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ON THE PHOTOGRAPHIC SPECTRUM OF β LYRÆ.

BY R. H. CURTISS.

PART I. HISTORICAL.

THE LIGHT VARIATION.

The Form of the Light Curve.—Since the discovery of the light variations of β Lyræ by Goodricke in 1784 there has been accumulated a great mass of data concerning its brightness, obtained almost exclusively through light intensity estimates on this star based upon Argelander's or a similar visual method. Notwithstanding all this work our knowledge of the form of the light curve of this object is still far from complete. The *general* nature of this curve is however well established, the well known principal feature being a pronounced but rounded minimum, followed in three or four days by a rounded maximum, in six or seven days by a second less marked minimum with no constant phase, in nine or ten days by a second maximum like the first, the whole cycle comprising 12.92 days. The accurate determination of the *true* form of the light curve has undoubtedly been complicated by actual changes in the curve, as well as by the limitations of visual methods.

Changes in the Form of the Light Curve.—Among these changes in the curve may be included the following long period or secular variations: the well known increase in the period; the slow decrease (0.1 magnitudes in fifty years) in the brightness at the secondary minimum, suspected by Luizet* in 1907; the flattening of the maxima announced by Roberts† (1907); and possible variations in the times of the principal phases referred to the principal minimum. Too great confidence should not be placed in the last three of these variations, since in their determination we are embarrassed by small irregularities in the curve.

In addition to the progressive variations in the light curve, capricious irregularities have been noted by different observers. Argelander recorded instances when the star remained for days or weeks below the mean brightness, and

* Bull. Soc. Astr. de France, 21, 30.

† Observatory, 29, 98.

Schmidt found the brightness of secondary minimum little below the maximum at certain times, and at others little above the principal minimum.

Minor Irregularities.—In spite of the difficulties which attend the study of the light curve of this star, certain minor oscillations, never greater in range than 0.15 magnitudes, persisting for indefinite periods and well distributed over the curve, have long been observed. Recently Luizet (1907) has examined the results of Goodricke (1784), Westphal (1818), Argelander (1840–44, 44–53, 54–59), Plassmann (1888–93), Pannekoek (1891–99), Menze (1895), Stratonow (1895–97), Glasenapp (1892–1902) and Luizet (1898–1906). As a result of his study he has found nine minor oscillations of the light-curve to be well established. Three of them were detected by all observers, even when the number of observations was relatively small. In addition to these nine, three others were found and a thirteenth has been suspected. In some cases the form of these oscillations seems to be roughly known. Nevertheless, the confirmation by photometric observations should be sought, in addition to the results obtained by Argelander's method, before the latter are unreservedly accepted.

The Period.—The best available formula for the prediction of principal minimal phase is the following of Pannekoek, as published in the Revision of the Elements of Chandler's Third Catalogue.

$$\text{Time of Principal Minimum} = 1855 \text{ Jan. } 6^{d}.604 \text{ G.M.T.} + 12^{d}.908009E + 3^{d}.855t^{2} - 0^{d}.047t^{3}, \text{ where } t = \frac{E}{1000}.$$

In 1907, however, this formula required a small positive correction according to Luizet. The amount of this correction was + 0.15 days according to the writer's observations in 1907.

Magnitude Range.—The visual magnitudes of this star at the principal phases are variously given, as the following will illustrate:

TABLE I.—MAGNITUDES OF β LYRÆ AT THE PRINCIPAL PHASES OF THE LIGHT-CURVE.

Source.	Prin. Min.	First Max.	Sec. Min.	Sec. Max.
	m.	m.	m.	m.
Argelander (1842–57)	4.50	3.40	3.9–	3.40
Harvard (1896)	4.10	3.37	3.73	3.31
Harvard (1896–8)	4.15	3.25	3.71	3.27
Markwick (1899–1905)	4.15	3.44	3.85	3.44

Principal Phases of the Light-curve.—The intervals of time from principal minimum to the other principal phases are tabulated for various epochs in Table II.

TABLE II.—INTERVALS BETWEEN PRINCIPAL MINIMUM AND THE OTHER PRINCIPAL PHASES.

Epoch.	First Max.	Sec. Min.	Sec. Max.
1784	3.58 days	6.38 days	9.58 days
1842–1870	3.12 ± 0.013	6.40 ± 0.017	9.54 ± 0.055
1870–1895	3.30 ± 0.036	6.48 ± 0.026	9.73 ± 0.055
1896–1908	3.42 ± 0.047	6.43 ± 0.028	9.72 ± 0.015

In the first epoch only Goodricke's observations are included; but the second combines results of Argelander, Schönfeld, Oudemanns and Schmidt; the third, Schmidt, Sawyer, Schur, Schwab, Plassmann, Pannekoek, Glasenapp and Menze; the last, Luizet, Lau, Markwick and the author. Apparently no variations in the above quantities are established.

ELEMENTS OF THE SYSTEM DERIVED FROM THE LIGHT CURVE.

The elements of the system of β Lyræ as expressed in the light curve have been investigated by Myers (Astrophysical Journal, 7, 1, 1898), Roberts (Monthly Notices, 65, 706, 1905), Stein (Verh. Konnink. Akad. Wetensch., Amsterdam, No. 10, 459, 1907), and v. Hepperger (Akad. Wiss. Berlin, Sitz., **118**, 923, 1909). In every case it has been assumed: (1) that the system comprises two discrete masses which are similar ellipsoids of revolution with uniformly bright surfaces, and major axes always, except for librations, in one line; (2) that the period of orbital revolution is identical with the light period (12.91 days). The results of the investigations are expressed in the following table, in which are used the symbols:

T, the time of periastron referred to the principal minimum;
ϖ', the longitude of periastron of the body eclipsed at principal minimum, measured in the plane of the orbit from the receding node in the tangent plane, in the direction of motion of the bodies in the system;
ι, the inclination of the orbital plane to the tangent plane;
e, the eccentricity of the relative orbit;
ϵ, the eccentricity of each ellipsoid;
h, the ratio of the major axes of the two components;
J, the ratio of the surface brightness of the larger star to that of the smaller;
A, the semi-axis major of the larger star in terms of the semi-axis major of the relative orbit;
S, the separation between the surfaces of the two stars in the same units.

The work of Myers was revised by Stein and later reviewed by v. Hepperger. Stein repeated the least-squares solution of Myers with corrected formulæ and thus derived his first set of elements from Argelander's curve. The second set was derived on the assumption that the longitude of periastron was 270°. The third set, which is based on Pannekoek's curve, was obtained upon the assumption of a circular orbit. v. Hepperger used the above curves and also a recent curve due to Stratonow. For each set of elements the representation of the observed light-curve was reasonably close.

TABLE III.—ELEMENTS OF THE SYSTEM DERIVED FROM THE LIGHT CURVE.

Light-Curve by	Argelander, 1850.						Pannekoek, 1895.			Stratonow, 1896.	
Investigator.	Myers.	Stein.		v. Hepperger.		Roberts.	Stein.	v. Hepperger.		v. Hepperger.	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
T	6.64	5.1	6.45	5.85	5.85	——	——	3.87	3.87	4.06	4.06
ϖ'	274°.9	231°.3	270°.0	256°.0	256°.0	——	——	195°.9	195°.9	200°.1	200°.1
i	90°.0	90°.0	82°.7	90°.0	90°.0	——	90°.0	65°.0	90°.0	65°.6	90°.0
e	0.020	0.017	0.04	0.038	0.038	——	0.00	0.037	0.037	0.028	0.028
ϵ	0.556	0.694	0.699	0.599	0.599	0.57	0.726	0.820	0.754	0.822	0.748
h	0.753	0.498	0.501	0.500	0.757	——	0.538	0.200	0.600	0.200	0.600
J	0.399	0.225	0.228	0.287	4.09	——	0.290	0.069	∞	0.071	∞
A	0.516	0.551	0.581	0.522	0.503	——	0.651	0.794	0.602	0.788	0.607
S	0.095	0.173	0.128	0.217	0.116	0.01	0.000	0.047	0.037	0.054	0.029

In the case of each light-curve, v. Hepperger has considered two assumptions: (1) that the smaller body has the greater surface brightness; (2) that the larger body has the greater surface brightness. The first assumption seems to accord best with Argelander's curve, while the second permits the better representation of the curve of Pannekoek and Stratonow. The derived ratio of the surface brightnesses under the second assumption indicates that the fainter star may be vanishingly dark according to the curves of Pannekoek and Stratonow.

Myers, André* and Roberts report strong evidence from the light-curve of a change in the small eccentricity of the orbit and of a revolution of the line of apsides. The above elements do not support the first effect, and though a change in the line of apsides seems to be indicated by the values in this table, neither v. Hepperger nor Stein considered the effect real.

According to all observers the inclination of the orbital plane is not far from 90°. The distance between the surfaces of the components is small. The two bodies are strongly ellipsoidal. All observations also concur in indicating a progressive approach both of surfaces and centers of the two bodies, together with an increasing eccentricity of figure, though this is difficult to harmonize with the known lengthening of the period. Again, from the above elements in connection with velocity results, we may infer that the density of the star is very low. But on the whole it must be admitted that the results determined in this way from the light-curve are rough approximations at best. Not only are widely

* Traité d'Astronomie Stellaire, 2, 259.

different alternative assumptions possible, but certain important considerations (such as light absorption in the system, surface reflections and secondary tidal effects) are not included in the investigations. We can infer only that the orbit is nearly circular, that the components are approximately ellipsoidal, of low density, relatively close together, and that they move in a plane making an angle of less than 25° with the line of sight.

Photometric Studies of the Continuous Spectrum.

Recently the continuous spectrum of β Lyræ has been studied photometrically by Nordmann* who has detected remarkable differences among the light variations of this star in the red, green and blue emissions of the continuous spectrum. The observations did not suffice to determine well the times of the principal phases nor the form of the curves for the three colors. But the following instructive facts were established:

(1) The range of light variation for the region covered is much greater for shorter wave-lengths than for longer ones. (2) The magnitude at first maximum is much greater (0.3 mag.) in blue light, and a little greater in green light, than the magnitude at second maximum; while in red light the brightness at these two phases is essentially the same. (3) At principal minimum the star's rays contain a greater proportion of red light than at secondary minimum.

The results of Nordmann's comparison of the spectra of β and γ Lyrae for the principal phases of the former are shown in the following table:

Table IV.—Comparison of the Spectra of β and γ Lyræ.

	Red.	Green.	Blue.
	m.	m.	m.
First Minimum...........	−0.70	−0.97	−1.36
First Maximum...........	+0.04	+0.03	+0.02
Second Minimum.........	−0.45	−0.61	−0.68
Second Maximum.........	0.00	−0.11	−0.26

These results suggest general absorption as a factor in the production of the variations in the light intensity of β Lyræ, and clearly introduce serious difficulties in the way of the investigation of the elements of the system on the basis of the visual light-curve.

Analyses of the Spectrum.

Early Observations.—Bright lines in the spectrum of β Lyræ were detected by Secchi in 1866 and were later studied visually by Von Gothard,† Vogel,‡

* Comptes Rendus, 146, 518, 1908.
† Astronomische Nachrichten, 111, 161, 1885.
‡ At Bothkamp.

Keeler,* and others. Extreme and erratic changes in these lines were noted by Von Gothard in 1883, but his attempt to connect them with the light variations led to seeming contradictions and to no conclusive result, while Keeler after three seasons' work (1889, 1890, 1891) with the star spectroscope used with the 36-inch refractor of the Lick Observatory, attributed them to personal errors due to the faintness of the spectra observed with small telescopes.

Though Keeler's visual observations are of much importance, the problem gave no promise of solution until the spectrum was studied photographically. It was the announcement by Pickering† in 1891 of relatively shifting bright and dark lines in the spectrum of β Lyræ that gave a great stimulus to the spectrographic study of this star, and led at once to the accumulation of a great mass of material bearing upon its spectral changes.

The Photographic Data.—The observations thus far discussed include the following:

29 plates in 1888 to 1891 made at Harvard with the objective-prism.

64 plates secured by Lockyer during a period beginning in July, 1891, and continuing probably three years, a few being made with a 6-inch objective-prism of 7°.5 refracting angle; the remainder with a prism of 45°.

156 plates in 1892 and 1893 made at Potsdam by Frost and Wilsing with a 13-inch refractor and slit spectrograph with a dispersion of 12 mm. from λ3720 to λ4900.

25 orthochromatic plates by Belopolsky, August to November, 1892, made with the 30-inch Pulkowa Refractor with a two-Halle-prism spectrograph; and 26 plates in the photographic region by the same observer with an improved spectrograph in 1897.

45 plates by Sidgreaves in 1892 and 1893 made with a slitless direct-vision spectrograph attached to an 8-inch telescope.

100 plates by Sidgreaves, May to October, 1895, with a slitless direct-vision spectrograph of dispersion 15.8 mm. from D to H, employing the 15-inch Perry Memorial Telescope.

About 65 plates by Sidgreaves in 1902 and 1903, made with a 4-inch objective-prism with a dispersion of 20 mm. between Hβ and Hγ.

12 plates from Hβ to H by McClean in the autumn of 1895, made with a 13-inch telescope of the astrographic chart pattern and an objective-prism.

Numerous plates made at Harvard college observatory with objective-prisms in connection with an 11-inch photographic refractor of 153-inch focal length; dispersions, Hβ to Hε, 20 mm. and 40 mm. according as one or two prisms were used.

* Astronomy and Astrophysics, 12, 114, 1893.
† Astronomische Nachrichten, 128, 39, 1891.

Qualitative Results.—It is not within the province of this paper to attempt an exhaustive discussion of the qualitative results contained in this mass of observational material. Nor is such a discussion opportune, since none of the above observers has complied with the conditions which are essential to a solution of the problem: namely, that several continuous series of spectrograms, including Hα if possible, distributed over the light period at intervals not exceeding a few hours, be taken at epochs separated by months and years, with slit spectrographs provided with comparison spectra.

In the above ensemble of material it is obvious that there is much that must be given small weight because of the limitations of instrumental efficiency and because of incorrect exposure and poor definition. On the other hand seeming contradictions among the results of different observers cannot be assumed to cancel the value of either, since in the variations of the spectrum of β Lyræ, long period changes (as well as capricious short period changes) have been suspected by several observers. A brief résumé of the early results could not do justice to all. To attempt to harmonize them for presentation through generalities would be premature. To reproduce them here in detail is unnecessary; but frequent reference to them will be made in connection with the development of the present paper.

Early Radial Velocity Measures and Elements.—Relative velocities from measures of lines in the spectrum of β Lyræ have been determined by Pickering,* Vogel,† Lockyer‡ and Sidgreaves.§ The first observed a relative displacement of the bright and dark lines corresponding to a velocity of 480 km. If the orbit be nearly circular, this leads to an orbital radius of 80,000,000 km. and high values for the mass of the system. Vogel determined the relative positions of bright and dark lines on two plates near second minimum and found the relative velocity to be 145 km., the bright line being displaced toward the violet. Lockyer observed the relative displacement corresponding to the epoch 1893 August 24.46, in the case of three pairs of dark lines (Hγ, Hδ and λ4026), and found relative velocities of respectively 249, 248, 254 km. These velocities correspond to a relative orbit of 42,000,000 km. radius. Sidgreaves measured the relative position of the red edge of the so-called Hζ emission with reference to the absorption component, and found a relative motion from the two lines of 60 km. per second on the assumption that the dark line was stationary and that the width of the emission line did not change.

These relative observations were impaired as a rule by the failure of the

* Loc. cit.

