PHYSICAL THEORY OF METEORS

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ABSTRACT

Heating of a meteor by molecular impacts.—When the *free path* of air molecules is not small compared to the dimensions of the meteor, the ideas of *fluid motion* are inapplicable, and the "friction" of the air is seen as a succession of individual molecular *impacts*. Assuming this input of energy to be balanced by radiation losses, the temperature may be calculated when the speed and the composition of the air are known.

Height of appearance.—When the *evaporation* of the solid body becomes *copious*, the *accession of energy* is much *increased* because of the greater effective size of the gas mantle. The meteor then becomes visible. The theory is shown to give results in good agreement with observed *heights* of appearance and with the data concerning the upper air which are furnished by the meteorologists.

Criticism of theory of Lindemann and Dobson.—The theory of these writers rests on the assumption of *adiabatic compression* of the air by the moving meteor. It leads to the assumption of pressures and temperatures in the upper air far higher than have been generally accepted. Their use of thermodynamic concepts is here seen to be unjustifiable, and hence their conclusions unwarranted.

Origin of Meteors.—That a large proportion of meteors come from *outside* the solar system is *contrary* to this theory, the *hyperbolic* velocities giving a height of appearance much greater than that of the Leonids, and such are relatively rare.

In the following paper an attempt is made to present a connected account, in physical terms, of what happens when a solid body enters the upper atmosphere of the earth at high speed. Such bodies may differ in composition, size, shape, and in their speed and the inclination of their path to the vertical. If to this list we add the composition and physical state of the upper air, we have enumerated all the independent variables at our disposal. In terms of these we must express the heights of appearance and disappearance, the kind and amount of radiated energy, and a number of minor peculiarities which the phenomena exhibit. The theory here offered can make no pretensions to affording a complete answer to all these questions, but it appears to account for all of them at least qualitatively, while showing good quantitative agreement with observation on those points at which such a test is possible. Since it has further the advantage of requiring no extensive modification of our current notions (particularly regarding the upper air), it is hoped that it will commend itself to astronomers and physicists as a basis for further work.

Before entering upon the development of the theory itself, we must clear the ground by a discussion of a few facts and data which enter into it. Of these the first in importance relate to the upper air. Table I, taken from W. J. Humphreys' *Physics of the Air*, may be regarded as a summary of the prevailing opinion of the meteorologists on this subject. The figures give the percentage composition and total pressure.

TABLE I

PERCENTAGE DISTRIBUTION OF GASES IN THE ATMOSPHERE

Height in km	Gases							Total Pressure in mm
	A	N_2	H_2O	O_2	CO_2	H_2	*He*	
140	....	0.01	....	....	....	99.15	0.84	0.0040
130	....	.04	....	....	....	99.00	0.96	.0046
120	....	.19	....	....	....	98.74	1.07	.0052
110	....	0.67	0.02	0.02	....	98.10	1.19	.0059
100	....	2.95	.05	.11	....	95.58	1.31	.0067
90	....	9.78	.10	0.49	....	88.28	1.35	.0081
80	....	32.18	.17	1.85	....	64.70	1.10	.0123
70	0.03	61.83	.20	4.72	....	32.61	0.61	.0274
60	.03	81.22	.15	7.69	....	10.68	.23	.0935
50	.12	86.78	.10	10.17	....	2.76	.07	0.403
40	.22	86.42	.06	12.61	....	0.67	.02	1.84
30	.35	84.26	.03	15.18	0.01	.16	0.01	8.63
20	.59	81.24	.02	18.10	.01	.04	....	40.99
15	.77	79.52	.01	19.66	.02	.02	....	89.66
11	.94	78.02	.01	20.99	.03	.01	....	168.00
5	.94	77.89	0.18	20.95	.03	.01	....	405.
0	0.93	77.08	1.20	20.75	0.03	0.01	....	760.

Our direct knowledge of the upper air extends only to about 30 km. For greater heights, the figures of Table I are an extrapolation based on the following assumptions: up to 11 km there is convective equilibrium, with mixing and consequent uniformity of composition (save for the diminishing water-vapor), and a temperature which falls uniformly with the height to the value 219° K. at 11 km. Above this point the temperature is assumed constant; there is no mixing, each constituent being distributed independently of the others, so that the whole may be regarded as a system of independent and interpenetrating atmospheres. The table being based on mean values allows a certain range of variation on account of seasonal and local conditions. Thus, at the equator, the tabulated value for 140 km would correspond to about 145 km.

