A Determination of the Photographic Magnitude Scale of the North Polar Sequence. By H. Spencer Jones, M.A., B.Sc.

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1. The North Polar Sequence consists of a series of stars, graded in magnitude, in the neighbourhood of the North Pole, which were selected by the late Professor E. C. Pickering to serve as a basis for the determination, by purely comparative methods, of the magnitudes of stars in other areas. The stars of the primary sequence were chosen as far as possible from amongst the stars whose spectra did not differ greatly from type Ao. This sequence was supplemented by another sequence, in which this condition was not adhered to, and also by a series of red stars of late spectral type.

Extensive investigations of the magnitudes of some or all of the stars of the combined sequence have been made at Harvard, Mount Wilson, Greenwich, Potsdam, and elsewhere. These investigations have served to confirm the photographic magnitudes originally determined at Harvard between the limits of magnitude 10 and 15. For both the brighter and fainter stars, however, systematic differences have been obtained. With the faint stars we are not concerned here. A determination of the magnitudes of the brightest stars of the sequence made by Seares at Mount Wilson with the 60-inch reflector showed discordances from the Harvard magnitudes amounting for some stars to as much as four-tenths of a magnitude. These bright stars were not included within the range of the Greenwich and Potsdam investigations, and have been fully covered only at Harvard and Mount Wilson. Further investigation of their magnitudes was greatly to be desired, in view of the importance of an accurate scale of magnitudes for statistical investigations and for other purposes. Professor Seares suggested that a determination of the magnitudes of the sequence stars brighter than 10^m, with a view to checking the Harvard and Mount Wilson scales, should be made the subject of an independent investigation. The investigation, the results of which are given in the present paper, covers all the stars of the North Polar Sequence, including the supplementary and red stars, down to a magnitude limit of about 13^m.

2. The telescope used for obtaining the photographs was the Astrographic Equatorial, of 13 inches aperture and 135 inches focal length. The observations were made by Messrs. Jones, Melotte, and Barton. The telescope was employed in conjunction with a parallel wire diffraction grating placed in front of the objective. The magnitude interval between the principal and first diffracted images obtained with this grating was determined theoretically from measurements of the mean diameter of the wire (d) and of the total grating interval ($a+d$), using the formula

$$\text{Magnitude interval} = 5\,(\log a\pi/a+d - \log \sin a\pi/a+d).$$

The determinations of d and of $a+d$, made on two separate occa-

sions by Messrs. Jones and Melotte, from a mean of measures made at a large number of points of the grating, were as follows:—

Date.	Measurer.	d. in.	$a+d$. in.	Mag. Interval. m
1920 Aug. 12	H.J.	·0634	·2770	2·831
,,	P.M.	·0631	·2769	2·841
1921 Jan. 3	H.J.	·0635	·2770	2·830
,,	P.M.	·0632	·2766	2·836

The value $2^m{\cdot}83$ was adopted throughout the investigation. The first diffracted images given by this grating are round and sharp, except when the image is near the limit of visibility. In such cases, the diffracted images were not measured. For bright stars, the second diffracted images may sometimes be used with advantage to supplement the information derived from the first diffracted image. The second image is slightly elongated, but its estimation presents no difficulties in the case of a bright star near the centre of the plate. The ratio of the brightness of the second to that of the first diffracted image is $\cos^2 a\pi/a+d$, corresponding to a magnitude difference of $-5\log\cos a\pi/a+d$. With the grating used in this investigation, this difference is $0^m{\cdot}62$, so that the difference in magnitude between the central and second diffracted images is $3^m{\cdot}45$. Direct comparison between first and second diffracted images gives a difference of $0^m{\cdot}65$, corresponding to a magnitude difference between central and second diffracted images of $3^m{\cdot}48$. The difference is probably due to a slight systematic error of measurement in the case of the elongated images. The value $3^m{\cdot}48$ has been used in the reductions.

3. The measurement of the plates consisted in estimating the sizes of the star images by comparison with a graded scale of images, photographed for the purpose with the astrographic telescope. The average magnitude difference between successive images of this scale is about $0^m{\cdot}55$. The micrometer employed was one which had been previously used at Greenwich in photometric investigations, but early in the course of the present investigation it was reconstructed, with a view to rendering the comparison between a field image and the scale images easier and more accurate. It consists essentially of a double microscope, the plate being viewed through one objective and the comparison scale through the other. A prism system is utilised to bring both objects into focus in the field of view of the eyepiece. In the reconstruction, the centre prism combination of a Barr and Stroud rangefinder was used for this purpose: with this combination, the field image and the scale images can be brought into close proximity on either side of the dividing line, rendering the estimation quick and accurate. The plate holder can be moved in two perpendicular directions, so that any desired star can be brought into the centre of the field of the eyepiece: the scale is movable by a fine screw, by means of which the images can be moved parallel to the dividing line in the field.

Each plate was measured by two measurers, one measuring the right-hand images in the micrometer and the other the left-hand. The measures were made throughout by Messrs. Jones and Jeffries.

