

A CRITERION FOR SPECTROSCOPIC BINARIES, WITH AN APPLICATION TO ρ LEONIS

By FRANK SCHLESINGER

It not infrequently happens that an observer of spectroscopic binaries has accumulated a large number of plates of a single star without being able to determine the period or even to decide whether the object is really a binary. This difficulty occurs when the amplitude of the star is small and of the same order as the accidental errors of observation.

The writer has found the following process to be of considerable service in such cases. The method consists in first constructing a frequency-curve for the velocities; this is done by dividing the total range exhibited by the measured velocities into successive groups of equal extent, say 3 km each, and then counting the number of velocities that fall within these groups. Regarding these numbers as ordinates, we plot them and join the ends by a smooth curve, which is the frequency-curve for these velocities. We next compare the shape of this curve with that of the well-known error-curve. If the two are reasonably the same, we conclude that the star is not a spectroscopic binary (within the limits that our measurements are capable of defining), but that the differences in the measured velocities are errors of observation. This conclusion entails the assumption that the orbits of all binaries, whatever their shape and situation may be, will yield frequency-curves for the velocities that differ decidedly from the error-curve. This assumption accords with the facts, for although the frequency-curves differ greatly from each other for various types of orbits, there is no type that corresponds to the error-curve.

If the frequency-curve is such as to indicate that we are not dealing solely with errors of observation but that the object is really a binary, we can in general go somewhat farther and infer something as to the character of the orbit. For this purpose we divide orbits into four classes:

1. *Circular orbits*.—In this case the velocity-curve (that is, the curve that shows the relation between the velocity and the time)

is a sine-curve as drawn in Fig. 1. Let us divide the total range of velocity into ten equal parts as indicated by the horizontal lines. For a large number of observations the frequency with which any

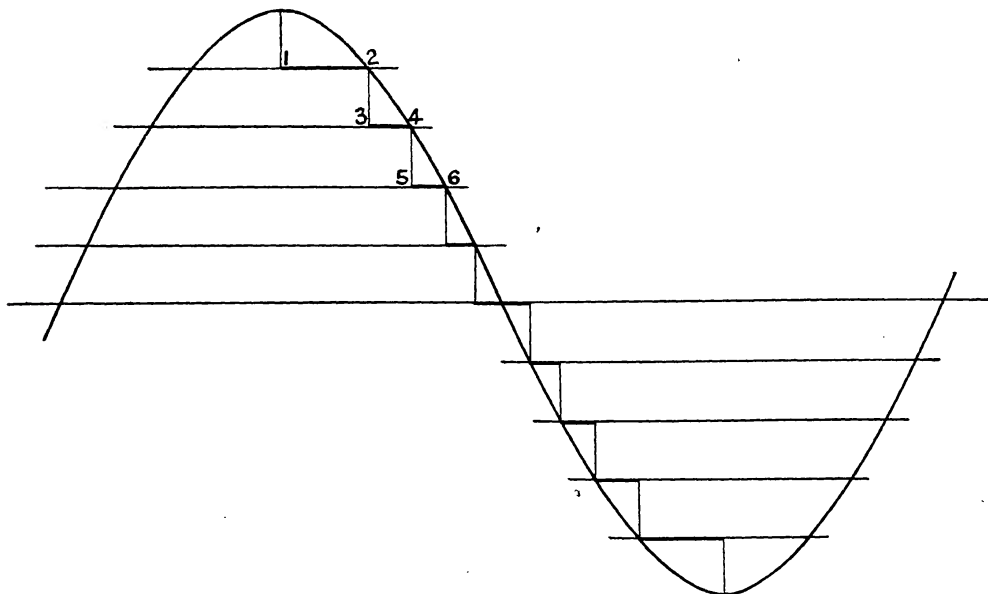


FIG. 1.—Velocity-curve for a circular orbit

velocity occurs will be proportionate to the time-interval during which the velocity is between these successive limits. Consequently the frequencies are proportionate to the lengths (1, 2),

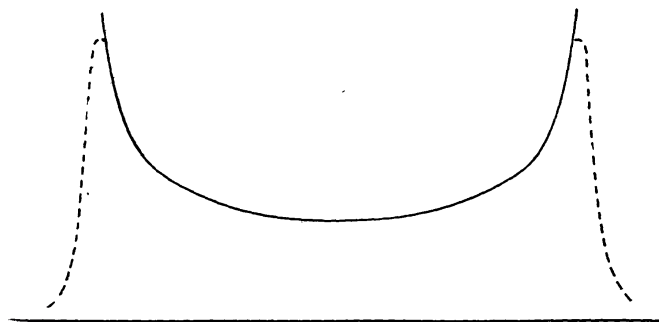


FIG. 2.—Frequency-curve for a circular orbit

(3, 4), (5, 6), etc. The frequency-curve is the full line in Fig. 2. Making allowance for the presence of accidental error, the kind of curve that we should expect in practice is shown by the dotted line.

