **68,** 308, 1928.

TEMPERATURE OF THE DARK SIDE OF THE MOON

Measurements of the radiation from the dark side of the moon 48″ from the east limb were made on July 5, 1927, when this point was nearly opposite the sun on the midnight meridian. The sensitivity used was such that a star of radiometric magnitude 0 would produce 19 mm deflection. Our result was a deflection of 0.09 ± 0.05 mm. This gives radiometric magnitude 10.6 per square second of arc, which corresponds to a temperature of about 120° K. The temperature corresponding to the deflection *plus* its probable error is 125° K, which may be considered its upper limit. Considerable observing will be required to establish a good value of this temperature.

We wish to acknowledge the assistance of Miss Richmond in the rather involved processes of reduction and also in much of the observing program.

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