4. The scale comprised 16 images numbered (commencing with the brightest) 1 to 16. Only the portion from images 7 to 16 was utilised, however, as sufficient material was not available for the calibration of the bright portion of the scale. For the portion of the scale that was used, preliminary corrections to reduce the successive scale intervals to a uniform sequence were first derived by measuring the diameters of the images in a filar micrometer and plotting the square roots of these values against a uniform scale. Assuming that a linear relation should result, corrections to the various scale readings, in decimals of a scale interval, were deduced. This assumption implies that a relationship of the form $m = a + b\sqrt{d}$ holds over the magnitude range involved. This is not strictly accurate. Supplementary corrections were therefore deduced from the measures of all the Pole plates: for this purpose the Mount Wilson magnitudes for the stars of the Polar Sequence fainter than 10^m were provisionally assumed and these magnitudes were plotted against the estimated scale readings, to which the provisional corrections had first been applied. A straight line was then drawn to represent the results as accurately as possible, and the residuals were read off. The exposures on the Pole Field ranged from a few minutes to half an hour, and were sufficiently varied to smooth out accidental errors in the assumed magnitudes. Collecting the residuals for each scale interval from all the plates, supplementary corrections were deduced, which were combined with the provisional values and used as definitive corrections throughout. The relationship between magnitude and corrected scale estimate for any plate should therefore be linear within the range of the scale employed. After plotting the measures of a plate, a straight line was drawn to represent all the results as accurately as possible.

No bias towards the Mount Wilson scale is introduced by the above procedure, since between magnitudes 10^m and 15^m the Mount Wilson and Harvard magnitudes are in essential agreement.

5. The plates exposed on the Pole Field alone were first measured. These comprised eleven plates, with the Pole at the centre and exposures ranging from 13^m to 30^m; four plates, with N.P.S. 11 at the centre and exposures of 10^m; four plates, with N.P.S. 12 at the centre and exposures of 15^m, and one plate, with N.P.S. 13 at the centre, exposure 15^m. The estimated scale readings of all the sequence stars fainter than 10^m, corrected as described in the preceding paragraph, were plotted against the Mount Wilson magnitudes: for each plate a straight line was drawn to represent the plotted points, and the magnitude of every star measured on the plate and of the diffracted images of the brighter stars was read off. The portion of each plate measured was limited in each co-ordinate by the réseau lines 6 and 22, the centre of the plate being at the réseau reading 13·5 in each co-ordinate, and the value of one réseau interval being 5 minutes of arc. Within this area the images given by the astrographic telescope are sensibly round and suitable for measurement, and the distance correction is negligible.

From the measures of the brighter stars, for which the diffracted images were not fainter than scale-image 15, the measured value of the grating interval was determined. Images fainter than scale image 15 are near the limit of the plate and cannot be estimated with very great

accuracy in terms of the scale: they were therefore discarded. From these twenty pole plates in this way was deduced a grating interval of $2^m{\cdot}81$, which compares with a calculated interval of $2^m{\cdot}83$. This agreement may be accepted as a justification of the use of the calculated interval. Alternatively, accepting the calculated value as necessarily correct, the agreement with the measurements confirms the light-ratio of the Polar Sequence between the range of 10^m and $13^m{\cdot}5$. The correctness of the light-ratio between 10^m and 15^m has been confirmed, as already mentioned, by various other researches. A subsequent determination of the grating interval, using the material available from all the plates, gave the value $2^m{\cdot}82$.

From the measurement of the exposures on the Pole Field, corrections to the assumed magnitudes of the sequence stars fainter than 10^m were deduced, and the corrected values were employed in the reductions of the measurement of plates which were exposed on other fields.

The exposures on the Pole Fields served also to determine the magnitudes of the stars N.P.S. 10 (B.D. $+89°$ 3) and 11 (B.D. $+89°$ 29), which are the two brightest stars within about 30′ of the Pole, having magnitudes between 9^m and 10^m. Their magnitudes were deduced from the magnitudes of the diffracted images, using the calculated grating interval of $2^m{\cdot}83$. For N.P.S. 11 the magnitudes of the principal images were also read off from the plotted lines, a very slight extrapolation being involved.

6. For the brighter stars of the sequence (with the exception of the few stars brighter than $5^m{\cdot}5$), magnitudes were deduced by comparing the star with the Pole Field. An exposure was first given on the Pole; then two equal exposures of the same length as the Pole exposure were given on the sequence star in question, and finally a second exposure on the Pole. The length of the exposure was so chosen as to give a conveniently measurable diffracted image of the sequence star. From the measures of the Pole Field, the straight line connecting the scale estimations with magnitude for the particular plate under investigation was constructed; this enabled the magnitude of the first diffracted image of the central star to be deduced; in some cases, when the second diffracted image was sufficiently well defined, its magnitude was also read off and used to supplement the magnitude-value deducible from the first diffracted image.

Occasionally two bright sequence stars are sufficiently close together to make it worth while measuring either on plates taken with the other at the centre. Examples of such pairs of stars are N.P.S. 1 and 4 (B.D. 86° 269 and 272); N.P.S. 7 and 2*s* (B.D. 88° 74 and 71); N.P.S. 8 and 9 (B.D. 88° 9 and 13); N.P.S. 2*r* and 4*r* (B.D. 88° 112 and 114); and N.P.S. 3*r* and 5*r* (B.D. 88° 76 and 89° 22). Additional material is thus obtained for deriving the magnitudes of many of the brighter stars.

A minimum of six good plates for each star was obtained; plates which on measurement were found to have unsatisfactory images, due to poor definition or errors in driving, or in which the Pole images were unequal, were discarded.