For a circular orbit, then, high and low velocities occur with the maximum frequency, while intermediate velocities occur more rarely.

2. *Eccentric orbits with periastron near the descending node.*—This velocity-curve is shown in Fig. 3. The frequencies are proportional to the lengths (1, 2), (3, 4), etc., and the frequency-curve therefore has the general appearance shown in Fig. 4. High velocities greatly predominate in number, while low and intermediate velocities are rare.

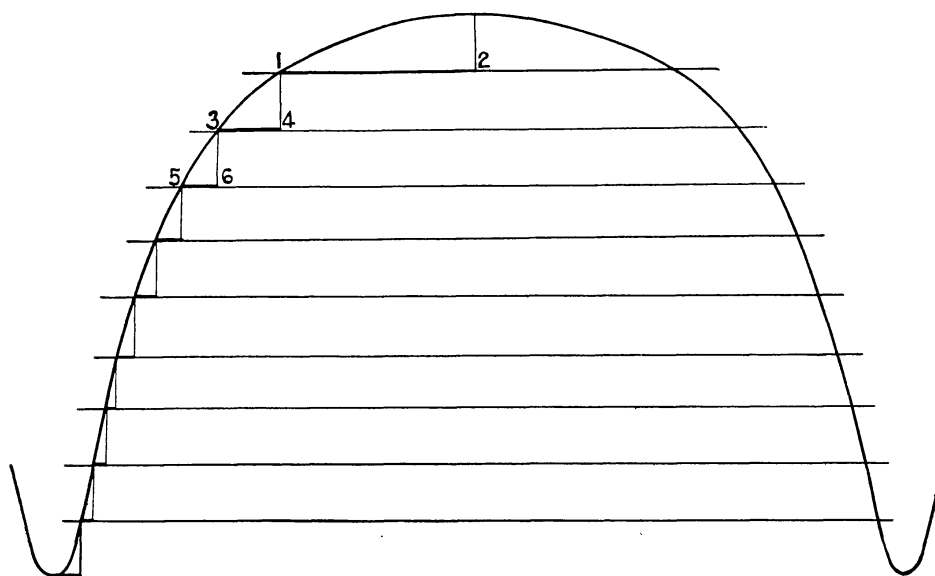


FIG. 3.—Velocity-curve for eccentric orbit with periastron at descending node

3. *Eccentric orbits with periastron near the ascending node.*—The velocity-curve for this case can be obtained by inverting Fig. 3, and the frequency-curve by turning Fig. 4 end for end. Low velocities predominate, while high and intermediate velocities are rare.

4. *Eccentric orbits with periastron removed 90° from the nodes.*—Fig. 5 shows the velocity-curve for the case in which periastron follows the ascending node by 90° , or, expressed in accordance with the usual conventions, the longitude of periastron is 90° . The frequencies are now proportional to (1, 2), (3, 4)+(5, 6), (7, 8)+(9, 10), etc. The corresponding frequency-curve comes out prac-

tically the same as Fig. 2. For the opposite kind of orbit, that in which the longitude of periastron is 270° , the velocity-curve can

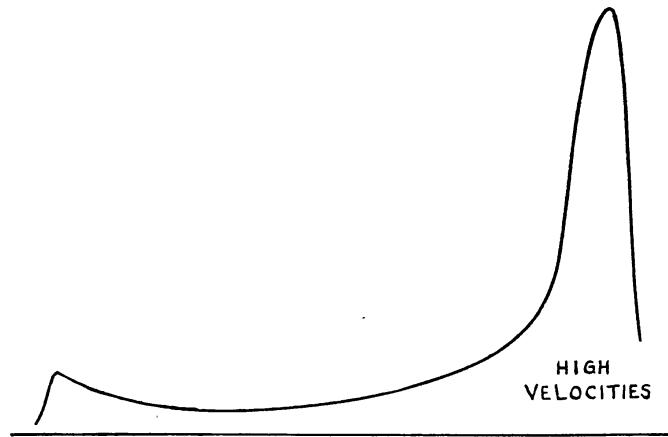


FIG. 4.—Frequency-curve for eccentric orbit with periastron at descending node

be obtained by turning Fig. 5 end for end, but the frequency-curve is precisely the same. The present criterion does not enable us to

