

ON THE ORBITS OF FOUR SPECTROSCOPIC BINARIES¹

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ABSTRACT

Orbits of spectroscopic binaries.—Four systems, η Orionis A, ϵ Hydrae C, Boss 4247, and B.D.+57°2309 are studied by means of slit spectrograms of one-prism dispersion made for the most part with the large reflectors at the Mount Wilson Observatory.

η Orionis A.—The velocity of the center of mass varies in a *period* of 9.5 years with a semi-amplitude of about 18 km/sec. Beal's earlier estimate of between 9 and 10 years for the period of variation in γ is thus confirmed. If explainable as a triple system, the velocity of the system is about +20 km/sec. The three *masses* would be comparable and of the order of 10 $\odot$ for each as a minimum. Radial velocities from the *D lines* of sodium are *constant*, thus agreeing with Slipher's observations for the H and K lines and exhibiting what has already been found for several other spectroscopic binaries of early B spectrum. Velocities for the triple system, for the D lines, and for the Orion nebula are in fairly good agreement, which may be significant since η Orionis is near the inner edge of the large ring of nebulosity surrounding the Orion nebula.

ϵ Hydrae C.—The orbit found for this member of the well-known visual triple has an *eccentricity* among the *largest* associated with spectroscopic binaries and comparable with that of the orbit of AB of the same system. The period is 9.9 days; the velocity of the system is about 6 km/sec. less than that found for A and B, which may be a consequence of the motion of C about the pair AB.

Boss 4247.—Since the lines of both spectra appear, it has been possible to determine the orbital elements of *both components* of the spectroscopic binary. The period is 2.3 days; the orbits are *circular*, and the two semi-amplitudes of velocity variation are 97.4 and 108.7 km/sec. These data give 1.11 and 0.99 $\odot$ as the values of $m \sin^3 i$ and $m_1 \sin^3 i$, respectively.

B.D.+57°2309.—This star is *imbedded in nebulosity*. Its spectral class is B3. The radial velocity varies in *two periods*: one 5.4 days, the other 225 days. Both orbits are elliptical, with small eccentricities; the semi-amplitudes of velocity variation are 40 and 22 km/sec., respectively. If interpreted as a *triple system*, the velocity of the center of mass would be -17.2 km/sec. The H and K lines are strong and sharp, and are provisionally accepted as showing a *constant* velocity of -19.4 km/sec., which, there is reason to believe, is the velocity of the surrounding nebulosity. Since the difference between this mean and the velocity for the assumed triple system can easily be accounted for by their respective probable errors, equality may be assumed to exist, whence it follows that the *triple system is at rest within the nebulosity*. Some evidence indicates that the D lines also do not oscillate with the lines which furnish the data for the two orbits found.

This paper derives orbits from the variable radial velocities of the four stellar systems listed in Table I. In three cases the data consist wholly of spectrograms made with the one-prism spectrographs attached to the 60- and 100-inch reflectors in Cassegrain form, while in the fourth they also include earlier measures, antedating the Mount Wilson series. The second, third, and fourth

¹ Contributions from the Mount Wilson Observatory, No. 317.

binaries were discovered at this Observatory.¹ The introduction to a previous paper² gives general details concerning instruments, observations, and methods of reduction.

η ORIONIS A AS A TRIPLE SYSTEM

η Orionis (β G.C. 2712) is a stellar system whose complexities have been resolved one by one by various observational methods as astronomical equipment has improved. Sir William Herschel in the latter part of the eighteenth century noted a tenth-magnitude companion, to which Burnham ascribes a proper motion identical with that of η Orionis. In the middle of the last century Dawes found the bright star to be a close double, the magnitudes of the

TABLE I

Star	H.D. No.	Mag.	α 1900	δ 1900	Spectrum
η Orionis A.....	35,411	3.7	5 ^h 19 ^m 4	— 2°29'	B ₁
ϵ Hydrae C.....		7.5	8 41.5	+ 6 47	F ₅
Boss 4247.....	150,682	5.9	16 37.5	+27 7	F ₂ , F ₂
B.D.+57°2309.....	203,025	6.4	21 14.6	+58 10	B ₃

components A and B being 3.7 and 4.9, respectively. Orbital motion has not been detected with certainty, the position angle and distance remaining about 80° and 1'', respectively.

Frost and Adams³ at the Yerkes Observatory found the radial velocity to be variable. About a year later, Adams⁴ published an orbit for the spectroscopic binary which is so nearly circular that it may be described by the period 7.9896 days; the semi-amplitude of velocity variation, 144.75 km/sec.; and the epoch of maximum negative velocity, J.D. 2415723.849. These give a very good representation of the Yerkes observations.

The pair is so close that integrated light from both stars passes into the slit of the spectrograph, but the difference of magnitude is such that the resulting spectrum is essentially that of star A. Its spectral class is B₁.

¹ *Publications of the Astronomical Society of the Pacific*, **36**, 137, 1924.

² *Mt. Wilson Contr.*, No. 201; *Astrophysical Journal*, **53**, 201, 1921.

³ *Publications of the American Astronomical Society*, **1**, 179, 1902.

⁴ *Astrophysical Journal*, **17**, 68, 1903.

In 1909 and again in 1915, a series of spectrograms of η Orionis A was made at the Allegheny Observatory. A. F. Beal¹ measured these and found good agreement with Adams' elements, provided it was assumed that γ , the velocity of the center of mass, varied in a sine curve with a period between nine and ten years. Many of the Allegheny plates showed the spectra of two components, from which Baker² obtained 10.6 and 11.2 $\odot$ as the two mass functions pertaining to the short period.

A half-dozen spectrograms made by Slipher³ at about the same epochs as the Allegheny series gave a constant velocity for the sharp H and K lines of calcium.

Next in order came observations by Stebbins⁴ with a photoelectric cell, which showed η Orionis to be an eclipsing binary with a principal minimum of at least 0.15 mag. below normal light and a secondary minimum of 0.02 or 0.03 mag. Stebbins states that "eclipses last probably less than half a day and seem to occur slightly before the predicted times." The period is so nearly an exact number of days that the complete photometric observation of the star becomes a slow process; but such results will be extremely valuable for the determination of the inclination of the orbit of the spectroscopic binary, and hence of the true semi-major axis and the masses.

A set of thirteen spectrograms was made at the Vienna Observatory in 1920, which Hnatek⁵ combined with the early Yerkes plates for a redetermination of the elements. The results agreed well with those by Adams, the period, however, being increased 0.0034 day.

