Further considerations relating to the systematic motions of the Stars. By J. Halm, Ph.D.

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In a paper entitled "On the velocity of the Sun's motion through space as derived from the radial velocity of Orion Stars," which appeared in vol. xxxii., No. 1, of the *Astrophysical Journal*, Messrs. Kapteyn and Frost point out some important anomalies in the radial motions of these stars, which appear to be intimately connected with similar results previously derived from a discussion of the radial motions of stars of more advanced types (*Monthly Notices*, vol. lxx., November 1909).

A more direct comparison of their results with those obtained by Mr. Hough and myself seemed therefore desirable in order to define more precisely the *rôles* played by these two groups of stars in the motions of the universe, and to point out both the common and diverse features of their motions.

This investigation was practically completed at the time of the publication of Professor Boss's Provisional General Catalogue. It was at once recognised that an important opportunity had arrived for testing some of the conclusions derived from the examination of the radial velocities by means of the transverse motions supplied by this exhaustive and reliable source of information. The principal question on which information from the transverse motions was desired referred to the particular features characterising the Orion type stars as distinguished from the others. An examination of the proper motions of the Boss Stars was well in hand when Mr. Eddington's paper appeared in the *Monthly Notices*, which to a large extent supplied the answer to this question. But on closer examination it was found that those particular features which distinguish the Orion stars from the others are shared by a much larger number of stars than has so far been assumed. The evidence on which this conclusion is based will form a special chapter of the present investigation.

A further point examined in this paper is the question of inequality of distribution of drift, and its bearing on the precessional constant derived from the stars of the Boss Catalogue.

Finally, some of the prominent facts of star motion are discussed from the point of view of the Maxwellian law of distribution, which establishes a necessary relation between the average masses and peculiar motions of the stars.

Examination of Radial Velocities.—A discussion of the radial motions of 491 brighter stars, *not* including those of the Orion type, was published in *M.N.*, vol. lxx., November 1909. It was found that, in addition to the motion of these stars which could be ascribed to a uniform relative motion with reference to the Sun, and which therefore analytically should be expressible by first order spherical harmonics, there was strong evidence of the existence of second order harmonics, which could be qualitatively accounted for by the assumption of an inequality in the mixture of the two drifts in various parts of the sky.

The examination of this point has now been taken up with special reference to those stars of our original list which are situated between $\pm 30°$ galactic latitude, the reason for this selection being that the Orion stars with which our results are to be compared show a marked preference for crowding in the vicinity of the Milky Way. After subtracting the solar motion represented by the numerical formula found in the paper (*M.N.*, vol. lxx., p. 99), viz.,

$$v_{\odot} = -0{\cdot}4 \cos\alpha \cos\delta + 18{\cdot}8 \sin\alpha \cos\delta - 9{\cdot}0 \sin\delta \text{ km/s} \quad . \quad (1)$$

the residuals were arranged in nine groups along the galactic circle, and within each group means were taken according to the weights of the individual data. The resulting mean group residuals are shown in the third vertical column of Table I.

TABLE I.

Mean R.A.	Mean Decl.	Mean Residuals.	Corrected Residuals.	Formula (3).	No. of Stars.
h	°	km.			
2·1	+48	+ 2·2	−0·6	−1·3	19
5·0	+16	+ 9·7	+4·7	+5·7	25
6·6	−10	+10·7	+6·1	+7·1	24
8·2	−30	+ 3·1	+0·9	+2·0	26
10·8	−56	− 1·6	−1·1	−1·1	13
16·1	−46	− 3·9	+0·5	+3·6	13
18·3	− 6	+ 2·8	+8·1	+7·1	29
20·1	+16	+ 2·4	+6·5	+5·5	31
22·8	+49	− 0·1	−0·5	−1·3	8

It is at sight evident that these residuals are of a highly systematic character, and the adopted solar motion quite insufficient to account for the observed motions. The first question is how far they can be reduced by changing the constants of equation (1), *i.e.* by altering the amount and direction of the solar motion. An analysis of the numerical data shows that corrections in the first order terms amounting to

$$-1{\cdot}3 \cos\alpha \cos\delta + 5{\cdot}0 \sin\alpha \cos\delta + 2{\cdot}5 \sin\delta$$

are required. The solar motion satisfying the galactic stars would therefore be

$$v_{\odot} = -1{\cdot}7 \cos\alpha \cos\delta + 23{\cdot}8 \sin\alpha \cos\delta - 6{\cdot}5 \sin\delta \quad . \quad (2)$$

Subtracting the corrections to solar motion from the residuals of the third column, the "corrected residuals" of the fourth column are obtained. They show a clearly pronounced double periodicity, which seems to be satisfied by a term of the form

$$2{\cdot}1 - 4{\cdot}8 \cos 2\alpha \cos^2 \delta - 1{\cdot}3 \sin 2\alpha \cos^2 \delta \quad . \quad . \quad (3)$$

the numerical values of which are entered in the fifth column of Table I.

Now let us compare these results with those derived by Kapteyn and Frost for the Orion type stars. Their investigation is limited to stars within moderate distances from the apex and antapex of the solar motion. It therefore does not lend itself to a complete analysis over the whole galactic circle, and all that could be achieved was a determination of the velocity of the Orion stars relatively to these two points. The solar apex assumed in Kapteyn and Frost's paper is $A = 269^\circ{\cdot}7$ and $D = +30^\circ{\cdot}8$. The principal result arrived at is the fact that the solar motion at the apex is 10 km. less than that near the antapex. To quote their figures, we have

$$\text{Orion Stars} \begin{cases} \text{at apex} & v = -18{\cdot}4 \pm 1{\cdot}4 \text{ km.} \\ \text{at antapex} & v = +28{\cdot}4 \pm 1{\cdot}4 \text{ ,,} \end{cases}$$

—*i.e.* the Orion stars are seen to approach our system at the apex at the rate of 18·4 km. per sec., and to recede from us at the antapex at the rate of 28·4 km. per sec.

On computing the corresponding velocities of the galactic stars as they would follow from the combined formulæ (2) and (3), we find

$$\text{Galactic stars in general} \quad \begin{array}{ll} \text{at apex} & v = -17{\cdot}2. \\ \text{at antapex} & v = +30{\cdot}4. \end{array}$$

The agreement being as close as we can reasonably expect, we may safely say that, with regard to the anomaly here under consideration, there is no striking difference between the behaviour of the galactic stars in general and the Orion type stars examined by Kapteyn and Frost. The phenomenon which they were able to trace is the local expression at the apices of the more general anomaly to which attention was drawn in *M.N.*, vol. lxx., November 1909, and which was there attributed to unequal mixture of the two drifts. They are indeed inclined to ascribe their results to such a cause, but fail to see the correctness of the above formula, which they maintain "gives an identical mixture for any two points of the sphere diametrically opposite." This, we need scarcely say, is an erroneous conclusion drawn from the analysis, which is based on the exactly opposite assumption, viz. that the two drifts are *unequally* mixed on opposite points of the sphere. The only way to explain the error into which they have fallen is perhaps to be found in the fact that they assign the negative sign to the motions of the stars both at apex and antapex (see (4), p. 87 of their paper), whereas in reality the one is a motion of recession and the other of approach, and they are therefore of opposite signs. The second order terms, introduced by unequal mixture, which certainly are equal in amount and sign at opposite points, must therefore tend to reduce the absolute amount of the velocities on the one side and increase it on the other.