† Astronomy and Astrophysics, 13, 358, 1894.

‡ Ibid., 13, 575, 1894.

§ Monthly Notices, 64, 94, 1903.

measurer correctly to analyse the spectrum. Spurious displacements of the lines due to the presence of the absorption components affected Pickering's velocities, while all of the results (with the exception of Lockyer's) are impossible of interpretation because of the complex character of the absorption lines. Lockyer's result expresses closely the separation of the dark lines at second maximum, but as will be seen later, it cannot be accepted as the true mean relative velocity, for the second lines of hydrogen are displaced about 56 km. toward the violet at all times when not blended. Thus Lockyer's result if diminished by this amount will give 192 km. as the velocity from the oscillating lines with reference to the center of mass of the system, a result agreeing very well with the value of 188 km., obtained in this paper.

The only published absolute velocities of the three sets of lines in β Lyræ, two dark and one bright, are due to Belopolsky*. On fourteen Pulkowa plates between September 23 and November 25, 1892, he determined the absolute velocity corresponding to his measured displacement of the bright and dark Hβ lines. On eleven of these plates he derived velocities from the dark λ4481 of magnesium. From twenty-six plates made between June 20 and August 2, 1897, he determined absolute velocities from the displacements of the dark magnesium line at λ4481. As later stated in this paper, the oscillating dark line of this star is very weak in Hβ so that Belopolsky measured a nearly single non-oscillating dark line, but an emission (E) component whose edges (and it was upon the edges that his measures were made) were somewhat affected by blending and by the encroachments of the weak oscillating dark component. On the other hand the oscillating line is exceptionally strong in λ4481, while the second dark component and the bright component of this line are weak and usually invisible. Thus Belopolsky's measures of these two lines contain a record of the displacements of each of the three spectra involved, under conditions that are nearly as favorable as any which I have been able to find by careful examination of each of the lines in the photographic region of this star's composite spectrum. Unfortunately these observations are too few in number to yield accurate values of the elements, but their importance is such that they have been reviewed and reduced by several investigators. They are plotted against the Allegheny curves in Figure 2.

Orbital elements from the above mentioned velocities have been determined by Tikhoff†, Myers, Stein, and Belopolsky and are contained in the following table, in which the notation of Lehman-Filhés is used and T is referred to principal minimum. Tikhoff's orbit of the star whose spectrum contains dark lines was determined from measures, communicated privately, of λ4481 of mag-

* Mem. d. Soc. Spett. Ital., 22, 101, 1893; Ibid., 26, 135, 1897. Astrophysical Journal, 6, 328, 1897.
† Mem. della Soc. Spett. Ital., 26, 107, 1897.

nesium on eleven of the Pulkowa plates of 1892, while Belopolsky's and Stein's are based upon the superior plates of 1897. Stein's elements were derived definitively from least-squares solutions, employing corrected phases and velocities, and may therefore be considered as the most reliable. It should be noted that Stein's elements resemble the others closely except in case of the uncertain time and longitude of periastron. Belopolsky's published values for T and ϖ have been diminished by P/2 and 180° respectively. The changes in the elements in Table V are discussed farther on in this paper in connection with the Allegheny results.

TABLE V.—ELEMENTS FROM EARLY RADIAL VELOCITY MEASURES.

Investigator.	Bright Line Star.				Dark Line Star.			
	Belopolsky.	Tikhoff.	Myers.	Stein.	Tikhoff.	Belopolsky.		Stein.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
P	d. 12.91	d. 12.91	d. 12.91	d. 12.91	d. 12.91	d. 12.908	d. 12.908	d. 12.908
e	0.0	0.1	0.108	0.075	0.1	0.04	0.07	0.0235
T	——	d. 0.8	d. 1.22	d. 7.89	d. 0.5	d. 6.70	d. 6.62	d 7.61
ϖ	——	270°	280° 43′	115° 20′	90°	270°.0	263.°4	300° 51′
K	90 km.	89 km.	90 km.	85.9 km.	200 km.	181.1 km.	181.1 km.	180.0 km.
$a \cdot \sin i$	15.8×10⁶km.	15.7×10⁶km.	15.8×10⁶km.	15.3×10⁶km.	35×10⁶km.	32×10⁶km.	32×10⁶km.	32.0×10⁶km.
γ	——	0.0km.	−0.8km.	−0.72km.	0.0km.	−14.8km.	−14.8 km.	−15.45km.
Mass	18☉	23.1☉	——	17.1☉	10.2☉	9☉	9☉	8.1☉

PART II. THE ALLEGHENY RESULTS.

THE SPECTRA.

The light of β Lyræ has long been known to be most complex; but in their analyses of the lines observers have differed widely, agreeing only in the conclusion that in addition to the continuous spectra both dark and bright lines are present.

Absorption Spectra.—Two dark-line spectra are clearly identified in this star. Of these one, resembling Rigel's spectrum, belongs to class B8. The second resembles Bellatrix and belongs to class B5. The first set of lines oscillates through a range of 369 km. in a period of one light cycle. The second set is apparently fixed in position within limits of ± 10 km. The presence of these two sets of dark lines was recognized by Lockyer and also by Miss Maury,* but through the lack of suitable comparison spectra they were both unable to measure absolute displacements. Belopolsky measured the oscillations of the B8 dark lines through their representative in λ4481 of magnesium. He also measured the B5 dark

* Annals, Harvard College Observatory, 28, 103.

line in Hβ in which the B8 line is relatively faint, and he saw representatives of both spectra in λ4472 of helium, but apparently did not recognize the presence of two complete dark-line spectra.

Emission Lines.—In addition to the dark-line spectra noted above, numerous bright lines are present. No bright lines exist without dark companions, but broad bright lines or bands accompany nearly all the hydrogen and helium lines between λ3800 and λ5017. Narrower emission lines accompany many of the dark lines, the existence of such a companion being denoted in Table VII by a small *b* appended to the wave-length. In this class occur lines of calcium, magnesium and iron, as well as lines of unknown origin.

Complex Lines.—Certain bright hydrogen and helium lines occur in blends. Among these should be mentioned Hε, from which only at first maximum can the H line of calcium be separated; and λ3888.8 of helium, which is blended with Hζ, producing a notably complex group. But all the hydrogen and helium lines are complex, since in them the lines of three different spectra are in general present. The K line of calcium, λ4481 of magnesium and a few other lines include representatives of all spectra with the B8 dark line strongly predominating. The remaining lines in Table VII include single dark components of the B8 spectrum with faint emission only in certain cases denoted as stated above.

The Dark Edges of the Emission Lines.—In a characterization of the spectrum of this star mention should be made of the dark borders of the emission lines. These have been noted in passing by Miss Maury, Belopolsky and others, especially in connection with Hβ, and also in γ Cassiopeiæ. They have been accounted for by Scheiner as due to absorption in the lowest layers of the star's atmosphere. In β Lyræ these dark borders apparently rise at times to the intensity of the principal absorption lines, vary in breadth and become resolved into several close lines, thus making plausible the conclusion that they are of great importance in the study of this star.

Changes in the Spectrum.—Changes in the spectrum of β Lyræ bearing some relation to the period of light variation were early recognized. Keeler (Astronomy and Astrophysics, 12, 114, 1893) announced that the fluctuations in the star's light were due to variations in the intensity of the continuous spectrum, a result which has been extended by Nordmann as stated above. Lockyer (ibid., 13, 574, 1894) concluded that the spectrum was constant at the same phase; while Vogel (ibid., 13, 358, 1894; and Astrophysical Journal, 2, 337, 1895) suspected long period changes in the characteristics of the bright bands, standing in no immediate relation to the light phase. The periodical changes in the lines include variations in the intensity, wave-length and sharpness of dark lines; and fluctuations in the width, intensity curve and possibly wave-lengths of the emission bands. As to the details of these changes there is an obvious lack of agreement as well as apparent errors in interpretation. The best descriptions are those of Sidgreaves (Monthly Notices, 54, 96, 1894; 54, 169, 1904; 57, 515, 1897) Miss Maury (*loc. cit.*) and Belopolsky (Astrophysical Journal, 6, 328, 1897). The elements of the last are given above, but the qualitative results of the first need not be reproduced here. Qualitative studies of the Allegheny plates are taken up later in this paper.

Journal of Observations and Measures.

In 1907 sixty-four spectrograms of β Lyræ were obtained with the Mellon Spectrograph attached to the Keeler Memorial Reflector. All but three of these photographs have been included in the present discussion. As far as practicable, Seed 23 or lantern slide plates were employed; but when conditions were adverse, Seed 27 plates were used to avoid underexposure. In the matter of recording details or bringing out small differences of intensity, the slow plates were found to be well nigh indispensable in the study of this star, whose spectrum is especially difficult to interpret. For this object a wide range of exposure was necessary because of the variation in the brightness of the star and because of the use of plates of widely different speed. Accordingly much care was required in timing the exposures. Because of the length of the exposure sometimes required to photograph the spectrum of this star, the author made a practice of introducing the comparison spectrum (titanium spark) at intervals not exceeding ten minutes throughout the exposures.

The phases in Table VI refer to the time of principal minimum as computed by Pannekoek's formula above cited and have been reduced to the sun. The velocities, weights and residuals in the table refer only to the lines of the B8 spectrum, which are here reduced separately in two groups: single lines from which the orbital elements are derived, and lines in complex groups which are measured and reduced for identification and for detection of blends, which last are here denoted by brackets enclosing the affected velocity. For comparison with the results of Belopolsky the individual measures of the component of the B8 spectrum in λ4481 of magnesium are set forth in columns (13), (14) and (15). The initials in column (2) are those of the observers at the telescope: Schlesinger, Baker and the author.

TABLE VI.—TABLE OF OBSERVATIONS WITH MEASURES OF LINES OF THE B8 SPECTRUM.

No. of Plate.		Date, G.M.T.	Phase from Prin. Min.	Single Lines. No. of Lines.	Single Lines. Wt.	Single Lines. Vel.	Single Lines. Resid. *O—C.*	Lines in Complex Groups. No. of Lines.	Lines in Complex Groups. Wt.	Lines in Complex Groups. Vel.	Lines in Complex Groups. Resid. *O—C.*	λ 4481 Mg. Wt.	λ 4481 Mg. Vel.	λ 4481 Mg. Resid. *O—C.*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
		1907 d. h. m.	days			km.	km.			km.	km.		km.	km.
547	C	June 3 18 34	2.190	4	3	−168.4	+ 4.4	6	5	[−127.7]		1	−153	+20
548	C	3 19 08	2.214	8	12	−173.5	+ 0.5	8	9	[−127.1]		2	−203	−29
549	C	3 20 05	2.253	17	18	−170.5	+ 5.3	10	12	[−134.6]		2	−183	− 7
550	C	3 20 52	2.286	12	13	−172.5	+ 4.8	7	6	[−121.1]		2	−190	+13
560	C	6 13 17	5.178	13	12	−137.4	+ 3.7	6	5	[−125.8]		2	−134	+ 7
561	C	6 19 08	5.214	22	25	−139.4	− 0.5	12	9	[−130.8]		3	−138	+ 1
562	C	6 20 08	5.255	19	33	−138.3	− 2.1	10	16	[−120.2]		3	−134	+ 3
571	C	8 17 40	7.153	22	24	+ 21.6	− 0.7	13	10	+ 25.4	+ 3.1	3	+ 14	− 8
572	C	8 18 36	7.192	23	33	+ 27.8	+ 1.9	9	13	+ 21.3	− 4.6	3	+ 14	+11
573	C	8 19 28	7.228	21	25	+ 31.3	+ 2.3	12	11	+ 32.8	+ 3.8	3	+ 28	0
582	S	9 19 47	8.241	10	7	+113.3	+ 4.3	5	3	+100.0	− 9.0	2	+115	+ 6
600	C	14 18 59	0.288	18	17	− 37.0	− 2.3	10	12	[− 54.4]		2	− 35	− 1
601	C	14 20 25	0.348	7	6	− 38.1	+ 1.9	5	4	[− 50.0]		1	− 15	+25
607	B	15 20 35	1.355	6	7	−134.4	−11.7	3	3	[−136.9]		1	−127	− 5
615	C	16 18 51	2.283	20	23	−168.9	+ 8.4	13	17	[−136.3]		2	−168	+ 9
627	B	20 19 52	6.325	12	9	− 51.0	+ 0.4	4	2	[− 43.1]		1	− 54	− 2
630	C	24 18 22	10.263	9	11	+168.8	+ 5.7	6	7	+167.7	+ 4.6	2	+166	+ 3
632	S	25 18 36	11.273	7	6	+124.3	+ 2.2	2	2	[+160.6]		1	+152	+29
637	S	27 17 56	0.325	13	12	− 44.3	− 6.4	11	9	[− 43.9]		1	− 33	+ 5
642	C	30 17 59	3.327	19	16	−201.4	+ 0.5	6	3	−187.0	+14.9	1	−204	− 2
643	C	June 30 19 05	3.373	9	8	−196.4	+ 5.5	4	2	−202.7	+ 0.8	2	−193	+ 9
652	C	July 4 18 11	7.335	22	22	+ 36.5	− 1.8	8	4	+ 43.9	+ 5.6	3	+ 33	− 5
653	C	4 19 06	7.373	9	9	+ 40.9	− 0.7	4	3	+ 30.5	+ 9.8	2	+ 39	− 2
656	C	6 18 31	9.349	20	18	+161.1	+ 0.5	13	12	+157.7	− 2.9	3	+151	−10
659	S	7 17 26	10.304	14	15	+162.7	+ 0.4	11	10	+164.8	+ 2.5	2	+172	+10
660	S	7 18 50	10.362	16	16	+167.0	+ 5.9	6	7	+169.0	+ 7.9	3	+163	+ 2
665	C	12 18 18	2.421	7	5	−180.5	+ 2.2	4	3	[−122.8]				
669	S	13 17 36	3.394	12	8	−200.9	+ 1.1	3	2	−199.3	+ 2.7	2	−202	0
670	S	13 18 18	3.421	8	8	−205.2	− 3.2	3	2	−200.3	+ 1.7	2	−209	− 7
671	S	13 18 48	3.442	4	4	−203.4	− 1.4	3	2	−202.1	− 0.1			
673	C	14 18 05	4.412	11	11	−184.5	− 2.9	9	6	−186.5	+ 0.9	1	−194	−12
675	C	18 15 37	8.309	14	16	+117.2	+ 4.0	10	9	+118.2	+ 5.0	1	+110	− 3
676	C	18 17 08	8.372	16	15	+114.9	− 2.4	11	11	+117.7	+ 0.4	2	+110	− 2
677	C	18 18 32	8.438	17	19	+119.5	− 2.0	9	8	+121.0	− 0.5	2	+116	− 5
680	C	20 17 30	10.388	14	15	+163.6	+ 3.3	13	14	+164.6	+ 4.3	2	+158	− 2
686	C	24 18 19	1.503	15	12	−146.0	−13.0	5	5	[−125.0]		1	−146	−13
689	C	26 16 48	3.440	18	16	−205.6	− 3.7	5	2	−215.4	−13.5	2	−216	−14
690	C	26 18 35	3.513	16	14	−205.1	− 3.3	9	6	−201.3	+ 0.5	2	−200	+ 2
694	B	27 18 16	4.501	13	17	−175.0	+ 2.9	10	8	−170.3	+ 7.6	2	−174	+ 4
695	B	27 19 04	4.535	16	16	−179.5	− 3.0	11	7	−176.3	+ 0.2	2	−179	+ 3
699	C	July 30 15 38	7.391	23	26	+ 41.6	− 1.4	12	8	+ 28.7	+14.5	3	+ 41	− 2
702	C	Aug. 1 15 02	9.366	16	18	+162.8	+ 1.7	13	15	+163.2	+ 2.1	3	+162	− 2
706	C	1 18 04	9.493	11	8	+158.7	− 5.0	8	7	+162.8	− 0.9	2	+171	+ 7
710	B	2 15 07	10.370	21	23	+156.7	− 4.1	13	16	+161.6	− 0.8	2	+158	− 3
713	C	3 15 22	11.380	17	22	+114.4	− 1.3	12	13	+123.2	− 7.7	3	+100	−16
715	C	3 19 06	11.535	15	18	+103.5	− 1.9	13	8	+109.5	− 4.3	3	+114	+ 9
718	B	4 15 15	12.375	13	10	+ 50.9	+11.5	9	6	+ 53.6	+14.2	2	+ 64	+25
721	B	4 18 32	12.512	8	6	+ 30.8	+ 3.4	4	2	+ 31.2	+ 3.8	1	+ 40	+13
723	B	6 15 37	1.470	22	22	−131.1	− 0.4	10	11	[−104.8]		2	−117	+14
729	C	9 16 26	4.505	22	22	−175.3	+ 2.5	8	7	−177.1	+ 0.7	2	−174	+ 4