The extrapolation from 30 to 140 km, involving an assumed diminution of pressure to about 1/2000 of the last experimental value, appears at first sight to be a very tenuous thread on which to hang any quantitative reasoning. It must be remembered, however, that this extrapolation is not empirical, but is based on a theory which has been verified between 11 and 30 km. On one point only have the data up to this height been seriously questioned; this point concerns the amount of hydrogen. The value given, 1 part by volume in 10,000 at sea-level, is that usually adopted; but Lord Rayleigh finds the amount to be less than 1 part in 30,000, Claude less than 1 in 35,000, while some deny its presence altogether. As the theory to be given throws some light on the question, we will defer a more detailed discussion,[1] but one point should be made clear at once. If we accept the view that there is no convection at high altitudes, the total amount of other gases present will be unaffected by the presence or absence of the hydrogen. Thus, if it be assumed altogether lacking, we should have, on the hypothesis underlying the table, an atmosphere which at 140 km would consist of about 1.2 per cent nitrogen and 98.8 per cent helium, with a total pressure of about 3×10^{-5} mm. If the helium also is denied, there would be practically pure nitrogen, but its total pressure would be only 4×10^{-7} mm. It is practically certain that meteors can appear as high as 160 km, at which the pressure would fall to about 1/25 of this value. Any substantial increase in the amount of nitrogen at these heights involves the assumption of higher temperatures. Though the assumptions of the meteorologist are intrinsically reasonable, the right to challenge them on the basis of other evidence cannot be denied. We are not, however, at liberty to play fast and loose, and to assign temperatures and pressures to suit our fancy; for the gas law and the hydrostatic equation impose limitations which we may not abrogate.

To see how the matter stands, we start from the two equations

$$p=R\rho T\,; \qquad \frac{dp}{dh}=-\rho g\,,$$

[1] For detailed references see Jeans, *Dynamical Theory of Gases* (3d ed.), p. 340 n.

from which, eliminating ρ, we get

$$\frac{dp}{dh}=-\frac{gp}{RT} \text{ or } \log\frac{p}{p_0}=-\frac{g}{R}\int_{h_0}^{h}\frac{dh}{T}\,.$$

If T is given as a function of h, the pressure at any height is determinate. As T is essentially positive, the absolute value of the integral on the right is greater than $(h-h_0)/T_{max}$, and hence

$$\frac{p}{p_0}\leq e^{-\frac{g(h-h_0)}{RT_{max}}}\,,$$

or, in other words, the actual pressure under a variable temperature will be less than that calculated on the assumption of an isothermal distribution at the maximum temperature existing below it. Since for constant T the density varies as the pressure, the density will satisfy the same inequality.

The form of the equation lends itself to easy orientation among the figures without extensive calculation, for we see that we can use the figures of Humphreys' table simply by changing the scale of heights proportionately to the absolute temperature. Assume, for instance, that while the tabulated values are correct up to 30 km, above this height the temperature changes so that the pressures and densities which it gives for 110 km actually exist at 150 km. The maximum temperature will then be at least as great as $219\times(150-30)/(110-30)$, or 328° K.

A theory of meteors was put forward a few years ago by Lindemann and Dobson[1] in which they reach conclusions regarding the density of nitrogen at high altitudes that are much at variance with current assumptions. Some of the reasoning on which these conclusions are based will be criticized later, but the writer finds himself unable to verify even the consistency of those conclusions. They assume, apparently, a temperature of about 300° K. The exact figures for their assumed density are not explicitly given. If the writer understands rightly the diagram on page 427 of their paper cited above, the value 219° K. is assumed to hold for T up to 40 or

[1] *Proceedings of the Royal Society of London* (A), **102,** 411, 1921–22.

50 km and the density at 150 km is assumed about equal to the accepted value at 90 km. This, however, would give a temperature of at least $219 \times (150-40)/(90-40)$, or 480° K. To turn the calculation the other way: a temperature of 300° K. would permit at 150 km a density not greater than that tabulated for 120 km; an increase by a factor of about 90, instead of the factor of 1000 which they give on page 428. Lindemann and Dobson appear either to have made an error in their calculations, or to have stated their conclusions in a form susceptible of misinterpretation.

This rather lengthy discussion of the atmospheric problem seems justified by its fundamental importance, and by the doubt which has been cast on the views hitherto prevailing. We assume in the calculations to follow, the substantial correctness of Humphreys' table, with a possible exception as to the amount of hydrogen, for which it is felt that a reduction of the assumed amount would be amply justified by existing evidence, should the theory seem to require it. The table in its present form does not lend itself, however, to easy use in calculation, and is more conveniently replaced by Table II, which gives the ratio of the density at a given height to the standard density. The minor constituents have been omitted, and the range of heights is extended to 200 km.

The relevant facts regarding meteors themselves may be more briefly considered. The composition varies considerably, the siderites being almost pure iron, while the aerolites show a great variety of minerals similar, but not always identical with, those constituting terrestrial rocks. The smaller the meteor, the greater is the presumption of homogeneity, but the particular mineral composing it is unknown to us, and the writer has not been able to find any data bearing on the behavior of meteoric minerals at high temperatures. It would be very convenient if the iron type could be assumed as the usual thing; but the statistics of recorded falls indicate the relative rarity of this type. The data to be used will be introduced as needed; as these values are largely guesses, the usual procedure will be to attempt an estimate of upper and lower limits, and to make alternative calculations.