Still another series of radial velocity determinations has been made by the writer at Mount Wilson about the mean epoch 1925.0, the details for which are given in Table II. Phase is reckoned from J.D. 2415723.849, the epoch found by Adams for maximum negative velocity.

Since both the Allegheny and Vienna plates indicate changes in the elements, it seemed best to discuss all the radial velocities to-

¹ *Publications of the American Astronomical Society*, **3**, 117, 1915.

² *Lick Observatory Bulletins*, **11**, 170, 1925.

³ *Ibid.*

⁴ *Publications of the American Astronomical Society*, **3**, 272, 1916.

⁵ *Astronomische Nachrichten*, **217**, 53, 1922.

gether. The values for the Yerkes, Vienna, and Mount Wilson Observatories were at hand. Dr. Curtis was kind enough to send the twenty-seven determinations comprising the Allegheny series of 1909 and the seventeen for 1915, with permission to make use of them. Dr. V. M. Slipher was so good as to send me his six plates for measurement. These have been used only as a confirmation of the much longer Allegheny series, made at the same epochs.

Each series of velocities was projected back upon Adams' velocity-curve with his period, from which it became apparent that

TABLE II
MOUNT WILSON OBSERVATIONS OF η ORIONIS A

Plate	Date	G.M.T.	Phase	Vel.	Corr. for Variable γ	Corrected Velocity
				km/sec.	km/sec.	km/sec.
γ 12196.....	1923 Oct. 21	0 ^h 28 ^m	0.4982	- 89	+22.5	- 66.5
γ 13034.....	1924 Oct. 17	0 17	3.428	+131	+28.5	+159.5
γ 13048.....	Oct. 18	23 28	5.397	+ 75		+103.5
γ 13060.....	Oct. 19	23 32	6.397	- 19		+ 9.5
γ 13088.....	Nov. 7	23 47	1.439	- 60		- 31.5
γ 13118.....	Dec. 2	19 26	2.291	+ 29		+ 57.5
γ 13124.....	Dec. 3	21 45	3.388	+119		+147.5
γ 13128.....	Dec. 8	19 2	0.285	-122		- 93.5
γ 13140.....	Dec. 15	17 42	7.130	- 93		- 64.5
γ 13171.....	1925 Jan. 6	18 25	5.292	+ 84		+112.5
γ 13173.....	Jan. 6	19 8	5.332	+ 82		+110.5
C 3121.....	Jan. 9	18 34	0.309	-134		-105.5
γ 13195.....	Jan. 10	18 48	1.318	- 77		- 48.5
γ 13769.....	Sept. 27	23 59	5.879	+ 16	+29.5	+ 45.5
C 3587.....	Dec. 2	20 7	7.802	-137		-107.5
γ 3591.....	Dec. 3	20 36	0.834	-118		- 88.5
γ 14052.....	1926 Feb. 24	18 21	3.804	+133		+162.5

only a slight shortening of his period would make all the series agree (except the Vienna observations), with γ alone variable. An increase of 0.0034 day is the minimum change in Adams' period which will reconcile the Yerkes and Vienna observations; and since this change is inadmissible for all the other series, the Vienna observations have been ignored, except in determining the value of γ . The new period, 7.98922 days, which harmonizes the remaining series, is 0.00038 day shorter than Adams' original period. This is not to be interpreted as a real change in the period, but as the improvement contributed by the long interval now available. In qualitative

agreement with this modification is the fact noted by Stebbins that eclipses occur before the epochs predicted with the old period.

With each series plotted against Adams' curve, the value of γ for each set of plates was easily found by the mean departure from Adams' γ . Two series of plates, which have been made since Beal found nine to ten years as the probable period for the change in γ , confirm his estimate, for all the values fall on a smooth curve with $P=9.5$ years, as shown in the lower part of Figure 1. Table III lists the separate values of γ .

The values do not define a velocity-curve with sufficient precision to distinguish between a circular and an elliptic orbit, although, as plotted, some eccentricity is indicated. This uncertainty affects to

TABLE III
VALUES OF γ FOR η ORIONIS A

No.	Epoch	Vel.	Series
		km/sec.	
1.....	1902.4	+35.5	Yerkes
2.....	1909.4	+24.8	Allegheny
3.....	1915.0	+ 9.4	Allegheny
4.....	1920.2	+37.0	Vienna
5.....	1925.0	+ 7.2	Mt. Wilson

some extent the mean γ , which was obtained in the usual way by finding the line which divides the γ -curve into equal areas. This line corresponds to +20 km/sec., which is probably a fair approximation to the velocity of the system, if triple. The semi-amplitude of velocity variation is about 17.5 km/sec.

The ordinates of the curve for γ give the corrections required to reduce each series of observations to Adams' curve. The upper part of Figure 1 shows this curve, the two series of Allegheny observations thus corrected and combined into normal places, and, finally, my individual values similarly corrected for variation in γ . Phase is reckoned from greatest negative velocity. Although minor improvements might still be made, the agreement is satisfactory. The Vienna observations, if plotted, would be well represented by Adams' curve displaced 3.1 days to the right, no adequate explanation for which has been found.

To predict the radial velocity of η Orionis A, it is therefore only necessary to use Adams' elements with the new period, 7.98922 days, and modify the velocity thus found by the difference between Adams' value of γ and that found from the curve in the lower part of Figure 1. This seems the safest procedure, since to give elements for the variation in γ would perhaps suggest a degree of accuracy certainly not at present attainable.

If we assume that the spectroscopic system involves three bodies and that the inclinations of both short- and long-period orbits are

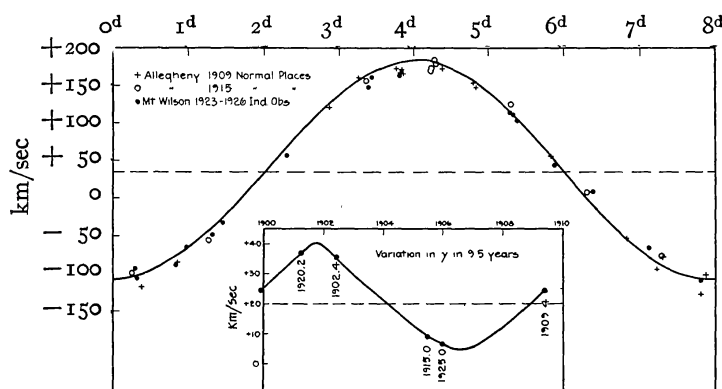


FIG. 1.—Radial velocity curves for η Orionis A

nearly 90° , then it follows that the mass function for the long-period orbit is

$$\frac{M_1^3 \sin^3 i}{(M + M_1)^2} = 1.9 \odot,$$

where M is the combined mass of the two stars whose period is eight days, and M_1 the mass of the third star. But if $i = 90^\circ$, $M = (10.6 + 11.2) \odot$, whence M_1 is of the order of $13 \odot$, so that we appear to be dealing with three comparable masses.

