ANALYSIS OF RADIAL VELOCITIES OF GLOBULAR CLUSTERS AND NON-GALACTIC NEBULAE[1]

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ABSTRACT

List of radial velocities of globular clusters and non-galactic nebulae.—Table I contains a list of all measured radial velocities of globular clusters and non-galactic nebulae, the great majority of the determinations being by Slipher.

Solar motion.—The sun's motion relative to the two classes of objects studied is given in Table II in rectangular equatorial co-ordinates $(-\xi, -\eta, -\zeta)$ and in polar co-ordinates (A_0, D_0, V_0). The value of the sun's motion relative to the nebulae is 344 km/sec. toward $A_0=305°$, $D_0=+56°$ and, relative to the clusters, 329 km/sec. in the direction $A_0=320°$, $D_0=+65°$.

Curvature of space-time.—On the basis of De Sitter's theory of curvature of space-time, we should expect to find for very distant objects a shift of the spectral lines toward the red end of the spectrum. According to Silberstein, we may expect a shift either toward the red or the violet. The correlation-coefficients between distance and radial velocities give no clear evidence of curvature in either De Sitter's or Silberstein's sense, at least up to the limit of distance studied. The only definite correlation is one between radial velocity and position in the sky, indicating a solar motion of 300 or 400 km/sec. in about the same direction for both classes of objects.

The determination of radial velocities of globular clusters and of non-galactic nebulae is very difficult on account of the faintness of the objects and the absence, in general, of bright lines in their spectra; but through the perseverance of Professor V. M. Slipher, a fairly large number of such velocities has been derived. Two reasons prompted the writer to study these velocities. One was the large solar velocity found from these objects, which, in connection with the asymmetry of stellar motions, indicated that a fundamental reference system could be defined by them. The second reason was the desirability of ascertaining whether the velocities give any evidence of a curvature of space-time.[2] Through the courtesy of Professor Slipher it has been possible to make use of his radial-velocity determinations up to a recent date.

In Table I are collected the data on which the computations are based. Slipher's determinations are given without references; for

[1] *Contributions from the Mount Wilson Observatory*, No. 292.

[2] After this paper was ready for printing, Lundmark's article, "The Determination of the Curvature of Space-Time in De Sitter's World," appeared in *Monthly Notices*, **84**, 747, 1924. Lundmark's conclusions as to space-curvature are about the same as those found in this study.