The size of meteors may of course vary enormously. A lower limit is obtained by equating the total radiated energy to the original kinetic energy of the particle. On the plausible view that the actual size is not a large multiple of this it has been concluded that the diameter of an ordinary shooting star, of magnitude 0 or fainter, does not exceed a few millimeters. We shall attempt later a more precise estimate, but this rough guess is sufficient for present purposes.

The speed is usually calculated on the assumption of a parabolic orbit about the sun, which gives a possible range of speeds from 16 to 72 km/sec. Another set of figures is obtained by dividing the length

TABLE II

VALUES OF $\frac{\rho}{\rho_0} \times 10^6$

HEIGHT IN KM	GASES		
	Oxygen	Nitrogen	Hydrogen
200	....	....	3.55
190	....	....	3.96
180	....	....	4.41
170	....	....	4.91
160	....	....	5.48
150	....	0.0002	6.10
140	....	.0008	6.80
130	0.0001	.0034	7.58
120	.0004	.0154	8.45
110	.0022	.0697	9.42
100	.0125	0.315	10.5
90	.0702	1.43	11.7
80	0.393	6.45	13.0
70	2.21	29.2	14.5
60	12.4	132.	16.2
50	69.4	597.	18.1
40	390.	2701.	20.1
30	2185.	12220.	22.4
20	12252.	55280.	25.0

of the path by its estimated duration. The results of these direct determinations show a considerable divergence from the theory; the values ranging from 10 to 160 km/sec. As the duration of visibility is usually a retrospective estimate, made by an observer who is attempting to note at the same time many other features of the phenomenon, it is at least conceivable that such subjective estimates are affected by very large systematic errors. Until such estimates can be checked by instrumental determinations it seems hardly safe to build upon them; a number of writers have, nevertheless, concluded that a large proportion of the observed meteors come from outside the solar system, and therefore move in hyperbolic orbits. We shall later on present some arguments which seem to make this supposition doubtful; but, in order not to prejudge the matter, a range of speeds from 10 to 120 km/sec. has been assumed in the calculations. The material for testing the theory, however, has been taken entirely from meteors belonging to definite showers; these being members of our own system, their velocities are deducible theoretically.

The height of appearance lies with few exceptions between 160 and 70 km, the general rule being that the faster meteors appear higher, but that for meteors of the same speed the height varies little, if at all, with the magnitude. The very large fireballs and meteorites form a possible exception to this last rule; but as these are usually of the sporadic type not belonging to known radiants, it is difficult to say whether the great heights of appearance sometimes recorded are due to their size or to their having greater velocity.

The height of disappearance is usually below 120 km. Since it is obvious that a meteor which enters the atmosphere horizontally must disappear at very nearly the same height at which it has appeared, this fact only means that at those places at which most observations of meteors have been collected the fastest meteors cannot enter the air from radiants on the horizon. The point is of some interest as indicating the gaps, from the standpoint of the physical theorist, in our meteoric data. There is no definite lower limit to the height of diasppearance. Few of the swifter meteors get below 85 km, the general rule being that the slow meteors appear lower, disappear lower, and describe longer paths than swift meteors of equal brightness.

The apparent temperature of meteors, as indicated by their color, varies from about 7000° K., for fast meteors, to 3000° K., or less, for the slow ones. As the radiation comes, in all probability, from a surrounding mantle of gas, we may not attach any precise quantitative significance to these figures.

We will now take up the details of the theory. In its main features it can be quite simply stated. We consider the meteor as at rest in a stream of gas molecules. At heights between 70 and 160 km the free path of an air molecule will be of the same order of magnitude as the assumed dimensions of the particle; there is thus no possible accumulation of air in front of the meteor, but a series of discrete impacts on the solid nucleus. An expression for the rate of input of energy is obtained, which, equated to the rate of radiation according to Stefan's law, gives the temperature of the solid nucleus. The meteor becomes visible when this temperature is high enough to produce copious evaporation.

The details of this theory had been fully worked out before the writer could get access to the paper by Lindemann and Dobson. On reading their paper, he was surprised to find, that, after considering the theory outlined above, they reject it as insufficient. As the whole need for higher atmospheric temperatures and pressures hinges upon this rejection, it would be interesting to have more of the calculations on which it was based. But even granting their postulate of higher pressures and temperatures, the writer is unable to accept the reasoning employed in the development of the theory which they offer as a substitute. Their calculations are vitiated, in his opinion, by a fundamental error. This error lies in their use of the equation of the gas adiabatic to calculate the rate of heating of the meteor. The equations of thermodynamics, including this equation, rest on the fundamental assumption of reversible processes, during which there is at every stage equilibrium of the working substance with its surroundings. To see how far the conditions of the actual problem depart from this requirement, we may consider the picture of an adiabatic compression which is afforded by the kinetic theory. The molecules which strike a moving piston (here the meteor) rebound from it with increased velocity, and by considering the statistics of such molecular impacts, the usual equation for the adiabatic may be deduced. The possibility of this deduction depends, however, upon the assumption that the velocity of the piston is small compared to the temperature velocity of the gas molecules. The use of the adiabatic equation by Lindemann and Dobson is therefore equivalent to the assumption that a velocity of 60 km/sec. is small compared to one of 0.5 km/sec. As a matter of fact, the total energy given to the gas by a definite displacement of the piston, which for small velocities V of the piston is independent of V

