Comète 1914 *a* (*Continued*).

Mai 15. La Comète est de 9.5 grandeur; la tête, ronde, est large de 1′.5.
22. L'éclat de la Comète apparaît plus faible: de 10^e^ à 11^e^ grandeur.
Juin 17. La Comète a augmenté d'éclat, elle est de 9^e^ à 9^e^.5 grandeur. La tête, ronde, mesure 2′ de diamètre, la condensation est centrale et mal définie.
26. La Comète est de grandeur 9.5; la tête est ronde et large de 1′; la condensation est centrale, bien prononcée, mais peu brillante.
29. L'éclat de la Comète est changeant et voisin de la 10^e^ grandeur.
Juil. 18. La Comète, de grandeur 11.5, a une tête ronde de 1′ de diamètre; le noyau est diffus et étalé.

Comète 1914 *b*

Mai 19. La Comète a une tête brillante, ronde, de 4′ de diamètre, avec forte condensation centrale et un noyau bien défini et 4″ à 5″ d'épaisseur, mais d'un éclat variable et voilé. Une queue principale, peu lumineuse, mince et rectiligne disparaît à 1° de la tête par un angle au pôle de 17°. La Comète est estimée de 5^e^ grandeur.
20. Le noyau stellaire est toujours très net; la queue est moins visible que la veille.
21. La Comète a peu changé d'éclat. La queue s'écarte en éventail par $p = 36°$.
Juin 1. La Comète est de 7^e^ grandeur. La condensation seule est visible: l'astre se trouvant dans le crépuscule, près de l'horizon et à proximité de la *Lune*.
3. Pour les mêmes causes que le 1^er^ Juin, la Comète est peu visible.

Observatoire de Besançon, 1918, Janvier 30.

GROUPS OF ASTEROIDS PROBABLY OF COMMON ORIGIN,

By KIYOTSUGU HIRAYAMA.

On examining the distributions of the asteroids with respect to their orbital elements, particularly to the mean motion (n), the inclination (i) and the eccentricity (e), we notice condensations here and there. In general, they seem to be due to chance. But there are some which are too conspicuous to be accounted for by the laws of probability alone.

As an example of such peculiar groups, I shall take the condensation near $n = 730''$. Out of 790 orbits given in the *Berliner Jahrbuch* for 1917, taking 37 between 720″ and 740″ of the mean motion, and classifying them according to the inclination, we count as follows:—

i	Actual No.	Total	Prop. No.	Diff.	Corr. Prop. No.	Diff.
0°— 4°	16	149	7	+9	5	+11
4 — 8	6	213	10	−4	7	− 1
8 — 12	6	194	9	−3	6	0
12 — 16	6	131	6	0	4	+ 2
16 — 20	3	55	3	0	2	+ 1
20 —	0	48	2	−2	2	− 2
Sum	37	790	37	0	26	+11

Sixteen orbits between 0° and 4° of i are surely out of proportion. Assuming the existence of a group physically connected and computing the proportional number according to the remaining $37-16=21$, we see that the probable number of the asteroids belonging to the group is eleven.

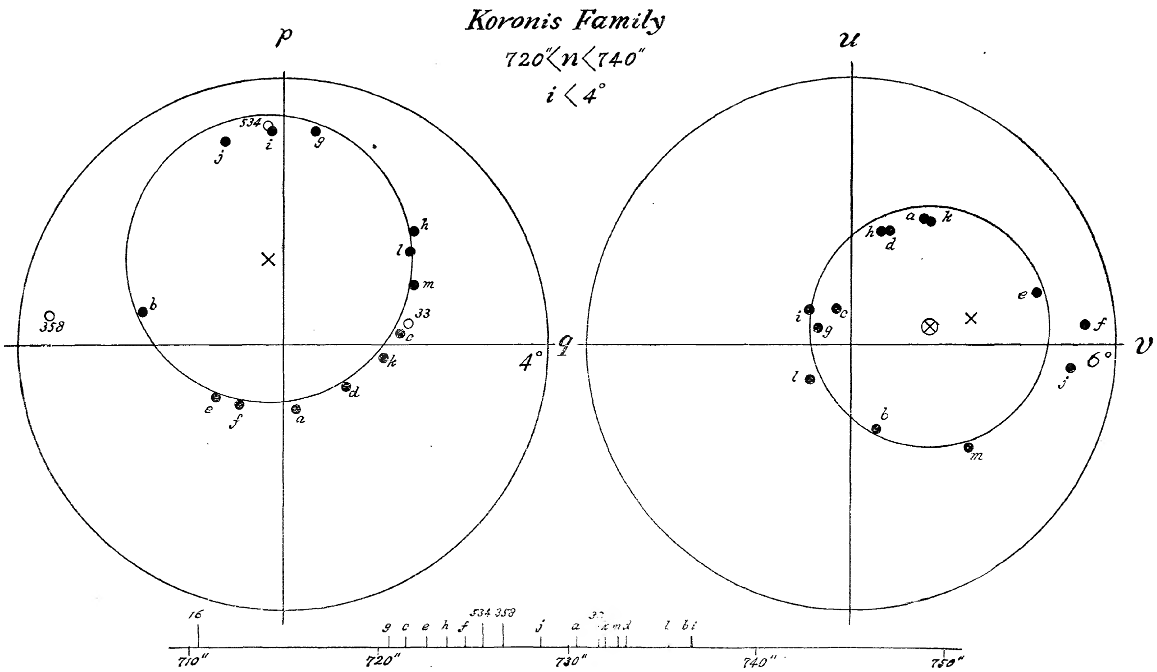
Classifying then the sixteen asteroids by the angle of eccentricity (φ), we get