TABLE I

RADIAL VELOCITIES OF GLOBULAR CLUSTERS AND NON-GALACTIC NEBULAE

N.G.C.	α (1900)	δ (1900)	V		
			Slipher	Others	Mean
			km/sec.	km/sec.	km/sec.
221	0^h37^m2	+40° 19′	− 300	.....	.....
224	0 37	+40 43	− 300	− 320*	−315
278	0 46.4	+47 1	+ 650::	.....	.....
404	1 3.9	+35 11	− 25	.....	.....
584	1 26.3	− 7 23	+1800:	.....	.....
598	1 28.2	+30 9	− 260	− 70†	− 70
936	2 22.5	− 1 36	+1300	.....	.....
1023	2 34.1	+38 38	+ 300	.....	.....
1068	2 37.6	− 0 26	+1120	+ 814‡	+ 916
1700	4 52.0	− 5 1	.....	+ 800§	.....
2681	8 46.4	+51 41	.....	+ 700‖	.....
2683	8 46.5	+33 48	+ 400	.....	.....
2841	9 15.1	+51 24	+ 600:	.....	.....
3031	9 47.4	+69 32	− 30	.....	.....
3034	9 47.6	+70 10	+ 290	.....	.....
3115	10 0.3	− 7 14	+ 600	.....	.....
3368	10 41.5	+12 21	+ 940	.....	.....
3379	10 42.6	+13 7	+ 780	+ 845§	+ 812
3489	10 55.0	+14 26	+ 600::	.....	.....
3521	11 0.7	+ 0 30	+ 730	.....	.....
3623	11 13.7	+13 38	+ 800	.....	.....
3627	11 15.0	+13 32	+ 650	.....	.....
4111	12 2.0	+43 37	+ 800:	.....	.....
4151	12 5.5	+39 58	+ 980	..+ 940¶	+ 950
4214	12 10.6	+36 53	+ 300	.....	.....
4258	12 14.0	+47 52	+ 500	.....	.....
4382	12 20.3	+18 45	+ 500:	.....	.....
4449	12 23.3	+44 39	+ 200	.....	.....
4472	12 24.7	+ 8 33	+ 850	.....	.....
4486	12 25.6	+12 57	+ 800::	.....	.....
4526	12 29.0	+ 8 15	+ 580	.....	.....
4565	12 31.4	+26 32	+1100:	.....	.....
4594	12 34.8	−11 4	+1100	+1180**	+1140
4649	12 38.6	+12 6	+1090	.....	.....
4736	12 46.2	+41 40	+ 290	.....	.....
4826	12 51.8	+22 14	+ 150	.....	.....
5005	13 6.3	+37 36	+ 900	.....	.....
5055	13 11.3	+42 34	+ 450	.....	.....
5194	13 25.7	+47 43	+ 270	.....	.....
5195	13 25.7	+47 43	+ 240:	.....	.....
5236	13 31.4	−29 21	+ 500:	.....	.....
5866	15 3.7	+56 9	+ 650	.....	.....
7331	22 32.5	+33 54	+ 500	.....	.....
Magellanic clouds	5 27	−68	.....	+ 287††	.....
	1 6	−73 44	.....	+ 168††	.....
1851	5 10.8	−40 9	+ 315	.....	.....
1904	5 20.1	−24 37	+ 235	.....	.....
5024	13 8.0	+18 42	− 170	− 200‡‡	− 180
5272	15 37.6	+28 53	− 125	− 140‡‡	130

TABLE I—*Continued*

N.G.C.	α (1900)	δ (1900)	V		
			Slipher	Others	Mean
			km/sec.	km/sec.	km/sec.
5904.....	15^h13^m5	+ 2° 27′	+ 10	.....	.....
6093.....	16 11.1	−22 41	+ 70	.....	.....
6205.....	16 38.1	+36 39	− 300	− 230§§	− 265
6218.....	16 42.0	− 1 46	.....	+ 160‡‡	.....
6229.....	16 44.2	+47 42	− 100:	.....	.....
6266.....	16 54.8	−29 58	+ 50	.....	.....
6273.....	16 56.4	−26 7	+ 30	.....	.....
6333.....	17 13.3	−18 25	+ 225	.....	.....
6341.....	17 14.1	+43 15	− 160	.....	.....
6626.....	18 18.4	−24 55	0	.....	.....
6934.....	20 29.3	+ 7 4	− 350:	.....	.....
7078.....	21 25.2	+11 44	− 95	− 93‡‡	− 94
7089.....	21 28.3	− 1 16	− 10	.....	.....
7099.....	21 34.7	−23 38	− 125	.....	.....

* Mean of several determinations.

† Slipher has measured a knot north following the nucleus, whereas the Mount Wilson determination is based on the spectrum of the nucleus; *A.S.P. Pubs.*, **28**, 33, 1916.

‡ Mount Wilson measures give $V=+765$; *A.S.P.*, **27**, 133, 1915; and Lick Observatory gives $V=+864$; *L.O. Pubs.*, **13**, 88, 1918.

§ Mount Wilson, *A.S.P. Pubs.*, **30**, 255, 1918.

‖ *Ibid.*, **34**, 222, 1922.

¶ *L.O. Pubs.*, **13**, 122, 1918.

** *Mount Wilson Comm.*, No. 32; *Proc. Nat. Acad. of Sciences*, **2**, 517, 1916.

†† *L.O. Pubs.*, **13**, 168, 1918.

‡‡ Sanford, unpublished.

§§ Mean of 4 stars, measured by Adams, Joy, and Humason.

other observers, references are given in footnotes. In the list 43 nebulae stand first, most of which have spiral structure, although several are irregular or globular. Then follow the Magellanic clouds and 18 globular clusters. The probable error of the radial velocities for the nebulae is probably about 50 km/sec. and for the clusters about 25 km/sec. The more uncertain values are marked by one or two colons.