and proportional to T, the temperature of the gas, is for large values of V independent of T, and proportional to V^2. The first value is that deducible from the adiabatic equation; the second corresponds to the theory of molecular impacts, either directly with the solid nucleus or with the outer boundary of a surrounding gas envelope. To see how large a difference this may make, consider a volume of hydrogen which is compressed to 0.9 of its original value, and which has initially the temperature 273° K. The adiabatic compression gives $T/T_0=(\rho/\rho_0)^{\gamma-1}$, from which we find $T=273\times(10/9)^{0.4}=285°$, a result which holds for all piston velocities which are small compared to 1.8×10^5 cm/sec. For a piston velocity of 60 km/sec., however, we may consider, in effect, that one-tenth of all the molecules in the gas are suddenly given this velocity. When this translatory energy has been distributed among all the molecules in all the degrees of freedom, the mean-square velocity will be $3/50\times(6,000,000)^2=2.16\times10^{12}$. We thus get $T=273\times2.16\times10^{12}/(1.8\times10^5)^2=18,200°$.

This collision of the air molecule with the meteor is a collision with a particular molecule of the meteor. The speed of the air molecule relative to the meteor is so great that the primary transfer of momentum on collision must be to the first thing that is able to start moving. We may go even farther and say that it is probably a question of the collision of atoms, for the energy available will in most cases be far in excess of that required for chemical dissociation. The energy required to dissociate or ionize a substance is $1.57\times10^{-12}\ P$ ergs, where P is the ionization or dissociation potential in volts. Taking for the mass of the hydrogen molecule $m=3.3\times10^{-24}$ gm, we thus get $P=0.0105\ V^2$, where V is in km/sec., as the potential equivalent to the speed V. This gives for $V=16$, $P=2.7$. This is greater than ordinary lattice potentials, and is of the same order of magnitude as Langmuir's value for the dissociation potential of the hydrogen molecule. For greater speeds or heavier molecules this disruption becomes, a fortiori, a practical certainty. When $V=30$ km/sec., we get $P=9.5$ volts; this is of the order of magnitude of the atomic ionization potentials. In considering the dynamics of impact, it seems therefore justifiable to regard it as practically inelastic, since this breaking up of the lattice, the molecule, or the atom provides the necessary means for the dissipation of the energy.

In formulating the dynamical problem, we make the usual simplifying assumption of spherical molecules, and consider a molecule of mass m_2 struck by one of mass m_1, which is moving relatively to it with velocity V (Fig. 1). Let the line of centers at the moment of impact make an angle θ with the direction of V. Let the component velocities along and perpendicular to the line of centers after impact be u_1, v_1; u_2, v_2. The component of the relative velocity perpendicular to the line of centers is unaltered by the collision, while that along the line of centers is reduced to zero. We thus have

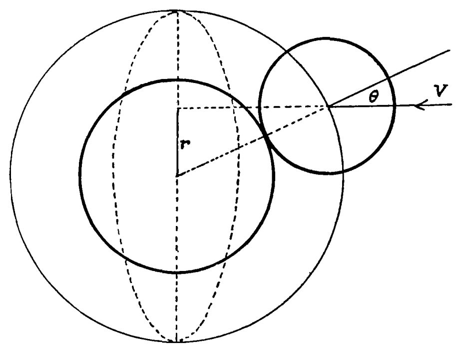

FIG. 1

$$u_1-u_2=0;\ v_1-v_2=V\sin\theta\ . \tag{1}$$

We have also two equations of momentum:

$$m_1u_1+m_2u_2=m_1V\cos\theta;\ m_1v_1+m_2v_2=m_1V\sin\theta\ .$$

From these we get

$$u_1-u_2=\frac{m_1+m_2}{m_2}u_1-\frac{m_1}{m_2}V\cos\theta=V\cos\theta-\frac{m_1+m_2}{m_1}u_2$$

$$v_1-v_2=\frac{m_1+m_2}{m_2}v_1-\frac{m_1}{m_2}V\sin\theta=V\sin\theta-\frac{m_1+m_2}{m_2}v_2\ .$$

Substituting the first set of values in (1), we get u_1 and v_1. We thus find for the energy of the first molecule after impact

$$\frac{1}{2}m_1V^2\left\{1-\frac{2m_1m_2+m_2^2}{(m_1+m_2)^2}\cos^2\theta\right\}\ . \tag{2}$$