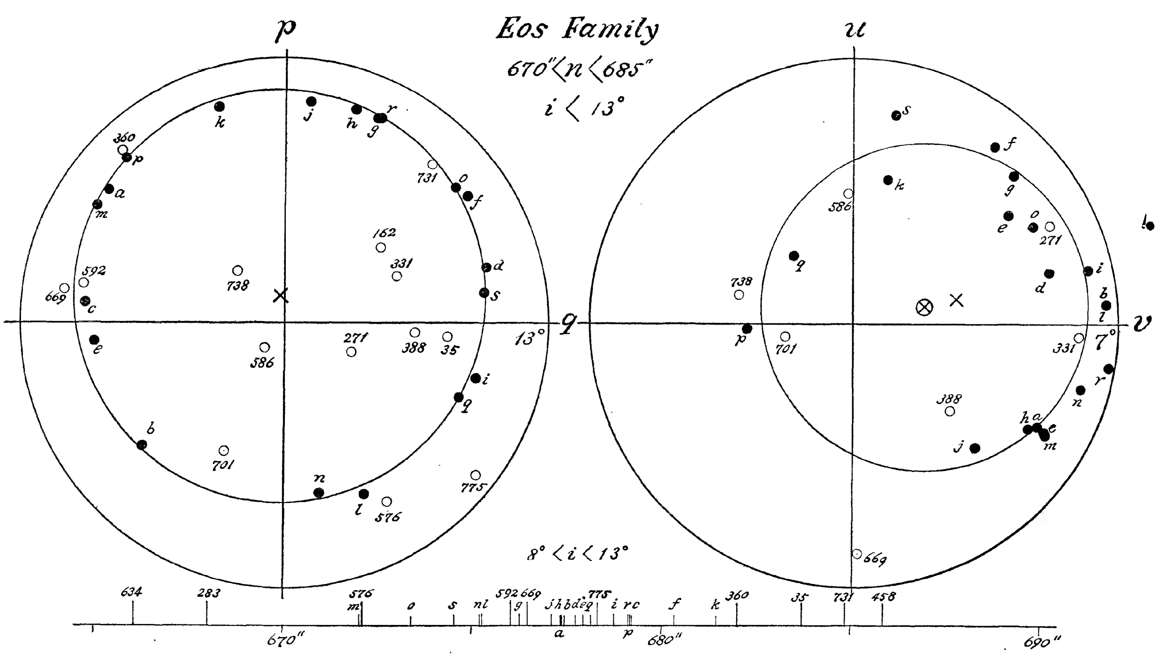
φ	Actual No.	Total	Prop. No.	Diff.
0°— 4°	10	123	2	+8
4 — 8	4	263	5	−1
8 — 12	1	235	5	−4
12 — 16	0	133	3	−3
16 — 20	1	28	1	0
20 —	0	8	0	0
Sum	16	790	16	0