The magnitudes of the stars in the Pole Field, used for comparison, range from a bright limit of $9^m{\cdot}1$. The magnitude of the comparison

sequence star can therefore be determined, without extrapolation, from the magnitude of the first diffracted image if the star is not brighter than $6^m{\cdot}3$, and from the second diffracted image if it is not brighter than $5^m{\cdot}7$.

For a few of the bright stars further material at the brighter end was obtained by photographing a sequence star of intermediate brightness near the centre of the plate. The exposures on the Pole Field were then given first and last, with two intermediate exposures on each of the two sequence stars. Thus, for instance, on some of the plates taken for N.P.S. 3 (mag. $5^m{\cdot}8$), two exposures were also given with N.P.S. 7 (mag. $7^m{\cdot}3$) near the centre. From the magnitude of the diffracted image of N.P.S. 7, derived from comparison with the Pole Field, the magnitude of the principal image is obtained. This image is also measured and can then be used to continue the plot to a magnitude much brighter than that of the first diffracted image of N.P.S. 3. In this way, also, N.P.S. 4 was compared on some plates with N.P.S. 7 and on others with N.P.S. 8 and 9; N.P.S. 5 was compared with N.P.S. 8 and 9, etc.

7. The magnitudes of the three brightest stars in the sequence—Polaris, δ Urs. Min., and 32 H Cephei—cannot be determined by direct comparison with the Pole Field using the grating employed in the present investigation. Their magnitudes may be determined, however, in a convenient manner by direct comparison with some of the stars of intermediate brightness. In order to secure as much material as possible for the plot connecting magnitude and scale estimation for any particular plate, advantage was taken of the groupings of certain of the bright stars. Polaris with stars N.P.S. 5, 8, and 9 form a group, and for the plates secured for the determination of the magnitude of Polaris the telescope was set on the centre of this group, so that these four stars are obtained for the same exposure and all within a short distance of the centre. On a number of these plates sequence stars 1 (δ Urs. Min.) and 4 were also photographed, the telescope being set for their mean R.A. and Dec. The magnitudes of sequence stars 4, 5, 8, and 9 having previously been determined, six points are available for the plotting of the scale estimation-magnitude relation, by using both principal and diffracted images of the two brighter stars. If, in addition, second diffracted images are also used, eight points become available. This number is amply sufficient for a good determination of the relation. From the plot the magnitude of the first diffracted image of N.P.S. 1 may be obtained, and that of the principal image used to extend the range beyond the point required for deducing the magnitude of Polaris. Plates with two exposures on the group N.P.S. 1*s*, 5, 8, and 9, and two on the pair N.P.S. 1 and 4, serve therefore to determine the magnitudes of both N.P.S. 1*s* (Polaris) and 1 (δ Urs. Min.).

For the determination of the magnitude of N.P.S. 2 (32 H Cephei) two exposures were given on the group N.P.S. 1*s*, 5, 8, and 9, two exposures on the pair N.P.S. 2*s* and 7, the telescope being set for their mean R.A. and Dec. and two exposures on N.P.S. 2. The length of each exposure being only one minute, the time occupied in obtaining a single plate was less than 10 minutes. These plates give the magnitude both of N.P.S. 2 and of N.P.S. 1*s*.

8. The deviations of individual points from the straight line representing as accurately as possible the measures of any one plate may be due to accidental error in estimation, to an error in the assumed magnitude, or to the use of incorrect scale corrections. The accidental error in estimation varies with the quality of the night: the scale used was obtained on a night of good definition and the images obtained under conditions of inferior definition are not exactly comparable with the scale images. To avoid, as far as possible, errors arising from this cause, observations were restricted to nights of reasonably good definition. With a sufficient number of plates the accidental error of measurement should be eliminated. With a good plate, with equal Pole images, the differences between the estimations of the two images of one star rarely exceed three-tenths of a scale interval, representing in magnitude about $0^m{\cdot}15$. The scale corrections used are believed to be accurate within very narrow limits; after their definite adoption, the accumulation of material from subsequent measurements did not indicate that the adopted values required any further revision. The mean deviation obtained from a large number of plates for any individual star may therefore be used as a correction to the assumed magnitude. Thus the plates taken primarily for the magnitudes of the brightest stars were utilised also for the revision of the magnitudes assumed for the intermediate stars, N.P.S. 4, 5, 7, 8, 9, and *2s*. In particular, the series of plates used for the determination of the magnitude of N.P.S. 2 give an accurate correlation of the magnitudes of N.P.S. 5 and *2s*. These two stars are nearly equal in photographic magnitude: the difference in the scale estimations for the two stars, converted into magnitude, gives a value of their magnitude difference almost independent of any possible errors in scale corrections or in the magnitudes assumed for other stars. Direct comparison was possible between their principal images, their first diffracted and on some plates their second diffracted images also.

It is hoped that in this way a homogeneous self-consistent series of magnitudes has been obtained.

In the reduction of the results, a correction has been applied to allow for differential atmospheric extinction whenever the fields compared on the same plate were not photographed at the same altitude. Had the endeavour to photograph each field when at the altitude of the Pole been too rigorously adhered to, it would not have been possible to complete the programme of observation within the desired time. Moreover, for the brightest stars, the condition could not be fulfilled.

The correction applied was $0^m{\cdot}01$ per 1° difference in altitude. This value was used by Schwarzschild in the *Göttingen Aktinometrie*. The correction is small, and as in general some plates of any given star were obtained above Pole and others below Pole, the mean correction for many of the stars is negligible.