If we substitute the second set of values, we find for the energy of the second molecule

$$\frac{1}{2}m_1 V^2 \frac{m_1 m_2}{(m_1+m_2)^2}\cos^2\theta\ . \tag{3}$$

The sum of these is less than the original energy by the amount dissipated, namely,

$$\frac{1}{2}m_1 V^2 \frac{m_2}{m_1+m_2}\cos^2\theta\ . \tag{4}$$

We may sum up these results by saying that the first molecule loses on impact a fraction $q_1 \cos^2\theta$ of its original energy; that it imparts to the second molecule the fraction $q_2 \cos^2\theta$; and that the fraction $q_3 \cos^2\theta$ is dissipated, where $q_1 = q_2 + q_3$. We form now the average $\cos^2\theta$ for all possible collisions. Through the center of m_2 pass a plane perpendicular to the direction of V. At the moment of impact the line of V, drawn through the center of m_1, meets this plane within a circle whose radius R is the sum of the molecular radii. This circle forms the "target" which m_2 offers to m_1, and all points within it are equally probable. The collisions for which V meets the target at distances from the center between r and $r+dr$ form a fraction $2rdr/R^2$ of the total. Putting $r = R\sin\theta$, we get $2\sin\theta\cos\theta\, d\theta$ as the fraction for which θ lies between θ and $\theta+d\theta$. The mean value of $\cos^2\theta$ for all collisions is thus

$$\int_0^{\frac{\pi}{2}} \cos^2\theta \cdot 2\cos\theta \sin\theta\, d\theta = 1/2\ .$$

From a number of analyses of aerolites given by Merrill[1] selected at random from the *Catalogue* of the National Museum, we find a mean atomic weight between 25 and 30. Taking the smaller value as a lower limit, and the atomic weight of iron, 56, as an upper limit, we get the distribution of the energy of collisions shown in Table III.

In the case of hydrogen, the energy of translation which is directly imparted is a very small fraction, so that the question of heating

[1] *Smithsonian Inst.*, *U.S.N.M. Bull.*, No. 94, 1916.

will depend primarily upon what part of the dissipated energy is available for this purpose. Lindemann and Dobson, in their examination of this point, apparently assumed: (1) that all of this dissipated energy immediately appears as radiation; (2) that none of this radiation can be effective in heating the meteor; (3) that even the energy actually imparted to the meteor is ineffective because there is not sufficient time for it to be transmitted to the adjacent molecules—the velocity of elastic waves in the meteor setting an upper limit to this rate of transfer. On this view the effect of the impacts would be to strip off a few molecules at velocities corresponding to an enormously high temperature, instead of using this energy to evaporate a very much greater number at velocities corresponding to the temperature of evaporation. We whall consider these objections in order.

TABLE III

	Hydrogen		Nitrogen	
	$m_2=25$	$m_2=56$	$m_2=25$	$m_2=56$
Gained by m_2	0.018	0.009	0.25	0.11
Dissipated	0.48	0.49	0.24	0.33

1. The dissipated energy may be divided into three parts: (*a*) the energy of dissociation of the atmospheric molecule, (*b*) the energy of dissociation of the meteoric molecule or of the disruption of its lattice structure, (*c*) the energy of atomic ionization. Of these, only (*c*) is responsible for immediate radiation, and we have seen that for speeds less than 30 km/sec. very little of the dissipated energy can come under this head. (*a*) is clearly not available for heating the meteor but the probably equal amount (*b*) should go entirely to its credit.

2. The energy radiated proceeds from points at or close to the surface of the meteor. The meteor is thus in the field of its own radiation, and should absorb approximately half of this radiated energy. This figure, which can easily be made to appear too high by arguments having a great deal of plausibility, seems nevertheless to the writer to be the assumption which would naturally be made

in the absence of any preconception as to the result. It should be remembered, moreover, that, in neglecting the possibility of a second or third impact, the figures for the amount of dissipated energy are certainly too low.

3. The assumption of the velocity of elastic waves as an upper limit for the rate of transfer of energy to adjacent molecules presupposes the existence of this energy as translatory molecular motion at speeds below the limit at which disruption of the lattice occurs. We have available for this transfer, however, not only much higher speeds, but free electrons and radiant energy, giving a speed of transfer comparable to the rapidity of metallic conduction. It is not

TABLE IV

	Hydrogen	Nitrogen
$m_2=25$	0.25	0.37
$m_2=56$	0.25	0.27

necessary, for the purposes of the theory, that the distribution of energy should extend to great depths.

We take, therefore, as the energy acquired by the meteor, the total energy of translation *plus* half the dissipated energy. We thus get as the efficiency k of the impact the values shown in Table IV.

The value of k for hydrogen is practically independent of m_2. For nitrogen we will assume the value 0.37, as the smaller values of m_2 are much more probable.

Assuming the meteor to be a sphere of diameter D, it will be struck in one second by all the molecules in the volume $\pi D^2 V$; that is, by $\pi D^2 V N \rho/\rho_0$, where N is the Avogadro number. From each of these it receives the energy $kmV^2/2$, where m is the mass of the atmospheric molecule. Since $mN=\rho_0$, we thus get for the rate of energy input $\pi k D^2 V^3 \rho/2$. If we may regard the meteor as a black body radiating at temperature T we have for the rate of loss $4\pi D^2 a T^4$, where a is Stefan's constant $(=5.3\times 10^{-5}$ erg/cm² sec.). Equating this, we get the relation

$$T^4 = kV^3\rho/8a. \qquad (5)$$

For a mixture of gases $k\rho$ is to be replaced by a summation.