The general result so far obtained from the examination of the galactic stars points to the conclusion that the systematic motions of these stars are satisfactorily expressed by the formulæ (2) + (3), viz.,

$$V = -1{\cdot}7\cos\alpha\cos\delta + 23{\cdot}8\sin\alpha\cos\delta - 6{\cdot}5\sin\delta + 2{\cdot}1 - 4{\cdot}8\cos 2\alpha\cos^2\delta - 1{\cdot}3\sin 2\alpha\cos^2\delta \quad (4)$$

Comparing the terms of this equation with the corresponding terms derived from the stars distributed over the whole sphere, viz.,

$$-0{\cdot}4\cos\alpha\cos\delta + 18{\cdot}8\sin\alpha\cos\delta - 9{\cdot}0\sin\delta + 2{\cdot}1 - 2{\cdot}2\cos 2\alpha\cos^2\delta - 2{\cdot}5\sin 2\alpha\cos^2\delta \quad . \quad (5)$$

we may perhaps hesitate to accept some of the differences as purely accidental. Considering that the galactic zone contains 2/5 of the total number of stars, it would follow that the motions of the extra-galactic stars are represented by the formula

$$V = +0{\cdot}5\cos\alpha\cos\delta + 15{\cdot}5\sin\alpha\cos\delta - 10{\cdot}8\sin\delta + 2{\cdot}1 - 0{\cdot}5\cos 2\alpha\cos^2\delta - 3{\cdot}3\sin 2\alpha\cos^2\delta \quad (6)$$

The mean solar motion derived from the first three terms of equations (4) and (6) would be

	For *galactic stars:*	For *extra-galactic stars:*
	$V = 24{\cdot}7 \pm 1{\cdot}5$ km/s.	$V = 18{\cdot}9 \pm 1{\cdot}3$ km/s.
Apex	$A = 274^\circ$	$A = 268^\circ$.
	$D = +15^\circ$	$D = +35^\circ$.

These differences in the amount and direction of the solar motion are worthy of attention. It will be noticed that the apex found from extra-galactic stars is in good agreement with that derived by Boss from the transverse motions, viz. $A = 270^\circ{\cdot}5$ $D = +34^\circ{\cdot}3$; but that of the galactic stars is considerably displaced towards the apparent apex of the first drift. The direction of the displacement and the increased velocity would jointly point to the conclusion that the first drift stars preponderate in the galaxy, a result which is at variance with that obtained in the discussion of the Bradley stars (*M.N.*, vol. lxx., June 1910), where the opposite conclusion was arrived at, and also with that of Eddington, whose results, derived from the proper motions of the Boss Catalogue, point to a uniformity of mixture inside and outside the galaxy. An explanation of this remarkable divergence of conclusions will be given later on.

Second Order Terms.—The conclusive evidence of the existence of these terms expressed by formula (3), and shown in their effect on the observed motions of the galactic stars in the 4th column of Table I., may be considered the most important result of this part of the investigation. It is obvious that residuals exhibiting such a marked regularity and an amplitude of about 1/4 of the amplitude of the solar motion cannot be passed over without the closest scrutiny.

It will be remembered that the same terms were previously evidenced (*M.N.*, vol. lxx., November 1909) when the observing material was arranged symmetrically with reference to the equator. Taking the means of the values of Table IV. of that paper for the northern and southern hemisphere, in order to eliminate the terms multiplied into sin 2δ, we obtain the following table of residuals arranged in order of right ascension :—

Table II.

h h	km.	h h	km.	h h	km.	h h	km.
0–2	+0·4	6– 8	+7·5	12–14	−0·2	18–20	+5·8
2–4	−3·5	8–10	+2·7	14–16	−1·8	20–22	+4·2
4–6	+3·8	10–12	+0·5	16–18	+2·8	22– 0	+3·5

Further, it was mentioned in paper (*M.N.*, vol. lxx., June 1910) that Newcomb had found terms of the same character in his discussion of the precession derived from the transverse motions of the Bradley stars. The appearance of Boss's General Catalogue and the detailed discussion of the proper motions in various papers in the *Astronomical Journal* by Professor Boss offered an opportunity for testing the validity of Newcomb's result. The data for this examination are to be found in Table I. of Professor Boss's first paper on "Precession and Solar Motion" in *Astron. Journ.*, vol. xxvi., No. 12, p. 97. In the table referred to, the mean components of the proper motions $\mu_\alpha \cos\delta$ and μ_δ are given in order of R.A. for trapeziums of equal area arranged in zones parallel to the equator with centres at $0°$, $\pm 20°$, $\pm 40°$, $\pm 60°$, and $\pm 80°$. In conformity with the arrangement of Table II. of the present paper, the observed $\mu_\alpha \cos\delta$ shown in the 7th column of Professor Boss's table of the trapeziums lying between 0^h–2^h, 2^h–4^h, . . . 22^h–0^h and $\pm 50°$ Decl., and again independently of the trapeziums lying between 1^h–3^h, 3^h–5^h, . . . 23^h–1^h and the same range of declination, have been combined. As the terms in question are multiplied by $\cos\delta$, the trapeziums near the poles may be left out of consideration.

The mean group values $\mu_\alpha \cos\delta$ thus obtained were then analysed with regard to the terms involving the component of the

Sun's motion on the assumption of a random distribution of the peculiar star motions, *i.e.* the terms $-V_1L\varpi \sin \alpha + V_1M\varpi \cos \alpha$ of equation (1) on p. 585 of the paper, *M.N.*, vol. lxx., June 1910. The residuals obtained after subtracting these quantities are contained in the 2nd column of Table III.

TABLE III.

Mean R.A.	Residuals.	Smoothed.	Mean R.A.	Residuals.	Smoothed.
h	″	″	h	″	″
0·1	−0·91	−0·76	12·1	−0·37	−0·43
0·9	−0·67	−0·78	13·1	−0·38	−0·48
2·2	−0·75	−0·60	14·2	−0·69	−0·57
2·8	−0·37	−0·39	14·8	−0·63	−0·41
4·1	−0·05	−0·15	16·1	+0·10	−0·11
5·0	−0·02	−0·04	16·8	+0·19	−0·01
6·0	−0·05	−0·10	18·0	−0·33	−0·11
7·2	−0·24	−0·18	19·0	−0·18	−0·19
8·1	−0·20	−0·29	20·0	−0·06	−0·22
9·3	−0·43	−0·36	21·1	−0·43	−0·28
9·9	−0·46	−0·48	21·9	−0·36	−0·50
11·2	−0·56	−0·46	22·8	−0·70	−0·56

The "smoothed" residuals of the 3rd column are obtained in the usual way by combining three neighbouring values into one.

Fig. 1 represents the graphs of the residuals of Tables II. and III. They present a striking similarity to one another, and fulfil very nearly the condition demanded by theory that the phase of the transverse motions should precede that of the radial motions by three hours. One peculiarity exhibited by both curves is deserving of special attention, viz. the steep and quick ascent from a minimum to the following maximum, and the slow and protracted descent to the following minimum.

To explain this, let us revert to the numerical expressions for the second order terms (including the constant) shown in equations (4) and (6) for galactic and extra-galactic regions. We found

$$+2\cdot1 - 4\cdot8 \cos 2\alpha \cos^2\delta - 1\cdot3 \sin 2\alpha \cos^2\delta \text{ for galactic regions,}$$
$$+2\cdot1 - 0\cdot5 \cos 2\alpha \cos^2\delta - 3\cdot3 \sin 2\alpha \cos^2\delta \text{ for extra-galactic regions,} \quad (7)$$

Now the galaxy intersects the equator at about 7^h and 19^h right ascension at an angle of approximately 67°. Roughly speaking, the stars between ±50° declination and R.A. 5^h–19^h and 17^h–21^h may be considered as predominantly galactic, and *vice versa*, the stars between R.A. 22^h–4^h and 10^h–16^h as predominantly extra-galactic.