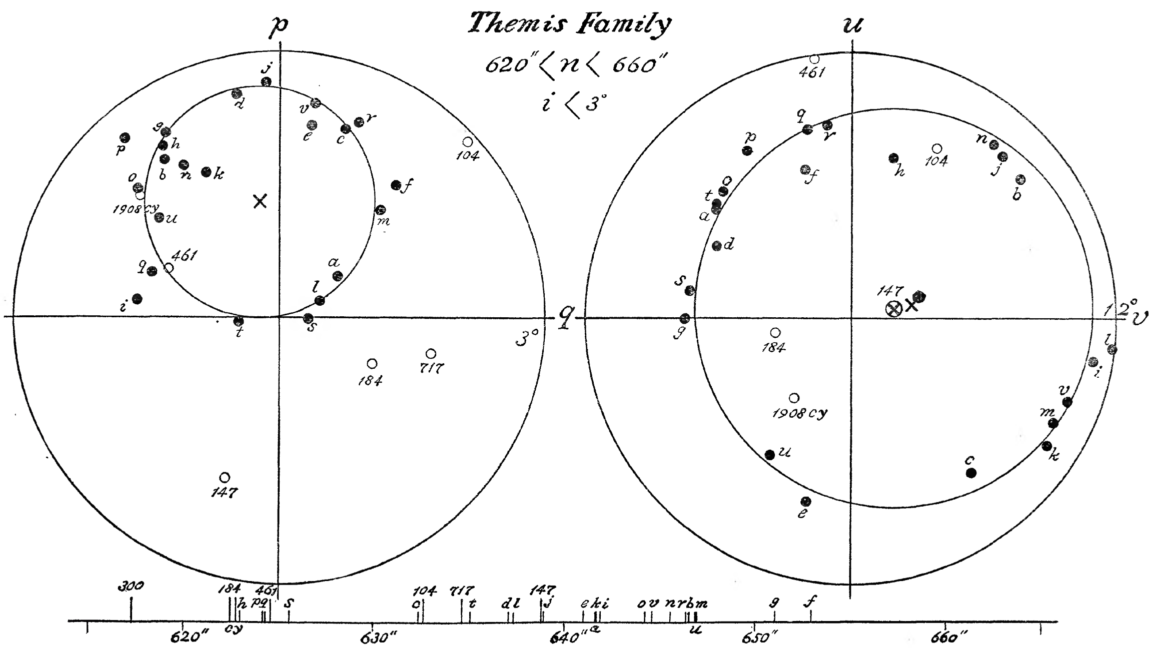
Ten orbits between 0° and 4° of φ are again out of proportion.

This is not all. Taking the sixteen asteroids, if we plot the poles of the orbital planes on a diagram, fifteen points, most curiously, are disposed on a circumference. The center very nearly coincides with the pole of *Jupiter's* orbit. This is certainly a remarkable coincidence.

Still curiously, if we draw similar diagram taking the eccentricity and the longitude of perihelion (ϖ) instead of the inclination and the longitude of the node (☊), thirteen points again are distributed on a circumference,







although not quite so marked as in the case of the inclination. The center in this case does not coincide with the corresponding point for *Jupiter* save the direction from the origin. A group of thirteen asteroids — two more than expected — is found in this manner.

Referring to the theory which will be mentioned below, there can be little doubt about the physical relation connecting these asteroids. So I venture to name the group the *Koronis Family*, associating the name of the asteroid (158) which was discovered first.

I find two more families which will be named the *Eos* (221) *Family* and the *Themis* (24) *Family*. The first contains 19 asteroids within the limits

$$671'' < n < 682'', \quad 8°.6 < i < 11.3, \quad 2°.3 < \varphi < 6°.9.$$

The second contains 22 within the limits

$$622'' < n < 653'', \quad 0°.3 < i < 2°.7, \quad 6°.8 < \varphi < 11°.8.$$

The explanation of the *asteroid families* seems not very hard. Considering the perturbations of *Jupiter*, the equations giving the secular variations of the orbital plane of the asteroid are: —

$$p = \tan i \sin \Omega = p' + N \sin (ht + \beta),$$
$$q = \tan i \cos \Omega = q' + N \cos (ht + \beta),$$

where p' and q' are the corresponding quantities for *Jupiter*, and, N and β are arbitrary constants depending on the initial values of p and q. The quantity h is a function of the semi-major axis (a) and is determined by the equations

$$\frac{h}{n'} = -\frac{3m'}{4\mu}\frac{n'}{n} S_1$$

$$S_1 = 1 + \frac{3}{2}\frac{5}{4}\left(\frac{a}{a'}\right)^2 + \frac{3\cdot5}{2\cdot4}\frac{5\cdot7}{4\cdot6}\left(\frac{a}{a'}\right)^4 + \cdots\cdots,$$

denoting by n' and a' the mean motion and the semi-major axis of *Jupiter* and by $\frac{m'}{\mu}$ the ratio of the masses of *Jupiter* and the *Sun*. Accordingly, if we regard p and q as rectangular coördinates, the point (p, q) describes a circle about the center (p', q') with the radius N and the angular velocity h. The motion is retrograde since h is negative.

