J.D.	Mag.	J.D.	Mag.	J.D.	Mag.
[2424]		[2424]		[2424]	
499·35	13·40	523·44	13·13	572·40	<13·0
501·34	13·13	525·41	13·23	578·36	13·67
502·29	12·96	526·25	13·05	584·37	13·67
506·30	12·74±	527·39	13·28	585·38	<13·5
508·27	12·59	528·25	13·31	596·38	<13·67
1926. Observations begun at Herne.		551·36	12·76	600·33	13·57±
		552·37	13·00	602·37	13·52±
519·25	13·08	555·40	13·3 ±	605·33	13·62±
520·26	13·08	560·37	13·28	607·36	13·62
522·25	13·08	563·44	13·64		

The magnitudes of the comparison stars are those given by Seares as used by Barnard and Steavenson.

All observations from J.D. 519 to 585 were made with the 5-inch refractor and should be regarded as second class, as the light grasp of the instrument is hardly adequate to deal satisfactorily with so faint an object.

Civil Date.	Julian Date.	Civil Date.	Julian Date.
1925 July 1	2424 333	1925 Dec. 1	2424 486
Aug. 1	364	1926 Jan. 1	517
Sept. 1	395	Feb. 1	548
Oct. 1	425	Mar. 1	576
Nov. 1	456	Apr. 1	607

Gorse Cliff School,
Herne, Kent.

A New Type of Measuring Engine for Photographic Plates.

By Frank Schlesinger.

The work of reobserving the zones of the *Astronomische Gesellschaft Catalogue*, now in progress at Yale Observatory, involves the identification on the plates of thousands of stars in each zone. To do this quickly and with certainty we have tried a number of devices. The engine here described was developed and constructed to facilitate these identifications, and it leaves little or nothing to be desired for such an application, enabling us to point quickly at an image by means of its right ascension and declination given in the Gesellschaft zone; or conversely to read directly from the plate the right ascension and declination of any object thereon.

We constructed this engine in our shop in rather rough form as this was sufficient for our immediate needs; but the principle involved applies to a more accurate engine, and I shall describe it here as though it were thus constructed.

A photographic plate is a central projection of a portion of the celestial sphere ; that is, points on the plate are obtained by prolonging the radius of the sphere through the corresponding points on the latter's surface, the plate being tangent at a point called its base. Great circles on the sphere are projected into straight lines on the plate. Thus all stars having the same right ascension lie on straight lines which radiate from the projection of the pole upon the plate. In the diagram, B is the base of the plate and P is the projection of the pole, the distance BP being equal to the focal length of the telescope divided by the tangent of the declination (δ) of the base. If S is a star image, X and Y are the usual rectangular co-ordinates referred to B as origin and BP as the axis of Y. The angle A is given by

$$\tan A = X/(\cot \delta - Y).$$

We have also the well-known relation

$$\tan \Delta\alpha = X/(\cos \delta - Y \sin \delta),$$

where $\Delta\alpha$ is the difference in right ascension between S and B ; and con sequently we have rigorously

$$\tan \Delta\alpha = \tan A \operatorname{cosec} \delta \quad . \quad . \quad . \quad (1)$$

or developing to include terms of the sixth order

$$\Delta\alpha = A \operatorname{cosec} \delta - \tfrac{1}{3} A^3 \operatorname{cosec}^3 \delta \cos^2 \delta + \tfrac{1}{15} A^5 \operatorname{cosec}^5 \delta \cos^2 \delta (1 + 2 \cos^2 \delta) \quad (2)$$

We see that A is proportionate to $\Delta\alpha$ except for terms of the third and higher orders.