Computing now the curve of the second order terms for the intervals 5^h–9^h and 17^h–21^h from the first of the expressions (7), we find two maxima at $6^h{\cdot}5$ and $18^h{\cdot}5$, while the curve for the intervals 22^h–4^h and 10^h–16^h shows maxima at $2^h{\cdot}7$ and $14^h{\cdot}7$. The interval between a minimum and following maximum is therefore $3^h{\cdot}8$, and between a maximum and following minimum $8^h{\cdot}2$, in close agreement with the evidence of the curves. We mention these details only to emphasise the very close resemblance of two sets of systematic residuals derived from entirely different material. The physical cause of the phenomenon will be demonstrated at a later stage of this paper. In the meantime let it be

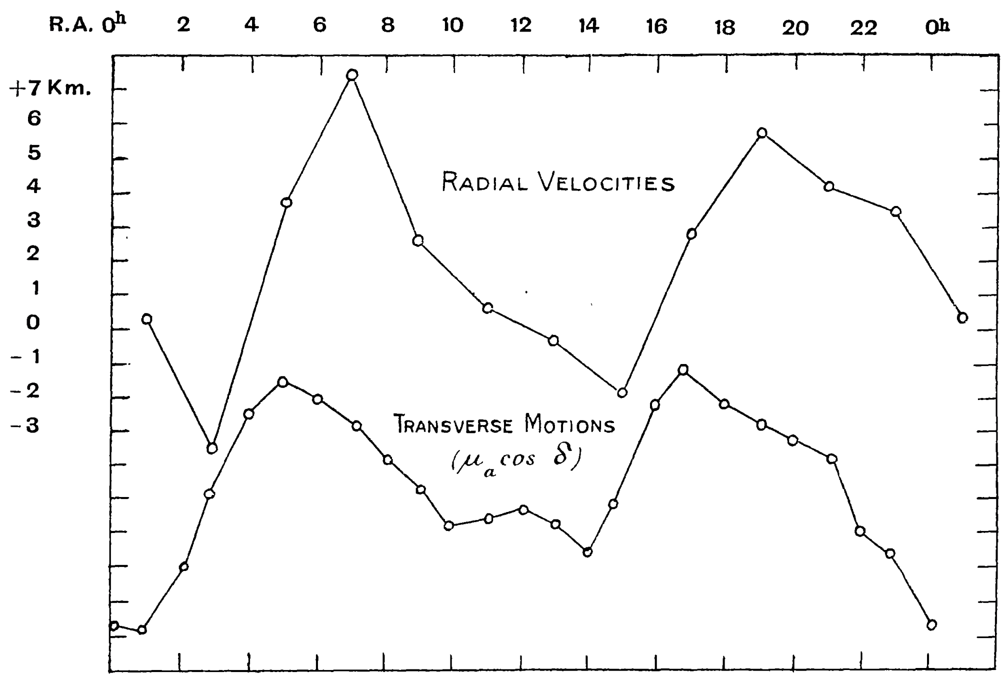

Fig. 1.—Residuals after eliminating Sun's Motion. (Stars grouped symmetrically to Equator.)

stated that the existence of such residuals, whatever their cause may be, is strongly opposed to the view of a random distribution of stellar proper motions.

Evidence of two Drifts from Radial Velocities.—In paper (*M.N.*, vol. lxx., November 1909), it was shown that the second order terms appearing in the radial velocities could be taken as evidence of two star drifts, which are mixed in unequal proportions in opposite parts of the sky. The investigation based on this assumption led to a determination of the true vertex of the two drifts in good accordance with that found by other and more direct methods. An alternative method of testing the two-drift theory by radial motions was suggested by Kapteyn, and applied by him with success in an examination of a limited number of bright stars. Its principle may be briefly stated as follows:—

Let V denote the observed radial velocity. This quantity, apart from accidental errors of measurement which may be neglected, is made up of (*a*) the radial component of the peculiar velocity of the star, ω, and the radial component of the systematic motion of the star relatively to our system, $v_\odot$; *i.e.*,

$$V = \omega + v_\odot,$$

where ω has the characteristics of an "accidental" error. Now, in the case of two drifts, where a star selected at random may belong to either of the two, we shall have either $V_1 = \omega + v_1$ or $V_2 = \omega + v_2$, where v_1, v_2, denote the radial components of the apparent systematic drift motion of Drifts I. and II.

If, among the number of stars contained in a certain area, there are s_1 stars belonging to the first drift and s_2 stars belonging to the second drift, we combine

$$s_1 V_1 + s_2 V_2 = (s_1 + s_2)\omega + s_1 v_1 + s_2 v_2,$$

and the resultant radial velocity which is obtained by dividing by the total number of stars

$$V = \omega + \frac{s_1}{s_1 + s_2} v_1 + \frac{s_2}{s_1 + s_2} v_2,$$

which, by assuming the total number of stars equal to unity, takes the form

$$V = \omega + v_1 + s_2(v_2 - v_1) \quad . \quad . \quad . \quad (8)$$

Now it is clear that the terms $[v_1 + s_2(v_2 - v_1)]$ represent the radial component of what is usually called the Sun's motion, *i.e.* the motion of our system relatively to all the stars combined. We therefore write

$$v_\odot = v_1 + s_2(v_2 - v_1),$$

and subtracting this quantity from the observed velocities of each star we obtain

$$\left.\begin{aligned} &\text{for Drift I. stars: } (V_1 - v_\odot) = \omega - s_2(v_2 - v_1) \\ &\text{for Drift II. stars: } (V_2 - v_\odot) = \omega + (1 - s_2)(v_2 - v_1) \end{aligned}\right\} \quad (9)$$

and hence in the mean of all stars of Drifts I. and II.,

$$(V_1 - v_\odot)^2 = \omega^2 + s_2^2(v_2 - v_1)^2$$
$$(V_2 - v_\odot)^2 = \omega^2 + (1 - s_2)^2(v_2 - v_1)^2.$$

Multiplying the first equation by its weight $(1 - s_2)$, the second by

s_2, and adding, we obtain the square of the residual of unit weight

$$V^2 = \omega^2 + s_2(1 - s_2)(v_2 - v_1)^2 \quad . \quad . \quad . \quad (10)$$

We have now still to consider that $(v_2 - v_1)$ expresses the radial component of the motion of the one drift with regard to the other. Its direction is indicated by the true vertex; and if we call v the relative velocity of the two drifts in that direction, we have the relation

$$(v_2 - v_1) = v \cos \lambda \,^* \quad . \quad . \quad . \quad (11)$$

where λ represents the angular distance between the vertex and the centre of the area under consideration. Substituting (11) into (10) we finally get

$$V^2 = \omega^2 + s(1 - s_2)v^2 \cos^2 \lambda \quad . \quad . \quad . \quad (12)$$

The meaning of this formula is obvious. If we subtract from the observed radial velocity of each individual star the radial component of the Sun's motion and square the residuals, these squares must attain maxima values at two opposite points of the sphere, viz. the vertex and antivertex, in case the stars move in two streams. They will be evenly distributed either for $s_2 = 0$, or $s_2 = 1$, *i.e.* when only one of the drifts exists.