THE SOLAR MOTION

The elements for the solar motion were first determined. The equations of condition used were of the following form:

$$\xi \cos\alpha \cos\delta + \eta \sin\alpha \cos\delta + \zeta \sin\delta + K = V \qquad (1)$$

where ξ, η, and ζ are the rectangular equatorial components of the group-motion as referred to the sun. The co-ordinates of the sun's

apex (A_0, D_0) and its velocity (V_0) relative to the groups of objects studied are determined by the equations[1]

$$\begin{aligned} V_0 \cos A_0 \cos D_0 &= -\xi \\ V_0 \sin A_0 \cos D_0 &= -\eta \\ V_0 \sin D_0 &= -\zeta . \end{aligned} \qquad (2)$$

Several solutions were made to test the effect of omissions, especially that of the large velocity +1800 km/sec. of N.G.C. 584. Whether we ought to include the Magellanic clouds among the non-galactic nebulae is doubtful, but solutions were made including these objects, both with and without K-terms. The results of the different solutions are given in Table II. The second, fifth, and seventh solutions for the nebulae were made after the radial velocity of N.G.C. 584 had been omitted. This omission seems to be justified on account of the small number of objects used. In the last solution for nebulae and clusters, 13 and 8 groups, respectively, were formed by combining objects near each other in the sky, the different groups being given equal weights. The probable errors are given below the numbers to which they belong. The bracketed probable errors for the right ascension of the sun's apex are reduced to a great circle. The quantity σ is the mean square residual velocity.

The first solution for the nebulae is identical with Lundmark's[2] values, which are based on practically the same material. His values are:

$$\xi = -\ 96 \pm 48; \qquad \eta = +138 \pm 108 \text{ km/sec.}$$
$$\zeta = -629 \pm 90; \qquad K = +793 \pm\ 59 \text{ km/sec.}$$

By omitting N.G.C. 584, the velocity of the sun decreases from 600 to 500, or to even less than 400 km/sec. The last solution for the nebulae is probably the best and agrees closely with that derived from the globular clusters. The position of the sun's apex as determined from the nebulae and clusters can be fixed at $\alpha = 315°$, $\delta = +62°$, with an uncertainty of about 10° in each co-ordinate. The velocity as determined from nebulae and clusters is 350 and 300 km/sec., respectively, the latter having the higher weight.

[1] Several writers omit the negative signs of the right-hand side, and instead give V_0 a negative sign. As the direction of the sun's motion is given by A_0 and D_0, it seems to the writer that V_0 must be used without sign.

[2] *Observatory*, 47, 279, 1924.

TABLE II

SUN'S MOTION RELATIVE TO NON-GALACTIC NEBULAE AND GLOBULAR CLUSTERS

No.	ξ	η	ζ	K	A_0	D_0	V_0	σ	Remarks
	Non-galactic Nebulae								
	km/sec.	km/sec.	km/sec.	km/sec.			km/sec.	km/sec.	
43...	− 98	+112	−610	+785	311°	+76°	628	345	Mag. clouds omitted.
	± 62	±140	±115	± 75	(±9)	±10	±115		
42*...	−196	+162	−450	+647	320	+61	517	302	
	± 58	±127	±106	± 72	(±12)	±11	±103		
45...	−170	+231	−246	+565	+306	+41	378	380	Mag. clouds included with K-term.
	± 66	±150	± 94	± 65	(±13)	±17	±121		
45...	−109	+121	−506	+737	312	+72	531	344	Mag. clouds included without K-term.
	± 60	±120	± 97	± 69	(±10)	±10	± 97		
44*...	−205	+162	−386	616	321	+56	468	308	
	± 56	±121	± 89	± 65	(±12)	±11	± 89		
45...	− 65	+142	−332	+668	295	+65	367	....	13 groups with equal weights. Mag. clouds included without K-term.
	±102	±188	±144	± 94	(±19)	±26	±120		
44*...	−111	+160	−284	+622	305	+56	344	....	
	± 89	±160	±124	± 82	(±22)	±21	±124		
	Globular Clusters								
18...	− 79	+108	−253	+ 25	306°	+62°	286	113	
	± 42	± 41	± 50	± 30	(±8)	± 9	± 49		
18...	− 81	+ 84	−261	0	314	+66	286	114	$K=0$.
	± 41	± 27	± 47		(±7)	± 7	± 45		
18...	−104	+108	−288	+ 21	314	+63	325	.....	8 groups of equal weight.
	± 45	± 43	± 60	± 32	(±8)	± 8	± 58		
18...	−106	+ 88	−299	0	320	+65	329	.....	8 groups of equal weight, $K=0$.
	± 41	± 28	± 53		(±6)	± 7	± 50		