A parallel of declination on the celestial sphere, that is the locus of points having the same declination, is projected on the plate as a conic ; an ellipse if the sum of the declinations of the base and the star ($\delta + \delta_1$) exceeds 90°, an hyperbola if this sum is less than 90°. The equation of these declination conics is

$$\sin^2 \delta_1 (1 + X^2 + Y^2) = (\sin \delta + Y \cos \delta)^2 \quad . \quad . \quad (3)$$

where δ_1 is the variable parameter. Differentiating this twice with respect to X and remembering that $\frac{dY}{dX}$ is zero when X is zero, we obtain for the radius of curvature at the vertex

$$\rho = \frac{\sin \delta \cos \delta + Y \cos \delta}{\sin^2 \delta_1} - Y \quad . \quad . \quad . \quad (4)$$

Or substituting from (3)

$$\rho = \frac{\cos \delta}{\sin \delta + Y \cos \delta}(1 + Y^2) - Y \quad . \quad . \quad . \quad (5)$$

Consequently the declination conic that passes through the base (Y=0) has its centre of curvature exactly at the projection of the pole upon the plane of the plate.

If, therefore, the positions of stars are measured in polar co-ordinates with P as a centre we obtain close approximations to their right ascensions and declinations.

In the diagram, M is a reading microscope sliding along the rigid arm MP, its position being read on a scale or a long micrometer screw. This arm is pivoted at P, and may be rotated around that point by means of a worm-sector WW and a micrometer worm TH. To locate a star image on the plate the worm is turned and the microscope moved along the arm until the image is bisected by both micrometer wires in M; the readings at M and H are then recorded. The scale on which the position of M is read should be long enough to cover the whole extent of declinations that it is desired to observe, or else this scale and the microscope should be constructed in a unit that can be attached to the rotating arm at any distance from P.

If we define D as the difference between PS and PB, we have

$$\cot\delta - Y = (\cot\delta - D)\cos A \quad . \quad . \quad . \quad (6)$$

or, all fourth order terms included,

$$Y = D + \frac{A^2}{2\tan\delta} - \tfrac{1}{2}DA^2 - \frac{A^4}{24\tan\delta} \quad . \quad . \quad . \quad (7)$$

Combining this with (2) and the well-known development

$$\begin{aligned} Y = {} & \Delta\delta - \tfrac{1}{2}\Delta\alpha^2 \sin\delta\cos\delta + \tfrac{1}{2}\Delta\alpha^2\Delta\delta(\cos^2\delta - \sin^2\delta) \\ & + \tfrac{1}{3}\Delta\delta^3 + \tfrac{1}{12}\Delta\alpha^4(6\cos^2\delta - 1)\sin\delta\cos\delta \\ & - \tfrac{1}{2}\Delta\alpha^2\Delta\delta^2 \sin\delta\cos\delta \quad . \quad . \quad . \quad . \quad . \quad (8) \end{aligned}$$

we obtain

$$\Delta\delta = D - \tfrac{1}{2}DA^2\cot^2\delta - \tfrac{1}{3}D^3 + \tfrac{1}{2}A^2D^2\cot\delta + \tfrac{1}{8}A^4\cot^3\delta \quad . \quad (9)$$

or, all fourth order terms included,

$$\Delta\delta = D - \tfrac{1}{2}X^2Y - \tfrac{1}{3}Y^3 + \tfrac{3}{8}X^4\tan\delta \quad . \quad . \quad (10)$$