9. *Results.* (i) *Primary Sequence.*—The results obtained can only be given in summarised form, on account of limitations of space. For the brighter stars, the nature of the material is given, the number of plates, the mean magnitude, the extreme range from individual plates, and the mean discordance. In arriving at these results, the magnitude interval between principal and first diffracted image has been adopted

as $2^m\cdot83$, and that between principal and second diffracted as $3^m\cdot48$. Corrections for differential atmospheric extinction have been applied.

N.P.S. 1. — Fourteen plates obtained. Magnitude deduced by comparison with adopted magnitudes of N.P.S. 4, 5, 8, and 9. First and second diffracted images measured on each plate. Mean magnitudes deduced from first and second diffracted images respectively are $4^m\cdot37$ and $4^m\cdot39$. The range in the magnitudes deduced from individual plates is $0^m\cdot15$ and the mean discordance $0^m\cdot03$. The adopted magnitude is $4^m\cdot38$.

N.P.S. 2.—Eleven plates obtained. Magnitude deduced by comparison with N.P.S. 2*s*, 5, 7, 8, and 9. First and second diffracted images measured on each plate: deduced mean magnitudes $5^m\cdot23$ and $5^m\cdot24$ respectively. The range in the magnitudes deduced from individual plates is $0^m\cdot12$ and the mean discordance is $0^m\cdot04$. The adopted magnitude is $5^m\cdot24$.

N.P.S. 3.—Fifteen plates obtained. On seven of these, the star was compared directly with the Pole Field: on the remaining eight, N.P.S. 7 was also photographed at the centre as an intermediary star. The second diffracted image was measured on two of the former seven plates and on the latter eight. The mean magnitude deduced from the former series is $5^m\cdot81$ and from the latter $5^m\cdot74$. The mean value from fifteen first diffracted images is $5^m\cdot79$, and from ten second diffracted images is $5^m\cdot70$. The fifteen plates give a range of $0^m\cdot36$, a mean discordance of $0^m\,08$, and a final magnitude of $5^m\cdot77$.

N.P.S. 4.—This star was compared directly with the Pole on eleven plates: on four of these plates N.P.S. 8 and 9 were also photographed at the centre. First and second diffracted images were measured on all these plates. The mean magnitudes given by them are respectively $5^m\cdot87$ and $5^m\cdot88$. The magnitude adopted from these plates is $5^m\cdot87$; the range is $0^m\cdot20$ and the mean discordance $0^m\cdot03$.

The star was also obtained on fourteen plates used for determining the magnitude of N.P.S. 1. The magnitudes read from the scale-magnitude relations for these plates—using principal, first diffracted, and for four plates the second diffracted image—have a mean value of $5^m\cdot86$, with a range of $0^m\cdot19$ and a mean discordance of $0^m\cdot04$.

The magnitude adopted for the star is $5^m\cdot87$.

N.P.S. 5.—This star was obtained at the centre of eight plates and compared directly with the Pole and with N.P.S. 8 and 9. First and second diffracted images were measured on all these plates, giving mean magnitudes of $6^m\cdot41$ and $6^m\cdot42$ respectively. The mean value for these plates is $6^m\cdot42$, with a range of $0^m\cdot21$ and a mean discordance of $0^m\cdot07$.

The star was measured also on four plates taken with N.P.S. 4, 8, and 9 at the centre and exposed also on the Pole. The images on these plates are about 35′ from the centre. First and second diffracted images were measured. The mean magnitude obtained is $6^m\cdot59$, with a range of $0^m\cdot06$.

The magnitudes of this star were also read off from the results obtained for the bright star plates. From the fourteen plates exposed for N.P.S. 1, a mean magnitude is deduced of $6^m\cdot47$, with a range of $0^m\cdot35$ and a mean discordance of $0^m\cdot09$. From ten plates exposed for

N.P.S. 2, a mean magnitude is deduced of $6^{m}\cdot42$, with a range of $0^{m}\cdot21$ and a mean discordance of $0^{m}\cdot06$.

The first series of eight plates receive the greatest weight, and the adopted mean value for the whole series is $6^{m}\cdot45$.

But on the ten plates exposed for N.P.S. 2, a direct comparison is obtainable between N.P.S. 5 and 2*s*, which are equidistant from the centres of the plates and of nearly equal magnitudes. This comparison is almost independent of any errors in the assumed scale corrections or of any uncertainty in the representation of the results of any single plate by a straight line, and therefore receive great weight. From the mean of the comparisons of principal images on ten plates, corrected for differential extinction, the difference of magnitudes of N.P.S. 5 and 2*s* is $-^{m}\cdot06$; from comparisons of first diffracted images on ten plates is obtained $-^{m}\cdot07$; from comparisons of second diffracted images on six plates is obtained $-^{m}\cdot03$. The adopted difference of magnitude is $-^{m}\cdot06$. Combining this with the results obtained for N.P.S. 2*s*, the magnitude finally adopted for N.P.S. 5 is $6^{m}\cdot46$.

N.P.S. 6 was obtained on fourteen plates, being compared directly with the Pole. The first and second diffracted images were measured on fourteen and nine plates respectively, and gave mean magnitudes of $7^{m}\cdot08$ and $7^{m}\cdot13$ respectively. The mean value from all plates is $7^{m}\cdot10$, with a range of $0^{m}\cdot27$ and a mean discordance of $0^{m}\cdot06$.