TABLE VI.—TABLE OF OBSERVATIONS WITH MEASURES OF LINES OF THE B8 SPECTRUM.—*Continued.*

No. of Plate.		Date, G.M.T.	Phase from Prin. Min.	Single Lines.				Lines in Complex Groups.				λ 4481 Mg.		
				No. of Lines.	Wt.	Vel.	Resid. O—C.	No. of Lines.	Wt.	Vel.	Resid. O—C.	Wt.	Vel.	Resid. O—C
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
		1907 d. h. m.	days			km.	km.							
735	B	Aug. 10 15 40	5.473	18	22	−125.5	− 4.5	7	6	[−111.6]		2	−129	− 8
743	C	11 15 06	6.469	24	27	− 36.7	+ 3.6	10	12	[− 37.0]		3	− 51	−11
744	C	11 16 17	6.498	25	28	− 35.9	− 0.1	12	10	[− 34.7]		3	− 39	− 3
747	C	11 18 49	6.604	14	14	− 29.7	− 2.7	8	6	[− 28.2]		2	− 29	+ 4
754	C	13 18 28	8.589	15	22	+123.4	− 6.9	9	13	+127.9	− 3.0	3	+111	−20
758	B	14 18 10	9.577	18	21	+160.9	− 4.0	10	7	+162.2	− 2.7	3	+164	− 1
768	C	17 19 10	12.618	13	12	+ 20.2	+ 2.1	5	3	+ 38.1	+20.0	2	+ 45	+27
771	C	19 17 06	1.612	14	15	−137.1	+ 3.0	6	7	[−114.4]		2	−134	+ 6
773	C	21 19 01	3.692	4	6	−198.9	+ 1.1	2	1	−201.6	− 1.6	3	−198	+ 2
777	B	24 17 20	6.622	15	13	− 18.9	+ 6.4	7	7	[− 19.8]		3	− 23	+ 2
805	C	Aug. 31 17 56	0.719	—	—	—	—	—	—					

IDENTIFICATION OF LINES AND DETERMINATION OF WAVE-LENGTHS.

Some knowledge of this system can be obtained by the identification of the elements represented by absorption and emission in its spectrum. Especially is this consideration important in connection with the body emitting both bright and dark lines. Accordingly, in the definitive quantitative study of sixty spectrograms of β Lyræ I have measured every available line, have eliminated the Doppler displacements by means of the known lines, and have then combined all the measures of each unknown line into a mean which has been transformed into wave-lengths through the application of a corrected Hartmann interpolation formula, based on a large number of measures of comparison lines. In addition to these unknown lines, Table VII contains all the known lines that I have been able to identify in the region considered.

The first column of Table VII contains the wave-lengths, followed by *B* in the case of lines which are especially bright, by *b* in the case of lines that are sometimes bright, by *d* in case the dark lines are double, and by *t* in case the dark lines are triple. The second column contains the probable errors of the wave-lengths determined by the author, computed on the basis of the probable error of an observation of weight unity determined in the least-squares solution for the computation of the orbital elements. The probable error for any determined wave-length was derived through the observed weight of the determination, the proper factor depending upon dispersion being introduced. Column (3) gives the number of Allegheny plates upon which the determined line occurs. All lines measured on one or two plates only were rejected, except in four special cases when some feature connected with the line excited interest. Column (4) gives the mean intensity of each dark and bright line on an arbitrary scale, in which a line clearly visible though on the limit of measurability is denoted by *1*, and a line twice as bright as this is denoted by *2*, etc. In the case of double lines the intensity of the line of the B8 spectrum is given first, the line of the B5 spectrum next, and the emission line last. Each intensity is a mean of all the estimates on that line. The intensities of the unknown lines are given to the nearest half unit, while those of the known lines are given to the nearest tenth, when sufficient estimates are available. A source of error in these intensities of the faint lines, is to be found in the circumstance that such a line will be included only when its brightness is sufficient to warrant measurement. Such lines will accordingly appear disproportionately bright, a consideration that will be taken into account in the studies of relative intensities. Columns (5) and (6) give wave-lengths determined by Belopolsky and Vogel for the lines here given. The new symbol, *D,* indicates that the line has a dark component. When the dark component was not noted the wave-length of the bright component has been given. The last column contains suggested identifications of unknown lines together with the elements to which the known lines are attributed. Identifications taken from Lockyer's *Tables of Enhanced Lines* are indicated by the wave-length according to Lockyer, followed by the symbol for the element and by the numbers indicating the relative intensities of the line in arc and spark. In the case of the helium lines *P* indicates that the line belongs to the so-called Parhelium series. The Roman numeral indicates the sub-series to which the line belongs, while *H* stands for "principal series." The first Arabic numeral indicates the number of the line in its series and the numeral in parentheses expresses the intensity of that line in the spectrum of the vacuum tube according to Runge and Paschen.

In the list of unknown lines especial attention should be called to λ3853.8, 3856.2, 4629.8 and 4635.0. These lines are relatively strong and are accompanied by emission bands. In the region of the second pair, lines have been observed in novæ and in the corona. The identification with two maxima of a strong group in the secondary spectrum of hydrogen seems probable. Other unknown lines that were measured frequently, and are thus well determined, are λ3862.73, 3995.17, 4067.22, 4179.07, 4296.92, 4390.96, 4417.27, 4515.63, 4520.19, 4555.83, and 4588.38. Of these λ4067.22 is near the mean of a strong pair in the secondary spectrum of hydrogen. Probably in several cases the identification suggested will be found to be correct.

Attention should be directed especially to the helium line at λ3964.875 and the blend at Hζ, for in these lines a third dark component is clearly measured. In each case the oscillating dark component is relatively faint. The component belonging to the B5 spectrum occurs with expected intensity, but in addition a third component, the strongest of all, appears in an unvarying position on the violet

TABLE VII.—LINES IN THE SPECTRA OF β LYRÆ.

λ	Prob. Error.	Plates.	Intensities.	Belopolsky.	Vogel.	Identifications, etc.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
3819.81 b	±0.064A	4	9.0; —		3820 B, D.	3819.781 He I 6(5).
3833.80	.12	1	4.0; —			3833.710 He PI 10(1)
3835.37	.12	2	4.0		3836 b, D	3835.60 $H\eta$
3845.27	.060	3	3.0			
3853.76 b	.021	24	4.0			
3856.16 b	.017	31	5.0		} 3860 B	
3862.73 b	.018	32	5.0			
3867.664 b, d.			3.5; 2.3;			He II 6(3)
3871.954			5.0; —			He PI 9(1)
3878.330			2.0; —			He PII 9(1)
3888.785 B } t						He H 3 (10) } Blend.
3889.14 B } t			2.–; 4.3; 4.7; 8.7		3889.9 B, D	$H\zeta$
3900.91	.053	3	2.0			3900.68 Ti 4–10.
3906.16	.034	8	2.0			3906.14 Fe 0–1.
3913.46	.044	5	2.0			3913.61 Ti 4–10.
3926.678 b, d			3.1; 2.2; 2 –		3927 B, D.	He PI 8(1).
3933.825 b, d			5.7; 2.6; 2 –		3934 D	K, Ca.
3936.064 b, d			2.7; 2.3; 2 –		3938 B	He PII 8(1)
3964.875 B, t			2.1; 2.8; 3.3; 4.8		3965 D	He PH 4(4).
3968.625 d			4.6; 13.		3970 D	H, Ca. } Blend
3970.177 b, d.			———		3975 B	$H\epsilon$ } Blend
3995.17	.024	23	2.0		3998 B	
4002.57	.065	5	2.5		4001 B	4002.77 Fe 0–1.5.
4005.73	.056	6	2.5			4005.75 Th 0–1.5.
						4005.85 V 7–10.
4009.417 B, d			3.8; 1.7; 2 –		4010 b, D	He PI 7(1)
4011.03	.065	3	2.0			
4012.61	.057	3	2.0			4012.63 Cr 2–6.5.
						4012.54 Th 0–2.
						4012.50 Ti 1–5.
4024.136 b, d			2.3; 2 – } 5.7			He PII 7(1)
4026.371 B, d			4.0; 2.7 } 5.7		4026 B, D	He I 5(6).
4028.57	.044	7	2.0			4028.50 Ti.
4054.11	.055	5	2.0			4053.98 Ti, tr–5.
4057.33	.055	4	3.5			
4064.55	.070	4	2.0		4062 B, D	
4067.22	.022	38	3.0		4068 B, D	4067.30 Ni 0–5. Sec. spect. H
4101.927 B, d			5.6; 5.7; 5.8		4102 B, D	$H\delta$
4121.015 B, d			2.6; 2.0; 3.0		4120 B, D	He II 5(4)
4123.01	.060	7	2.0			
4128.204			3.6		} 4130 B, D	Si.
4131.040			3.7			Si.
4143.919 B, d			4.0; 1.8; 2.5		4143 B, D	He PI 6(2)
4153.16	.041	12	3.0			
4163.14	.049	8	3.0			4163.17 Sn (0)–1.
4169.131 b			2.7; 0 –; 1.–			He PII 6(1)
4173.765			3.4			
4179.07	.023	36	3.1			
4198.50	.081	4	3.0			
4224.56	.052	9	2.0			
4233.328 b, d			3.0; 1 –			Fe.
4242.535			2.5			Cr.
4258.36	.056	8	2.0			
4267.66	.052	8	2.0			4267.30 C.
4282.00	.099	2	2.0			4282.25 Th 2–5.
4294.38	.052	8	2.0			4294.20 Ti 3–7.
4296.92	.035	20	2.0			
4300.14	.071	7	2.0			4300.211 Ti 1.5–6.
4340.634 B, d.			5.4; 4.5; 7.0		4340 B, D	$H\gamma$
4352.083 b			2.6			Mg.
4385.144			2.6			Cr.

TABLE VII.—LINES IN THE SPECTRA OF β LYRÆ.—*Continued.*

λ	Prob. Error.	Plates.	Intensities.	Belopolsky.	Vogel.	Identifications, etc.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
4388.100 B, d.			3.0; 3.0; 4.5		4388 b, D	He PI 5(3).
4390.96	.048	16	2.0			4391.19 Ti, tr–1.5.
4395.67	.078	6	2.0			
4417.27	.034	28	2.5			
4419.72	.120	4	3.0	4419.2 D		
4437.718 b, d			2.3; 1 –; 1.–	4435.9 D		He PII 5(1)
4471.676 B, d			2.5; 3.6; 9.0	4471.5 B, D	4470 B, D	He I. 4(7).
4481.397 b, d.			3.3; 1.–	4480.9 B, D	4481 b, D	Mg.
4484.43	.099	7	1.0	4484.1 D		
4507.92	.12	4	2.0	4506.3 D		
4508.455			2.5	4510.4 D		Fe.
4512.16	.068	8	1.5	{ 4511.8 D		
4515.63	.046	23	1.5	{ 4513.9 D		4515.51 Fe, tr–4.
4520.19	.046	20	1.5	4520.2 D		4520.40 Fe 1–3.
4522.77			1.5			Fe.
4525.28	.087	6	1.5	4525.6 D		
4534.02	.088	6	2.0			4534.14 Ti 2–5.
4535.92	.13	4	2.0	4536.4 D		
4549.642 b			2.4	4547.8 D		Fe.
4555.83	.047	18	2.0			4556.09 Fe 0–5.
4558.827			1.6			Cr.
4563.33	.13	2	2.0			
4575.70	.13	2	1.5	4577.7 D		
4584.018 b			3.2	} 4586.7 D		Fe.
4588.38	.050	27	2.5			4588.38 Cr 1–10.
4621.32	.096	7	2.5			
4629.79 B	.061	15	4.0 } 4.0	4630.5 B, D		{ Also in Novae; the Corona. Sec. spect. H.
4634.96 B	.091	11	4.0 }			{ 4634.25 Cr 0–8. 4635.50 Fe 0–3.
4656.98	.13	3	2.0			
4663.29	.13	3	2.0			
4681.98	.13	3	2.0			4681.8 In. 0–8.
4713.308 B, d			2.1; 2.0; 5.3	4714.2 B, D		He II 4 (4).
4861.527 B, d			2.–; 3.8; 8.7			Hβ
4922.096 B, d			2.7; 3.3; 3.5	4923.5 B, D		He PI 4 (4)
5015.732 B			0.–; 4.2; 4.6	5014.0 B, D		He P.H. 3 (6)

side of the B5 dark line. The constants of these two quadruple lines are considered below in connection with the quantitative studies.

LINE INTENSITIES.

The matter of determining the apparent intensities of absorption and emission lines at different phases of the light period is of singular importance. In the case of single lines such a study will bear strongly on the question of eclipse and the relative strength of the continuous spectra if more than one are present. In the case of complex lines, intensity variations in the bright and dark lines will not only assist in the study of changes due to eclipse, but will contain information regarding the relative position of absorbing layers, as well as other physical conditions in the system.

Unfortunately the Allegheny plates do not warrant an intensive study of the strength of spectral lines. The use of plates giving varying contrast and the difficulties in the way of securing accurately timed exposures present great obstacles to such an investigation. However at the time of measuring I have estimated directly the intensity of each absorption line on all the plates on the scale described above and in so doing have taken into account variations due to overexposure and underexposure. From these estimates the following studies have been made.

Thirteen single B8 spectrum lines (λ4128, 4131, 4174, 4179, 4233, 4242, 4352, 4385, 4481, 4508, 4523, 4550 and 4584) were first selected. Two of these lines (λ4233 and λ4481) are known to have faint dark companions, too weak however to produce a noticeable effect upon the intensity of the stronger component. For each line the intensity estimates were combined into thirteen normal places, one for each day of the light period, and the resulting means were plotted for each line separately, with phase times as abscissæ and intensities as ordinates. In each case the observations followed a horizontal straight line with no deviation great enough to be considered real. For a more exacting test the intensity of each line was found by forming the mean of all the intensity estimates of that line. (These means appear in Table VII.) Residuals for each normal place were then formed and the residuals of all these lines for each normal place were combined into final means. The thirteen resulting mean deviations from the mean intensities for eleven single lines are given for the thirteen normal phases in Table VIII. The probable error of each mean deviation is ±0.3. The mean intensity of all the lines on all plates is 2.94 ± 0.08. With this quantity the proportional deviation of the group of eleven single lines from a constant intensity at each normal phase is also given in Table VIII. The probable error of each of these deviations is ±0.10 or ±10 per cent.