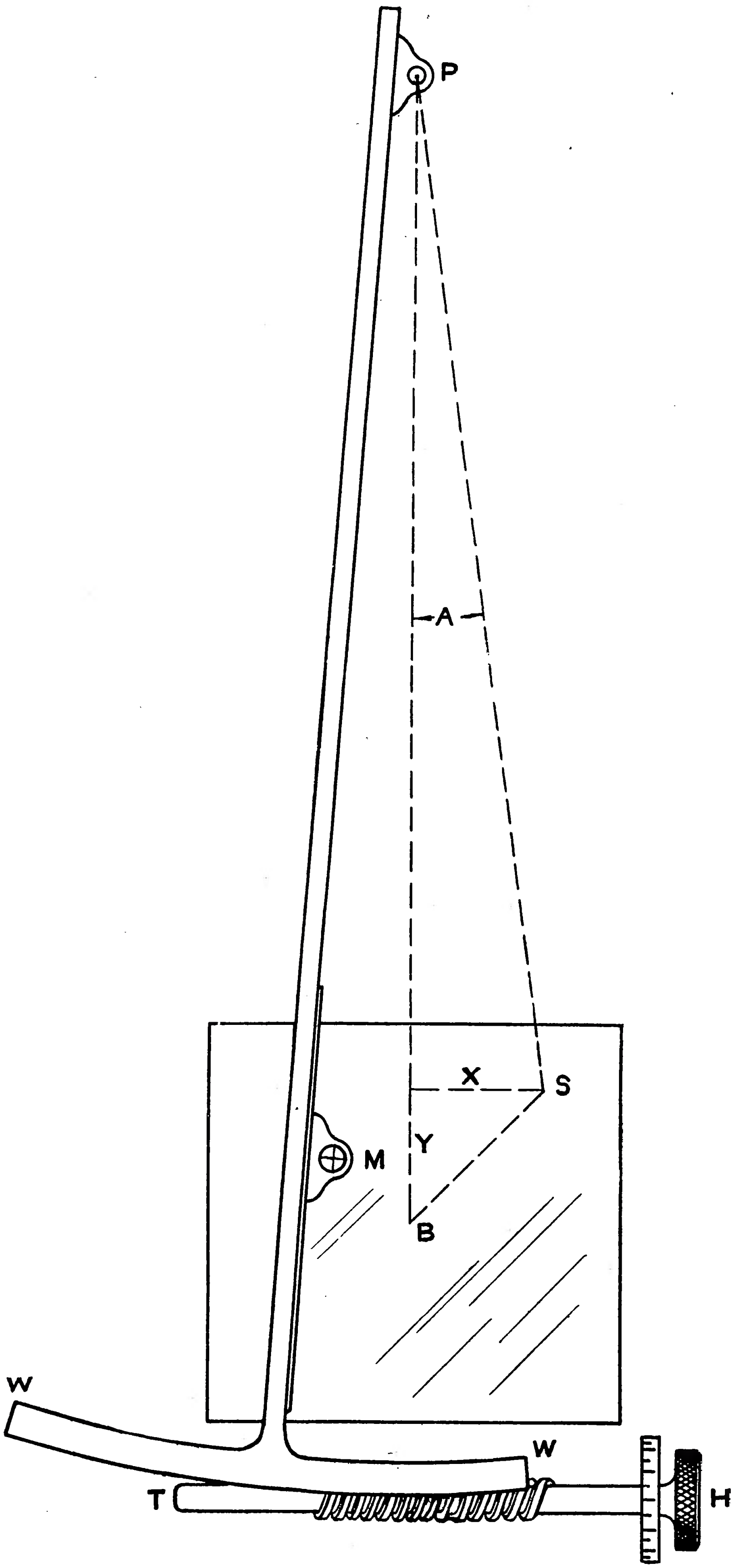
We learn from (9) and (10) that D differs from $\Delta\delta$ only by terms of the third and higher orders.

From (10) we see also that the third order corrections do not contain δ and depend only upon the rectilinear co-ordinates of the stellar image. For plates of the size employed for the Astrographic catalogue the maximum value of X or of Y is 1°, and consequently the third order terms cannot exceed 0″·91. The fourth order term is equal to 0″·0072 tan δ, for X=1°, and may be neglected on plates of this size.

As the third order terms can never exceed 0″·91, they may conveniently be applied by making a diagram on the same scale as the plate itself, showing by contour lines the amount of the correction in various parts of the plate. If this is placed parallel to the plate, a long and light pointer attached to M and reaching to the diagram will automatically indicate the correction, and this can be recorded when the measurement itself is made. It is possible to use the same correction diagram for all the plates taken with any particular camera, but the description of the details of such an arrangement would be out of place here.