N.P.S. 7.—This star was compared with the Pole and photographed at the centre of the plate on fifteen plates, on eight of which N.P.S. 3 was also photographed at the centre. First diffracted images were measured on all the plates and second diffracted images on six of the latter plates. The mean magnitude deduced from fifteen first diffracted images is $7^{m}\cdot30$, and from six second diffracted images $7^{m}\cdot37$. The individual plates give a mean value of $7^{m}\cdot32$, with a range of $0^{m}\cdot29$ and a mean discordance of $0^{m}\cdot07$.

The magnitude of the star may also be deduced from the six plates taken for the magnitude of N.P.S. 2*s*, with the latter star at the centre and compared with the Pole. First diffracted images only were measured. The mean magnitude so obtained is $7^{m}\cdot33$, with a range of $0^{m}\cdot32$ and a mean discordance of $0^{m}\cdot09$. On these plates N.P.S. 7 is at a distance of $35'$ from the centre of the plate.

From the plates taken specially for N.P.S. 2 the magnitude of N.P.S. 7 may also be read off. The principal image only was available. The mean magnitude thus deduced from ten plates is $7^{m}\cdot35$, with a range of $0^{m}\cdot33$ and a mean discordance of $0^{m}\cdot07$.

N.P.S. 8 and 9.—These stars are close together and do not differ greatly in magnitude. Their magnitudes were therefore deduced from the same plates.

The pair of stars were photographed at the centre and compared directly with the Pole on six plates, N.P.S. 4 being also photographed at the centre of four of these plates. For N.P.S. 8 both principal and first diffracted images were measured, the magnitude of the principal image being obtained by a slight extrapolation. The mean magnitude deduced from these six plates is $8^{m}\cdot33$, with a range of $0^{m}\cdot27$ and a mean discordance of $0^{m}\cdot07$. For N.P.S. 9, the first diffracted image

was measured on one plate only. The mean magnitude from the six plates is $8^m\cdot95$, with a range of $0^m\cdot20$ and a mean discordance of $0^m\cdot07$.

These stars were photographed also with N.P.S. 5 on the eight plates, on which the latter star was obtained at the centre and compared with the Pole Field. On these plates, the two stars are about 40′ from the centre. For N.P.S. 8, measuring both principal and first diffracted images, the mean magnitude deduced is $8^m\cdot35$, with a range of $0^m\cdot10$ and a mean discordance of $0^m\cdot02$. For N.P.S. 9, measuring second diffracted images on two plates only and principal images on all, the mean magnitude deduced is $9^m\cdot03$, with a range of $0^m\cdot21$ and a mean discordance of $0^m\cdot07$.

The magnitudes were also read off from the plots for the bright star plates (taken for N.P.S. 1 and 2). The principal images only were shown on these plates. From the mean of twenty-five plates, the magnitude of N.P.S. 8 was found to be $8^m\cdot37$, with a range of $0^m\cdot35$ and a mean discordance of $0^m\cdot06$. N.P.S. 9 was measured on nineteen plates, which gave a mean magnitude of $9^m\cdot00$, with a range of $0^m\cdot38$ and a mean discordance of $0^m\cdot06$. The adopted magnitudes are—for N.P.S. 8, $8^m\cdot35$, and for N.P.S. 9, $8^m\cdot98$.

10. (ii) *Supplementary Sequence Stars. N.P.S.* 1*s* (Polaris). — Polaris was measurable on most of the plates obtained for the two bright stars N.P.S. 1 and 2. A common exposure was given on the group N.P.S. 1*s*, 5, 8, and 9, with the telescope set for a mean R.A. and Dec. Polaris is about 15′ from the centre of the plate. On a few plates, only the second diffracted image was measurable, the first diffracted image being too bright and merging into the principal image. From fourteen plates exposed for N.P.S. 1, the magnitude deduced from measurements of nine first diffracted images is $2^m\cdot56$, and from fourteen second diffracted images is $2^m\cdot49$. The combined results give a mean value of $2^m\cdot52$, with a range of $0^m\cdot23$ and a mean discordance of $0^m\cdot06$. After correction for the variation of Polaris the mean value becomes $2^m\cdot53$.

From the plates obtained for N.P.S. 2, the magnitude of Polaris is most accurately determined by direct comparison between the principal image of N.P.S. 2 and the first diffracted image of Polaris. From nine plates, the mean difference in magnitude is found to be $0^m\cdot11$, N.P.S. 2 being the brighter, with a range of $0^m\cdot16$ and a mean discordance of $0^m\cdot05$. This difference becomes $0^m\cdot13$, after correction for the variation of Polaris. The magnitude of N.P.S. 2 has previously been adopted as $5^m\cdot24$, whence it follows that the magnitude of Polaris (at mean brightness) is $2^m\cdot54$.

The value $2^m\cdot54$ is adopted for the mean magnitude.

N.P.S. 2*s*.—This star was photographed at the centre of the plate and compared directly with the Pole on six plates. First diffracted images only were measured. The mean magnitude found from these plates is $6^m\cdot49$, with a range of $0^m\cdot27$ and a mean discordance of $0^m\cdot06$.