We have attempted to examine from this point of view the galactic stars which are particularly suitable as the vertex appears to be situated in the Milky Way. Unfortunately, however, the material is not advantageous for another reason. The velocities are not given individually for each star, but on the average two or three neighbouring stars have been combined into groups. The chances are, therefore, that stars belonging to different drifts have been united into one and the same group. This, no doubt, would tend to diminish the effect of the second term of equation (12). The separation into individual stars would have been possible for those of the Cape list, by far the smaller number; but no data being available for the northern stars, we had to be satisfied with the existing material. Of course, if we find a pronounced accumulation of errors near the vertices even under these unfavourable conditions, the existence of two drifts would be proven *a fortiori*.

The result of this examination is shown in Table IV. A double calculation has been made, in the first case by neglecting the second order terms, and in the second case by taking them into account. The tabulated quantities represent the residual mean errors of the radial velocity of a single star, as determined from the weighted groups, after elimination of the systematic motion.

* The quantity v is twice the quantity which Kapteyn defines as the "cloud velocity."

TABLE IV.

Mean R.A.	Mean Decl.	Mean Error of Single Star: (Second Order Terms Excluded.	Included).
h	°	km.	km.
2·1	+48	±14·8	±12·3
5·0	+16	22·6	15·3
6·6	−10	29·0	22·6
8·2	−30	23·4	22·9
10·8	−56	13·1	13·9
16·1	−46	16·9	13·9
18·3	−6	25·8	24·0
20·1	+16	23·4	23·4
22·8	+49	15·8	15·7

The figures show distinct accumulations of the residual errors at two opposite points of the galactic circle, approximately where the galaxy intersects the equator. The result would therefore prove that the motions of the stars are not reconcilable with the assumption of a single drift, but point to the existence of two star streams, a result in agreement with that found by Mr. Hough and myself in our first paper, and with the conclusions arrived at by Kapteyn from the discussion of a more limited number of bright stars.

The error due to peculiar motion of the star is represented by the minimum value in the above table. It amounts to about ±15 km. in the case where second order terms are excluded, and this value may be adopted as more comparable with similar values derived by Kapteyn. This being the "mean" error, the average error is $\frac{15}{1{\cdot}25} = \pm 12$ km., which is again in good agreement with the corresponding values for the same class of stars obtained by Kapteyn and Campbell.

The above results are in marked contrast with the conclusions at which Kapteyn and Frost have arrived regarding the motions of the stars of the Orion type. Not only have they found that the average peculiar speed of these stars amounts to only ±6·3 km., *i.e.* about half the amount shown by the other stars, but they have conclusively shown that the cloud velocity of these stars is practically nil. The Orion type stars appear to form a *single* drift, the motion of which is directed neither towards the apparent apex of the first nor towards that of the second drift, but towards the apparent apex of the two drifts as a whole, *i.e.* towards the solar antapex.

Statistical evidence of drift motion derived from the Stars of Professor Boss's General Catalogue.

The extraordinary and exceptional position of the Orion type stars appeared of such vital importance for the progress of our knowledge of the systematic motions of the universe that the publication of Professor Boss's General Catalogue was anxiously awaited in order to examine the transverse motions of this class of stars in relation to other spectral types. Unfortunately the work, which was taken in hand on arrival of Professor Boss's publication, was unavoidably delayed, and in the meantime Eddington's valuable paper "On the systematic motions of the Stars in Professor Boss's Provisional General Catalogue" appeared in the *Monthly Notices*, in which, partly at least, the answer which we searched for was given.

Eddington's researches seem to confirm Kapteyn and Frost's conclusions that the Orion stars form a drift by themselves, moving in the direction of the solar antapex.

But the most noteworthy result of his investigation is that, even after eliminating the Orion stars, the numbers show distinct tendencies to accumulate in this particular direction, tendencies shown in his diagrams by a marked bulging of the curves.

Naturally this phenomenon did not escape his attention, and he accounts for it—justly, we think—by the presence of stars which, though not of the Orion type, nevertheless share in that peculiar motion by which the former are characterised. Where we differ from him, however, is in the estimate of their numbers. Eddington asserts that they are but few, whereas the outcome of the present investigation ascribes to them numbers considerably in excess of those of the "principal" two drifts. The point which Eddington

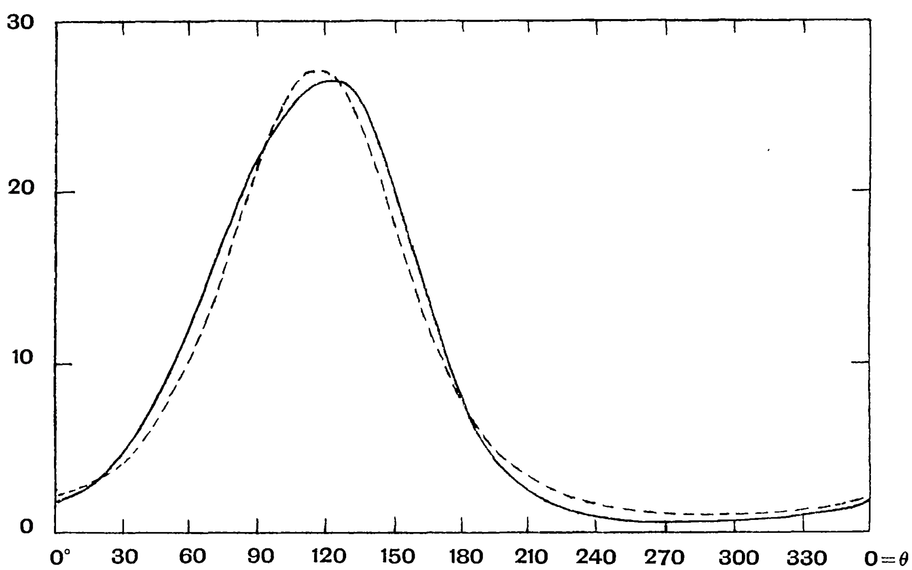

Fig. 2.—Showing the difficulty of separating closely intermingled Drifts.

has not considered is, that when two drift directions are always so close together as the first drift, and the solar drift appear on his diagrams, the resultant curve, although both drifts may be equally strong in number, presents itself with great approximation as a one-drift phenomenon. A fictitious case may illustrate the point. Suppose two drifts, one of which is moving in the direction

$\theta_0 = 90°$ with a velocity $hV = 1{\cdot}05$, the other in the direction $\theta_0 = 135°$ with a velocity $hV = 1{\cdot}23$. Each drift is supposed to contain 100 stars; the interval $d\theta = 15°$. With these data, Eddington's formulæ enable us to compute the number of stars contained in each angular spacing $d\theta$. The resultant numbers are shown in the drawn-out curve of fig. 2. Now there is undoubtedly a lack of symmetry in this curve, which shows that we have not to deal with a *pure* one-drift phenomenon. But the deviations are so slight that, not knowing the real nature of the phenomenon, we should certainly conclude that the curve was due to one single star-drift, and would try to represent it by a theoretical one-drift curve, such as the dotted curve of our figure. Our empirical result would thus be a single drift moving in the direction $\theta_0 = 116°$ with a velocity $hV = 0{\cdot}90$, and containing 200 stars, a result widely different from the actual premises of the case. It was doubtless due to such failures of the analytical method in disentangling closely interwoven drift motions that we arrived at the erroneous conclusion with regard to the distribution of Drift II. stars pointed out by Eddington.