* N.G.C. 584 omitted.

The reality of this large solar motion can perhaps best be judged from the correlation diagram in Figure 1. This shows the correlation between observed radial motion and $\cos\lambda$, where λ is the angular distance from the sun's antapex, which is assumed to be the same for the two classes of objects. If we omit N.G.C. 584, the correlation coefficients between V and $\cos\lambda$ for nebulae and clusters are +0.54 and +0.73, respectively, and the equations for the regression-lines of V on $\cos\lambda$ are, respectively,

$$V=452\cos\lambda+648 \qquad \text{and} \qquad V=295\cos\lambda+4\,.$$

The Magellanic clouds are indicated by crosses in the diagram for the clusters, but are not used in the derivation of the regression-

lines. All indications are that we are justified in our assumption of a vanishing value for the K-term for the clouds. The dotted horizontal line corresponds to $K = +616$ km.

The assumption of a constant K-term for the nebulae is probably only approximately correct; the K-term is probably much larger for N.G.C. 584 and smaller for the Andromeda nebula than for the rest of the objects. A variable correction to the wave-lengths would account for the fact that the dispersion in radial velocity is larger

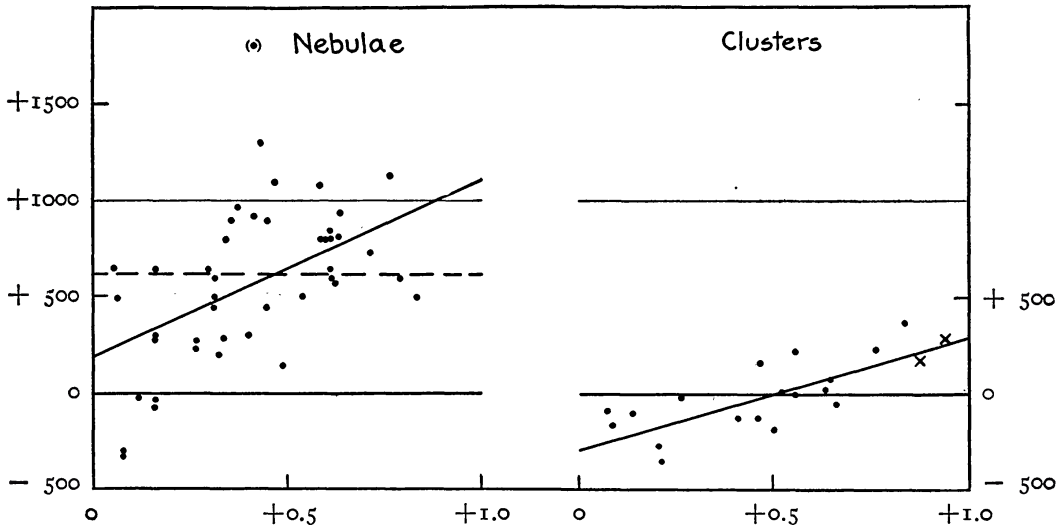

FIG. 1.—Diagram showing correlation between radial velocities (ordinates) and cos λ (abscissae) for nebulae and clusters.

for the nebulae than for the clusters. The vanishing value of the K-term for the clusters can be seen directly from the second scatter-diagram in Figure 1.

