

THE ÅNGSTRÖM COMPENSATION-PYRHELIOMETER AND THE PYRHELIOMETRIC SCALE¹

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I propose to discuss here a small source of error which has appeared in the construction of the Ångström compensation-pyrheliometer. The correction involved is shown not to exceed 2 per cent, but since the instrument has been accepted as a standard instrument by the International Congress at Oxford, and since this correction is of particular interest in the construction of instruments of this type, a somewhat detailed discussion may be desirable.

In this paper, therefore, I intend (1) to deduce the magnitude of the correction theoretically, and (2) to give an experimental determination of this correction.

As is well known, the principal part of the Ångström pyrheliometer consists of two equally blackened manganin strips, to the middle points of which are connected the two junctions, m and n , respectively, of a thermo-element. The two metal strips are soldered at both ends to the metal supports O and P , O' and P' .

The measurement of the radiation is made as follows: One of the strips is exposed to the radiation, the other screened from it. We then measure the current which must flow through the screened strip in order to keep the attached junction of the thermo-element in the middle of the strip at the same temperature as the junction at the middle of the illuminated strip. The two strips are then interchanged so that the strip previously covered is illuminated, and vice versa. The intensity of the radiation is computed from the mean value obtained from three such measurements of the current.

If the distribution of temperature in the two strips is exactly the same, this procedure will naturally give entirely correct results. But this will not be the case if the distribution of temperature is not the same. Such a difference in distribution is due principally to two causes.

¹ This paper will appear also in the *Meteorologische Zeitschrift*.

1. The illuminated strip is warmed from the surface, whereas the shaded strip is heated throughout its entire volume by the electric current. The consequence is that the temperature-gradient, taken from the surface inward, is not quite the same in the two cases. It is easy to show that in practice (length of the manganin strips 2 cm, thickness 20 μ , thickness of the coating of soot 10 μ) the error resulting from this cannot exceed one-half of 1 per cent.¹ This is on the assumption that the conductivity of

¹ We consider first the problem of ascertaining the illumination necessary to impart a temperature t_2 to the back surface of a thin strip consisting of two portions laid face to face. We obtain easily

$$I = t_2(\alpha + \beta + a\beta\mu)$$

where

$$\mu = \frac{c_2 d_1 + c_1 d_2}{c_1 c_2}$$

and α is the rate of efflux of heat per element of area of the illuminated face and β is the rate of efflux of heat per element of area of the back surface. d_1 and d_2 are the thicknesses, c_1 and c_2 the thermal conductivities, of the two substances 1 and 2.

In the case where the second portion is heated by an electric current, producing a quantity of heat W in an element of volume determined by the element of area and the thickness of the strip, we can deduce

$$W = t_2 \frac{\alpha + \beta + a\beta\mu}{1 + a\mu - (ad_2/2c_2)},$$

whence

$$\left(\frac{I}{W} = 1 + a\mu - \frac{ad_2}{2c_2} \right)$$

If now, as in the compensation-pyrheliometer, the first substance is soot, the second manganin, c_1 may be neglected relatively to c_2 , and we get

$$\left(\frac{I}{W} = 1 + \frac{ad_1}{c_1} \right)$$

If then we set: $a = 0.00026$ (MacFarlane); $d_1 = 0.001$; $d_2 = 0.002$, and $c_1 = 0.57 \times 10^{-4}$ (conductivity of air), we obtain

$$\left(\frac{I}{W} = 1.0046 \right)$$

wherein we have taken the conductivity of soot as equal to that of air, which probably gives much too small a value for c_1 , and hence too great a value for ad_1/c_1 . We therefore conclude that the correction to W required by (1) alone cannot exceed one-half of 1 per cent.

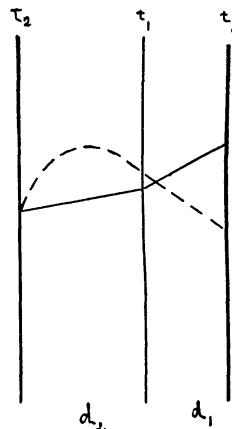


FIG. 1

the soot is not less than that of the air. Such a result can also be deduced from the empirical investigations of Kurlbaum.