Evidently, then, the analytical method by which a mixture of drifts is separated into its individual components is not always reliable, and therefore an attempt to trace the existence of a third drift, always closely associated with one of the two others, by an analysis of the numbers of stars in each individual area, is bound to prove abortive.

But a more definite answer may be given, if we approach the question in the opposite direction, by making a definite assumption with regard to the direction and velocity of the third drift, and then find how far the representation of the star numbers in the separate areas is improved by its inclusion.

Suppose we have two star drifts and we know the positions of their apices as well as their velocities hV. Then, for any area, the number of stars of Drift I. lying between θ and $\theta + d\theta$, according to Eddington, is expressed by

$$Bd\theta\left\{\tfrac{1}{2} + \tau e^{\tau^2}\int_{-\tau}^{\infty} e^{-x^2}dx\right\} = Bd\theta\,.\,f(\tau),$$

where $Bd\theta$ is a constant depending on the total number of stars of the drift and the width of the angular spacing within which the stars are counted. θ is the inclination of the centre line of any particular spacing to the direction of the drift motion. The latter being known from the assumed position of the apex, θ and also τ are known, since $\tau = hV\cos\theta$. We may therefore compute $f(\tau)$ for all values of θ, and obtain for each θ an equation of condition

$$\text{Observed number} = af_1(\tau) + bf_2(\tau),$$

in which the only unknowns are a and b, which are readily determined by a least square solution. We may even reduce the number of unknowns from two to one if we express the observed numbers in percentages of the total number of stars, and multiply the $f_1(\tau)$ and $f_2(\tau)$ by appropriate factors so that $\Sigma f_1(\tau) = \Sigma f_2(\tau) = 100$.

This makes $a + b = 1$, and leads to the equation of condition

$$[\text{Obs.} - f(\tau_2)] = a[f(\tau_1) - f(\tau_2)].$$

Similarly, in the case of three drifts, the equation of condition assumes the form

$$[\text{Obs.} - f(\tau_3)] = a[f(\tau_1) - f(\tau_3)] + b[f(\tau_2) - f(\tau_3)]$$
$$a + b + c = 1$$

involving now two unknowns, a and b; c, the proportional number of stars of the third drift, being determined by the second relation. Now, as regards the apex of the supposed third drift, it was decided, in view of the results derived by Kapteyn and Eddington with regard to the Orion type stars, to assume the solar antapex as their point of convergence. Further, the theoretical value of hV was found by the following consideration. The most probable value of the average peculiar speed of the spectroscopic stars (excluding the Orion type) was found above to be $\pm 13 \times 0{\cdot}8 = \pm 10{\cdot}4$ km. per second; for the Orion stars by Kapteyn and Frost $\pm 6{\cdot}3$ km.; the ratio $= 1{\cdot}65$. On the other hand, Eddington found the velocity of the motion towards the solar antapex for stars (excluding the Orion type) $hV = 0{\cdot}9$. Hence, for the Orion stars and those physically associated with them, we should expect $hV = 0{\cdot}9 \times 1{\cdot}65 = 1{\cdot}5$. The hypothetical constants of the third drift, which in future will be denoted by O, are therefore

Drift O: Apex R.A. $= 90°$, Decl. $= -36°$; $hV = 1{\cdot}5$.

With these constants the values of $f(t)$ were computed for each area for all values of θ by means of Eddington's table on p. 37 of his paper, "The systematic motions of the Stars" (*M.N.*, vol. lxvii., November 1906).

As regards the apices of the principal two drifts, it is clear that if Drift O really exists, and has passed unrecognised in former investigations, the apices formerly derived cannot be strictly correct. The tendency of the Drift O stars would be to displace the direction of either drift motion towards their own. The mean of all former researches (see Eddington, *M.N.*, vol. lxxi. p. 41) places the apex of

Drift I. at R.A. $= 89°$, Decl. $= -13°$;
and Drift II. at R.A. $= 272°$, Decl. $= -55°$.

In the present work the positions of these apices and the values of hV were found to be

Drift I. R.A. $= 90°$, Decl. $= 0°$; $hV = 1{\cdot}5$.
Drift II. R.A. $= 270°$, Decl. $= -49°$; $hV = 0{\cdot}9$.

We need not enter here upon the method of trial and error by which these apices were found. The points we wish to set forth are—

(*a*) The three-drift hypothesis represents the observed distribution of stars with greater accuracy than the assumption of either one or two drifts.

(*b*) The distribution of the Drift O stars points to a close relationship with the Orion type stars.

(*c*) The inequalities in the distribution of the drift stars as found from the present hypothesis account qualitatively and quantitatively for the systematic character of the residuals in the radial and transverse motions which have been demonstrated above.

It might be urged that the introduction of a third drift, by involving additional unknowns, was bound to lead to a better agreement between observation and theory. But it must not be forgotten that the more rigorous geometrical conditions imposed on the problem lead to a considerable reduction in the number of quantities which have to be evaluated from the observed facts. Thus the representation of the observed distribution in the six standard areas, which will presently be discussed, is based on twenty-one unknowns, viz. two coordinates of each of the three apices, three velocities hV, and two further unknowns a and b for each area. Eddington's analysis based on the two-drift hypothesis involves five unknowns for each area, if the star numbers are expressed in percentages of the total number, *i.e.* thirty unknowns for the six areas.

Of the areas to be examined, those in $\pm 50°$ declination were taken from Eddington's paper. Their centres are situated at 0^h, 4^h, 8^h, 12^h, 16^h, and 20^h R.A. For the equatorial areas, whose centres lie at 1^h, 3^h . . . 23^h and $0°$ declination, the star counts which had already been made here before the appearance of Eddington's paper were utilised. The Orion stars are included, but stars with proper motions greater than $30''$ per century were omitted, the reason being to make the material as nearly as possible comparable with the material used in the determination of the precessional constant.

For a general test of the theory it is, of course, advisable to reduce as much as possible the purely accidental disfigurements of the observed curves by combining such areas in which the apical directions are either the same or can be made identical by counting the θ's in the opposite sense. Owing to the fact that the R.A. of the three apices are very nearly 6^h or 18^h, we may thus combine

Decl. $0°$:	1^h, 11^h, 13^h, 23^h	Decl. $\pm 50°$:	0^h, 12^h
	3^h, 9^h, 15^h, 21^h		4^h, 8^h
	5^h, 7^h, 17^h, 19^h		16^h, 20^h