TABLE VIII.—THE DEVIATIONS OF THE SINGLE LINES FROM A CONSTANT MEAN INTENSITY DURING THE LIGHT CHANGE.

	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
Phases from Prin. Min.	0.3	1.6	2.3	3.5	4.5	5.4	6.6	7.3	8.5	9.5	10.4	11.5	12.6
Deviations in intensity units	+0.2	−0.4	+0.2	+0.2	−0.4	+0.2	−0.1	+0.5	0.0	0.0	+0.3	−0.3	−0.5
Intensity of group	3.1	2.5	3.1	3.1	2.5	3.1	2.8	3.4	2.9	2.9	3.2	2.6	2.4
Deviations in terms of group intensity	+0.07	−0.13	+0.07	+0.07	−0.13	+0.07	−0.03	+0.17	0.00	0.00	+0.10	−0.10	−0.17

The table indicates that no strong variation in the intensities of the single lines is established. At and near principal minimum, when the continuous emission of the B5 spectrum should be relatively strong, there is a slight preponderance of negative residuals. But the average residual for three days during principal minimum is —0.08 ± 0.04, a not very pronounced effect at most. Further, at secondary minimum, when the continuous emission of the B5 spectrum should be at its faintest according to the eclipse theory, the dark lines of the B8 spectrum have gathered little strength ($+0.07 \pm 0.05$). Apparently then if the eclipse hypothesis is to hold, the continuous spectrum of the star which eclipses the other at principal minimum must be relatively faint, a condition which is also indicated by the relative depressions of the principal and secondary minima of the light curve.

The intensities of the dark lines occurring in complex groups were next investigated. Blends, bracketed in column (11) of Table VI, were of course frequent at and near conjunction, and had to be carefully distinguished from resolved lines by consideration of velocities, though the abnormal intensities of the blends themselves were often sufficient to betray them. An apparent decline in the intensity of the B5 lines before and after secondary minimum with a small increase in the corresponding intensities of the B8 component was observed in some lines, especially in Hδ, Hγ and λ4471. For Hβ and λ5016, where the B8 component is absent, the same decline in intensity is also noted at secondary minimum. This is discussed later on in connection with velocity measures of the lines of the B5 spectrum and is attributed to a blending of the B8 component with part of the B5 component, or to an actual broadening of the B8 component simultaneously with a narrowing of the B5 component in some manner connected with the secondary minimum.

Eliminating the blends from consideration, there is found no certain evidence that there are, in the intensities of dark lines in complex groups, variations connected with the light change, though at the phases when such variations might be greatest, blending prevents any clear interpretation. It has therefore been considered wise to determine the intensities of each of the dark lines in complex groups by forming the simple mean of all the intensity estimates of that line when found resolved. The results of this procedure are found in the intensity column of Table VII, which may now be discussed in this connection.

The intensities of the helium lines are of especial interest. Fortunately there are in the region considered twenty available lines due to this element, including representatives of all six helium series except the principal one, the only member of which in this region is blended with Hζ. The comparison of the relative intensities of the three components of these twenty lines, series by series and series with series, in connection with the relative intensities in the vacuum tube, is of great importance as bearing upon the different conditions under which these lines have been produced. Accordingly the data are set forth in Table IX in a manner calculated to facilitate comparison. In the first column the approximate wave-lengths are given. Each of the successive columns contains the data for a given series. In the first row the Roman numerals refer to the number of the subordinate series in the corresponding column. The data for any line include: (1) the number of the line in its series, (2) the intensity of the component of the B8 spectrum, (3) the intensity of the component of the B5 spectrum, (4) the intensity in the vacuum tube according to Runge and Paschen, (5) the intensity of the emission on a similar scale. A dash in place of an intensity means that the line was not measured; a dash following an intensity indicates that the number is approximate and probably too large; a zero indicates that the line was too faint to measure. When the tenths place is omitted the line has been measured a few times only.

TABLE IX.—INTENSITIES OF THE HELIUM LINES IN β LYRÆ AND THE VACUUM TUBE.

Wave Lengths.	Helium I.					Helium II.					Parhelium.					Parhelium I.					Parhelium II.				
	No.	B8.	B5.	Vac	E.	No.	B8.	B5.	Vac.	E.	No.	B8.	B5.	Vac.	E.	No.	B8.	B5.	Vac.	E.	No.	B8	B5	Vac.	E.
3819.8	6	9	—	5	—																				
3833.8																10	4	—	1	—					
3867.7						6	3.5	2.3	3	—															
3872.0																9	5	—	1	—					
3878.3																					9	2	—	1	—
3926.7																8	3.1	2.2	1	2.–					
3936.1																					8	2.7	2.3	1	2.–
3964.9											4	2.1	2.8	4	4.8										
4009.4																7	3.8	1.7	1	2.–					
4024.1																					7	2.3	2.–	1	—
4026.3	5	4.0	2.7	6	5.7																				
4121.0						5	2.6	2.0	4	3															
4143.9																6	4.0	1.8	2	2.5					
4169.1																					6	2.7	0	1	1
4388.1																5	3.0	3	3	4.5					
4437.7																					5	2.3	1.–	1	1
4471.7	4	2.5	3.6	7	9.0																				
4713.3						4	2.1	2.0	4	5.3															
4922.1																4	2.7	3.3	4	3.5					
5015.7											3	0	4.2	6	4.6										

In studying the intensities in the above table it must be remembered that the scale used by Runge and Paschen differs from my own, so that a comparison between the relative intensities of the helium lines in the star and the vacuum tube must be based upon ratios and not upon direct differences. If this be borne in mind, and if all lines of intensity *1* by Runge and Paschen be omitted (since the assigned value in such cases may be very greatly in error), the striking resemblance between the intensities of the vacuum tube lines and those of the spectrum of class B5 is at once evident. The ratio between the intensities of the lines of the vacuum tube and of the B5 type spectrum is practically a constant for any series; and for both the vacuum tube and the dark lines of the class B5 spectrum in any series, except Parhelium Series II in which the lines are too faint to consider, the absolute intensity of the lines increases toward the red or remains nearly constant.

But the nearly constant ratio between the intensities of the vacuum tube and B5 spectrum lines of any series increases slightly with each succeeding series from left to right in Table IX. Thus the average values of these ratios for the above series are: for HI, 0.48; HII, 0.58; P, 0.70; PI, 0.9—; PII,—. On the other hand a comparison between the ratios of the intensities of the helium lines in the B8 spectrum and the vacuum tube reveals most striking differences. In any series except in case of the Parhelium Series II, which permits of no conclusion since all its lines are of intensity *1*, this ratio increases rapidly toward the violet. In other words, the lines of the B8 spectrum are relatively strong toward the violet, while toward the red, notably in λ5015.7, they are relatively weak. Again in the case of the B8 lines, as in the companion B5 spectrum, the star lines for any wave-length become relatively brighter as compared with the vacuum tube lines, in the higher series, this effect being even more pronounced in this case than before. Finally the intensities of the emission lines follow those of the bright lines of the vacuum tube spectrum within the limits of error.

Again in the case of hydrogen the relative faintness of the Hβ line of the B8 spectrum as compared with its companion of the B5 spectrum, and the relative increase in the brightness of the line of the B8 spectrum in Hγ and Hδ, is in accord with the evidence from the helium lines. Apparently the lines of the B5 spectrum are produced by an absorbing medium similar in condition to the gas in the vacuum tube; whereas those of the B8 spectrum are produced under conditions quite different, indicating probably higher temperature in the absorbing medium producing the latter set of lines.

A special low-power survey of forty-seven of the best plates was made for the study of bright line intensities. The intensity of each of the lines, Hζ, λ3927, 3936, 3965, Hε, λ4009, 4026, Hδ, λ4121, 4144, 4169, Hγ, λ4388, 4438, 4472, 4630, 4713, Hβ, λ4922 and 5016, was estimated on each plate on a basis similar to that employed for the absorption lines. The results were grouped into normal places as before and were plotted with respect to phase when the observations were sufficiently continuous. These intensity estimates express the intensity of the lines relative to the continuous spectrum, and in assigning an intensity the attempt was made to eliminate the apparent variation of intensity due to the variations in width of the absorption components of the line. This, however, was difficult and in the case of Hδ, Hγ and λ4472 especially, small depressions in the intensity curves were noted at light maxima when the absorption is most extensive. But in addition depressions were observed at the light minima when the above effect is negligible. To investigate the extent of this reduction in the intensity of the bright lines at minimum, I have selected the lines Hβ, Hγ, λ4472 and Hδ, as being best suited for study. The intensity estimates of these lines have been treated exactly as in case of the single dark line of the B8 spectrum, the results being shown in Table X. The depressions at the minimum are seen to be sharp and distinct, indicating a range of variation referred to the continuous spectrum amounting to about thirty per cent. of the mean brightness of this group of lines. But the loss of intensity of the continuous spectrum in this region at principal minimum is about seventy per cent. (and at secondary minimum about thirty per cent.) of the intensity at maximum. Hence we may conclude with Keeler* that variations in the intensity of the bright lines is in the same direction as that of the continuous spectrum but probably greater in range.

TABLE X.—THE DEVIATIONS OF THE EMISSION LINES FROM A CONSTANT INTENSITY DURING THE LIGHT CHANGE.

	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
Phases from prin. min.	0.3	1.6	2.3	3.5	4.5	5.4	6.6	7.3	8.5	9.5	10.4	11.5	12.6
Mean intensity of group	6.5	7.9	8.6	8.3	8.1	7.3	6.7	8.3	8.3	7.5	7.5	8.2	8.3
Deviations in terms of the mean intensity	−0.17	+0.04	+0.10	+0.06	+0.04	−0.06	−0.14	+0.06	+0.06	−0.04	−0.04	+0.05	+0.06

The mean intensity of the emission lines is given (when observed) in Table VII. The decrease in intensity toward the violet of the bright lines of any helium series has already been noted.† The same is true of the hydrogen series. Bright Hε is weak and the other lines of hydrogen beyond Hζ seen on some plates nearly to 3700, show no bright components. I am therefore inclined to infer that the bright Hζ line is really weak and that the strong emission of this group at Hζ is due to λ3888.785 of helium which is the only line of the principal series of helium between λ3200 and λ10800, and by far the brightest helium line in the vacuum tube in the region here studied.

QUALITATIVE STUDIES.

In the preceding paragraphs intensity changes in the continuous spectra, the absorption lines and the emission bands have been discussed. It now remains to describe briefly further spectral changes occurring on the Allegheny plates during the light period, in certain of the typical complex lines in β Lyræ.

The complex lines which because of their brightness are best available for study are the so-called Hζ line at λ3889.1, λ3964.9, λ4024.1, Hδ, Hγ, λ4388.1, λ4471.7, λ4713.3, Hβ and λ5015.7. Of these Hζ is undoubtedly a blend with the close strong line λ3888.8 of helium; λ3964.9 is closely bordered on the red by H of calcium and Hε; λ4026.4 is blended with λ4024.1 of helium; λ4388.1 has close

* Astronomy and Astrophysics, 12, 114, 1893.

† Also observed by Frost, Astrophysical Journal, 2, 383, 1895.

dark lines at λ4385.1 and λ4391.0; λ4471.7 is flanked on the red side by the bright λ4481.4 of magnesium; λ4713.3 is near the maximum of photographic action and is often overexposed and the region of λ5015.7 is often underexposed. It would seem best therefore to confine the discussion to Hδ, Hγ and Hβ, but in order not to overlook any important divergences in the behavior of the helium lines I shall advert to the other groups noted above.

In this description as well as elsewhere in this paper reference is made to the dark components of the B8 spectrum, as the B8 lines, and the dark components of the B5 spectrum, as the B5 lines.

It will make the following qualitative notes clear if it be stated here, that quantitative studies described later indicate that the B5 lines are essentially fixed in position, nearly in the center of the emission lines, that the B8 line oscillates across the B5 line, remaining always inside of the limit of the emission lines, though sometimes blending with the absorption lines at the edges of the emission. The latter are probably complex lines with components in essentially fixed positions but varying in intensity and blending often, especially in case of the groups in the region of lower dispersion.

It proves best to begin the qualitative study at some point where the spectral changes are least rapid. Accordingly a time near secondary maximum is chosen, but the phases are reckoned from the principal minimum.

Phase, 9.5 Days.—Dark lines sharp; resolved when both components are present. Emission is stronger to red of B5 line in case of λ3889, 3965, 4922 and 5016; of equal strength on either side in case of λ4026, 4472, 4713 and Hβ and stronger to violet in case of Hγ and Hδ. This effect in the cases of the last two lines is only apparent however, being due to the absorption of the strong B8 lines which are now on the red side of the B5 lines. The dark borders on the red edge of the emission bands are strong in the cases of Hζ, Hγ, Hβ and λ5016. In the last two cases the violet border is fairly strong. The intensity of emission bands rises sharply at the edges to a nearly constant wide maximum.

Phase, 10.4 Days.—Dark lines unchanged. Emission to violet of B5 dark lines gaining strength though the red half is still brighter on the average. The dark border on the red edge is increasing in strength. In some lines it rivals the strong dark lines. The intensity curve of the emission bands rises sharply on the red edge, continues at constant maximum across the B5 line, then slopes gradually to the continuous spectrum, except in case of Hβ where the violet edge is marked by strong absorption.

Phase, 11.5 Days.—Dark lines closer together, otherwise unchanged. Violet half of emission now seems brighter on the average even in Hβ. The dark border to violet is becoming stronger, apparently narrowing the violet half of the emission, and the intensity curve of the emission band apparently falls more sharply at the violet edge.

Phase, 12.6 Days.—The dark components are resolved with difficulty, though the B5 component is clearly resolved in the case of λ4232 and 4481. The red half of the emission line is in general brighter than the violet, especially in case of Hζ and Hβ. One or more of the inner components of the absorption border to the red have risen greatly in intensity, rivalling the principal dark lines in strength; but the emission lines apparently extend beyond this absorption. The inner component of the violet border is clearly seen, though not strong.

Phase, 0.4 Days.—Dark lines losing sharpness. Components of double dark lines now coincident. Strong emission with sharp maximum close to the red edge of the absorption lines and fading slowly to red across a series of faint dark border lines, the strong dark border line of the previous day having weakened greatly. The violet half of the emission line is relatively weak, in most cases even weaker than the surrounding continuous spectrum, but the component of the violet dark border seen on the previous day is now stronger in the same position, indicating that the bright line source is still emitting a line symmetrical with respect to the B5 lines, but that some absorbing agency has altered the intensity curve of the resulting band.

Phase, 1.6 Days.—Single dark lines less sharp. Dark doubles widened but not resolved except at the violet end of the spectrum. Maximum of the emission lines still to red of B5 lines but much less sharp. Emission now clearly present again on violet side of B5 line, but not as strong or as extensive as the red component. No strong borders are present though dark lines are visible both at red and violet edges.

Phase, 2.3 Days.—Single dark lines diffuse. Doubles wide but blended in many cases. Emission stronger to red of B5 lines, narrower to violet because of presence of B8 lines on that side. No strong border lines.

Phase, 3.5 Days.—All dark lines diffuse. Doubles resolved. Emission strong to red usually fading off gradually. When the B8 line is present no emission is seen to violet of the B5 line, though doubtless present as on the previous day. The B8 line blends with the violet border in some cases.