Equation (2) may be put in the form, third order terms included,

$$\Delta\alpha = A\,\mathrm{cosec}\,\delta - \tfrac{1}{3}X^3\sec\delta \quad . \quad . \quad . \quad (11)$$


P
A
X
S
Y
M
B
W
W
T
H

The error in right ascension (measured in arc of a great circle) caused by the neglect of third order terms is therefore

$$-\tfrac{1}{3}X^3,$$

which is again independent of the declination and might be read from a correction-diagram like that just described, or from the same diagram with contour lines of a different colour from those used for the declination. For plates of the astrographic size, however, this correction may be omitted. To be sure its maximum amount is $0''\cdot36$, and this is usually more than we should wish to neglect; but if we omit it altogether the least-squares solution (or other process), by which the scale of the plate is determined from the comparison stars, will automatically take into account the greater part of this term. Let us assume that the comparison stars are uniformly distributed on the plate, then the least-square solution will so determine a (the scale-value correction) as to make a minimum the quantity

$$\int_0^{+\cdot01745}(aX-\tfrac{1}{3}X^3)^2dX,$$

the limit being 1°, expressed in radians. Performing the integration, differentiating with respect to a and equating the result to zero, we obtain

$$a=+\cdot0000609,$$

and consequently the neglect of the third order term involves the errors

$$+\cdot0000609X-\tfrac{1}{3}X^3.$$

The amounts of these errors for various values of X are as follows :

X.	Error.	X.	Error.
°	″	°	″
0·0	·00	0·6	+·05
0·1	+·02	0·7	+·03
0·2	+·04	0·8	−·01
0·3	+·06	0·9	−·07
0·4	+·06	1·0	−·14
0·5	+·06		

We see then that on plates of this size we may assume without serious error that A is proportionate to $\Delta\alpha$. This conclusion would not be modified if some other process than least-squares were employed for deducing the scale value from the comparison stars.

The necessity for multiplying A by cosec δ in order to obtain $\Delta\alpha$, involves, in general, no additional labour; for with ordinary engines we must multiply readings on scales and screws by appropriate factors that reduce them to arc or time, and in the present case the two multiplications can be combined into a single operation.

We next inquire what errors are incurred if the centre of rotation P is not at the correct distance from the base of the plate. We do not need to consider in this connection the error due to the uncertainty in determining the position of the base, since this gives rise to errors that are inherent in the plate and do not depend on the method of measurement. We need only study the effect of an uncertainty in the focal length of the telescope at the time the plate was taken. If the error in the focal length is ΔF then the point P is misplaced by

$$\Delta F \cot \delta,$$

and the error in A is

$$\Delta A = A\Delta F + AD \tan \delta \Delta F.$$

The first term is large, but may be disregarded since it is a pure scale-value error. The second term, expressed in seconds of arc and substituting X and Y for A and D, is

$$XY\Delta F \sin \delta \tan \delta.$$

This error is small except for plates in very high declinations ; thus if ΔF is one millimetre, then the amount of this error in the *corners* of astrographic plates is as follows :

$\delta = 40^\circ$	0·010″
50	0·017
60	0·027
70	0·047
80	0·10
85	0·21

The corresponding error in the other co-ordinate D is

$$\Delta D = \Delta F \cot \delta - \frac{\Delta F}{2} \cot \delta\, A^2.$$

The first term is a constant and therefore introduces no real error. The second term may be put into the form

$$-\tfrac{1}{2}\Delta F \tan \delta X^2.$$

It has the following values on the *edges* of plates of the astrographic size, if ΔF is again one millimetre :

$\delta = 40^\circ$	0·008″
50	0·011
60	0·016
70	0·025
80	0·052
85	0·104

We see that on these plates F may be uncertain to the extent of several millimetres unless the plate is one of high declination. It ought always to be possible to determine F within a millimetre. For example, if the distance between two stars whose images appear on opposite edges of the plate is known within 1″, then F can be found within about half a millimetre.

To sum up : *the geometry of this engine is such that right ascensions and declinations can be read off with practically no more labour than is required to secure rectilinear co-ordinates with engines of the ordinary type.*

The actual construction of such an engine allows much choice in the matter of details. For example, the worm-screw may be replaced by an arc of a graduated circle and may be read with a reading microscope ; or, again, an eye-piece scale, such as has been employed at Greenwich, Oxford, and elsewhere to measure astrographic plates, can readily be adopted here.