The star numbers in these six standard areas were now treated

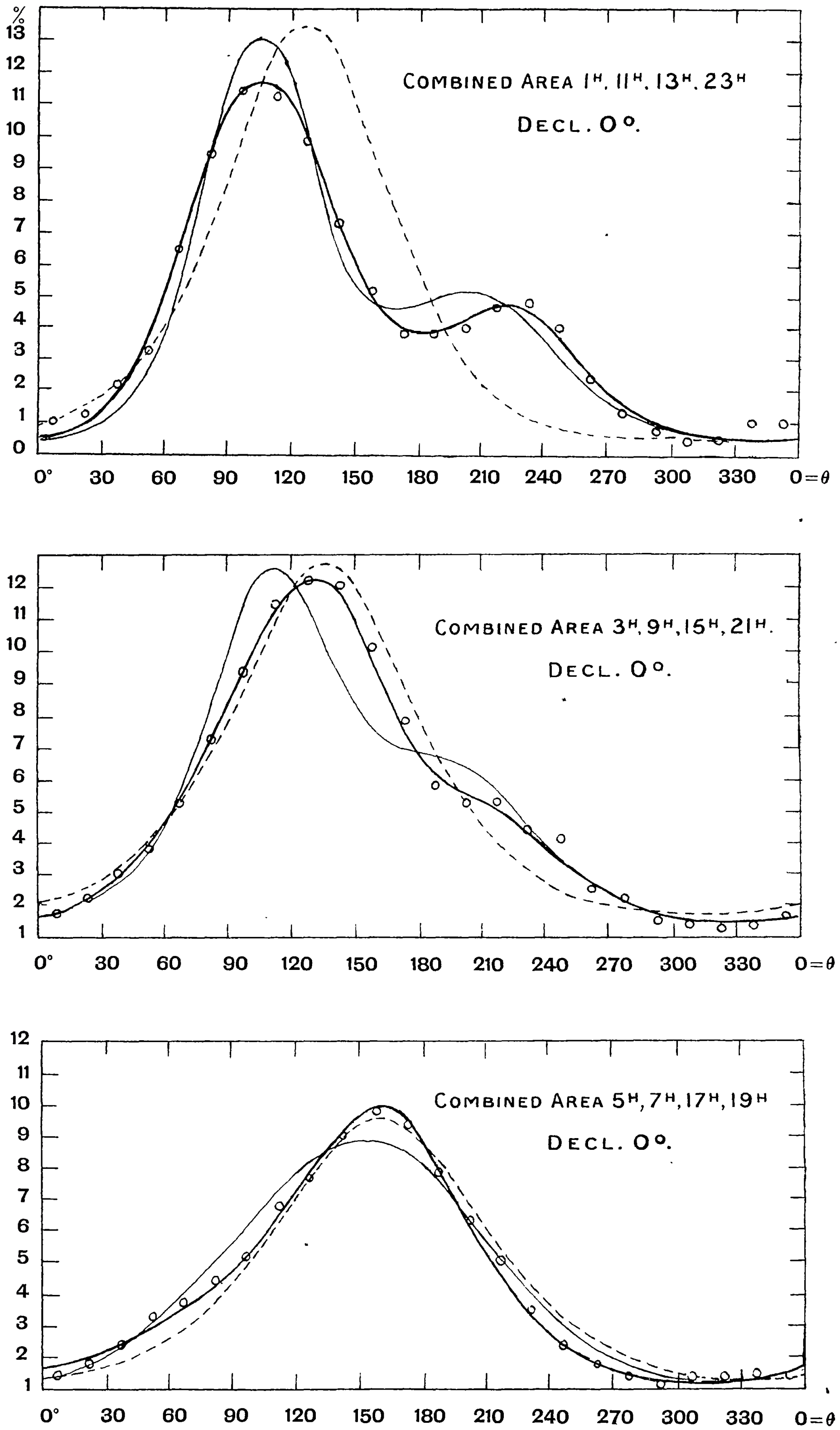
%
Combined Area 1H, 11H, 13H, 23H
Decl. 0°.
0=θ
Combined Area 3H, 9H, 15H, 21H.
Decl. 0°.
0=θ
Combined Area 5H, 7H, 17H, 19H
Decl. 0°.
0=θ

Fig. 3.

in the manner described above, the values of $f(\tau)$ being computed from the assumed positions of the apices and values of hV,* and the unknowns a and b, expressing the proportional numbers of stars of Drifts I. and II., being evaluated by a least square solution.

The outcome of these calculations is condensed in a diagram-

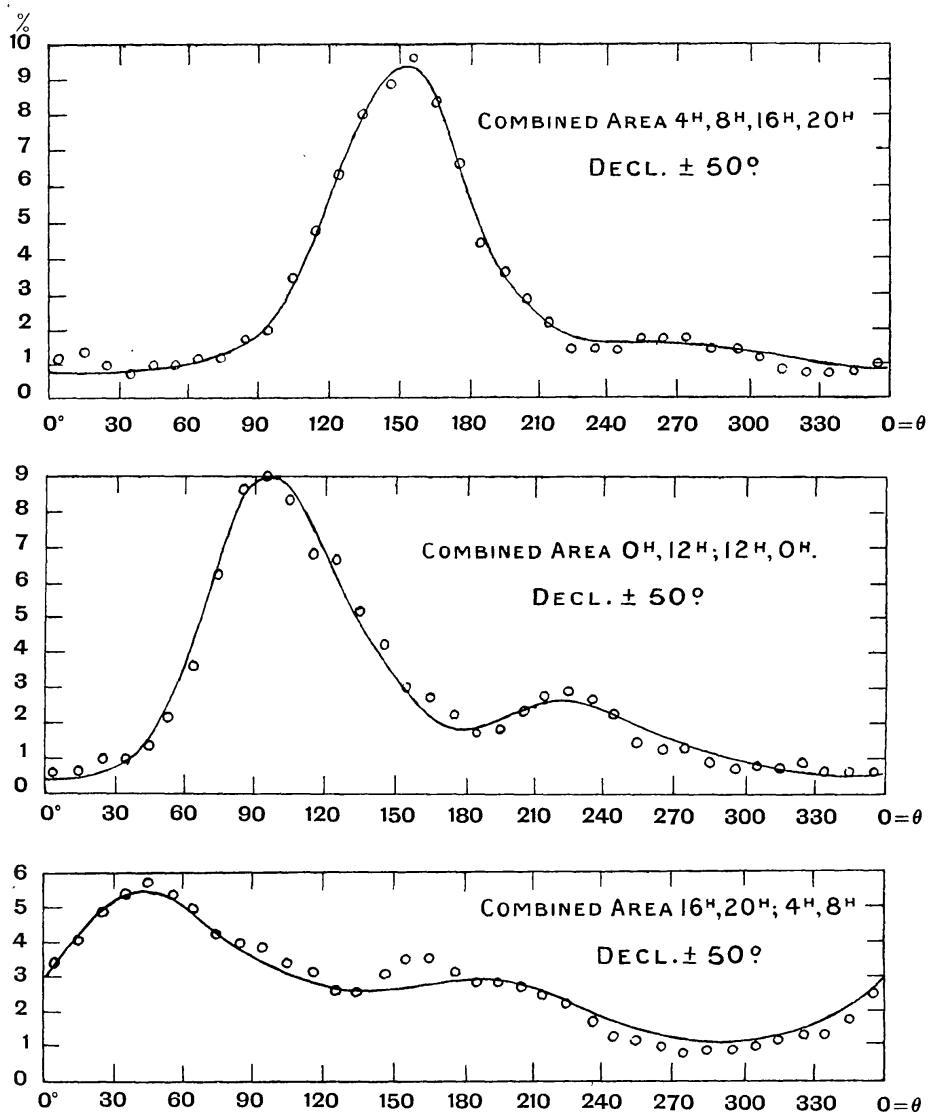

Fig. 3 (*continued*).

matical form in fig. 3. The axis of abscissæ represents the angles θ counted from north over east, and the ordinates show the theoretical percentage number of stars contained within each angular spacing $d\theta$; the latter being 15° wide in the equatorial areas, and 10° in Mr. Eddington's curves. The observed percentage

* It must be mentioned that the value of this quantity employed in the calculations is not the hV given in the text, but $hV \sin \alpha$, where α is the angular distance between the centre of the area and the apex.

numbers are represented by small circles, while the strongly drawn curve shows the numbers resulting from calculation.

It will be admitted that the agreement between theory and observation is, on the whole, as close as can be expected, except a distinctly local disturbance in the last curve at $\theta = 160°$, of which none of the hypotheses can give an account.