These three components comprise star A of the visual system, magnitude 3.7, which is 1 mag. brighter than the value usually assigned to star B. It is of interest that spectra of two of the stars in A are shown on many spectrograms without serious interference from the spectrum of B. Of course the latter may be fainter than estimated, at least photographically. At any rate, the differences in magnitude must be most favorably distributed to insure the selec-

tion actually found upon spectrograms resulting from the combined light of all four stars. It is hoped that with first-class seeing conditions the spectrum of B may be obtained free from that of A.

Spectroscopic binaries of type B₃, or earlier, are quite numerous in which the H and K lines of calcium are sharp and give velocities which do not seem to vary. In several such cases, Miss Heger¹ has found a similar behavior for the D lines of sodium. Since Slipher finds that the H and K lines in η Orionis A give a constant velocity, I have obtained two plates covering the region of the D lines. The dispersion of the plane-grating spectrograph used for this purpose is 65 Å per millimeter. A neon comparison spectrum was used. From

TABLE IV
 η ORIONIS A—D-LINE VELOCITIES

Plate No.	Date	Computed Vel.	Meas. Vel. D ₃	Meas. Vel. D ₁ and D ₂
G113.....	1926 Jan. 27	km/sec. -137	km/sec. -134	km/sec. +26.8
127.....	Mar. 27	+ 82	+ 51	+ 6.6
Mean...				+16.7

two measures of each plate, I have derived one velocity from D₃ of helium and another from the D lines of sodium, and have also computed the velocity from the elements already found. The results are in Table IV. The agreement between the computed velocities and those from D₃ is quite as good as one could expect from a single rather weak line and such dispersion. Although the phases are such as to give a large difference in the orbital velocities, the results for the D lines are not greatly different, and their mean, +17 km/sec., is not far from +20 km/sec., which has been found for the velocity of the triple system. It is of interest that η Orionis is located on the inner edge of the great ring of nebulosity surrounding the Orion nebula, and that the velocities from the D lines as well as for the triple system are nearly the same as the mean of many determinations for the Orion nebula itself (+17 km/sec.) as published by Campbell and Moore.²

¹ *Lick Observatory Bulletins*, 10, 59, 1919; *ibid.*, p. 141, 1921.

² *Lick Observatory Publications*, 11, 107, 1918.

THE ORBIT OF THE SPECTROSCOPIC BINARY

 ϵ HYDRAE C

The multiple system, ϵ Hydrae (β G.C. 4771), is of more than ordinary interest, for it includes a visual pair whose data were complete enough in 1912 to enable Aitken¹ to derive both its visual and spectrographic orbits, and thus obtain the linear dimensions of the orbit and the parallax. In addition, there is a third star, C, at a distance of 3'', of about 7 mag., which has described an arc of some 40° about AB during the one hundred years since its discovery. The spectrum obtained from the close visual pair (separation 0".23) is F8, and is that of the brighter component A. The spectral class of star C is F5.

To obtain the spectrum of C free from that of AB, it is necessary to keep the image very nearly at the same point on the slit, thereby producing a very narrow spectrum. Furthermore, the seeing must be good; otherwise the enlarged images merge and the spectrum becomes a composite of A, B, and C. Effort has always been made to meet these requirements, but some of the measures may be affected by blending with the brighter star, either because the rate of the driving clock made close guiding very difficult or because the seeing was variable throughout the exposure.

The first two spectrograms early in 1917 gave velocities which differed from each other and from the value of the velocity of the center of mass for AB so much and in such a direction that the binary character of C seemed certain. The seventh and eighth plates, taken in December, 1922, definitely settled the matter, after which the star was placed on a selected list of spectroscopic binaries for regular observation.

In all, forty-four spectrograms were obtained, covering satisfactorily the entire velocity-curve. Less than one-sixth of the period is required for the drop from maximum to minimum velocity; this and the other conditions already referred to delayed considerably the completion of the observations covering this branch of the velocity-curve.

The final adjustment of the observations gave $P = 9^d.9047$, which is judged to be correct within 0.001 day, since any greater change

¹ *Publications of the Astronomical Society of the Pacific*, 24, 216, 1921.