The star was also measured on fifteen plates taken for N.P.S. 7, this star being at the centre and N.P.S. 2*s* about 35′ away. Second diffracted images were measured on eight of these plates. The mean magnitude derived from the fifteen first diffracted images is $6^m\cdot62$, and from the eight second diffracted images is $6^m\cdot55$. The mean magnitude from

the fifteen plates is $6^m{\cdot}60$, with a range of $0^m{\cdot}34$ and a mean discordance of $0^m{\cdot}08$.

From the nine plates obtained for N.P.S. 2, it was deduced—as explained under N.P.S. 5—that the difference of the magnitudes of N.P.S. 5 and 2*s* is $0^m{\cdot}06$. These plates also give a mean magnitude for N.P.S. 2*s* of $6^m{\cdot}48$.

From the whole of this material, a final mean magnitude of $6^m{\cdot}52$ is deduced.

N.P.S. 3*s*.—The magnitude of this star is deduced from nine plates, on each of which it was photographed at the centre and compared with the Pole. First and second diffracted images were measured on each plate, giving mean magnitudes respectively of $6^m{\cdot}66$ and $6^m{\cdot}64$. The adopted mean is $6^m{\cdot}65$, with a range of $0^m{\cdot}18$ and a mean discordance of $0^m{\cdot}04$.

11. (iii) *Red Stars. N.P.S.* 1*r*.—Eleven plates were obtained of this star, which was photographed at the centre of the plate and compared directly with the Pole. First and second diffracted images were measured respectively on eleven and six plates, and gave mean magnitudes of $6^m{\cdot}73$ and $6^m{\cdot}70$. The magnitude adopted from the eleven plates is $6^m{\cdot}72$, with a range of $0^m{\cdot}44$ and a mean discordance of $0^m{\cdot}09$.

N.P.S. 2*r* and 4*r*.—These two stars are close together and their magnitudes were deduced from the same plates, the telescope being set at their mean R.A. and Dec. Ten plates were obtained, the Pole Field being photographed on each.

For N.P.S. 2*r*, first and second diffracted images were measured respectively on ten and eight plates, giving mean magnitudes $7^m{\cdot}89$ and $7^m{\cdot}93$. The magnitude adopted is $7^m{\cdot}91$, with a range of $0^m{\cdot}30$ and a mean discordance of $0^m{\cdot}10$.

The principal image of N.P.S. 4*r* was measured on ten plates and the first diffracted image on eight plates. The resulting mean magnitudes are $9^m{\cdot}07$ and $9^m{\cdot}14$ respectively. The magnitude adopted is $9^m{\cdot}11$, the ten plates having a range of $0^m{\cdot}28$ and a mean discordance of $0^m{\cdot}06$.

N.P.S. 3*r* and 5*r*.—The magnitudes of these two stars, which are close together, were also obtained from the same plates. Six plates were obtained on which the pair were compared directly with the Pole. Each star was at a distance of about 20′ from the centre of the plate. The principal images only were measured.

The magnitude deduced for N.P.S. 3*r* is $8^m{\cdot}93$, with a range from the six plates of $0^m{\cdot}16$ and a mean discordance of $0^m{\cdot}03$; for N.P.S. 5*r*, the value found is $10^m{\cdot}10$, with a range of $0^m{\cdot}22$ and a mean discordance of $0^m{\cdot}08$.

These two stars were also measured on certain of the bright star plates. From nine of these plates, a mean magnitude for N.P.S. 3*r* is found of $9^m{\cdot}03$, with a range of $0^m{\cdot}28$ and a mean discordance of $0^m{\cdot}06$. N.P.S. 5*r* was measured on five plates, which give a mean magnitude of $10^m{\cdot}07$, with a range of $0^m{\cdot}17$ and a mean discordance of $0^m{\cdot}06$. On these plates, the stars were about 45′ from the centre of the plate.

N.P.S. 5*r* was also measured on nine plates taken primarily for N.P.S. 1*r*, the star being at a distance of 45′ from the centre of the plate. These measures give a mean magnitude of $10^m{\cdot}27$, with a range of $0^m{\cdot}25$ and a mean discordance of $0^m{\cdot}07$.

The magnitude adopted for N.P.S. $3r$ is $8^m\cdot97$, and for N.P.S. $5r$ is $10^m\cdot13$.

12. *Pole Field.*—The stars of the Polar Sequence after N.P.S. 9, $3s$, and $5r$ are all obtained from the Pole Field itself. The magnitudes of N.P.S. 10 and 11, which are brighter than 10^m, were primarily determined from the long-exposure Pole plates, by measuring the diffracted images. Provisional magnitudes were thus deduced, which were assumed in subsequent reductions. From all the plates on which the Pole Field was photographed, the magnitudes of the sequence stars in the field were read off. The mean values of the magnitudes so determined, with the number of plates utilised for each star, are given in the summary below. They carry the determination of the magnitudes of the Polar Sequence down to a limit of $13^m\cdot4$; on a few of the longer exposure plates some fainter stars were shown, but they were too close to the threshold of the plate to be measurable with accuracy. The present investigation, therefore, covers a range of about eleven magnitudes.