2. A second cause of the difference in temperature-distribution between the strips arises if the exposed strip is illuminated along a part of its length only. This in fact is the case. In a pyrheliometer (No. 158) used by me in expeditions to Algiers and to Mount Whitney, it was found that about one millimeter at the edges of the illuminated strip (the length of which was 2 cm) was in the shadow of the diaphragm.

An idea of the magnitude of this border-effect, which is an important source of error in all instruments of similar construction, will be gained from what follows. I will assume the following simplifications: that the temperature is the same throughout each cross-section, and that the temperature of the supports O , P , and O' , P' is the same as that of the surroundings. If, further, we assume that the rate at which each surface-element loses heat is proportional to its difference of temperature from the surroundings ($-kt$), we obtain for the determination of the thermal condition of the electrically heated strip

$$Wadx = ktadx - abc \frac{d^2t}{dx^2} dx \quad (1)$$

where W is the heat evolved per element of area of the strip, a the breadth of the strip, b its density, and c its internal thermal conductivity.

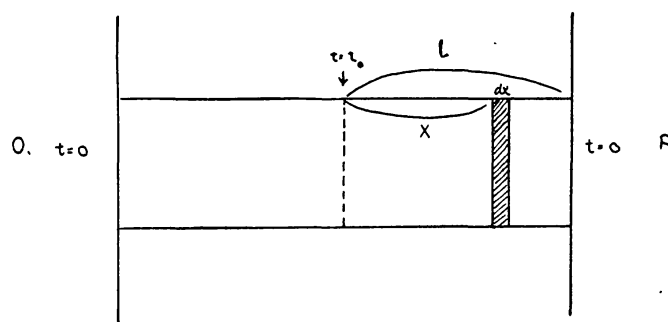


FIG. 2

From (1) we obtain

$$\frac{d^2t}{dx^2} - \frac{kt}{bc} + \frac{I}{bc} = 0 \quad (2)$$

The general solution is of the form:

$$t = Ae^{-\mu x} + Be^{\mu x} + S$$

where

$$\mu = \sqrt{k/bc} \quad S = W/k$$

If the constants A and B are determined from the conditions $t=0$ when $x=l$ and $dt/dx=0$ when $x=0$ (the latter follows from symmetry), we obtain

$$A = B = -\frac{S}{e^{-\mu l} + e^{\mu l}}$$

For the temperature t_0 at $x=0$, i.e., the temperature at the junction, we have

$$t_0 = S \left[1 - \frac{2}{e^{-\mu l} + e^{\mu l}} \right] \quad (3)$$

This temperature t_0 must be equal to the temperature of the middle point of the illuminated strip. For the illuminated part of this strip there holds, in analogy with (1), the equation:

$$t = A_1(e^{-\mu x} + e^{\mu x}) + S_1 \quad (4)$$

where

$$S_1 = I/k,$$

I being the intensity of radiation per element of area. For the part not illuminated we have

$$\frac{d^2 t'}{dx_1^2} - \frac{k}{bc} t' = 0 \quad (5)$$

which gives the solution:

$$t' = Me^{-\mu x'} + Ne^{\mu x'}$$

where A_1 , M , and N are determined by the conditions that at $x=d$, $t_1=0$, and at $x=0$, $t_1=t$, and the third condition,

$$\left(\frac{dt}{dx} \right)_{x=l-d} = \left(\frac{dt'}{dx_1} \right)_{x_1=0}$$

whence we obtain

$$\left. \begin{aligned} Me^{-\mu d} + Ne^{\mu d} &= 0 \\ -M + N &= A_1[e^{\mu(l-d)} - e^{-\mu(l-d)}] \\ M + N &= A_1[e^{\mu(l-d)} + e^{-\mu(l-d)}] + S_1 \end{aligned} \right\} \quad (6)$$

and from (6)

$$A_1 = \frac{S_1}{2} \left[\frac{e^{-\mu d} + e^{\mu d}}{e^{-\mu l} + e^{\mu l}} \right],$$

which together with (4) gives

$$t_0 = S_1 \left[1 - \frac{e^{-\mu d} + e^{\mu d}}{e^{-\mu l} + e^{\mu l}} \right]. \quad (7)$$