Phase, 4.5 Days.—Single dark lines unchanged. Doubles often resolved and diffuse. The B8 lines leave only partly visible the violet half of the emission lines except in case of Hζ, λ3965 and λ5016, in which the B8 line is weak or absent. Red half of emission as on previous day.

Phase, 5.4 Days.—Dark lines sharper. Doubles still resolved. B5 component appears faint. B8 lines still obscure emission on the violet side of the B5 lines, making the red half of the bright lines appear relatively stronger and more extensive, especially in case of Hγ and Hδ.

Phase, 6.6 Days.—Dark lines sharper. Doubles all blended. Emission apparently equally extensive on either side of B5 lines but stronger to red, though λ4713 and λ4922 are exceptions to this. Strong dark border lines terminate the emission lines at both edges. Several single dark lines show bright borders.

Phase, 7.3 Days.—Dark lines sharp. Doubles beginning to separate, though B5 component is fainter than normal. The emission now appears stronger and more extensive on the violet side of the B5 lines partly because the B8 lines are now cutting off the strongest part of the emission to red. Well defined absorption lines are visible near the edges of the bright lines. In the case of Hβ the border lines are strong, wide or double to violet, double to red, the emission extending over the inner one.

Phase 8.5 Days.—Dark lines possibly less sharp than on previous day. Doubles well resolved. For the same reason as on the previous day the emission appears stronger to violet and equally extensive on either side; if any thing, more extensive to red as in case of λ4472, Hβ and λ5016. Edges of emission usually well marked by absorption in case of Hβ.

Radial Velocities and Elements.

In the study of the radial velocities obtained from measures of the lines in the spectrum of β Lyræ I have grouped the measurable features into four classes; the simple lines of the B8 spectrum, the lines of the B8 spectrum occurring in complex groups, the dark lines of the B5 spectrum, and the emission lines with their dark borders. In the following discussion these four classes will be considered separately.

Simple Lines of the B8 Spectrum.—From the large number of absorption lines included in Table VII, there were selected twenty-six which were considered available for accurate measurement. The wave-lengths of these lines are italicized in Table VII. In quality these lines compare well with their counterparts in the spectrum of Rigel or of Algol. For fourteen of them, wave-lengths were assumed, and upon these fourteen lines the velocity of the center of mass depends. For the other twelve lines wave-lengths in Table VII were determined, as described above, by taking the mean of all the measures of each line after the measured wave-lengths had been reduced to Rowland's zero, by eliminating the velocity given by the known lines. The measures of the unknown lines were then employed to strengthen the relative values of the final velocity from each plate. These final velocities are included in Table VI, column 7.

To avoid the possible introduction of unknown blends in the lines for which wave-lengths were assumed, these wave-lengths were again determined from my own measures on the basis of the velocities determined from all fourteen lines.

In this way the assumed wave-lengths were represented within the limits of error of the determination. And as a further precaution to avoid possible confusion, measures of the two lines, λ3933.825 of calcium and λ4385.144 of chromium, were rejected on all plates when blending was possible with the close helium lines at λ3936.064 and λ4388.100.

The final velocities thus found for each plate were combined into thirteen normal places with the phase as the basis of grouping. In forming these combinations the grouping of the observations was a natural one resulting from the rough commensurability of the period with the day. For greater ease in the computation a simple system of weighting the normal places was as usual adopted, depending not only on the sum of the separate plate weights determined from the individual measures, but also upon the number of plates and nights involved. As stated above the phases are reckoned from principal minimum as determined by the formula above given.

TABLE XI.—NORMAL PLACES.

No.	Single Lines of the B8 Spectrum.						B8 Line in λ 4481 Mg.			
	Phase from Prin. Min.	Limits of Phase.		Vel.	Resid. $O-C$.	Wt.	Phase from Prin. Min.	Vel.	Resid. $O-C$.	Wt.
(1)	(2)	(3)		(4)	(5)	(6)	(7)	(8)	(9)	(10)
	d.	d.	d.	km.	km.		d.	km.	km.	
1	0.309	0.28 to	0.35	− 39.60	−3.16	0.3	0.309	− 39.5	− 3.1	0.3
2	1.504	1.35	1.62	−136.50	−3.53	0.5	1.506	−129.3	+ 2.7	0.4
3	2.272	2.18	2.42	−171.70	+4.87	0.5	2.246	−182.5	[− 6.8]	0.6
4	3.436	3.32	3.70	−202.90	−0.92	0.7	3.478	−203.0	− 1.0	1.0
5	4.494	4.41	4.51	−178.00	+0.27	0.5	4.515	−177.4	+ 0.8	0.5
6	5.285	5.17	5.48	−135.60	−1.53	0.6	5.267	−134.3	+ 0.8	0.7
7	6.505	6.32	6.63	− 33.90	+1.75	0.6	6.523	− 37.6	− 0.4	0.9
8	7.264	7.14	7.40	+ 32.30	+0.27	1.0	7.270	+ 27.6	− 5.3	1.3
9	8.426	8.23	8.60	+119.10	−1.67	0.6	8.418	+112.9	− 8.4	0.7
10	9.446	9.34	9.58	+161.40	−1.36	0.5	9.443	+161.4	− 1.4	0.8
11	10.344	10.26	10.47	+162.90	+1.52	0.7	10.338	+163.5	+ 2.1	0.8
12	11.428	11.27	11.54	+112.20	−0.33	0.4	11.431	+113.6	+ 1.1	0.5
13	12.513	12.38	12.62	+ 32.80	+5.48	0.2	12.501	+ 51.6	[+22.8]	0.4

From preliminary considerations these approximate elements were adopted:

$$\left.\begin{aligned} P &= 12.9194 \text{ days. (Light Period.)} \\ e &= 0.02 \\ \varpi &= 0^\circ.00 \\ T &= 9.867 \text{ days referred to Principal Minimum} \\ K &= 184.2 \text{ km.} \\ \gamma &= -21.18 \text{ km.} \end{aligned}\right\} \text{Preliminary elements.}$$

Following the procedure described in Volume I, page 33, of these publications, normal equations were obtained; τ has been omitted, since, on account of the small value of the eccentricity, it is useless to attempt to evaluate both τ and π.

$$\begin{aligned}
7.100\Gamma - 0.146\kappa - 1.196\pi + 3.115\epsilon &= + 2.239 \\
+ 3.657 \quad - 0.162 \quad - 0.024 &= + 0.823 \\
+ 3.443 \quad - 0.967 &= - 2.232 \\
+ 2.126 &= + 1.758
\end{aligned}$$

The normal equations yielded:

$$\Gamma = - 0.094, \qquad \kappa = + 0.205, \qquad \pi = + 0.205, \qquad \epsilon = + 0.735.$$

From these the final elements resulted:

$P = 12.9194$ days.	(Assumed.)		Final elements
$e = 0.018$	± 0.0027		
$T = 9.867$ days after Principal Minimum.			
$\varpi = 0^\circ.15$	$\pm 0^\circ.17$		
$K = 184.40$ km.	± 0.48 km.		
$\gamma = - 20.95$ km.			
$A = 187.61$ km.			
$B = 181.19$ km.			
$a \cdot \sin i = 32{,}750{,}000$ km.	$\pm 85{,}000$ km.		
$\dfrac{m_s^3 \sin^3 i}{(m + m_s)^2} = 8.41 \odot$			

Upon the velocities computed according to these elements are based the residuals that appear in the table of observations and also those in the table of normal places. The latter may also be computed by substituting the quantities Γ, $\varkappa$, ε, etc., in the equations of condition for the least-squares solution. A comparison between these two sets of residuals shows no differences greater than 0.06 km., and only one greater than 0.04 km., hence a second solution is not necessary. The probable errors here given were derived from the residuals which appear in the table of observations. The probable error of a normal place of weight unity on this basis proved to be ±0.89 km., the probable error of an average plate, ±2.6 km., the probable error of the best plate, ±2.0 km. The small probable error of ϖ is of course due to the rejection of τ as an unknown in the solution.

The center of mass of the system is approaching us at the rate of 20.95 km. per second. The true orbital period of the star is therefore somewhat longer than the observed period. During one revolution the whole system has moved 23,500,000 km., a distance of 78 light seconds. Hence the true period is 0.0009 days longer than the observed one.

In view of the unusual scale of this orbit the equation of light of the system should be considered. However, its effect may be eliminated from the observation by corrections never exceeding ±0.0013 days and may therefore be neglected in the case of a star of this character.

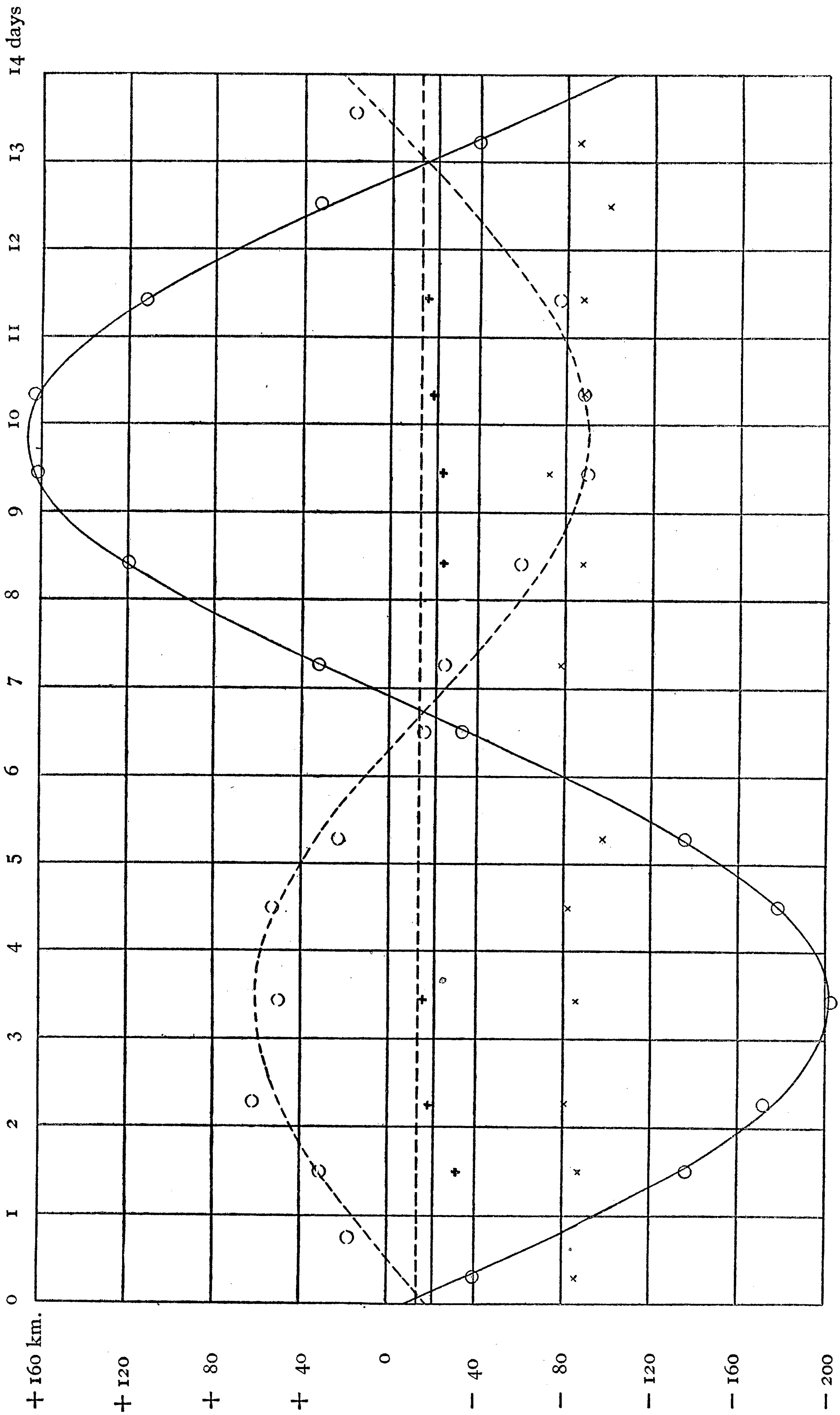

FIGURE 1. Velocity Curves of β Lyræ.

○ = Single Dark Lines. ◯ = Bright Hβ Line. + = Dark Reversal, K. × = Dark Reversal, λ5016.

From the definitive elements it is evident that $u = 90°$ and $270°$ when the phase according to Pannekoek's revised elements is 0.13 and 6.68 days respectively, referred to principal minimum, a result not in strict accordance with the eclipse theory. A series of magnitude estimates made by the writer and as yet unpublished furnish a correction of $+0.15$ days to Pannekoek's predicted time of principal minimum for this epoch. This leaves a discrepancy of only 0.02 days between principal light minimum and conjunction of the components of the system, thus bringing the velocity observations into close accord with the eclipse theory. According to Table II above, secondary minimum in 1907 occurs 6.43 days after principal minimum, comparing closely with the instant of second conjunction, 6.48 days after principal minimum determined from the velocity observations, and again verifying the applicability of the eclipse theory to the case of the double minima of β Lyræ.

The definitive elements are represented by the velocity curve in Figure 1, in which are plotted as small circles the thirteen normal places employed in the discussion of the orbit. In Figure 2 the same curve is compared with Belopolsky's observations of λ4481 in 1892 and 1897. To the former a correction of — 20 km. was applied, to reduce them to the zero of 1907.

In comparing the above elements with those deduced by Stein, the close accordance of the eccentricity is especially striking. The marked difference in the elements, T and ϖ, may be attributed to one or all of three sources: (1) an actual change in the form of this orbit between 1897 and 1907; (2) a difference between the results obtained from λ4481 and the remaining lines; (3) uncertainties, always encountered when e is small, in one or both of the determinations.

The last possibility should receive first consideration; indeed, from the great difference between the results of Belopolsky and Stein obtained from the same observations, it would seem that the true explanation is to be found here. But to test the chance of a real difference between Belopolsky's results and those of this paper, I have in Figure 2 plotted Belopolsky's observations against my final curve, which apparently does not satisfy them along the first quarter of the cycle. It must not be overlooked, however, that the only observations by Belopolsky which show a divergence from my curve are included between one and four days after principal minimum, and here according to Belopolsky's remarks as well as to the writer's experience, the dark lines of this star are diffuse and difficult. Indeed this fact is well borne out by the wide discrepancies between Belopolsky's velocities from plates on this section of the curve. Bearing in mind the obstacles to be met by him in determining this part of the curve from one line (λ4481) there remains little difficulty in harmonizing his velocities with my curve.