TABLE V
 OBSERVATIONS OF ϵ HYDRAE C

Plate	Date	G.M.T.	Phase	Velocity	O-C	Wt.	Notes
				km/sec.	km/sec.		
γ 5361....	1916 Dec. 12	22 ^h 31 ^m	5 ^d 962	+38.7	- 4	$\frac{1}{2}$	(1)
γ 5419....	1917 Jan. 4	21 54	9.128	+56.7	+ 3	I	
γ 5573....	Feb. 9	20 35	5.454	+43.7	+ 4	I	
γ 7771....	1919 Jan. 15	20 08	7.201	+44.8	- 5	I	
γ 7946....	Mar. 11	20 11	2.774	+20.7	- 2	I	
γ 8038....	Apr. 11	17 15	3.938	+30.2	- 1	I	
γ 11456....	1922 Dec. 5	00 25	0.103	- 5.8	- 1	$\frac{1}{2}$	(2)
γ 11463....	Dec. 5	22 51	1.038	-10.0	- 9	I	
γ 11498....	1923 Jan. 2	22 01	9.193	+43.9	- 9	$\frac{1}{4}$	(2), (3)
γ 11543....	Jan. 8	22 01	5.290	+34.0	- 6	$\frac{1}{2}$	(3), (4)
γ 11576....	Jan. 27	20 18	4.407	+42.2	+ 8	I	
C 2100....	Jan. 31	23 07	8.520	+51.8	- 3	$\frac{1}{4}$	(3), (4)
γ 11616....	Feb. 7	18 35	5.431	+40.6	0	$\frac{1}{4}$	(3), (4)
γ 11625....	Feb. 24	18 39	2.626	+25.8	+ 5	I	
γ 11645....	Mar. 1	20 48	7.711	+46.3	- 6	$\frac{1}{2}$	
γ 11655....	Mar. 5	17 18	1.663	+ 7.4	- 2	I	
γ 11748....	May 2	16 30	0.203	-16.3	- 6	I	
C 2236....	May 4	15 18	2.153	+16.4	+ 1	I	
γ 11759....	May 5	15 45	3.172	+25.4	0	I	
γ 11766....	May 6	15 20	4.154	+26.6	- 6	I	
γ 11772....	May 7	15 48	5.173	+36.5	- 2	I	
C 2280....	May 31	15 42	9.361	+54.9	+ 6	I	
C 2546....	Nov. 22	0 55	5.459	+43.1	+ 2	I	
γ 12304....	Nov. 25	21 35	9.416	+52.6	+ 6	I	
C 2599....	Dec. 28	21 28	2.696	+18.6	- 3	$\frac{1}{2}$	(1), (3)
γ 12367....	1924 Jan. 12	21 52	7.810	+51.7	- 1	I	
C 2606....	Jan. 12	23 54	7.884	+49.0	- 4	$\frac{1}{2}$	(1)
γ 12407....	Jan. 17	22 40	2.937	+27.6	+ 4	I	
C 2643....	Jan. 20	19 46	5.817	+50.4	+ 8	I	
γ 12449....	Jan. 26	18 57	1.878	+ 8.7	- 3	I	
γ 12565....	Apr. 16	15 16	3.486	+22.1	- 6	I	
γ 12578....	Apr. 18	15 21	5.491	+55.7	+15	$\frac{1}{2}$	(2), (3)
γ 13174....	1925 Jan. 6	19 40	1.243	+ 8.3	+ 5	I	
γ 13382....	Apr. 14	15 14	0.012	- 6.8	-10	I	
γ 13417....	May 6	15 43	2.222	+16.8	0	I	
γ 13430....	May 14	15 53	0.324	- 0.9	+12	I	
γ 13996....	1926 Jan. 25	21 54	8.959	+52.1	- 3	I	
γ 14002....	Jan. 26	17 20	9.768	+20.9	+ 2	I	
γ 14006....	Jan. 26	21 21	0.031	- 4.8	- 6	I	
γ 14014....	Jan. 27	21 22	1.031	- 2.9	- 2	I	
C 3677....	Feb. 5	19 08	0.033	+ 8.2	+ 7	I	
γ 14126....	Mar. 27	17 26	0.438	-14.3	- 2	I	
C 3765....	Apr. 25	14 58	9.527	+37.7	+ 4	I	
C 3767....	Apr. 25	16 03	9.572	+29.4	- 1	I	

- (1) Short camera.
 (2) Poor focus.
 (3) Not used in forming the normal places.
 (4) Poor plate.

would seriously upset the assemblage of velocities on the very steep and therefore critical branch of the curve. This element has been assumed to be known.

Because of the uncertainties already alluded to which may affect individual radial velocity determinations, it was deemed best to use normal places for the derivation of the preliminary elements and their subsequent correction by the method of least-squares. The

TABLE VI
NORMAL PLACES FOR ϵ HYDRAE C

No.	Limits of Phase	Phase	Velocity	O—C	Wt.	Plates Included in Normal Place
			km/sec.	km/sec.		
1.....		0 ^d 103	— 5.8	— 0.5	$\frac{1}{2}$	γ 11456
2.....	0 ^d 019–0 ^d 331	.163	— 3.8	+5.1	2	γ 13382, 13430
3.....		.203	—16.3	—6.0	1	γ 11748
4.....		0.438	—14.3	—1.5	1	γ 14126
5.....		1.038	—10.0	—8.7	1	γ 11463
6.....	1.031–1.243	1.137	+ 2.7	+1.6	2	γ 13174, 14014
7.....	1.663–1.878	1.771	+ 8.0	—3.0	2	γ 11655, 12449
8.....	2.153–3.172	2.647	+22.1	+1.0	6	C2236, γ 7946, 11625, 11759, 12407, 13417
9.....	3.486–4.154	3.859	+26.3	—4.3	3	γ 8038, 11766, 12565
10.....	4.407–5.459	5.123	+41.4	+3.1	4	γ 5573, 11576, 11772, C 2546
11.....	5.817–5.962	5.865	+46.5	+4.0	$1\frac{1}{2}$	C2643, γ 5361
12.....	7.201–7.884	7.603	+48.0	—3.7	3	γ 7771, 11645, 12367, C2606
13.....		8.959	+52.1	—3.1	1	γ 13996
14.....		9.128	+56.7	+2.9	1	γ 5419
15.....	9.361–9.416	9.388	+54.8	+7.2	2	C2280, γ 12304
16.....	9.527–9.572	9.550	+33.6	—5.7	2	C3765, C3767
17.....	9.768–0.033	9.881	+ 8.1	+1.0	3	γ 14002, 14006, C3677

data for the forty-four plates are in Table V; the normal places are given in Table VI. Notes to the tables indicate that a few of the spectrograms were not used for the normal places and give the reasons for rejection. Individual observations with phases such that they cover an essentially linear portion of the velocity-curve have been combined to form a normal place. Although there are objections to combining into one normal place observations separated by many periods, that liberty has been taken because the period was already rather well determined.

The preliminary elements used are the mean of sets obtained by the methods of Russell and Lehmann-Filhés. These have been

once corrected by least-squares, which yielded appreciable corrections only in the case of the velocity of the center of mass, γ , and the semi-amplitude of velocity variation, K . The preliminary elements, corrections, and final elements are in Table VII.

The eccentricity, 0.62, is among the very large values for spectroscopic binaries, and is much like the value 0.65 found for both the visual and spectroscopic orbits of components A and B.

If the plane of the orbit of C around AB were normal to the line of sight, we should expect the value of γ for the spectroscopic binary C to be the same as that found by Aitken for AB, provided neither system contained a third body. The velocity of the center of mass

TABLE VII
ELEMENTS FOR ϵ HYDRAE C

	Preliminary Elements	Corrections	Final Elements
P			9 ^d 9047
e	0.62	0.000	0.62
ω	117°7	-0°1	117°6
K	36.2	-1.2	35.0 km/sec.
T	J.D. 2423800.000	+0.007	3800.007
γ	+30.0	+1.2	+31.2 km/sec.
$a \sin i$			3,700,000 km
$m_1^3 \sin^3 i$ ($m+m_1$) ²			0.0206 $\odot$

for C is, however, about 6 km/sec. less than that derived for AB. Campbell's K-term cannot be responsible, since the two spectra are too nearly alike. There may be a third body in either system; to decide this would mean a great deal more observing. But the orbit of C around AB may not be normal to the line of sight, and the velocity difference of 6 km/sec. may be the component of the orbital velocity of C around AB. Assuming this to be the case, and that the observed difference is the component at the descending node, and further, that the angular separation of C from AB converted into linear measure by the parallax is the projected radius of a circular orbit, we find that the period of C about AB would be of the order of six hundred years. The angular rate of revolution of C about AB thus far observed gives six hundred and fifty years as the period, on the assumption of a circular orbit. Hence the difference in the veloci-

ties of the center of mass for C and AB is compatible with the known motion in position angle.