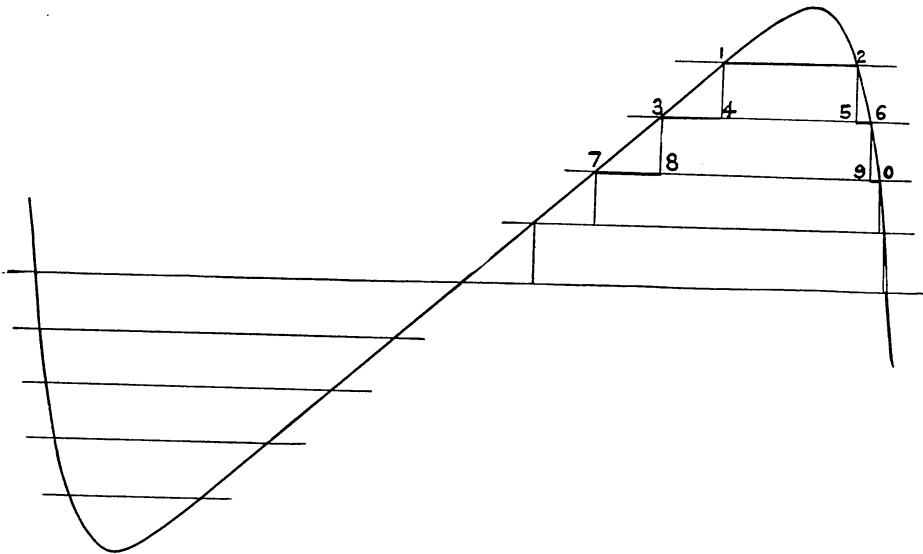


FIG. 5.—Velocity-curve for eccentric orbit with periastron at 90°

distinguish between these two orbits, nor to distinguish either of them from a circular orbit. Consequently if we obtain in any particular case a frequency-curve like that in Fig. 2, we infer either

that the orbit is circular or else that the longitude of periastron is about 90° from the nodes.

The writer has applied this criterion to advantage in a number of cases; the most instructive of these is that presented by ρ Leonis ($10^h28^m, +9^\circ49'$), a helium star of the fourth magnitude. This star was first suspected of varying in its radial velocity by the Lick observers, but later it was removed from their list of spectroscopic binaries. In 1910 and the two following years Mr. Harper obtained sixty-five plates of the star with the single-prism spectrograph of the Ottawa Observatory. From these he was unable to decide whether the object is a spectroscopic binary, though he inclines to believe that it is. Not succeeding in finding a period that will fit the observations, he has (in a very commendable spirit) published¹ his measures and the other data for the plates, in order that they may be available to other astronomers.

The measured velocities for these sixty-five plates range from $+26$ km to $+57$ km. Dividing them into groups with 3 km limits, and counting the number of velocities that fall within each group, we have:

3 velocities less than $+30$ km				
1 velocity between $+30$ km and $+33$ km				
3 velocities	"	$+33$	"	$+36$
6	"	"	$+36$	" $+39$
8	"	"	$+39$	" $+42$
14	"	"	$+42$	" $+45$
20	"	"	$+45$	" $+48$
6	"	"	$+48$	" $+51$
3	"	"	$+51$	" $+54$
1 velocity between $+54$ and $+57$				

The frequency-curve that these numbers give is very different from the error-curve. The greatest number of velocities is found between 45 km and 48 km; below these limits there are altogether thirty-five velocities, while above them there are only ten. We conclude that the star is a spectroscopic binary, and furthermore, since this is an example of what we have described as Case 2, that the orbit is one of high eccentricity with the longitude of

¹ *Publications of the Dominion Observatory*, 1, 337, 1914.

periastron (always reckoned from the ascending node) in the neighborhood of 180° .

If this conclusion is correct, a search for the period would most profitably concern itself only with the lowest velocities. Computers of the orbits of spectroscopic binaries are only too familiar with the difficulty in deciding upon which branch of the curve certain velocities belong, when they are trying to ascertain the period. In the present instance this additional difficulty is removed for the lowest velocities, because these must all be in approximately the same phase, as the reader may see by referring to Fig. 3.

Acting upon this hint, I examined the seven velocities lower than $+33$ km and found that with the period 12.28 days all seven of them fall within a day of the phase 7.3 days, the date of Mr. Harper's first plate being taken as the initial epoch. Furthermore, I found that not one of the highest eighteen velocities falls within these same limits. There can therefore be no doubt that 12.28 days is a close approximation to the period of this binary.

An inspection of the velocity-curve obtained by plotting the Ottawa results on this period enables us to estimate that the semi-amplitude of oscillation is 10 km or more, that the eccentricity is probably greater than 0.5, and that the longitude of periastron probably lies between 150° and 210° . Closer approximations to these elements could doubtless be deduced from the present material by a thorough computation, but this work would hardly be repaid. Now that the period of the binary has been established it is possible to secure additional plates near periastron, and these are indispensable for a more accurate determination of the orbit.

In conclusion I may remark that the present criterion is of wider application than the title of this paper implies. For example, we may by its help sometimes infer the character of the variations in the light of a star before the period has been determined. The test is in fact applicable to all periodic phenomena.

ALLEGHENY OBSERVATORY
UNIVERSITY OF PITTSBURGH
November 21, 1914