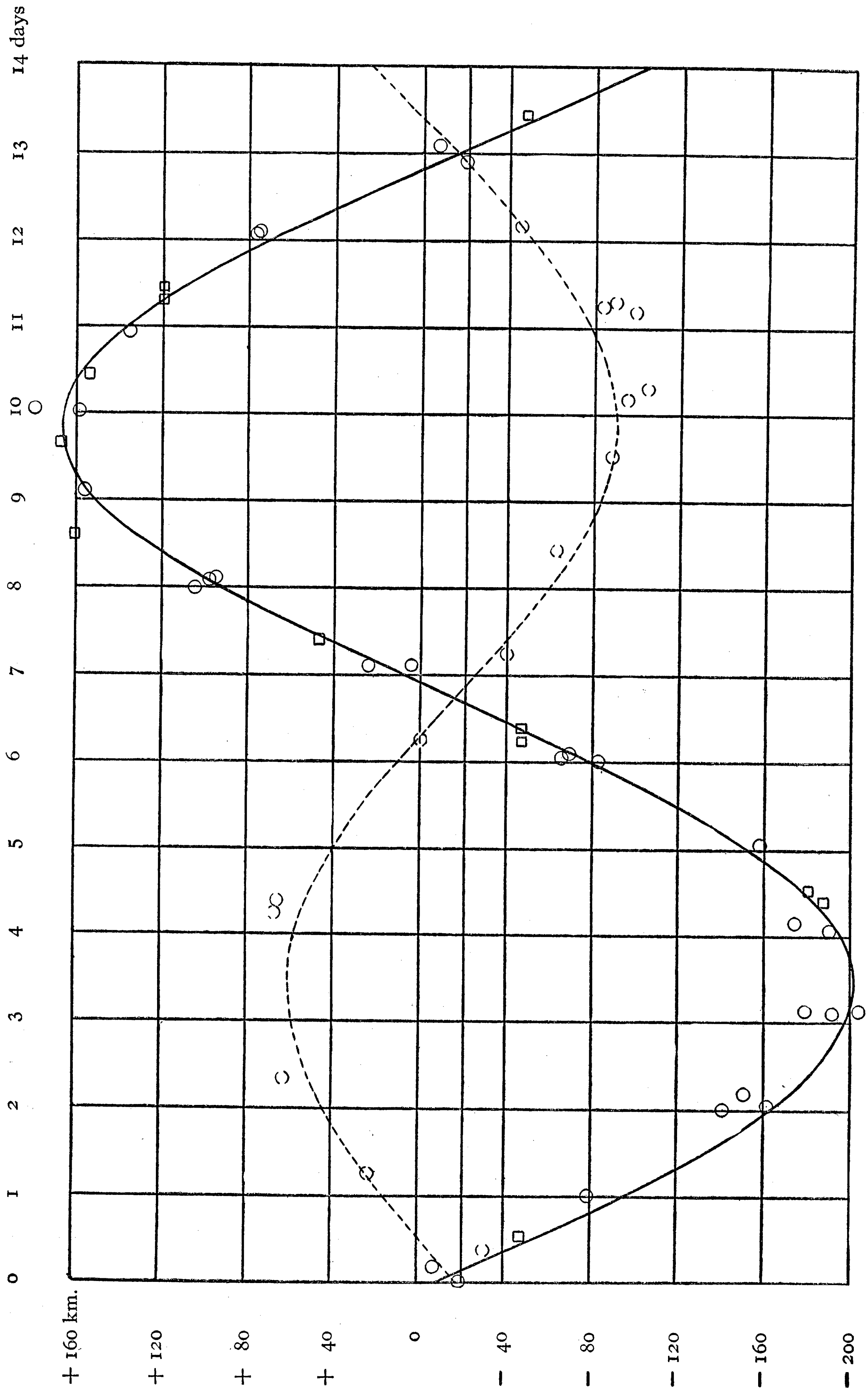

FIGURE 2. Comparison of Pulkowa Observations with the Allegheny Curves.
○ = Measures of λ4481 in 1897. □ = Measures of λ4481 in 1892. ⊂⊃ = Measures of Bright Hβ in 1892.

There remained however the possibility that some part of the discrepancy between Belopolsky's results and those of the present paper might be assignable to a real difference due to his exclusive use of λ4481. For this reason I have isolated and reduced my measures of this line. These are included in Table VII and have been combined into thirteen normal places which appear in Table XI. The results do not agree with Belopolsky's measures of the same line any more closely than do the velocities obtained from twenty-six single lines, especially at phases 2.3 days and 12.5 days. During phases one to two days, there is probably more or less blending of the two components of λ4481. This would be a small effect and may not have been noted by Belopolsky. From the second to the fourth day the oscillating component of λ4481 is near the edge of, or immersed in, the emission of λ4472 of helium. At normal place 2.3 days, there is close absorption on the violet side of λ4481 attributable perhaps to a component of the red dark border of λ4472 of helium, and this accounts for large deviations noted at this phase. At phase 12.5 days, a similar deviation is caused by the close B5 component. There is also evidence of a systematic displacement of the velocities from λ4481 with reference to the curve from all the lines. If these facts be taken into account as well as the known diffuseness of all lines in this spectrum during phases one to four days, it would seem that the differences between Stein's elements (T and ϖ) and my own, are largely due to uncertainities involved in the exclusive use of this one line by Belopolsky. These differences then permit of no possible conclusion as to variations in ten years in the orbital elements expressing the position of the line of apsides for this component of the system.

As to the remaining orbital elements, it will be noted that the three best values of K for the principal star are 200 km., 180.0 km., and 184.4 km. for epochs 1892, 1897 and 1907 respectively. The early value, depending upon eleven measures of λ4481 near the edge of the plates, can be given only small weight. Indeed these observations are well satisfied by the Allegheny curve in Figure 2 with the exception of one discordant place. The probable error of my value for K is ± 0.5 km. and that of Stein is probably ± 1.0 km. Thus the discrepancy between the last two values for K is no greater than accidental and systematic errors would account for. The same may be said of $a. \sin i$. The values of γ for the principal star are 0.0 km., -15.4 km. and -20.95 km. for epochs, 1892, 1897 and 1907 respectively. Since a different instrument was employed in each case it is impossible to determine to how great an extent these values are affected by systematic errors. It seems probable however that a progressive change in γ is actually present, which may prove on further investigation to be periodic. If so, this will account for some of the irregularities in the light period. A small increase in the period, and therefore in $a. \sin i$, has been established through the light variations, but this effect is too small to be brought out by the velocity observations here considered.

A possible systematic displacement of the velocities derived from λ4481 with reference to the definitive curve suggests an effect depending upon wave-length, similar to that assigned by some investigators to the dispersion of light in space. To test this further I have selected two groups of lines, the one of shorter wave-lengths (λ3933, 4128 and 4131) and the other of greater wave-lengths (λ4481, 4508, 4523, 4550, 4559, 4584). The results of the separate reductions of these groups are included in Table XII, in which the weights of the velocity

TABLE XII.—VELOCITIES FROM SELECTED GROUPS OF SINGLE DARK LINES.

No.	Mean λ4050.			Mean λ4520.			Phase Difference Red-Violet.	Wt.
	Phase.	Vel.	Wt.	Phase.	Vel.	Wt.		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	days	km.		days	km.		days	
1	0.308	− 45.2	0.1	0.307	− 31.7	0.1	+0.15	0.1
2	1.482	−137.0	0.1	1.505	−135.7	0.1	+0.04	0.1
3	2.277	−162.7	0.2	2.145	−175.1	0.1	−0.40	0.0
4	3.444	−200.8	0.2	3.464	−204.3	0.2	—	0.0
5	4.495	−173.4	0.2	4.496	−182.8	0.1	+0.22	0.05
6	5.297	−132.7	0.2	5.266	−137.0	0.2	+0.10	0.2
7	6.508	− 32.4	0.2	6.523	− 34.8	0.2	+0.03	0.2
8	7.257	+ 33.2	0.3	7.265	+ 30.8	0.4	+0.05	0.4
9	8.426	+126.3	0.1	8.418	+109.8	0.2	+0.26	0.2
10	9.464	+161.4	0.1	9.430	+160.9	0.2	—	0.0
11	10.335	+162.7	0.2	10.341	+163.6	0.2	—	0.0
12	11.420	+116.5	0.1	11.431	+111.4	0.1	−0.09	0.1
13	12.523	+ 31.6	0.1	12.520	+ 35.1	0.1	+0.05	0.1

observations are on the same basis as those in Table VII. The velocities corresponding to the lines toward the red tend below the velocities corresponding to the lines of shorter wave-length in the ascending branch of the curve, and above them in the descending branch. Of the thirteen normal places employed there are but two exceptions to this rule; they are furnished by the third normal place (which has been discussed above and is here given zero weight), and the twelfth, into which only three plates enter. Assuming that the velocity curves corresponding to the two groups of lines are similar in form and dimensions, I have determined for each normal place the phase difference between the velocities from the two groups. These are given in column (8) in the sense $R-V$ with weights in column (9) based upon the number of lines and the position on the curve, observations at maximum and minimum receiving zero weight. The final mean of $+0.088 \pm 0.021$ days, if it may be considered real, indicates that the velocity curve derived from the group of lines toward the violet precedes in each of its phases by nearly a tenth of a day the similar curve derived from the group of lines toward the red.

The significance of the above effect is not clear. Further investigation is desirable in this and other stars. It seems probable that there is here an actual relative retardation of phase in different parts of the spectrum, the effect observed in other stars by Schlesinger, Tikhoff and others and attributed by some to relative retardation of light in space. Since in this case however (as also pointed out by Schlesinger in connection with the Algol variables observed by him) the relative retardation of the different colors is opposite to that which would be caused by light dispersion in space, it follows that if these two cases are related their true explanation has not yet been found. In this connection it should be noted that no strong relative retardation of phase is indicated in the comparison between the times of the minima of the visual light curve and the instants of conjunction as expressed in the photographic velocity curve.

Lines of the B8 Spectrum Occurring in Complex Groups.—The dark lines in complex groups were measured at all times, whether blended or not. This was done to effect the analysis of these difficult groups, to succeed in which it was necessary to determine without question those phases at which the B8 dark lines, the B5 dark lines and the dark borders of the emission lines were blended or discrete. Even when demonstrably single the B8 dark lines in complex groups were not included in the determination of the elements, since it remained to be shown whether the emission in which they lay affected their positions.

The eighteen lines included in this group are all those marked with a *B* in Table VII with the exceptions of the lines near λ4630, the blended Hζ and λ5015.7; in the last the B8 line if present was too weak to be seen. In addition the following lines marked *b* were included; λ3867.7, 3926.7, 3936.1 and 4024.1.

The velocities for each plate determined from these lines are included in column (11) of Table VI. It will be seen at once that on all plates of phases between 12.8 days and 3.0 days and between 4.8 and 5.3 days, the means are displaced by blending; and on plates of phases between 6.0 days and 6.4 days all lines are certainly blended if the B5 lines are present, but are probably nearly coincident, since no apparent displacement results. Those velocities which are affected by blending are enclosed in brackets and the corresponding residuals omitted. In all other parts of the curve no blending is observed except with the dark edges of the emission lines, and the observations follow the definitive velocity curves as closely as the simple B8 lines do when the weights are taken into account.

But the behavior of the eighteen lines of this group was by no means uniform, for the absolute and relative intensity as well as the dispersion were factors affecting the blending, which varied greatly for different lines. The details of the blending for each complex line need not be included here. Their study has assisted greatly in the analysis of the second set of dark lines.

The Dark Lines of the B5 Spectrum.—With the exception of λ4169 and 4438, all of the B5 dark lines were measured at least twice in each of the hydrogen and helium lines between λ3889.7 and λ5016.7 on the Allegheny plates. However, in the cases of λ3868, 3889, 3936, 3970, 4024 and 4026, the B5 dark lines were either weak or blended, or both weak and blended with other lines, with the result that they could not be considered to advantage in determining radial velocities. Lines at λ3927, 4388, 4481, 4713 and 4922 were measured at intervals, but no reasonably complete series of measures throughout the cycle of variation could be obtained. Lines at λ3934, 4009, 4102, 4144, 4341, 4472, 4862 and 5015 proved best available for this study, since the character of these lines permitted a fair degree of continuity in the results. In the cases of all the lines of this class however the measures were impaired at certain phases by blending with the accompanying B8 lines.

As attempts to combine the radial velocity results from these B5 lines soon made it apparent that each line should be considered separately in the reductions, I departed from the usual methods, and the velocities from each line were combined separately into normal places. In Table XIII the results of these reductions are given with phases recorded only to tenths of a day, since no greater accuracy is here required. The plates are grouped in normal places. All blends resulting in single lines are omitted from the table except in case of Hβ, for which the first three velocities are pure blends. The velocities enclosed in brackets near conjunction at phases 4.5, 5.2, 6.5 and 12.5 days, are affected lines due to blending, to the encroachment of the B8 lines or to some similar effect. Weighted means of the velocity displacements excluding those in brackets for each line are given in the last row of the table.

TABLE XIII.—NORMAL PLACES FOR THE LINES IN THE B5 SPECTRUM.

No.	Phase	λ 3933.825=K.		λ 4009.417.		λ 4101.927=Hδ.		λ 4143.919.		λ 4340.634=Hγ.		λ 4471.676.		λ 4861.527 Hβ.		λ 5015.732.	
		Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.
	days	km.		km.		km.		km.		km.		km.		km.		km.	
1	0.4													[−72.8]	2	−85.5	2
2	1.5	−31.2	3	−59.0	½					−51.7	2			[−90.0]	2	−87.0	3
3	2.3	−18.4	3			−30.0	1	−31.0	½					[−86.5]	2	−80.3	4
4	3.4	−16.0	3	−30.5	1	−56.3	4	−53.7	1½	−54.0	3	−83.4	4	−60.3	5	−85.4	5
5	4.5	[−10.8]	4	−25.0	½	[−22.0]	2	−42.0	1½	[−11.0]	2	[+14.3]	1	[−39.4]	2	−82.2	2
6	5.2	[+ 5.2]	3	−14.5	1	[+13.3]	1	−50.0	1	[− 0.3]	1	[+12.4]	1	[−45.7]	2	−97.5	1
7	6.5													[−41.1]	2	[−49.0]	2
8	7.3	[−50.5]	1	[−88.0]	½	[−81.2]	2			[−70.0]	3	[−63.7]	6	[−56.5]	2	−78.2	2
9	8.4	−24.0	3	−12.3	2	−60.0	5	−49.3	1	−55.0	6	−48.9	5	−78.0	2	−87.9	3
10	9.5	−23.0	2	−21.7	2	−67.7	3	−43.0	1	−48.6	5	−53.8	7	−61.7	3	−72.3	3
11	10.3	−20.0	5	−37.0	½	−57.1	3	−56.0	1	−52.6	7	−63.8	7	−52.8	3	−88.1	4
12	11.5	−16.2	3	−38.0	1	−59.0	1	−43.0	1	−39.5	1	−58.7	4	−47.3	2	−87.5	2
13	12.5	[−79.0]	½	[−65.0]	½	−49.2	1	−73.0	½	−44.5	3	−66.6	4	−61.5	2	−99.0	2
Mean vel.		−21.1		−24.9		−57.6		−48.5		−51.1		−61.4		−59.9		−85.0	

A study of the results in this table leads to the following most interesting conclusions:

(1) At all phases excepting those near the light minima where blending occurs, the velocities corresponding to the measured displacements of any given line show no variations greater than accidental errors will account for. (2) The average displacements of the hydrogen lines (Hδ, Hγ, Hβ) in km. per second show no progressive differences. (3) The negative displacement of the other lines in km. per second decreases with smaller wave-lengths. (4) The negative velocities of all these lines are greater in absolute magnitude than the center of mass velocity determined from the measures of the lines of the B8 spectrum. (5) The K line of calcium possesses a velocity practically identical with that of the center of mass of the system.

In addition to the eight lines whose measures are included in Table XIII, three others were measured on nine or more different plates and all are included in Table XIV. Of all the helium lines herein included, λ4713 alone does not belong to the Parhelium series or the first subordinate helium series, a fact which may account for the small divergence from the above third conclusion in the case of this line. Other helium lines of which a few measures could be made did not always accord with this third conclusion which, however, does hold for all the lines which have permitted reliable measures well distributed in phase.

The cases of λ3964.875 and the blend at Hζ are of great interest. Each possesses an oscillating dark component but in addition there are in each group two

TABLE XIV.—MEAN VELOCITIES FROM DARK LINES OF THE B5 SPECTRUM.