From (3) and (7) follows the relation desired:

$$\frac{I}{W} = \frac{\psi(l) - 1}{\psi(l) - \psi(d)}, \quad (8)$$

where

$$\psi(l) = \frac{e^{-\mu l} + e^{\mu l}}{2}.$$

Knowing l , d , and μ , we can now determine from (8) the relation between the quantity of heat generated by the current per element of area, and that imparted by the radiation. We thus obtain a value $(1 + \delta)$, where δ is the correction which must be made upon the value for the quantity of heat computed from the strength of the current. If c is set = 0.52 (from Jaeger and Diesselhorst), and k is set = 0.00018 + 0.00026 = 0.00044 (from MacFarlane), the computed value for μ is 0.65, since the thickness of the strip is 20 μ .

The length of the strip is 2 cm; hence $l = 1$ cm. Hence we have

$$\frac{I}{W} = \frac{0.2188}{1.2188 - \psi(d)}$$

The following table is instructive:

d	I/W
0.05.....	1.0025
0.1	1.011
0.15.....	1.023

The correction increases rapidly as d increases. In our special case d was estimated at about 1 mm. The corresponding correction, according to the theoretical considerations, is 1.1 per cent.

Experimentally the correction was determined by replacing the diaphragm belonging to the instrument by another, which permitted the illumination of the whole strip. In both cases the

readings were compared with those of a pyrheliometer belonging to the Smithsonian Institution (A.P.O. 9). The mean value of five series gave in the first case the relation

$$\frac{\text{A.P.O. 9}}{\text{Ångström}^1} = 1.0430$$

and in the second case

$$= 1.0299$$

The difference of 1.3 per cent is, therefore, to be ascribed to a border-effect, the sign of which is such that the readings give values too small. In my opinion, therefore, all measurements made hitherto with the Ångström pyrheliometer should be corrected by +1.3 per cent. As I have since found that my pyrheliometer No. 158 gives values about 1 per cent lower than most Ångström pyrheliometers, the difference between the International scale and that of the Smithsonian Institution becomes after this correction about 2.5 per cent (± 1 per cent).¹

As regards this difference, it is perhaps too early to make a definite statement. It must be conceded, however, that Abbot's method (the water-flow pyrheliometer) has a distinct tendency to give values too high. Thus in 1908 the difference between the two scales was 9.2 per cent,² in 1911 it was 5.5 per cent,³ and in 1913 it was 3.9 per cent,⁴ whence it is evident that the Abbot scale after revised measurements has gone considerably downward. On the other side, all the sources of error in the Ångström method tend to result in too small values, and the possibility of a necessary extra per cent of correction is not excluded. It is, therefore, probable that the true pyrheliometric scale lies between limits which do not differ by more than 2.5 per cent of the quantity of heat measured.

In this connection it may be of interest to mention briefly an experimental result which my friend Professor C. G. Abbot has

¹ Vol. 3 of the *Annals* of the Astrophysical Observatory of the Smithsonian Institution gives the difference as 3.9 per cent.

² *Ibid.*, 2, 47.

³ Abbot and Aldrich, *Astrophysical Journal*, 33, 125, 1911.

⁴ *Annals*, 3, 72.

kindly placed at my disposal. It follows from this that the measurements with the compensation-pyrheliometer are practically independent of pressure. Abbot inclosed the Ångström pyrhelimeter in a glass tube which could be evacuated, and compared the readings at different pressures with those of a pyrhelimeter outside. He found that at very low pressures (0.1 mm) the instruments gave values differing by 4.9 per cent from those obtained at atmospheric pressure. At a pressure of 34 cm nothing remained of this effect. When the ratio of the readings of the two pyrhelimeters is set = 1 at atmospheric pressure, the value of the ratio at 34 cm comes out 1.0001 (eight measurements).

The erroneous result at very low pressures is probably to be ascribed to a magnification of the above-mentioned border-effect.

SUMMARY

Theory and experiment show that the readings of the Ångström pyrhelimeter must be corrected by about +1.3 per cent. The difference between this scale and that employed by the Smithsonian Institution appears, therefore, to be 2.6 per cent.

Experiments show that the constant of the instrument is independent of pressure within the limits of practical work.

I am much indebted to Professor C. G. Abbot for valuable discussions with him.

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¹ *Ibid.*