The meteor will become visible when heated to the temperature of copious evaporation. The writer agrees fully with Lindemann and Dobson that this view seems the only one tenable. If we know this temperature and also the distribution of the atmosphere as a function of the height, we can calculate with the aid of equation (5) the height of appearance corresponding to a given speed. Taking $V=7\times 10^6$ cm/sec. and $h=160$ km, values which agree well with the observed maximum height of Leonids, and using the values of ρ/ρ_0 tabulated above, we get $T=3200°$. This is higher than any probable temperature that the substance of the meteor could attain without completely disintegrating. The temperature of rapid evaporation is not definitely known; it is probably in no case above 2500° K., though not less than 2000° K. At these levels, the hydrogen alone is effective; if we assume the value 2500° K., we find for the density of hydrogen 0.4 of the tabulated value; the lower limit 2000° K. requires only 0.16 of the tabulated value. If for brevity we call this temperature the "flashing point" of the meteor, we may sum up these figures thus: On the assumption that the flashing point is between 2000° and 2500° K., and that the height of 160 km corresponds to a speed of 70 km/sec., the theory indicates an amount of hydrogen at this height corresponding to a surface value of between 1 part in 60,000 and 1 part in 25,000 by volume.

The varying composition of meteors will produce corresponding variations in their flashing point. The maximum height would then belong to the fastest and most volatile meteors. If we assume $T=2000°$, $V=70$, and $H=160$ to be corresponding values, the amount of hydrogen in the atmosphere is thereby fixed, and equation (5) predicts a definite height of appearance corresponding to any other values of T and V. We thus get the values for h in kilometers as shown in Table V.

If we take Lindemann and Dobson's estimate of 2000°–2300° as the probable limits of T, the range of altitudes will be given by the first two lines. It will be noticed that this range increases with V.

As a rough check on these results we may take Newton's figures for Leonids, 154 km, and for Perseids, 115 km. The maximum velocities are 76 km/sec., and 54 km/sec., respectively. As the figures are averages, the mean velocity would be less than this by an unknown amount, but the relative values are in good agreement with the theory. Another more definite figure is represented by the mean height of 3 Geminids, observed by Olivier and Alden.[1] These gave a mean height of 97±2 km, with a computed speed of 33 km/sec. It does not seem worth while to multiply instances, as the assumed values, from which the table of heights (Table V) was derived, have no great claim to accuracy. The writer would emphasize again the need, for the purposes of a physical theory, of observations restricted to meteors which are known members of the solar system, and to the necessity of using theoretical velocities. This is borne out by the result of the calculations for speeds greater than 70 km/sec. Such meteors, if they exist, should appear at heights much greater than the maximum height of the Leonid meteors. The fact, therefore,

TABLE V

T	*V* (km/sec.)											
	10	20	30	40	50	60	70	80	90	100	110	120
2000°.....	75	89	98	105	113	126	(160)	197	232	259	285	309
2250.....	72	86	94	101	107	114	126	157	186	215	242	261
2500.....	69	83	91	98	103	108	115	125	147	174	201	225

that the Leonids appear about as high as any negatives the reality of the high values of some of the direct determinations of speed.

This increase of the height of appearance with the speed seems to be a consequence, not only of this particular theory, but of almost any which could be proposed, besides being well established by observation. The conclusion does not entirely negative the possibility of hyperbolic orbits with radiants of high celestial latitude, but it impugns an important portion of the evidence by which the hyperbolic theory is supported.

When we turn to a consideration of what happens after the meteor becomes visible, the problem is more intricate. Certain rather obvious conclusions may be drawn at once. The light of the visible meteor arises from the excitation of ionization of the escaping vapor by collision with the air molecules. The total amount of radiation emitted will therefore be proportional to the rate of evapo-

[1] C. P. Olivier, *Popular Astronomy*, **32**, 591, 1924.

ration; while the character of the radiation will depend only upon the levels of excitation or ionization and will be independent of the density of the atmosphere. If a mass E evaporates in unit time, this mass loses by collision the energy $EV^2/2$, a certain fraction of which appears as radiation. It is usually assumed that this fraction must be very nearly unity; this question will be examined later. To confine ourselves for the moment to the qualitative aspects of the question: At heights below the altitude of appearance, the density of the air is increasing. The increasing number of impacts with the naked nucleus would evaporate the meteor almost instantly, but the nucleus is screened from these impacts by the envelope of vapor. When this envelope has reached a sufficient density, few or none of the atmospheric molecules will be able to penetrate directly to the nucleus, but must communicate the necessary energy through the envelope, either by radiation or conduction. It is clear, however, that there will always be evaporation, for as it falls off the rate of supply of energy to the nucleus at once increases because of the relaxation of its defenses. Only in exceptional cases can we imagine it to cease temporarily. A meteor which is moving nearly horizontally might reach, for instance, a temperature of 2000°, at which the more volatile constituents would distil off in sufficient quantity to make it visible. If these should be exhausted before it had reached the level of appreciably greater density, the evaporation would cease for a while, to be renewed lower down. Cases are not unknown in which meteors have thus disappeared and reappeared, but whether the inclination of the path was ascertainable for them, the writer is unable to say. The "spindles," or periodic variations in brightness which are characteristic of photographic meteor trails, might perhaps be similarly explained. The more volatile constituents, distilling off from the surface, would leave a "glaze" of more refractory material. With the reduced rate of evaporation, the temperature would rise, resulting in the evaporation of the refractory film and a sudden increase in the amount of vapor as the underlying fresh material became exposed. It is not clear, however, that the spindles represent variations in the instantaneous brightness of the meteor, or whether they represent dilatations of the luminous train subsequent to the passage of the meteor, so that the explanation just given may be fanciful. Similar variations would perhaps be produced by the rotation of a fragment of irregular shape. In the "bursts" and even explosions of the larger fireballs, we seem to be on safer ground, for the known heterogeneity of large meteors makes variation of the rate of evaporation extremely probable.