It may be well to point out that all but the first six observations of the radial velocity of C have been made at times when the position angles of B referred to A were in the same quadrant as those of C referred to AB. Further, since aphelion in the system AB occurs in that quadrant, the configuration mentioned persists longer than for B in any one of the other three quadrants. It would be of interest to redetermine the orbit for star C when star B is in the quadrant opposite to C.

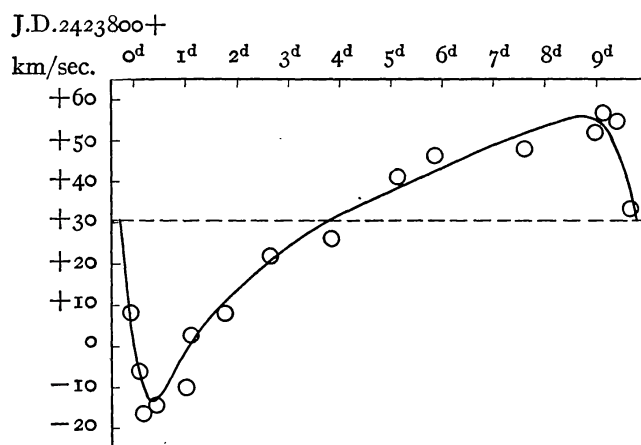


FIG. 2.—Radial velocity curve of ϵ Hydrae C

Table V gives the residuals for the individual observations as scaled from the velocity-curve based upon the final elements. Table VI shows the differences O—C between the velocities for the normal places and those computed by the final elements. When the practical difficulties are considered, the representation in both tables is quite as satisfactory as could be expected. In Figure 2 the normal places only have been plotted with the computed velocity-curve.

Besides stars A, B, and C, there is a fourth star, D, $19''$ distant and of $12.5 \pm$ mag., which has the same proper motion as the brighter stars and is therefore presumably a member of the system. Since its absolute magnitude is some 8 mag. fainter than that of star A, or about $M=9$, it would be of interest to know something of its spec-

trum. It is hoped that a spectrogram may be obtained in the near future with a spectrograph of low dispersion.

TABLE VIII
OBSERVATIONS OF BOSS 4247

PLATE No.	DATE	G.M.T.	PHASE	VELOCITY		O-C	
				Prim.	Second.	Prim.	Second.
				km/sec.	km/sec.	km/sec.	km/sec.
γ 11848....	1923 June 6	19 ^h 46 ^m	od ^d 108	+ 84.0	-105.5	+ 3.3	+11.1
γ 11859....	June 21	21 11	1.323	- 98.7	+ 95.9	+ 1.2	+10.9
γ 11899....	June 28	18 04	1.270	- 99.6	+ 87.5	+ 5.6	- 3.2
γ 11905....	June 29	19 10	0.008	+ 88.0	-122.2	+ 3.2	- 0.8
γ 12004....	Aug. 5	15 20	2.234	+ 81.9	-132.6	- 1.0	-13.5
γ 12087....	Sept. 1	14 52	1.524	- 72.0	+ 50.2	- 7.5	+ 4.8
C 2425....	Sept. 2	14 57	0.219	+ 67.2	-116.2	- 0.7	-13.6
C 2431....	Sept. 4	14 51	2.215	+ 82.0	-113.4	+ 0.3	+ 4.5
C 2432....	Sept. 4	15 05	2.224	+ 81.8	-116.4	- 0.5	+ 2.1
γ 12534....	1924 Mar. 18	22 30	0.080	+ 81.3	-117.9	- 1.2	+ 0.9
γ 12541....	Mar. 19	22 20	1.073	-108.4	+ 82.0	- 0.8	-11.4
γ 12564....	Apr. 15	21 49	0.360	+ 54.0	- 71.8	+12.4	+ 1.3
γ 12569....	Apr. 16	21 20	1.340	- 93.1	+ 93.0	+ 4.7	+10.0
γ 12575....	Apr. 17	21 58	0.058	+ 76.2	-112.8	- 6.3	+ 7.2
C 2753....	Apr. 18	22 08	1.065	-101.7	+ 97.2	+ 5.5	+ 4.2
γ 12640....	May 15	18 39	0.229	+ 58.6	- 98.6	- 7.8	+ 2.3
γ 12646....	May 16	19 26	1.262	-112.6	+ 90.7	- 6.8	- 0.7
γ 12651....	May 17	20 05	2.289	+ 80.6	-118.9	- 4.1	+ 2.3
C 2785....	May 18	22 46	1.093	-109.7	+ 98.0	- 1.1	+ 3.4
γ 12664....	May 19	22 24	2.077	+ 68.0	-102.8	+ 1.8	- 2.2
γ 12738....	June 17	19 50	0.973	- 94.8	+ 85.2	+ 3.7	+ 1.9
γ 12743....	June 18	17 25	1.872	+ 16.8		- 7.2	
γ 12782....	July 13	15 42	1.416	- 95.2	+ 56.4	- 9.0	-13.2
γ 12789....	July 14	15 52	0.116	+ 84.6	-111.4	+ 4.7	+ 4.5
γ 12795....	July 15	16 35	1.145	-112.9	+ 95.6	- 2.9	- 0.6
C 3313....	1925 June 5	18 15	0.842	- 71.8	+ 63.7	+ 5.2	+ 4.3
γ 13438....	June 7	17 32	0.505	- 5.4		-11.7	
γ 13444....	June 7	23 26	0.750	- 67.7	+ 41.3	-10.9	+ 4.5
γ 13445....	June 8	15 30	1.420	- 86.5	+ 61.5	- 0.9	- 7.4
γ 13450....	June 8	23 28	1.752	- 13.8		- 6.8	

ORBITS OF THE TWO COMPONENTS OF THE SPECTRO-
SCOPIC BINARY BOSS 4247

The first spectrogram of this star showed the lines of two stars, both of spectral class about F2, and established its binary character.