The direction of the sun's motion is very nearly the same as that derived from the asymmetry in the distribution of stellar velocities on the assumption of the existence of a velocity-restriction in a fundamental reference-frame.[1] The direction of the sun's motion as derived from the asymmetrical velocity-distribution of stars in our neighborhood is $\alpha = 327°$, $\delta = +61°$, and its velocity relative to the stars of highest velocity-dispersion is about 280 km/sec.

[1] *Mt. Wilson Contr.*, No. 275; *Astrophysical Journal*, **59**, 228, 1924.

RADIAL VELOCITIES AND CURVATURE OF SPACE

In Professor de Sitter's[1] theory of space-time of cosmical extent, a constant radius of curvature has been postulated, the expression for the line-element ds in space-time being given by the equation

$$ds^2 = -R^2[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)] + \cos^2\chi\, dt^2 . \qquad (3)$$

This expression is the equation of a hypersphere in five dimensions, the distance along a geodesic to a point being given by $\rho = R\chi$, where R is the curvature-invariant. Equation (3) differs from Einstein's expression for an elliptical (or cylindrical) space-time by the factor $\cos\chi$ for dt. In Einstein's world, the time-dimension is not curved, and the difference in curvature between space and time makes it possible to single out a fundamental space in which all the matter in the universe is at rest, and an absolute time, which is the time for an observer at rest in this absolute space. De Sitter's world, on the contrary, is perfectly isotropic.

When the distance to a star is very large, the variable $\chi = \rho/R$ may differ appreciably from zero, and when $\chi = \pi/2$, the interval ds becomes independent of dt. This is interpreted as meaning that the time appears to slow down at large distances, and we should accordingly expect an apparent slowing down of the atomic clock, i.e., a shift of the spectral lines to the red end of the spectrum. Such an effect would produce a fictitious positive radial velocity.

In Silberstein's[2] opinion, if we make the assumption that the motions are so adjusted that they are necessarily small when the object comes near the sun, we may expect a shift either to the red or to the violet.

The present collection of radial velocities can be used to ascertain whether the measured radial velocities are dependent upon the distances of the objects. The velocities of some globular clusters have been used for this purpose by Silberstein, but the result cannot be regarded as conclusive. One thing is obvious, however. If the nebulae studied are at about the same mean distance, or if they are nearer than the globular clusters, we cannot regard the large positive K-term as a De Sitter effect, as the K-term for the clusters must be

[1] *Monthly Notices*, **78**, 3, 1917.

[2] *Monthly Notices*, **84**, 363, 1924; *Phil. Mag.*, **47**, 907, 1924; **48**, 619, 1924.

very small. On the basis of De Sitter's theory, we may expect an increase of radial velocity (V) with distance; on Silberstein's theory, an increase of radial velocity, without regard to sign ($|V|$), with increasing distance from the sun.

The writer has derived coefficients of correlation between distance, on the one hand, and radial velocity with and without sign (V and $|V|$), on the other, and similar coefficients after the velocities have been corrected for a solar motion of 300 km/sec. (V' and $|V'|$). The data for these correlations are collected in Table III. The distances for the globular clusters (ρ in kiloparsecs) are Shapley's[1] determinations. For the nebulae we do not know the distances, but we can probably get some rough indication of their relative distances by assuming that they have nearly the same total brightness. As a measure of the distance I have consequently used the quantity $\rho' = 10^{0.2m}$, where m is the total apparent brightness taken from Wirtz's[2] study. The correlation coefficients (r), together with the equations for the two regression-lines, are given in Table III. As an illustration, the scatter-diagram and the regression-lines for the correlation between $|V|$ and ρ are given in Figure 2. They may be compared with the cor-