As we increase V, the ionization or excitation will proceed to deeper and deeper levels, resulting in a general shift of the emitted light toward shorter wave-lengths. The general character of the known relation between speed and apparent temperature is thus easily accounted for. The maximum amount of energy which can be dissipated by a single molecular impact is, according to the collision formulae given earlier, 0.98 times the energy of a hydrogen atom, or 0.66 times the energy of a nitrogen atom; the mean values being half this. By equating these values to $h\nu$, where h is the Planck constant, we obtain a maximum frequency corresponding to a given velocity. Expressed in wave-lengths we thus get in angstrom units:

$V=$	10	20	30	40	50	60	70
Hydrogen......	23,800	5950	2645	1490	950	660	485
Nitrogen.......	2550	640	283	160	102	70	52

These figures, even as minima, should probably be doubled, as the energy is divided between the two colliding atoms. They indicate that for hydrogen the speeds below 30 km/sec. would be capable of producing little or no visible radiation. At the levels at which hydrogen predominates, a considerable proportion of the kinetic energy of the vapor would in the course of its collisions become distributed among the hydrogen molecules at speeds too small to excite visible radiation. For the higher velocities, on the other hand, the figures indicate the probability of a quite intense ultra-violet radiation. A portion of this, consisting of wave-lengths less than 1000 A, would be reabsorbed by the hydrogen, and would reappear in part as visible light. Meteoric masses have usually been calculated by taking the total radiated energy corresponding to a given stellar magnitude, assuming the sun as a comparison star, and equating this to the kinetic energy of the meteor. This virtually assumes an efficiency of radiation equal to that of the sun; if the sun is Q times as efficient, these masses should be multiplied by Q. The value of Q cannot be accurately determined, but an approximate figure may be found by comparing the efficiency of the best artificial sources with that of the sun. The flaming arcs form a fair experimental analogue of the processes of meteoric luminescence, but the efficiency of the best of them (2.5 candles/watt) is only one-third that of the sun (7 candles/watt). The efficiency of the meteor's luminescence may very well be less than this, as, owing to the high velocities of collision, its spectrum would approach the spark type, at least in the case of collisions with nitrogen. It would seem safe to assume, therefore, values of Q at least as great as 5 and perhaps even 25.

Since the sun, a star of magnitude -26.7, sends to the earth about 1.4×10^6 erg/cm^2 sec., we get for a star of magnitude M about $2.8\times10^{-5}\times(2.5)^{-M}$ erg/cm^2 sec. This gives, for the total radiation of a meteor of this magnitude at r km distance, $3.5\times10^6\times(2.5)^{-M}r^2$ erg/sec., which we shall call Br^2. The total kinetic energy required to produce this radiation is QBr^2, so that we have $QBr^2=2\pi\rho' V^2D^2dD/dt$ (ρ' = density of meteor). It is generally agreed that the smaller meteors show little, if any, change in apparent brightness during their entire course; this would require the constancy of $(D^2dD/dt)/r^2$, since V is known to change very little. D is, however, diminishing; r probably also diminishes somewhat for most observed meteors, but not rapidly, so that dD/dt should increase. It is difficult to conceive any process by which the constancy of D^2dD/dt is assured under all circumstances. The rate of evaporation of unit area is measured by dD/dt. It is reasonable to suppose that it increases with the density of the air, but it is difficult to see why it should depend upon anything else except V. We might imagine, in a particular case, that the diminution of D and the increase of dD/dt were so adjusted as to give a roughly constant rate of mass diminution, but then the assumption of a greater initial diameter would give a rapid increase in brightness, while the assumption of a more inclined path would result in a diminution. It must be admitted that the constancy of magnitude cannot be accurately affirmed. The estimates of brightness by different observers frequently vary by a whole magnitude. If we exclude from consideration the faintest meteors whose paths are too short to afford any accurate judgment, and the larger meteors which undoubtedly do show variations of brightness, the question hinges on the facts concerning magnitudes between 4 and 0, and it may be that the constancy is best for meteors of the second or third magnitudes, and even then not so good as has been supposed. If this constancy should be confirmed, it would then be necessary to understand why, as the mass of the nucleus diminishes, the surface rate of evaporation increases. One or two other points which bear on this question may be mentioned, which will serve mainly to show the extreme complexity of the problem. The rate of evaporation is a very sensitive function of the temperature. For the larger meteors, the conduction of heat to the colder interior of the nucleus tends to lower this; the smaller meteors tend to get heated through, thus diminishing this two-sided loss of the heated surface layer. If all the vapor emitted were effective in screening the meteor, this would perhaps be unable to increase evaporation, but only that which comes off from the advancing face can protect the nucleus, so that for the smaller meteors there would be a larger proportional loss of mass.