Thirty spectrograms with one-prism dispersion were made with the 60- and 100-inch reflectors between June, 1923, and June, 1925. The data for these plates are in Table VIII. Both the phases and the residuals refer to the adopted elements. The period, which was

soon found to be near 2 days, turned out to be 2.3076 days, and satisfactorily represents all observations in an interval which includes about three hundred and thirty revolutions in the orbit. The distribution of velocities would be very seriously affected were this period changed as much as 0.0005 day, which may be taken as an upper limit of the possible error in P . A freehand velocity-curve through the assembled observations was not distinguishable from a sine curve, and the orbit has been assumed to be circular. The elements are therefore period, velocity of the system, semi-amplitude of velocity variation for both components, and an arbitrarily

TABLE IX
ELEMENTS FOR BOSS 4247

	Preliminary Elements	Corrections	Corrected Elements
P			2 ^d 3076
K	97.5	-0.11	97.4 km/sec.
K_1	107.5	+1.22	108.7 km/sec.
T	J.D. 2423923.862	-0.006	3923.856
γ	-12.5	-0.09	-12.6 km/sec.
$(a+a_1) \sin i$			6,540,000 km
$m \sin^3 i$			1.1065 $\odot$
$m_1 \sin^3 i$			0.9915 $\odot$
$\frac{m}{m_1} = \frac{K^2}{K_1^2}$			1.11

chosen epoch, in this case the time when the primary attains its maximum positive velocity. These were determined by the graphical method of Lehmann-Filhés, and once corrected by the method of least-squares. Table IX gives these elements, the corrections, and the corrected elements which have been adopted. The representation is quite satisfactory as attested by the residuals in Table VIII and the fit of the observed velocities to the curve as shown in Figure 3.

The values of $a \sin i$ and $a_1 \sin i$ are 3,114,300 and 3,425,700 km, respectively. If the radii of the two stars are of the order of that of the sun, any inclination between 70° and 90° will result in a partial eclipse twice each revolution at phases corresponding to the intersections of the two curves in the radial velocity diagram. It might prove of interest to test this point by photometric observations.

Measures of the trigonometric parallax of this system are un-

known to me, and the determination of the absolute magnitudes by spectral criteria is made difficult on account of the presence of the two sets of lines.

Boss's proper motion is small. The system appears, however, to consist of two stars not greatly different in absolute magnitude from that of the sun.

$$\text{B.D.} + 57^{\circ} 23' 09''$$

Barnard found this star to be involved in nebulosity, as may be seen by reference to Plate 82, *Publications of the Lick Observatory*, **II**,

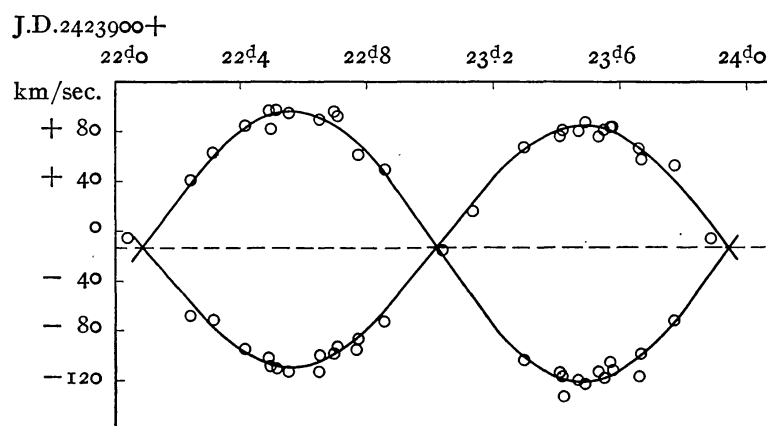


FIG. 3.—Radial velocity curves of Boss 4247

1913, where it appears as the first object listed in the appended table. This led Hubble, in 1920, to observe its spectrum with the one-prism spectrograph and 60-inch reflector. His five plates gave certain evidence of variability in the radial velocity of the star, and velocities from the H and K lines of calcium which did not agree with those from the other lines. He noted that $H\beta$ was an abnormally weak absorption line, and found from objective-prism plates that $H\alpha$ appeared as an emission line.¹ The character of $H\alpha$ has been verified by slit spectrograms.

The star was placed on the program of spectroscopic binaries in 1921, and by the end of 1925 forty-two usable spectrograms had been obtained. These have furnished the data for the investigation

¹ Adams, Joy, and Sanford, *Publications of the Astronomical Society of the Pacific*, **36**, 137, 1924.

of the orbit. Considerable difficulty was encountered in deriving a period until it was noted that observations which evidently established maxima and minima could not be harmonized with a single

TABLE X
NORMAL PLACES; B.D.+57°2309; $P=225^d.44$

No.	Julian Day	Phase	Velocity Deviation from 5-Day Orbit	O-C	Wt.	Nos. from Column 1, Table XII
			km/sec.	km/sec.		
1.....	2424490.13	9 ^d .73	+19	+4	2	40, 41
2.....	3599.42	20.88	+4	-3	4	10, 11, 12, 13
3.....	4514.61	34.31	-8	-4	1	42
4.....	4310.45	55.59	-23	-7	2	30, 31
5.....	3636.63	58.10	-11	+6	3	14, 15, 16
6.....	4339.67	84.81	-19	+4	6	32, 33, 34, 35, 36, 37
7.....	3665.82	87.29	-27	-4	5	17, 18, 19, 20, 21
8.....	3014.72	112.50	-31	-9	1	6
9.....	3696.80	118.26	-34	-12	1	22
10.....	3923.97	119.99	-22	-1	3	27, 28, 29
11.....	3720.73	142.19	-10	+6	4	23, 24, 25, 26
12.....	2609.45	158.11	-12	-2	4	1, 2, 3, 4
13.....	4420.18	165.32	-11	-4	2	38, 39
14.....	3570.64	217.54	+22	+1	3	7, 8, 9

TABLE XI
B.D.+57°2309—ELEMENTS OF LONG-PERIOD VARIATION

	Preliminary Elements	Corrections	Adopted Elements
P			225 ^d .44
e	0.095	+0.131	0.226
ω	0°	+16°6	16°6
K	26.4	-4.5	21.9 km/sec.
T	J.D. 2423797.50	+6.48	3803.98
γ	-4.5	-1.6	-6.1 km/sec.
$a \sin i$			65,988,600 km
$\frac{M_1^3 \sin^3 i}{(M+M_1)^2}$			0.3956 $\odot$

constant period. An attempt to establish a periodicity for these changes indicated a cycle of about 225 days. Further observation seemed to confirm this, so that a tentative curve having a period of 225 days was used as a starting-point for the correction of the observed velocities.

When the corrected velocities were studied, it was found that $P=5^d.41364$ provided an assembly which, in view of the nature of the spectrum from which the velocities were obtained, was fairly satisfactory. As the lines of only one star are visible, and their breadth and diffuseness make them somewhat difficult to measure, the velocities are subject to rather large probable errors. The calcium lines H and K are sharp and fairly strong, however, and special reference will be made to them later. The proximity to $H\epsilon$ makes the H line less satisfactory than the K line.