Now suppose that an asteroid was broken into a number of fragments at a certain epoch. Suppose also that the additional velocities of the fragments are small compared with the original velocity. Then if we denote the elements of the original asteroid by a, e etc., those of the fragments will be denoted by

$$a + \delta a, \qquad e + \delta e, \quad \text{etc.}$$

The corresponding values of N, β and h become $N + \delta N$, $\beta + \delta\beta$ and $h + \delta h$. The effect of δN and $\delta\beta$ is always small; but that of δh becomes larger and larger as the time increases. The result after a very long duration will be that the points (p, q) of the fragments are distributed irregularly on the whole circumference of a circle whose radius is approximately equal to N and whose center occupies the point (p', q').

As for the eccentricity we have the equations

$$u = e \sin \varpi = ku' + M \sin (gt + \alpha),$$
$$v = e \cos \varpi = kv' + M \cos (gt + \alpha),$$

where u' and v' are the corresponding values for *Jupiter*. The only difference compared with the case of the inclination is that the quantities u' and v' are multiplied by a constant k which depends on a or e. The expression of k is

$$k = \frac{5}{4} \frac{a}{a'} \frac{S_2}{S_1}$$

where

$$S_2 = 1 + \frac{3}{2} \frac{7}{6} \left(\frac{a}{a'}\right)^2 + \frac{3 \cdot 5}{2 \cdot 4} \frac{7 \cdot 9}{6 \cdot 8} \left(\frac{a}{a'}\right)^4 + \cdots .$$

We have also $g = -h$ and therefore the motion is direct.

The approximate mean values of h, k, etc., are computed as follows:

Family	*Koronis*	*Eos*	*Themis*
n	725″	675″	638″
$-n'/h$	1710	1440	1260
k	0.660	0.692	0.716
Period in years	20300	17100	15000

The centers of the motion determined by these values of k are marked on the diagrams with ⊗. It may be seen that these points represent the centers of the circular distributions pretty closely. Our theory is thus verified.

The numbers of the asteroids belonging to each family are as follows:—

Family	*Koronis*	*Eos*	*Themis*
a	158	221	24
b	167	320	62
c	208	339	90
d	243	450	171
e	263	513	222
f	277	520	223
g	311	529	268
h	321	562	316
i	452	573	379
j	462	579	383
k	658	590	431
l	720	608	468
m	761	633	492
n	...	639	515
o	...	651	526
p	...	653	555
q	...	661	561
r	...	742	621
s	...	766	637
t	...	...	656
u	...	...	710
v	...	...	767

Astronomical Observatory, Tokyo, May 16, 1918.

NEW ASTRONOMICAL WORK.

Measures of Rectangular Co-ordinates and Diameters of 63436 Star-Images. Astrographic Catalogue 1900.0, Hyderabad Section, Dec. –16° to –21°, by R. J. POCOCK. Nizamiah Observatory, 1918.

Eighth Report of the Section for the Observation of Variable Stars 1910–1914. Memoirs of the British Astronomical Association, Vol. XXII, 1918.

The Harvard Map of the Sky and the Milky Way, by H. NORT. *Recherches Astronomiques de l'Observatoire d'Utrecht*, VII. Utrecht, 1917.

On the Determination of the Principal Laws of Statistical Astronomy, by W. J. A. SCHOUTEN. W. KIRCHENER, Amsterdam, 1918.

Observaciones con la Ecuatorial de 433 Milimetros de Abertrua, por BERNHARD H. DAWSON. *Observatorio Astronomico, Universidad Nacional de la Plata.* Tome IV (Parte 1), La Plata, 1918.

Observations de Nebuleuses (decouvertes par M. STEPHAN), par E. ESMIOL. Travaux de l'Observatoire de Marseille, IV. Toulouse, 1916.

Formules et Tables Nouvelles Relatives a l'Etude du Mouvement des Cometes et a Differents Problèmes de la Theorie des Orbites, par H. ANDOVER. Paris, 1918.

Latitudine della Prima Sala Meridiana del R. Osservatorio di Pino Torinese, by G. BOCCARDI. Torina, 1916.

CONTENTS.

EDITOR, BENJAMIN BOSS, ALBANY, N. Y.; ASSOCIATE EDITORS: E. E. BARNARD, ERNEST W. BROWN, F. R. MOULTON AND R. S. WOODWARD.
PUBLISHED BY THE DUDLEY OBSERVATORY, ALBANY, N. Y., U. S. A., TO WHICH ALL COMMUNICATIONS SHOULD BE ADDRESSED. PRICE, $5.00 THE VOLUME.
PRESS OF THOS. P. NICHOLS & SON CO., LYNN, MASS. *Closed, October 7, 1918.*