Let us consider now some details of the formation of the luminous envelope. The quantity QBr^2, which we equated above to the kinetic energy of the vapor lost per second, can be also considered as the rate of input of energy into this vapor by the collision of the air molecules. Each of these has relative to the meteor the speed V; to contribute the necessary energy a certain number is required. Since the meteor traverses the distance V, we may consider these molecules as contained in a cylinder of diameter D' and length V. D' may be called the effective diameter of the envelope; it is the least diameter of the track within which the luminous molecules must lie. We thus get

$$(\pi/8)\rho D'^2 V^3 = QBr^2 \ .$$

The difficulty in calculating D' lies in the uncertainty of Q. Even for small values of Q, however, the equation gives values of D' much larger than the diameter of the nucleus as calculated in the usual way. This would indicate that the actual envelope takes the form of a short spreading tail, like the tail of a comet, since in the time required for the lateral migration of the vapor it would be left behind by the meteor. Again we may compare the values of D' with the diameter of a circular disk, which, radiating like a black body of the same color temperature, would appear of equal stellar magnitude. Even assuming that the envelope is spherical, this gives for the solid disk diameters from a third to a fourth of the corresponding values of D'. This is as it should be, for at such low densities we should not expect the envelope to be perfectly opaque.

That the meteor does not sensibly slow up throughout its course is an immediate consequence of the foregoing. If L is the length of the meteor's path, its total energy is roughly equal to that of the air molecules in the volume $(\pi/4)D'^2L$, considered as moving with the speed V. Only those in the volume $(\pi/4)D^2L$ can possibly be effective in stopping the nucleus, for the other molecules collide with the vapor to the side and rear. The direct loss of energy is certainly not greater than the fraction D^2/D'^2 of the whole. It must, in fact, be considerably smaller, for the collisions in the line of the nucleus are not direct impacts on the nucleus, but with its vapor, which is being swept aside. The effect of evaporation is to provide, as it were, a slippery sheath within which the nucleus is able to progress with very slight retardation.

Let us return now to a consideration of the process by which, after the formation of the envelope, the evaporation of the meteor is maintained. The writer's attempts at a mathematical formulation of this problem have so far been unsuccessful; a clear statement of the physical picture will be all that can be attained. The outer part of the envelope, which receives the impacts of the atmospheric molecules, will consist of a mixture of air and vapor. Referred to the meteor, the mean velocity of these molecules is high, but it is only in a loose sense that we can identify this mean velocity with a temperature, since the law of distribution departs widely from Maxwell's law. There is nevertheless a transition, as we go from these outer parts of the envelope in to the nucleus, from a region of high mean molecular energy to a region with a very much lower value. There will thus be a transfer of energy by molecular collisions by a process identical, from the point of view of kinetic theory, with that of heat conduction. It differs from heat conduction only in that the ordinary formulae will not apply, for the energy of the molecular motion cannot be called heat. As the same mechanism of transfer is invoked, however, we should expect the rate of transfer to be independent of the density of the vapor, and to be proportional to the difference in mean molecular energy at the boundary and at the surface of the nucleus, and to vary inversely as the thickness of the nucleus. This quasi-conductivity of the envelope will be much increased by the free electrons present, which by virtue of their greater free path and high speed will carry most of the energy. In addition, we have to include the energy received by the nucleus from the radiation of the envelope. This is perhaps not large; for the radiation comes for the most part from that portion of the envelope for which the density of the vapor is small, while close to the surface of the nucleus the denser and colder layers of vapor would probably absorb most of the ingoing radiation.

To complete this picture we must examine the process at heights below 60 km, where the free path is small enough to permit the formation of an air-cap. The main differences thus introduced seem to be two. First, the screening of the nucleus is not dependent, as before, on the rate of evaporation; and second, the luminescence is more a matter of the ionization of air by air, for it is the piled-up air that now receives the impacts of the stream of fresh molecules. Whether this would result in any essential reduction of the rate of evaporation is difficult to say. At this point, also, the effects of a true viscous drag between the gas-cap and the surrounding air would begin to be felt, and would increase as the height diminished, so that the deceleration would be more marked. But in calculating the rate of input of energy the same principles should apply as before; the idea of adiabatic compression being no more applicable here than at the greater heights, we have still a rate of input which is proportional to V^3, and which is practically independent of the temperature of the atmosphere.

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November 15, 1925