13. *Light Variation of Polaris.*—After the magnitudes of Polaris had been obtained from the bright star plates, an examination of the results was made as to whether they revealed any evidence of variation. The elements assumed were

$$M = \text{J.D. } 2418985\cdot936 + 3^d\cdot9681n$$
$$M - m = 1^d\cdot984.$$

No attempt had been made when taking the plates to secure a distribution well-balanced about the mean brightness, and it was found that unfortunately a large majority of the plates had been taken when the brightness was above the mean. Of the fourteen plates taken for N.P.S. 1 on which Polaris was measured, eleven were obtained when the star was brighter than the mean. Assuming a range of variation of $0^m\cdot10$, the results of these plates may be summarised as follows:—

	First Diff. Image.	Second Diff. Image.
Mean mag. from eleven plates	2·26 (6 plates)	2·20 (11 plates)
Correction to reduce to mean mag.	+ ·04	+ ·04
Mean mag. from three plates	2·35 (3 plates)	2·28 (3 plates)
Correction to reduce to mean mag.	− ·04	− ·04

The three plates obtained when Polaris was fainter than mean give a magnitude difference compared with those obtained when brighter than the mean of $+0^m\cdot09$ and $+0^m\cdot08$, using first and second diffracted images respectively. These differences, after applying the correction for variation, are reduced to $+0^m\cdot01$ and $0^m\cdot00$.

Polaris was also compared directly with N.P.S. 2 on nine plates. Of these, six plates were obtained when it was brighter than the mean.

	Magnitude of First Diffracted Image of Polaris *minus* Magnitude of N.P.S. 2.
	m
Mean of six plates	+ ·08
Correction to reduce to mean mag.	+ ·04
Mean of three plates	+ ·14
Correction to reduce to mean mag.	− ·04

The difference of $+^{m}\cdot06$, as between the plates taken when Polaris was faint and bright respectively, is therefore reduced, after correction for variation, to $-^{m}\cdot02$.

These results indicate that a range of variation in photographic magnitude is about $0^{m}\cdot10$.

14. *Summary of Results.*—The magnitudes obtained in the present investigation are summarised in the table below. The first column of this table gives the Harvard North Polar Sequence number; the second column gives the B.D. numbers of the brighter stars; the third and fourth columns give respectively the right ascension and declination for the epoch 1900; the fifth column contains the magnitudes adopted from the present work; and in the next two columns are given the differences between these magnitudes and those determined at Mount Wilson (F. H. Seares, *Mount Wilson Contributions*, No. 97) and the final revised values obtained at Harvard (*Harvard Annals*, **71**, part 3) respectively. The eighth column gives the colour index determined at Mount Wilson. The ninth column gives the number of plates, from which the magnitudes of each star were determined in the present investigation. The numbers of plates for the bright stars are fewer than for the brighter stars in the Pole Field, but for these stars the weight of a single plate is greater.

Catalogue of Magnitudes.

N.P.S. No.	B.D. No.	R.A. 1900.	N. Dec. 1900.	Greenwich Mag.	Greenwich *minus* M.W. (Unit $0^{m}\cdot01$.)	Greenwich *minus* H.C.	Colour Index (M.W.).	No. of Plates.
		h m	° ′	m			m	
1	86°·269	18 4·6	86 36·8	4·38	− 1	−10	0·02	14
2	85·383	22 21·3	85 36·3	5·24	− 6	− 4	0·02	11
3	86·344	23 27·8	86 45·3	5·77	− 6	− 4	0·27	15
4	86·272	18 7·8	86 59·6	5·87	− 4	−12	0·07	25
5	88·4	0 55·6	88 29·3	6·46	+ 1	− 3	−0·02	26
6	89·13	7 58·0	88 56·0	7·10	− 1	− 1	0·06	14
7	88·64	11 4·2	88 11·0	7·35	− 6	+ 4	−0·11	31
8	88·9	2 14·2	88 42·1	8·35	+ 3	+12	0·19	39
9	88·13	2 42·2	88 34·1	8·98	+10	+15	0·07	33
10	89·3	3 18·9	89 41·2	9·12	0	+10	0·05	108
11	89·18	11 35·0	89 29·1	9·82	+ 9	+27	0·20	114
12	89·25	13 51·5	89 28·9	10·09	0	+23	0·29	113
13	89·29	15 13·6	89 38·4	10·51	− 2	+21	0·16	116
14	89·1	0 34·0	89 51·2	10·88	−10	+23	0·44	109
15		7 43·2	89 45·3	11·27	+ 5	+19	0·33	96
16		20 55·2	89 43·2	11·56	− 6	+16	0·39	81
17		18 1·2	89 40·2	11·89	+ 2	+26	0·59	56
18		18 43·6	89 43·6	12·27	0	+21	0·38	37
19		17 33·6	89 52·2	12·66	− 3	+24	0·41	15
20		19 46·8	89 55·0	13·02	0	+29	0·51	11
21		17 21·6	89 44·6	13·39	+ 6	+43	0·85	9

Catalogue of Magnitudes —continued.