Not so satisfactory is the manner in which the facts are represented by the two-drift hypothesis. Calculations based on Eddington's determination of the two apices and of his values for hV were made for the three equatorial areas, and are shown in the thin-line curves of the diagrams.

Lastly, similar calculations were made on the supposition of one drift only, and based on the determination of the solar antapex R.A. = 90°, Decl. − 36°, and Eddington's value $h\text{V} = 0{\cdot}9$. These calculations are represented by the dotted line curves.

A comparison of these curves is interesting. In the 1^h area the one-drift curve appears to be hopelessly wrong, the two-drift curve a fair approximation, but not nearly so good as the curve calculated from the present theory.

In the 3^h and 5^h areas, however, the one-drift curve, while distinctly inferior to the three-drift curve, is even better than the two-drift curve. This fact is remarkable in so far as it may help to explain Professor Boss's conclusion that the one-drift hypothesis is sufficient to account for the observed facts in the greater part of the sky, although we must confess that we have not been able to ascertain on what grounds his conclusion is based.

The analogy between the present results and those obtained from the radial motions is apparent. From the latter we found that there are two distinct families of stars moving in different directions. They are closely, though not evenly, intermixed, and form a star cloud of unknown dimensions. If the Sun's motion with regard to this cloud as a whole is eliminated, the systematic motions of the individual members of the cloud are not reduced to rest, but one part of the cloud is observed to move with considerable velocity in one direction and the other part in the opposite direction. But it was found that not all the stars could be classified among these two star drifts. A certain class of stars, spectroscopically characterised as the B type or Orion type, showed a notably different behaviour. A precise geometrical idea of their peculiar motions may be gained from the following consideration, the *dynamical* probability of which will, however, not be advocated.

Suppose the density of the two drifts to be the same, also assume that certain stars of the one drift have collided with stars of the other drift, and have become permanently united. According to mechanical principles, the motion of the centre of gravity of the pair is not affected by the collision, and hence the motion of the Sun with reference to the double system after union must be identical with its motion with reference to the two drifts as a whole. The apparent motion of the combined stars is therefore directed towards the same point towards which the stars as a whole converge, viz. the solar antapex. But the translatory motion of the two stars relatively to their common centre of gravity being destroyed by the collision, the combined stars are obviously reduced to rest by eliminating the solar motion. This is an exact geometrical interpretation of the systematic motions of the Orion type stars. They form a *single* drift, whose apparent motion is directed towards the solar antapex. The fact that the Orion type stars exhibit this peculiar drift in perhaps its purest form does, however, not preclude the presence of stars of the same characteristic motion in other types as well, where they cannot be separated from the more prevalent members of the two other drifts.

We have now traced with tolerable certainty the same phenomena in the transverse motions by demonstrating the better agreement between theory and observed facts if such a drift as is represented by the Orion type stars is taken into consideration. We shall proceed to strengthen the evidence in favour of the existence of this third drift by pointing out a close relationship in distribution between the Drift O and the Orion type stars.

Distribution of the Drifts.—In order to examine how the stars of the three drifts are mixed in different regions of space, we have to revert to the individual areas and to ascertain by the method above described the quantities a, b, c, the proportional number of stars belonging to each drift, and further, by multiplication into the total number of stars, the absolute star numbers of each drift. The result of our calculations is shown in Table V. The absolute star numbers are reduced to the same unit area, which comprises 1700 square degrees.

TABLE V.

Declination +50°.				Equator.				Declination −50°.			
Area. h	Drift I.	II.	O.	Area. h	I.	II.	O.	Area. h	I.	II.	O.
0	135	75	63	23, 1 *	82	88	46	0	52	77	66
4	160	62	9	3	38	62	88	4	59	30	127
8	0	111	129	5	113	19	177	8	10	31	272
12	58	61	78	7	146	6	129	12	56	62	112
16	67	57	78	9	59	31	112	16	42	38	112
20	85	85	151	11, 13 *	72	48	47	20	118	59	0
Mean	84	75	85	15	22	38	137	Mean	56	49	115
				17	34	22	159				
				19	94	127	55				
				21	77	78	84				
				Mean	74	55	94				

* In counting the stars, these two areas had been treated jointly before it was decided to subdivide into smaller groups.

The first striking fact is the large percentage of Drift O stars. In the general mean we find that 32 per cent. of the stars belong to Drift I., 25 per cent. to Drift II., and 43 per cent. to Drift O.

Another remarkable feature is that when the Drift O stars increase, it is at the expense of the two other drifts. Notice the gradual increase in the number of the former from north to south, and the corresponding decrease in the latter. The impression at first sight is that the O stars have been formed by a combination of stars of the Drifts I. and II. This feature is specially marked in the southern hemisphere, where we notice a most pronounced inequality in R.A. in the O stars, which is faithfully accompanied by the inverse phenomenon in the other stars.

The areas grouped along the parallel $-50°$ are specially in-

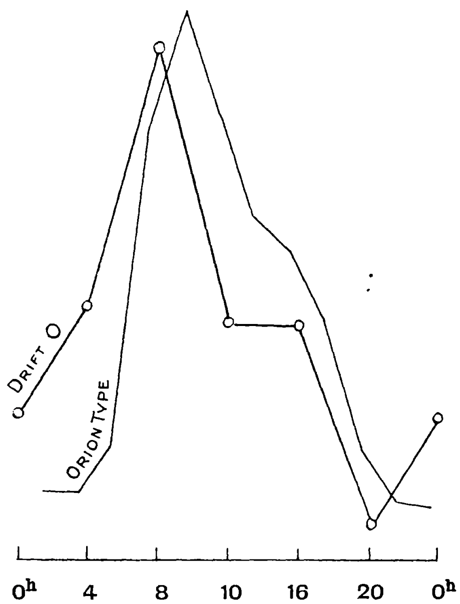

FIG. 4.—Relative Frequency of Drift O and Orion Type Stars along the parallel of $-50°$ Decl.

teresting because they cover regions where the Orion stars are particularly abundant.

The frequency of the Orion stars along that parallel has been examined on the basis of the data supplied by the Harvard Southern Catalogue of brighter spectroscopic stars, and we exhibit the parallelism between the frequency of the Orion stars and stars of the O Drift in fig. 4. A similar correspondence on a lesser scale obtains at the equator, where both Orion and Drift O stars are most frequent in the galactic regions near 6^h and 18^h R.A. Along the parallel $+50°$ the Orion stars show only slight accumulations near 20^h and 4^h R.A. The distribution of Drift O stars is erratic, though there are maxima at 20^h and 8^h R.A.

On reviewing all these facts, the association of the two classes of stars in distribution cannot but point to a relationship. We can hardly avoid the conclusion that the Orion stars are members of a much greater family, with which they share the particular features of motion which distinguish them from the remainder of the stars.