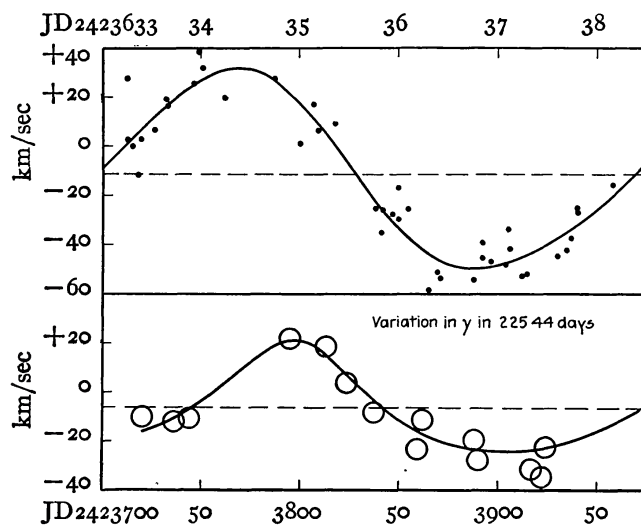


FIG. 4.—BD+57° 2309

The adjustment between the two velocity-curves with periods of 225 days and 5 days, respectively, was one of successive approximations that led finally to $P=225^d.44$ for the long-period oscillation. Normal places for this variation were then formed, which are given in Table X. The weights are the numbers of plates used in forming the normal places. The spectrograms included in each normal place are indicated by the numbers in the last column, which refer to the successive plates in Table XII. The velocity deviations in Table X are residuals from the best approximation that could be gotten for the short-period velocity-curve. The velocity-curve which results from a plot of these data was used to derive preliminary elements by the method of Lehmann-Filhés. One least-squares solution for

TABLE XII
OBSERVATIONS OF B.D.+57°2309

No.	Plate No.	Date	G.M.T.	Phase	Obs. Vel.	Corr. from Long- Per. Var.	Corr. Vel.	O - C	H and K Vel.
		1920			km/sec.	km/sec.	km/sec.	km/sec.	km/sec.
1....	γ 9609	Oct. 2	18 ^h 27 ^m	5 ^d 048	+13.4	+14.5	+27.9	+1	
2....	γ 9619	Oct. 3	18 29	0.635	-39.2	+14.0	-25.2	-3	
3....	γ 9626	Oct. 20	14 58	1.248	-57.6	+6.5	-51.1	-5	
4....	γ 9627	Oct. 20	15 45	1.280	-60.3	+6.5	-53.8	-7	
5....	γ 9717	Nov. 19	17 46	4.296	+42.3	-10.5	+31.8	+5	
		1921							
6....	γ 10651	Nov. 20	17 21	2.152	-75.0	+23.0	-52.0	-8	
		1923							
7....	γ 11814	May 28	23 30	4.210	+45.6	-20.0	+25.6	0	
8....	C 2279	May 30	23 15	0.793	-7.4	-20.0	-27.4	+3	
9....	C 2283	May 31	23 07	1.787	-26.8	-20.0	-46.8	+2	-15.7
10....	C 2325	June 26	22 18	0.685	-26.6	-8.5	-35.1	-10	-13.9
11....	C 2330	June 27	22 30	1.693	-36.8	-8.5	-45.3	+4	
12....	γ 11903	June 28	21 51	2.665	-18.8	-6.0	-24.8	+6	
13....	γ 11908	June 29	21 14	3.569	+6.0	-6.0	0.0	-3	-11.8
14....	γ 11996	Aug. 3	20 16	0.704	-41.8	+16.0	-25.8	0	-11.2
15....	γ 12002	Aug. 4	20 05	1.707	-54.9	+16.0	-38.9	+10	
16....	γ 12007	Aug. 5	19 29	2.672	-43.7	+17.0	-26.7	+5	
17....	γ 12090	Sept. 1	18 15	2.552	-65.6	+23.5	-42.1	-7	
18....	γ 12091	Sept. 1	19 13	2.593	-60.6	+23.5	-37.1	-4	
19....	C 2428	Sept. 2	19 56	3.622	-35.5	+23.5	-12.0	-16	*
20....	C 2429	Sept. 2	20 42	3.655	-21.1	+23.5	+2.4	-3	-14.2
21....	C 2435	Sept. 4	20 27	0.230	-14.2	+23.5	+9.3	+8	-16.1
22....	C 2478	Oct. 3	19 5	2.105	-74.6	+22.0	-52.6	-7	-15.5
23....	C 2493	Oct. 21	17 40	3.805	-11.2	+18.0	+6.8	-5	-28.5
24....	γ 12245	Oct. 28	18 20	0.006	+1.3	+16.0	+17.3	+6	+4.2
25....	γ 12249	Oct. 29	16 44	0.939	-41.5	+16.0	-25.5	+11	-18.0
26....	γ 12257	Oct. 30	16 59	1.950	-49.3	+15.5	-33.8	+12	-38.6
		1924							
27....	γ 12649	May 16	23 31	0.856	-51.6	+22.0	-29.6	+3	
28....	γ 12653	May 17	23 32	1.918	-70.0	+22.0	-48.0	-1	
29....	C 2792	May 19	23 56	3.934	-4.5	+21.5	+17.0	0	-16.6
		1925							
30....	γ 13443	June 7	22 52	3.522	-13.0	+15.5	+2.5	+2	+2.3
31....	γ 13449	June 8	22 50	4.520	+3.8	+16.0	+19.8	-11	
32....	γ 13518	July 5	18 18	4.263	+15.9	+23.0	+38.9	+12	
33....	γ 13524	July 6	18 28	5.270	-22.2	+23.0	+0.8	-16	
34....	γ 13527	July 6	22 50	0.038	-16.5	+23.0	+6.5	-3	
35....	γ 13531	July 7	17 46	0.827	-40.4	+23.5	-16.9	+16	
36....	γ 13539	July 8	21 18	1.974	-65.1	+23.5	-41.6	+5	
37....	γ 13545	July 9	22 14	3.013	-38.6	+23.5	-15.1	+4	
38....	γ 13738	Sept. 24	14 56	3.918	+11.1	+8.0	+19.1	+3	
39....	γ 13760	Sept. 27	17 39	1.617	-60.8	+6.5	-54.3	-5	
40....	C 3588	Dec. 3	14 42	3.532	+43.8	-16.0	+27.8	+27	-34.4
41....	γ 13912	Dec. 6	15 20	1.144	-44.6	-14.5	-59.1	-15	
42....	γ 13942	Dec. 29	14 34	2.458	-46.8	+2.5	-44.3	-7	

* Red sensitive plate.

corrections to these elements was made, and Table XI gives preliminary elements, corrections, and corrected elements. The element called γ in this case must be considered as a correction to the γ corresponding to the short-period orbit, so that the velocity of the triple system is the sum of these two gammas.