N.P.S. No.	B.D. No.	R.A. 1900.	N. Dec. 1900.	Greenwich Mag.	Greenwich *minus* M.W. (Unit $0^m{\cdot}01$.)	Greenwich *minus* H.C.	Colour Index (M.W.).	No. of Plates.
	°	h m	° ′	m			m	
1*s*	88·8	1 22·6	88 46·4	2·54	0	−15	0·46	23
2*s*	88·71	12 14·4	88 15·3	6·52	+ 7	+ 4	0·12	30
3*s*	87·107	12 13·9	86 59·5	6·65	+ 1	0	0·29	9
4*s*	89·12	8 30·5	89 32·2	10·37	+12	+25	0·42	127
5*s*	89·37	15 29·3	89 52·8	11·03	− 6	+22	1·03	108
6*s*	89·26	12 53·2	89 41·7	11·36	− 2	+25	0·65	95
7*s*		15 24·0	89 53·0	12·55	− 6	+10	0·51	22
1*r*	87·51	6 53·7	87 12·3	6·72	+11	− 8	1·52	11
2*r*	88·112	19 22·5	88 59·3	7·91	− 3	+ 4	1·59	10
3*r*	88·76	13 4·5	88 11·2	8·97	+ 1	+22	1·40	15
4*r*	88·114	19 43·3	88 41·1	9·11	−11	+ 1	0·96	10
5*r*	89·22	12 51·5	88 54·1	10·13	0	+22	1·48	20
6*r*	89·9	6 38·9	89 28·1	10·58	+12	+28	1·25	116
7*r*	89·35	15 55·1	89 46·2	10·96	+ 2	+26	1·04	111
8*r*	89·31	16 49·3	89 38·6	11·43	− 1	+23	1·00	93
9*r*		14 51·2	89 34·2	11·95	...	+25	...	48
10*r*		13 8·0	89 42·7	12·60	...	+25	...	20
11*r*		14 58·4	89 53·3	13·29	+ 7	+32	1·16	8

15. The zero of the magnitudes given in the fifth column of the preceding table was chosen to give agreement with the Mount Wilson scale for the principal and supplementary sequence stars between $10^m{\cdot}0$ and $13^m{\cdot}5$. The zero of the Mount Wilson series was chosen so as to agree with the Harvard scale at the sixth magnitude.

The differences in the sixth column in no case exceed $0^m{\cdot}12$, and indicate a very close parallelism between the results obtained at Mount Wilson and in the present investigation. The differences from Harvard magnitudes given in the seventh column show a progressive variation. The two series agree at about the sixth magnitude, but for brighter stars the Harvard magnitude is too faint, whilst for stars fainter than sixth magnitude the Harvard magnitude is too bright.

The present investigation, therefore, confirms the Mount Wilson photographic scale between the limits of magnitude $2^m{\cdot}5$ and $13^m{\cdot}4$.

The magnitudes of the sequence stars between 10^m and 16^m have previously been determined at Greenwich with the 26-in. refractor; Seares found that the differences between these magnitudes and those found for the same stars with the 60-in. reflector at Mount Wilson showed a close correlation with the colour of the star. If a red star with colour index of $1^m{\cdot}0$ is measured with the 26-in. refractor $0^m{\cdot}27$ brighter than a blue star with colour index $0^m{\cdot}0$, the two stars are found with the reflector to be of equal magnitude.

No marked colour difference between the results obtained with the

13-in. refractor and the 100-in. reflector is shown by the residuals in the sixth column of the above table. A least-squares solution was made, giving equal weight to each star, and representing the residuals in the form $a + b(m - 10) + c\mathrm{C}$, C being the colour index and m the magnitude of the star. The second term would allow for a difference in light-ratio between the two series of results such as might have occurred, for instance, had an incorrect value of the grating interval been adopted in the present investigation. The following relation was thus found:—

$$\mathrm{M.W.} - \mathrm{Gwh.} = +0^{m}{\cdot}01 + 0^{m}{\cdot}001(m - 10) - 0{\cdot}04\mathrm{C}.$$

The smallness of the coefficient of the second term indicates that the two scales are in almost perfect agreement as regards light-ratio. There is a small colour coefficient, in a sense opposite to that obtained with the 26-in. refractor but similar to—though smaller than—the value found by Seares for the Potsdam 80-cm. refractor ($-0{\cdot}06$).

The colour equation of the 13-in. refractor is therefore not greatly different from that of the reflector, a result of value in photometric work. For a difference in colour index of one magnitude there is a relative difference between the two instruments of only $0^{m}{\cdot}04$.

Applying this correction to the residuals (Greenwich minus Mount Wilson), to eliminate the small relative colour equation, the sum of the squares of the residuals was reduced by about 10 per cent.

16. *Summary.*—The photographic magnitudes of all the stars in the North Polar Sequence, down to a limit of $13^{m}{\cdot}4$, have been determined. The 13-in. astrographic refractor was used in conjunction with a parallel wire grating: this grating gives magnitude differences between central and first and second diffracted images of $2^{m}{\cdot}83$ and $3^{m}{\cdot}48$ respectively. The results obtained are summarised in the table accompanying this paper: they confirm the magnitude scale determined at Mount Wilson with the 60-in. reflector. Comparison of the two series of results indicates very close agreement between the two series in light-ratio. It is also found that the colour equation of the 13-in. refractor does not differ greatly from that of the reflector.

Stellar Parallaxes determined at the Royal Observatory, Greenwich.

(Communicated by the Astronomer Royal.)

The following list is a continuation of the results obtained with the 26-inch telescope.

The method of measurement, by means of an auxiliary scale plate, as described in the *Monthly Notices* for Nov. 1920, has been continued and appears to be quite satisfactory.

In the column Proper Motion the first is derived from meridian observations; the second, in R.A. only, is the relative proper motion derived from the parallax plates. (Both are expressed in arc of a great circle.)

In calculating the parallax, the relative proper motion is used, *not* the meridian proper motion.