TABLE III

CORRELATION TABLE

Objects	No.	Variates	r	Regression-Lines
Clusters	18	V, ρ	−0.06	$V = -1.338\,\rho + 2$; $\rho = -0.0029\,V + 19$
	18	$\|V\|, \rho$	+0.26	$\|V\| = +3.281\,\rho + 77$; $\rho = +0.0200\,\|V\| + 16$
	18	V', ρ	+0.02	$V' = +0.326\,\rho + 18$; $\rho = +0.0016\,V' + 19$
	18	$\|V'\|, \rho$	+0.33	$\|V'\| = +2.770\,\rho + 43$; $\rho = +0.0403\,\|V'\| + 15$
	18	V, cos λ	+0.73	$V = +295 \cos\lambda + 4$; $\cos\lambda = +0.001820\,V - 0.05$
Nebulae	29	V, ρ'	+0.28	$V = +1.323\,\rho' + 545$; $\rho' = +0.0596\,V + 53$
	29	V', ρ'	+0.23	$V' = +0.960\,\rho' + 639$; $\rho' = +0.0566\,V' + 52$
	41	V, cos λ	+0.54	$V = +452 \cos\lambda + 648$; $\cos\lambda = +0.000647\,V - 0.54$

[1] *Mt. Wilson Contr.*, No. 152; *Astrophysical Journal*, 48, 154, 1918.

[2] *Lunds Observatorium, Meddelanden*, Serie II, No. 29, 1923.

relation data for V and cos λ as given in Table III and Figure 1. For the nebulae, the correlation between $|V|$ and ρ can be regarded as equivalent to the correlation between V and ρ, as nearly all the radial velocities are positive. The same holds for $|V'|$, which is nearly identical with V', as the K-term obviously must not be subtracted.

From Table III we see that the only correlation which can be accepted with any confidence is that between radial velocity and position in the sky (V and cos λ). For the nebulae, the correlation

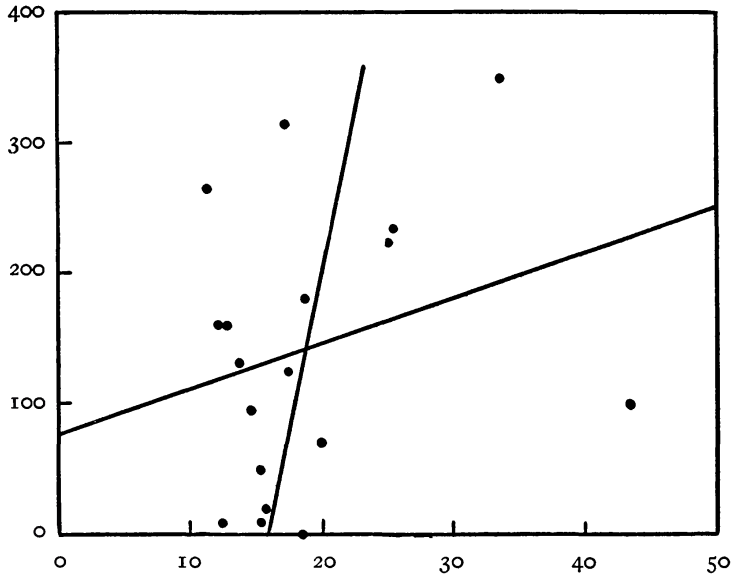

FIG. 2.—Scatter diagram showing correlation between radial velocities without regard to sign (ordinates) and distances in kiloparsecs (abscissae) for globular clusters.

coefficient is only +0.54, but the fact that the solar motion is nearly the same for the nebulae as for the clusters strengthens our belief that the correlation is real.

De Sitter's effect can be regarded as disproved by the clusters if their distances are of the same order as those of the nebulae. Silberstein's effect seems possible, but cannot be established by the data. It is significant, however, that the regression-line for the clusters, $|V| = 3.281\ \rho + 77$, does not go through the origin as expected from the theory. Silberstein's correlation is slightly improved if the radial velocities are corrected for the sun's motion.[1]

[1] This inclusion of the Magellanic clouds among the clusters does not materially alter the size of the correlation coefficient.

In conclusion we may say that we have found no sufficient reason to believe that there exists any dependence of radial motion upon distance from the sun. The only dependence fairly well established is one that is a function of position in the sky. This indicates a solar motion of 300 or 400 km/sec. in the direction $\alpha = 315°$, $\delta = +62°$, which agrees with the solar motion as determined from the asymmetrical velocity-distribution of stars in our neighborhood.

MOUNT WILSON OBSERVATORY
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