The lower part of Figure 4 shows the curve defined by the corrected elements of Table XI and the normal points from which it was derived. It is doubtful whether these elements are quite right, but the limited data hardly warrant further refinement.

TABLE XIII
NORMAL PLACES; B.D. $+57^{\circ}2309$; $P=5^d41364$

No.	Phase	Velocity Freed from Long-Period Oscillation	O-C	Wt.	Plate Nos. from Table XII
		km/sec.	km/sec.		
1.....	0 ^d 118	+13.3	+ 7	2	21, 24
2.....	0.795	-28.7	+ 2	5	8, 10, 14, 25, 27
3.....	0.986	-38.0	- 1	2	35, 41
4.....	1.054	-43.4	- 3	3	2, 3, 4
5.....	1.796	-48.0	0	2	36, 39
6.....	1.860	-44.2	+ 4	6	9, 11, 15, 22, 26, 28
7.....	2.152	-52.0	- 8	1	6
8.....	2.620	-32.7	0	4	12, 16, 17, 18
9.....	2.740	-29.7	- 1	2	37, 42
10.....	3.693	- 0.7	- 8	4	13, 19, 20, 23
11.....	3.758	+ 9.8	0	2	29, 30
12.....	4.088	+26.4	+ 6	4	31, 32, 38, 40
13.....	4.548	+28.4	- 2	3	1, 5, 7
14.....	5.391	+ 3.6	-10	2	33, 34

The lower curve in Figure 4 was then used to derive the corrections to the observed velocities to free them from the superposed long-period variation. Table XII gives the various data for the forty-two plates. The phases depend on the finally adopted elements of the short-period orbit; the corrections in the sixth column are those just referred to. The residuals are scaled from the upper part of Figure 4.

The assembly of the corrected velocities based on $P=5^d41364$ was so satisfactory that the period could be assumed to be known.

On account of the rather low weight for a single observation, it seemed best to form normal places, a procedure which also greatly

diminished the labor of correcting the preliminary elements. These normal places are given in Table XIII. The weights are equal to the numbers of plates forming a normal. The phase and residuals refer to the final elements.

The freehand curve drawn through the plot of the normal places of Table XIII, assembled with the period 5.41364 days, gave by Russell's method a set of preliminary elements, which, as in the other case, were once corrected by least-squares. Table XIV gives the preliminary elements, corrections, and corrected elements. In this

TABLE XIV
B.D.+57°2309—ELEMENTS OF SHORT-PERIOD VARIATION

	Preliminary Elements	Corrections	Adopted Elements
P			5 ^d 41364
e	0.10	+0.014	0.114
ω	63°5	-5°0	58°5
K	41.3	-1.3	40.0 km/sec.
T	J.D. 2423635.137	+0.003	3635.140
γ	-11.3	+0.2	-11.1 km/sec.
$a \sin i$			2,959,900 km
$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$			0.0353 $\odot$

case, the corrections are satisfactorily small. By combining the γ thus derived with that from Table XI, the velocity of the triple system is found to be -17.2 km/sec.

The values for $a \sin i$ and the mass function have both been added to Tables XI and XIV. In the former M_1 is the mass of the third body, and a and K refer to the semi-major axis and projected semi-amplitude of the velocity variation of the pair of stars forming the short-period system about the center of gravity of the triple system; and M is the combined mass of the two stars forming the short-period system and therefore the sum of m and m_1 which appear in Table XIV. If we assume that the mass of the primary in the short-period system is twice that of the secondary and that i , the inclination of the orbit plane to the plane normal to the line of sight, is the same for both orbits, it is readily shown that we are dealing with moderate and entirely reasonable masses, whatever the value of i between 30° and 90°. Further discussion of this point

is hardly warranted, since the assumptions have no observational foundation.

The last column of Table XII gives the velocities derived from the H and K lines of calcium whenever these lines have been of satisfactory quality for measurement. There is considerable range in these velocities; but it has not been possible to find any smooth variation based on either the long or short period or a combination of the two. This does not preclude the possibility of periodicity, but the present data can be just as well represented by an invariable velocity whose apparent variability is only accidental. Hence the weighted mean is used as the best value of the velocity based upon the H and K lines, and this will probably not be far from the truth even if there is a periodic variation. This mean is -19.4 km/sec., which is not greatly different from the velocity of the triple system, -17.2 km/sec., or from the reflex of the solar motion in the direction of this system, namely, -15 km/sec. If it may be identified with the velocity of the triple system, which seems reasonable, and if the source of the H and K absorption is assumed to be in the immediate vicinity of the system, which is closely surrounded by nebulosity, it would be natural to infer that the velocity of the triple system is also that of the nebulosity, and that such nebulosity has ionized calcium as one of its constituents. So far as known, the spectrum of the nebulosity is continuous in character. The sodium lines D_1 and D_2 are present, a point of interest, because, except when these two lines manifest a behavior similar to the detached H and K lines, they do not ordinarily appear until later in the stellar spectral sequence.

Since, as already mentioned in connection with η Orionis A, several spectroscopic binaries which have detached H and K lines also have detached sodium lines, a spectrogram covering the D region of B.D. $+57^\circ 2309$ was made on June 22, 1926. The photographic density is such that the spectrum can be measured from K of calcium to the D lines. The strength of the D lines is in itself sufficient to show that these lines are of the detached variety, since, otherwise, they would be almost vanishingly weak, inasmuch as the spectrum of the star is B₃. The measured velocities also support this conclusion. The epoch was so chosen that the star's velocity was at

the maximum for both the short and long periods, and hence deviated a maximum amount from the velocity of the triple system. The velocities for the K line and for the D lines and the mean velocity from the remaining lines are -2 , -10 , and $+60$ km/sec., respectively. The third velocity is to be compared with a computed velocity of $+50$ km/sec. The D-line velocity certainly differs markedly from this value, and is of the order of the previously determined mean for H and K and of the value for the single K line on the same plate. The evidence from this single plate is therefore for a similarity in behavior between the sodium lines and the H and K lines. In order to settle the question definitely, however, more plates are needed, especially such as have a greater precision for the D lines.

MOUNT WILSON OBSERVATORY
July 1926