The greatest disadvantage of this form of engine is its size ; thus if it were employed to measure plates at $\delta=45^\circ$ the length of the arm PM would be equal to the focal length of the telescope. With astrographic instruments, whose length is 3·4 metres, it would hardly be practicable to use the engine at lower declinations than 45° ; and even with the shorter cameras that are now in use at many observatories it would seldom be advisable to measure at declinations lower than 30°. At first sight the use of a long arm appears to introduce the difficulty of changes in temperature. But if the arm is constructed of the same metal and with approximately the same cross-section as that which unites the plate with the socket under P, changes of temperature are made harmless. For example, if iron is used a difference of 1° C. changes the length of a 3·4 metre arm by 0·4 mm. ; as the plate will move by the same amount the effect is equivalent to an error in ΔF, similar to the one that we have discussed. In any case the observer will assure himself, as is customary with all other forms of measuring engines except that in which the reseau is employed, that the plate and scale have maintained their relative position ; this can best be done by measuring several " zero " stars at the beginning and the end of each session of measurement.

Considering the geometrical aspect alone of this type of engine, it is clear that we do not need actually to connect M with P by means of a long arm, but may instead employ a system of links that will permit the point M to move in accordance with the conditions. Whether such systems can be constructed in a way that would be mechanically satisfactory is doubtful, and this question would probably have to be answered by constructing an experimental engine.

I have discussed this engine with special reference to plates of the astrographic size because these are, generally speaking, the largest that are in use for the determination of precise positions. In fact I believe the only important exception to this statement is presented in the work now going on at Yale Observatory where plates of much larger angular dimensions are being employed to reobserve the Gesellschaft stars. With plates considerably larger than the astrographic, this type of engine could not be used without sacrificing something of its accuracy or something of its convenience. However, for the location of objects that cannot or need not be measured with the highest precision (such as asteroid trails and images of comets) this engine could be used to advantage on plates much larger than two degrees square.

Yale University Observatory
1926 *March* 15.

Note on the Radiation from a Pulsating Star. By J. J. M. Reesinck.

(*Communicated by the Secretaries.*)

In M.N., 1926 January, Dr. Jeans considers the radiation from a pulsating star, and arrives at the conclusion that the luminosity and the temperature should be in phase with r. Professor Eddington has demonstrated (*M.N.*, **79**, 17, 1918) that a difference in phase can only occur in the outer layers. An investigation on the atmospheres of the Cepheids will soon be published by the author. Here I make some remarks relating to Dr. Jeans' article.

Dr. Jeans' treatment of the equations (14) and (15) becomes illegitimate in the outer layers. On page 88 it is proved that λ is small. But now Dr. Jeans excludes the possibility of T_2 becoming large of the order of $\frac{1}{\lambda}$. So, in connection with (10), any difference in phase is excluded *a priora*.

At the boundary $\frac{d}{dr}(r^2H)=0$, but r^2H remains positive. Neglecting in (14) and (15) the terms with λ, but not those with λT_2, the equations become :

$$-H\frac{d}{dr}\left[(6+n)\frac{T_1}{T}+4\frac{r_1}{r}-\frac{\rho_1}{\rho}+\frac{\frac{dT_1}{dr}}{\frac{dT}{dr}}\right]=\rho c_v q\lambda T_2$$

$$-H\frac{d}{dr}\left[(6+n)\frac{\lambda T_2}{T}+\frac{\frac{d(\lambda T_2)}{dr}}{\frac{dT}{dr}}\right]=-\rho c_v q T_1+qP\frac{\rho_1}{\rho}.$$

Let x be the optical depth. Suppose, for convenience, that the absorption-coefficient remains constant during the pulsation (although it may vary with x). The equations reduce to :

$$H\frac{d}{dx}\left[3\frac{T_1}{T}+\frac{\frac{dT_1}{dx}}{\frac{dT}{dx}}\right]=-\frac{c_v q}{k}\lambda T_2 \quad . \quad . \quad . \quad (14a)$$

$$H\frac{d}{dx}\left[3\frac{T_2}{T}+\frac{\frac{d(\lambda T_2)}{dx}}{\frac{dT}{dx}}\right]=\frac{c_v q}{k}T_1-\frac{q}{k}\frac{P}{\rho^2}\cdot\rho_1 \quad . \quad . \quad (15a)$$

In an atmosphere we have, according to Schwarzschild, $H=\mu T_b{}^4$,