Helium and other Lines.				Hydrogen Lines.		
λ	Series.	Vel.	Wt.	λ	Vel.	Wt.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
		km.			km.	
5015.73	P, H, 3	−85.0	8	4861.53	−59.9	4
4713.31	II, 4	−58.−	1	4340.63	−51.1	7
4481.40	Mg —	−72.−	1	4101.93	−57.6	5
4471.68	I, 4	−61.4	8			
4388.10	P, I, 5	−57.−	1	Mean	−55.3	
4143.92	P, I, 6	−48.5	2			
4009.42	P, I, 7	−24.9	2			
3933.82K	Ca —	−21.1	6			

non-oscillating dark components. In the former group these lines, of intensities 2.8 and 3.3, are displaced toward the violet by amounts corresponding to 32.3 km. and 102.3 km. respectively. The first of these two lines occupies a position which identifies it with the set of lines of the B5 spectrum in Table XIV. In the Hζ group, the two non-oscillating dark components are of intensities 4.3 and 4.7, and are displaced toward the violet by amounts corresponding to 69.2 km. and 150.2

km. respectively; and the first of these two lines is displaced by an amount corresponding well with the mean displacements of the B5 component in the measured hydrogen lines. But the more refrangible non-oscillating dark components in these two groups is not clearly identifiable with any component of the other complex lines, though on a few plates the B5 line in Hδ gives evidence of duplicity. Possibly this line is present as an unresolved component of many of the complex lines. If so, its variation in intensity might explain the progressive change in position of the helium lines in Table XIV as well as the displacements noted in the following paragraph.

The bracketed velocities in Table XIII, noted previously as blended or affected velocities, are of sufficient interest to merit brief discussion. At phases 0.4, 1.5 and 2.3 days after principal minimum, there was extensive blending between the two sets of dark lines as already noted in connection with the B8 lines. Such palpable blends I have omitted from Table VIII except in case of Hβ where their effect was very small though always in the direction of the weak unresolved lines of the B8 star. In the case of λ5015.7 no displacement due to blending was apparent, because of the faintness of the B8 line. On the second day (phase 1.5 days) some of the lines to violet were resolved, and by the fourth day (phase 3.4 days) resolution was general throughout the spectrum, although the two sets of dark lines were at this phase wide and diffuse, as indicated in their intensity curves. On the next day (phase 4.5 days) the B8 lines were measured practically in their normal positions, but with the exception of λ5015.7, λ4143.9 and possibly a few measures of other lines, the B5 lines were uniformly displaced toward the red. On the average this displacement amounted to some 25 km., accompanied probably by a small decrease in the intensity or width of the measured B5 lines. It was as if the strong B8 lines had blended with a part of the B5 lines, producing an apparent narrowing and displacement of the latter toward the red. On the following day (phase 5.2 days) this effect had increased. All the lines in Table IX excepting λ5015.7 were involved. The effect of the addition of the intensity curves of the overlapping lines now resulted in a maximum between them which, interpreted as their common edge, narrowed the B5 lines artificially, broadened the B8 lines and led to apparent displacements of both sets of lines toward the red, an effect averaging about 15 km. for the stronger B8 lines, and about 45 km. for the B5 lines. On the next day (phase 6.5 days) there was a universal blending amounting almost to a coincidence between lines. Even λ5015.7 was affected, being displaced toward the red for the only time during the light period. At phase 7.3 days, the B8 lines had crossed to the red side of the B5 lines and were again apparently resolved, but now the displacements due to overlapping were toward the violet, and the B5 lines thus displaced were quite narrow. In the cases of λ5015.7, Hβ and λ4471.7 however, the B5 lines predominated and, when not measured as palpable blends, were found to appear in their normal positions as their velocities indicate. Finally for the rest of the period no blending or overlapping occurred until phase 12.5 days, less than one day before conjunction, when in several cases the effect noted at phase 7.3 days was again observed.

The blending at and after principal minimum was easily recognized as such, but the effect of the possible overlapping near secondary minimum, when two resolved displaced lines resulted, could easily be misinterpreted. The measures indicated in the otherwise horizontal lines representing the velocities in the lines of the B5 spectrum, a symmetrical oscillation rising about 45 km. above the normal velocity and lasting from phase 3.5 days to phase 7.0 days, and falling a little below the normal for a fraction of a day after the latter phase. Such an effect might be construed as an actual displacement of the B5 lines or an effect due to their doubling, one component only being blended with the B8 lines. In the former case we must assume that the B5 lines narrow unsymmetrically by contracting their violet edges before secondary minimum while the B8 lines broaden by extending their red edge, thus displacing both lines toward the red, the opposite effect being observed in less degree after secondary minimum. Such an effect associated with eclipse is worthy of consideration, but a more probable explanation of the displacements here observed is found in the assumption that the B5 dark lines are really double, or single with a bright reversal. In the cases of λ3965 and the blend at Hζ this duplicity is actually established as noted above and possibly would also be observed in the cases of other hydrogen and helium lines under like conditions of sharpness and dispersion. Possibly also the resolution of these pairs if they exist is more complete at secondary minimum. If so, when the B8 dark lines approach the double B5 line at secondary minimum, the violet component of the B5 line is first blended with the B8 line producing the effect noted above; and when after this minimum the B8 lines shift to the red they blend with the red component of the B5 line producing the second effect noted above. The displacement of the dark line in λ5015.7 to the red at the phase 7.3 days, as well as its simultaneous loss of intensity, is possibly due to a temporary fading of the violet component of this line. In any case the absence of displacements which might indicate real measurable variations in the wave-lengths of these B5 lines, due to changes of radial velocity, seems well established.

The mean of Belopolsky's fourteen measures of the B5 line in Hβ on his plates of 1892 is — 63.0 km. Though affected by blends this result agrees well with my value of — 59.9 km. given above, but is hardly reliable enough to permit of any conclusion regarding a variation in the position of this line in the interval of fifteen years between the observations of Belopolsky and the Allegheny results. As in the case of the Allegheny measures Belopolsky's results indicate no periodic variations in the velocities derived from this line, and show the effect of blending after principal minimum and near secondary minimum.

Velocities obtained from the Allegheny measures of the B5 component in λ5015.7 of helium and *K* of calcium are plotted in Figure 1.

The Emission Lines and Their Dark Borders.—The study of the bright bands of β Lyræ involves the greatest difficulties. Whatever the method of attack, whether qualitative or quantitative, the meaning of the observed data is often in doubt. The difficulties encountered in determining changes in the width and intensity of these bands have led to much uncertainty, while the obstacles in the way of position measures have led to a variety of results. Possibly some of the difficulties would have been removed if the dark borders of the emission lines had always been recognized, though their study would have required plates of the very best quality.

As the attempt to measure the position and width of the emission lines under the conditions which were best suited to the measures upon dark lines proved a failure, measures upon the bright lines were repeated with low power (6) and especial care was taken in the work. It was found that at each edge of each bright line there was a more or less marked region of absorption containing faint dark maxima which, at times when blended, rivalled in intensity the principal dark components in the line. It was impossible under the circumstances to locate definitely the true termination of the emission components with an accuracy necessary in this case. The only method promising success seemed to require an exhaustive study of the dark borders of the emission lines; and accordingly thirty-seven plates were measured for this purpose. On them, seven bright lines promising the best results (Hδ, Hγ, λ4472, λ4713, Hβ, λ4922 and λ5016) were selected for study, and on each of the selected plates measures were made upon each observed absorption line, at or near the estimated edges of these emission lines.

The displacement of each measured dark component of the border from the normal position of the line was first determined and later reduced to velocities. All the results were then plotted on one graph in the usual manner with symbols to distinguish the components of each of the seven lines. A study of this graph led to the following conclusions upon the basis of which the observations have been reduced:

(1) The displacement in Ångströms of any given component of the absorption at the edge of a bright line is proportional to the wave-length of the line. (2) On either edge of any given bright line there is a region of absorption containing

at least four distinct components which are always present though varying in intensity. (3) The strong absorption lines which are observed at times at the edges of Hζ, Hβ and λ5015.7, and also in other cases, are due to blending of two or more of these four components.

The first of these conclusions was arrived at from the graph of velocities, for it was found that the corresponding component in any two lines (*e. g.*, Hβ and Hδ) was displaced from a normal position by the same amount when expressed in kilometers per second. Such a conclusion was however not obvious, for it was necessary to consider carefully the confusion due to blends which were often present and usually recognizable because of their intensities. On the basis of this conclusion the results from different lines were compared, the better to eliminate blends from the lines in the region of lower dispersion by comparison with the measures in the higher dispersions of Hγ, Hδ and Hζ. By this means the measures of single lines could be picked out and the existence of at least four dark components at each edge of each bright line probably established. On certain plates blending was especially prevalent giving rise to strong dark lines. Such confusion of lines was probably associated with actual changes, possibly in the width of lines, depending upon phase and also upon underexposure or related considerations.

TABLE XV.—POSITIONS OF DARK COMPONENTS OF ABSORPTION BORDERS OF THE BRIGHT LINES, REFERRED TO NORMAL CENTER OF LINES, FOR THE WAVE-LENGTH OF Hγ.

Phase.	Violet Edge.								Red Edge.							
	V_4	I	V_3	I	V_2	I	V_1	I	R_1	I	R_2	I	R_3	I	R_4	I
days	Å		Å		Å		Å		Å		Å		Å		Å	
0.51	−7.06	2	−5.46	2	−4.70	3	−3.26	2	+2.32	1	+3.18	2	+4.92	1	+7.08	2
1.53	−7.73	2	−6.01	2	−4.72	1	——	–	+2.40	1	+3.36	2	+4.78	2	+6.58	2
2.27	−7.13	2	−5.81	1	——	–	——	–	+1.90	1	+3.47	2	+4.98	2	+6.84	2
3.43	−7.37	2	−5.92	1	−4.34	1	——	–	——	–	——	–	+4.89	2	+7.01	1
4.51	−7.47	2	−5.69	2	−4.63	1	——	–	——	–	——	–	+4.67	1	+6.32	1
5.21	−7.19	–	−5.91	2	−4.47	1	−3.17	2	——	–	——	–	+4.82	2	+6.60	2
6.52	−7.31	2	−6.09	2	−4.49	2	−2.92	1	+2.06	1	+3.34	1	+4.81	1	+6.61	2
7.28	−7.28	2	——	–	−4.50	2	——	–	+2.21	1	+3.26	2	+4.81	2	+6.95	2
8.43	−7.35	1	−6.08	2	−4.54	2	——	–	——	–	+3.23	1	+4.76	2	+6.71	2
9.43	−7.38	1	−5.98	2	−4.81	2	——	–	——	–	+3.20	3	+4.98	2	+6.58	2
10.33	−7.35	2	−6.16	2	−4.41	2	——	–	——	–	+3.39	2	+4.92	2	+6.85	2
11.45	−7.11	2	−5.62	2	−4.66	2	——	–	——	–	+3.53	2	+4.63	2	+6.37	2
12.51	−7.11	1	−5.96	2	−4.47	2	−3.33	1	+2.32	2	+3.26	4	+4.91	2	+7.02	2
Mean	−7.30	1.8	−5.89	1.8	−4.56	1.8	−3.17	1.5	+2.20	1.2	+3.32	2.1	+4.84	1.8	+6.73	1.9

The position of each component of each line was found when possible on each plate, and the results have been combined into thirteen normal places as above. These positions are expressed as displacements from the normal center of the line in Ångströms for the wave-length of Hγ, and in the reductions appear to be in-variable throughout the period. Table XV contains these values together with the intensity of the lines on the scale used throughout this paper, phases being given in the first column. At certain phases some components are missing from the table. This does not indicate their absence from the spectrum. They may be blended on all plates of a given normal place either with a neighboring border line or with the dark line of the B8 spectrum. In general it seems probable that all the lines are present on all the plates but that their intensities are subject to change. In the borders of the emission at Hζ the line at + 4.84A is apparently resolved into two persisting lines displaced + 4.20A and + 5.36A from the normal position of the line. It seems possible that a higher resolving power would separate the dark borders even farther. My measures of the Hζ borders are not included in Table XV because of the uncertainties of this blend.

If the absorption groups measured in the above table be considered the edges of the emission lines, they may be employed to determine the position of those lines on the assumption that the various pairs beginning with the two inner components of the edges of each line are due to absorption in one and the same stratum, or more generally that the mean of each group marks an edge of an emission line. In either case the mean position of the bright line would be the same. On the first hypothesis the positions of the centers of the four pairs of lines, using now the mean values at the end of Table XV and beginning with the inner pair, would be —33, —42, —36 and —20 km.; and in the mean, —33 km., which may then be considered the radial velocity corresponding to the emission lines of β Lyræ. As might be anticipated, the mean of the outer pair, presumably corresponding to higher pressure, is farther to the red. But no great weight can be given to these results, since in the pairing of lines as done above the assumptions involved are arbitrary.

In the case of Hβ, as stated above, the blending of the lines at the edges is often encountered, and probably at times the oscillating dark line when at maximum displacement blends into the edges, producing in the latter even more complex conditions. During the first half of the period the components of the edges at + 4.84A and + 6.73A are often blended, and the resulting line with its superior strength at once attracts the measurer's attention, both because the inner components of this edge appear relatively weak and also because the bright band appears to terminate at this line. In the middle of the second half of the period the inner component at + 3.32A is measured very consistently. When strong it is blended with the lines on either side. On some plates it is probably resolved. Now if an oscillating sine curve be drawn using the velocity corresponding to the first-mentioned line as the maximum ordinate in the middle of the first half of the period, and the velocity corresponding to the last-mentioned line as the minimum ordinate in the second half of the period, observed blended or single lines can be found which follow along such a curve at secondary minimum and to some extent at principal minimum. Similarly at the violet edge the observations can be interpreted as a similar parallel oscillation though with a slightly smaller amplitude; and the conclusion is at once suggested, if the blending be overlooked, that these shifting lines mark the edges of an oscillating bright Hβ band.

On this hypothesis the measures of these two oscillating edges of Hβ have been reduced, and the results combined in Table XVI. The second column contains the phases of the normal places; the third column, the corresponding velocity of the emission line of Hβ defined by the edges as described above; the fifth column, the corresponding weight from the measures; the fourth, the residual from the curve defined by the elements below; the sixth, the width of the line in Ångströms. At phases 2.26 days and 5.21 days, since no appropriate absorption line could be found at the violet edge, the velocity was determined from the red edge by assuming the mean value for the width of the line. At phases 0.31 days and 12.51 days, the large residual may be ascribed to the irregularities during eclipse.

TABLE XVI.—VELOCITIES DERIVED FROM THE BRIGHT COMPONENT OF Hβ.

No.	Phase from Prin. Min.	Velocity.	Residual *O—C.*	Wt.	Width of Line.
(1)	(2)	(3)	(4)	(5)	(6)
1	0.31 days	[+30] km.	[+37] km.	5	12.32 A.
2	0.73	+18	+ 8	3	11.28
3	1.53	+31	− 1	5	11.60
4	2.26	+62	+12	2	11.48
5	3.43	+50	−10	6	11.57
6	4.51	+53	+ 3	6	11.23
7	5.21	+23	− 9	3	11.48
8	6.52	−16	− 8	7	11.58
9	7.28	−25	+10	8	11.26
10	8.43	−61	+10	7	11.88
11	9.43	−90	− 2	7	10.39
12	10.33	−88	±10	8	10.49
13	11.45	−77	− 9	3	11.05
14	12.51	[−65]	[−32]	3	11.62

The elements have been derived graphically on the assumption that the form of the curve is the same as that derived from the measures of the oscillating B8 dark lines. The corresponding velocity curve is compared with the observations in Figure 1, and with Belopolsky's observations in Figure 2 in which a correction of — 13 km. has been applied to each one of Belopolsky's observations to reduce them to my zero.

Elements from the Hβ Bright Line.

$P = 12.9194$ days (assumed).
$e = 0.018$ (assumed).
$T = 9.867$ days after Principal Minimum.
$\varpi_s = 180^\circ.15$ (assumed).
$K_s = 75.0$ km.
$\gamma_s = -13.0$ km.
$A_s = 67.8$ km.
$B_s = 82.2$ km.
$a_s \cdot \sin i = 13{,}300{,}000$ km.

$$\frac{m}{m_s} = 0.407$$

$$m = \frac{6.77\odot}{\sin^3 i}$$

$$m_s = \frac{16.64\odot}{\sin^3 i}$$

The close resemblance between these elements and Belopolsky's from the same line would indicate that it was by a method similar to the above that his results were obtained, since the variations in a. sin i, A_s, B_s and K_s are no greater than we should expect under the circumstances. Some corroboration of this supposition is contained in the following translation from his paper:* "Under the supposition that the two bright lines of Hβ were nothing more than parts of a single broad band which belonged to the spectrum of one body, we have attempted to find the radial velocity of this body by measuring the differences in wave-length between the artificial line and its bright stellar counterpart. We have made the settings on the *outer edges* of these lines with all the precaution possible. The measures have been made upon the original spectrograms and also upon positive copies and have been repeated under various conditions with different magnifications."

The difference of 13 km. between Belopolsky's value of γ and my own does not necessarily establish a difference between our interpretations of the Hβ bright line. Between Belopolsky's measures of λ4481 in 1892 and 1897 a still greater disparity exists, which seems to indicate a systematic difference of 15 km. or more between the earlier and the more recent measures. Possibly there is a progressive change in this element. But the difference of 8 km. between my own values of γ and γ_s may be considered as indicating the degree of uncertainty of the value of γ_s, since γ is well determined. However such differences may be otherwise accounted for. The explanation is at once suggested that this disparity is due to relative line displacements caused by differences in pressure or other physical conditions under which the lines are produced. And the character of the two

* Mem. della Soc. Spett. Ital., 22, 101, 1893.

sets of lines is quite in accord with this explanation, for it might be expected that the broad bright lines whose edges are probably produced under higher pressure should be displaced toward the red more than the narrower dark lines of the star giving the spectrum like Rigel. Possibly then we find here an actual case of relative pressure displacement if we accept the reality of the oscillating bright line.

But the writer is inclined to reject these results from the Hβ bright line for several important reasons. In the first place the Hβ line is the only one in which this oscillation can be traced, a fact attributable, I think, to a fortuitous combination of blends due to the particular dispersion and the variation in the components of the dark edges of the Hβ emission line. Further, in the Hβ emission line other dark components at the edges are measurable in addition to those selected. Thus the blend which has been adopted as the red edge of the oscillating bright line of Hβ during the first half of the period continues clearly in the same position during the second half of the period, and there seems to be no good reason for selecting a new border line for the edge during this time. Again the bright line of Hβ probably extends over the adopted violet edge during the first half of the period and clearly overlaps the adopted red edge during the second half of the period. To be sure, this might indicate that a second bright line component were present or that the emission of one component were spread over the absorption edges by something akin to a Doppler effect, but the more plausible conclusion would seem to be that the true edges of this line were not variable in position. Finally the results of Table XV, in which the existence of eight non-oscillating absorption lines is probably established, indicate that the suspected periodic variations of the Hβ bright line are illusory. It must not be overlooked, however, that the interpretation of bright Hβ as an oscillating line leads to results which accord with the published deductions from the light variations.

Under any hypothesis the question of the true limits of the emission lines of β Lyræ remains undecided, and I do not see how the point can be settled with any great certainty. As stated above, the emission lines certainly extend at times over some or all of the inner components of the dark edges, but whether they extend to and terminate at the outer component on each edge, it is impossible to decide. There are times when there seems to be no emission on the violet side of the principal dark lines, as for example just after principal minimum; but the edges are clearly measured and on the next day the emission reappears on the violet side, showing that the temporary disappearance was due probably to selective absorption of the continuous spectrum at the phases near eclipse. The best lines on the Allegheny plates for a study of the position of the emission components without analysis of the dark borders are λ5015.7, Hβ and Hζ, for these are usually better defined than the other lines. But special attempts to study and to measure them have resulted only in the conclusion that the bright lines may oscillate in a complex manner, but more probably remain fixed in position though changing greatly in the distribution of light in their various parts, these changes being due to variations in the continuous spectrum on which they lie, as well as to variations in their own intensity curves. The best method of attack seems to consist in a study of the absorption edges of the bright lines as effected above.

SUMMARY.

The Allegheny observations consist of sixty-four spectrograms made between June 3 and August 31, 1907, with the Mellon Spectrograph, covering the region from λ3800 to λ5016. The results from the study of these plates may now be reviewed.

The spectrum of β Lyræ is a composite one comprising: (1) a set of oscillating dark lines resembling those of Rigel or of a star of the B8 type, and containing representatives of hydrogen, helium, calcium, magnesium, iron, silicon, chromium, titanium and probably other elements, and also many unidentified lines whose wave-lengths are here determined; (2) a set of non-oscillating dark lines comparable with those of a B5 spectrum, of a sharpness resembling the corresponding components in the oscillating spectrum and containing representatives of hydrogen, helium, calcium, magnesium and possibly iron; (3) a non-oscillating bright-line spectrum of emission bands with more or less marked non-oscillating dark borders, accompanying dark lines of hydrogen, helium, calcium, magnesium, iron and possibly other elements. In the case of the hydrogen lines and the brighter helium lines the emission bands are broad, often with strong dark borders. The two dark line components lie always in the emission bands, when the latter are present, the one fixed at the center of the group, the other oscillating from one edge to the other of the bright bands. In the case of Hζ and λ3965 of helium, another dark line is observed on the violet side of the non-oscillating component of the B5 spectrum. Changes in the intensity curves of the various lines, both bright and dark, proceed simultaneously with the light variations.

The wave-lengths of fifty-four dark lines, several with bright companions, were determined from the measures and suggestions for identification made in many cases.

The relative intensities of all established lines were estimated during the measures or through special surveys. Variations in the intensities of the single lines indicated that in addition to the continuous spectrum of the B8 star, there was present in this region a second continuous spectrum, much fainter than the

first, belonging to the star eclipsed at secondary minimum. The intensities of lines in complex groups could not be followed through minimum phases on account of blending. The relative intensities of the two sets of dark lines of helium in complex groups indicated that the lines of the B5 spectrum were produced under conditions similar to those in the vacuum tube while those of the B8 spectrum originated in absorbing strata in a different state, probably at a higher temperature. The relative intensities of the emission lines of helium also follow those of the lines of the vacuum tube spectrum as observed by Runge and Paschen. The intensities of the emission lines vary in the same direction as the continuous spectrum but with greater range.

Qualitative studies show that the dark lines are more diffuse and blend more freely during the first half of the period from principal minimum. At secondary minimum the dark lines of the B5 spectrum become relatively faint. The intensity curves of the emission bands undergo great changes especially during principal minimum. The maximum of the emission bands is as a rule on that side of the non-oscillating dark line not occupied by the absorption of the oscillating dark component, but exceptions to the rule are frequent. The dark borders of the emission bands vary in intensity, at times rivalling the principal dark components.

Radial velocity determinations from measures of the oscillating dark lines follow closely an elliptic velocity curve corresponding to an orbit of small eccentricity (0.018) and of semi-axis major greater than 32,750,000 km.; with periastron passage at second maximum, with a period of revolution equal to the light period, and with a velocity for the center of mass of -21 km. The agreement of the times of orbital conjunction with the phases of the minima is established within the limits of error of the determinations. Old observations by Belopolsky conform to this velocity curve except for certain phases where the divergence is probably not due to the Doppler effect. But a variation of the velocity of the system's center of mass is strongly indicated. Velocities from groups of lines of different wave-lengths indicate a relative retardation of phase with increasing wave-length.

Radial velocity determinations from measures of the dark components belonging to the B5 spectrum indicate that these lines are fixed in position within limits of ± 10 km. The B5 lines of hydrogen are all displaced by -55.3 km. on the average. The lines of helium vary progressively in displacement from -24.9 km. for $\lambda 4009$ to -85.5 km. for $\lambda 5016$, the negative displacement increasing with wave-length. The radial velocity of this component in the K-line of calcium is practically identical with that of the center of mass of the system. Blending at secondary minimum indicates that the lines of the B5 spectrum are double at this phase or narrow unsymmetrically.

It was found impossible to measure the positions of the emission lines with the accuracy desired in radial velocity measures. But the absorption regions at the edges of the bright bands were found to be made up of several close dark maxima approximately fixed in position probably within ±10 km. The displacement for the wave-length of Hγ was determined for each of four of these maxima at either edge of the emission bands. When these were combined in pairs beginning with the two inner maxima, the mean of each pair being considered to indicate the center of the emission line corresponding to that pair, the resulting mean velocity of the emission lines was found to be −33 km. per second. Measures of the edges of the Hβ emission line reproduced Belopolsky's results for the same line as closely as might be expected in view of the difficulty of the determinations, but the measured velocity variations were probably attributable to blends and variations in the relative intensities of the component dark maxima in the absorption regions at the edges of this line. The corresponding elements were nevertheless derived. And in them is indicated a variation in the mean position of all lines corresponding to an increase in the absolute value of the negative velocity of the center of mass of the system since the early observations of Belopolsky.

Systematic displacements of sets of similar lines with reference to other sets of lines in the spectrum of β Lyræ were attributed to pressure or other physical differences.

CONCLUSION.

It has been the writer's purpose in this paper to review the more important results of research upon β Lyræ and to develop the evidence contained in the Allegheny plates of 1907 with necessary explanatory remarks. An exhaustive discussion is not contemplated at this time; but a few suggestions or interpretations of results may be useful in indicating the lines along which further study might be prosecuted.

Two hypothetical binary systems, each of which is consistent with the light variations, may profitably be considered in connection with the results of this paper. In the first system of two bodies, one is an extensive gaseous mass with a nucleus in which there exist high pressure conditions, great density and probably high temperature. The nucleus of this mass possesses a photosphere, or a degree of density compatible with the emission of a continuous spectrum. The second body is not more than one-twentieth as massive as the first. It possesses a well defined photosphere contributing in the photographic region a continuous spectrum considerably stronger than that of the gaseous star. Its reversing layer, resembling that of Rigel, is also well developed. The second body possesses its own envelope of gas producing bright lines too faint to recognize. The gaseous envelope of the more massive nucleus is by far the more extensive and produces essentially alone the broad bright bands in the spectrum of β Lyræ.

In this system the oscillating dark lines are attributable to the less massive body, the broad emission bands to the massive gaseous star, the non-oscillating dark lines to reversal in the outer atmosphere of the gaseous star, the complex borders or dark regions at the edges of the emission bands, to the absorption of the denser layers of the gaseous star about the central nucleus, the separate maxima in these dark borders being ascribed to abrupt falls in the density gradient at increasing distances from the center of this mass. The variations in the intensity curves of the emission lines are due to varying degrees of eclipse and to oscillation of the absorption lines of the smaller body. The variations of the continuous spectrum are due to eclipse involving general absorption, as well as to varying presentment of ellipsoidal bodies. The great width of the emission lines is due largely to high pressure and other extreme conditions in the inner layers of the gaseous star, but partly also to the rotation of this body. The varying positions of the helium reversals are due to different physical conditions in the layers predominating in the production of any particular line, or perhaps to unsymmetrical double reversal, or to variations in the intensities of close component lines.

In the second system the two bodies are of nearly equal mass. One is gaseous, giving a bright line spectrum and a relatively faint continuous spectrum. The second is a dark line star resembling Rigel and furnishing normally a large percentage of the light in the continuous spectrum. About both of these stars is an extensive gaseous envelope shared about equally by each.

In this system the oscillating dark lines are attributable to the Rigelian star, the broad emission bands to the gaseous star and the gaseous envelope of the system, the non-oscillating dark lines to reversal in the outer layers of the gaseous envelope. The bright lines of the gaseous star are in this case relatively narrow, the great width of the emission bands being due to rotation of the gaseous envelope of the system. The variations in the intensity curves of the emission lines are due to occultation and absorption at eclipse, to oscillations of the absorption lines of the Rigelian star and to oscillations of the narrower bright lines of the gaseous star within the emission bands due to the gaseous envelope. The varying positions of the helium reversals and the variations of the continuous spectrum may be accounted for as in the case of the first system, but the dark borders of the emission bands are here unexplained.

The great range in the variation of the intensity of the emission bands may be accounted for on the basis of either of the above systems. It is an effect attributable to a combination of the variation of the bright lines due to eclipse with the variations of the continuous spectrum upon which these bright lines lie. Thus at principal minimum, when the gaseous star is eclipsing the brighter star, we should expect the bright lines to be especially conspicuous. That they are not is due to complete selective absorption in the gaseous star of those emissions in the bright continuous spectrum of the eclipsed star which correspond to the bright lines of the gaseous star. At secondary minimum during the partial eclipse of the gaseous star, the variations of the bright lines exceed those of the continuous spectrum because in the latter the light of the gaseous star is relatively faint, whereas in the strong emission lines considered in this discussion the light of the gaseous star is especially bright, impressing its variations strongly, even though these variations during eclipse be proportionately less than those of the continuous spectrum of this star.

It now remains to examine the findings of Nordmann as to the variations of the continuous light in different colors, in connection with the systems here proposed. At principal minimum the always predominating light of the Rigelian star must pass in part at least through the gaseous star, and thus by the stronger general absorption of the gaseous star for the shorter wave-lengths, the deeper minimum in the violet is produced. At secondary minimum the Rigelian star is unobscured except for the absorption of its own atmosphere including the general gaseous envelope: thus the violet light is present in normal proportion or perhaps in more than normal proportion since the continuous spectrum of the gaseous star with its probable excess of red light is now partly occulted at least. Hence the basis of Nordmann's discovery that "at principal minimum the star's rays contain a greater proportion of red light than at secondary minimum."

At first light maximum the Rigelian star is approaching the observer at a maximum rate, and if its photosphere is brightened through heat energy gained by encounters with particles of the gaseous envelope, or is revealed through a displacement of its atmosphere because of its motion through the gaseous envelope, its spectrum will be relatively rich in violet light at this phase, as in the probably similar case of Cepheid variables discussed by Duncan* and the writer.† And since the continuous spectrum of the Rigelian star predominates in β Lyræ, the greater relative amount of violet light in the spectrum at first maximum may be accounted for. At second maximum the proportion of violet light is again about normal or about that at secondary minimum. Probably whatever increase in violet light may attend the maximum approach of the gaseous star is small or is overcome by the loss in violet light which may attend the recession of the brighter star. This may explain Nordmann's observation: "The magnitude at first maximum is much greater in blue light and a little greater in green light than the magnitude at second maximum."

Finally in this discussion the reasons are suggested for Nordmann's first observation: "The range of light variation for the region considered is much greater for shorter wave-lengths than for longer ones."

It would be possible to consider more of the known facts regarding β Lyræ in connection with the system here proposed and modifications might be introduced where such seem necessary. But it would seem best to defer the treatment of any further details until more evidence is at hand.

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* Lick Observatory Bulletins, 5, 82, 1909.
† Ibid., 3, 19, 1904.