Of special interest for the subsequent part of our investigation is the drift distribution along the equatorial belt. In fig. 5 the plane of the paper is supposed to represent the equator, the lengths of the radii drawn from S the Sun towards the centres of the areas marking the frequency of drift stars in that particular direction. It will be noticed that Drift I. stars are most frequent in galactic regions. In the case of the O stars the diameter of greatest frequency does not quite coincide with the line passing through the opposite galactic centres. It is perhaps more than coincidence that they are most abundant in those regions where

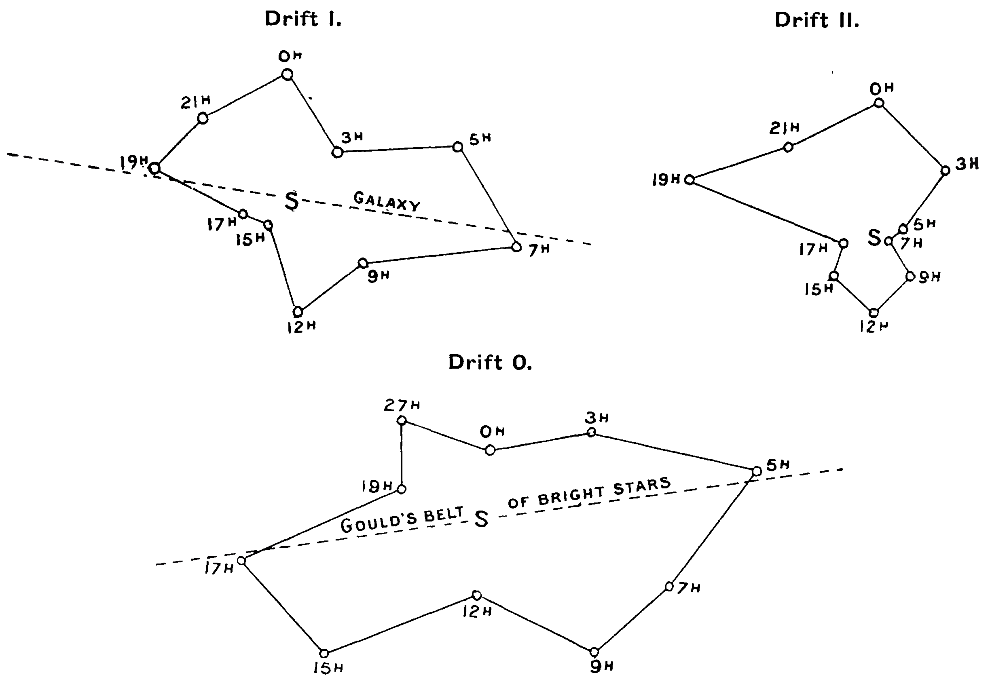

FIG. 5.—Distribution of Drifts along the Equator.

the belt of bright stars known as Gould's belt intersects the equator. The most remarkable feature, however, is presented in the highly unsymmetrical distribution of the stars of the second drift. It is this pronounced inequality to which attention was drawn in the previous paper (*M.N.*, vol. lxx., November 1909), and which there led to the conclusion that the systematic character of the residuals in the radial motions may be due to inequalities in the distribution of the stars of the second drift, a conclusion borne out by the results of the next paragraph. We notice again the "inverse" relation between Drift II. and Drift O, the diameter of *greatest* frequency of the latter coinciding with the diameter of least frequency of the former. This relation, however, is not marked in the first drift, where apparently a preponderating influence of the galaxy exerts itself.

Effect of inequalities in the distribution of the Drift Stars on the radial and transverse motions of the Stars as a whole.

Let a, b, c be the proportional numbers of the drift stars for a given area, subject to the condition

$$a+b+c=1.$$

Let a_0 be the mean value of a taken over the whole sky and write

$$a=a_0+\Delta a,$$

where Δa is an unknown function of the position of the area, and similarly

$$b=b_0+\Delta b$$
$$c=c_0+\Delta c.$$

The systematic part of the mean radial motion of all the stars contained in the area is then

$$\begin{aligned} V = & [(a_0+\Delta a)V_1\xi_1+(b_0+\Delta b)V_2\xi_2+(c_0+\Delta c)\xi_3]\cos\alpha\cos\delta \\ & +[(a_0+\Delta a)V_1\eta_1+(b_0+\Delta b)V_2\eta_2+(c_0+\Delta c)\eta_3]\sin\alpha\cos\delta \\ & +[(a_0+\Delta a)V_1\zeta_1+(b_0+\Delta b)V_2\zeta_2+(c_0+\Delta c)\zeta_3]\sin\delta \end{aligned}$$

where ξ_1, η_1 and ζ_1 are the direction cosines of the apparent motion of Drift I., etc.

Subtracting the mean solar motion, we are left with the residuals

$$\begin{aligned} \Delta V = & [\Delta a V_1\xi_1+\Delta b V_2\xi_2+\Delta c V_3\xi_3]\cos\alpha\cos\delta \\ & +[\Delta a V_1\eta_1+\Delta b V_2\eta_2+\Delta c V_3\eta_3]\sin\alpha\cos\delta \\ & +[\Delta a V_1\zeta_1+\Delta b V_2\zeta_2+\Delta c V_3\zeta_3]\sin\delta. \end{aligned}$$

The first term may be neglected, since $\xi_1=\xi_2=\xi_3=0$, and the last term vanishes if the centre of the area is chosen on the equator. Hence in this case

$$\Delta V=[\Delta a V_1\eta_1+\Delta b V_2\eta_2+\Delta c V_3\eta_3]\sin\alpha.$$

Now the preceding investigation supplies the numerical values of the bracket term for a number of such equatorial areas, and thus enables us to find the residuals ΔV which would arise from unequal distribution. If these theoretical values of ΔV agree with those directly obtained from observation, our assertion that the observed anomalies are due to that cause attains a high degree of probability.

Table VI. shows the percentage numbers a, b, c, as found from star counts in the area 1^h, 2^h, 5^h . . . . . . 23^h, and between $\pm 30°$ declination, after smoothing out the more accidental fluctuations by the usual process of combining three neighbouring values into one.

TABLE VI.

Area.	*a.*	*b.*	*c.*	Δ*a.*	Δ*b.*	Δ*c.*
h						
1	0·319	0·381	0·300	+0·001	+0·132	−0·133
3	0·315	0·266	0·419	−0·003	+0·017	−0·014
5	0·362	0·137	0·501	+0·044	−0·112	+0·068
7	0·393	0·079	0·528	+0·075	−0·170	+0·095
9	0·400	0·144	0·456	+0·082	−0·105	+0·023
11	0·355	0·224	0·421	+0·037	−0·025	−0·012
13	0·294	0·236	0·470	−0·024	−0·013	+0·037
15	0·217	0·185	0·598	−0·101	−0·064	+0·165
17	0·200	0·249	0·551	−0·118	0·000	+0·118
19	0·270	0·295	0·435	−0·048	+0·046	+0·002
21	0·345	0·396	0·259	+0·027	+0·147	−0·174
23	0·360	0·381	0·259	+0·042	+0·132	−0·174
Mean	0·318	0·249	0·433			

In each area the conditions $a+b+c=1$ and $\Delta a+\Delta b+\Delta c=0$ must, of course, be strictly fulfilled.

Expressed in the unit adopted in the preceding investigation we assume

$$V_1 = 1{\cdot}5, \quad V_2 = 0{\cdot}9, \quad V_3 = 1{\cdot}5,$$

and also

$$\eta_1 = \sin A_1 \cos D_1\,; \quad \eta_2 = \sin A_2 \cos D_2\,; \quad \eta_3 = \sin A_3 \cos D_3\,;$$

A_1, D_1, ... being the right ascensions and declinations of the apparent apices. Hence we find

$$\eta_1 = 1{\cdot}000\,; \qquad \eta_2 = -0{\cdot}656\,; \qquad \eta_3 = 0{\cdot}809\,;$$

and $\quad V_1\eta_1 = +1{\cdot}50\,; \quad V_2\eta_2 = -0{\cdot}59\,; \quad V_3\eta_3 = +1{\cdot}21.$

Multiplying these latter respectively into Δ*a*, Δ*b*, Δ*c*, the numbers of the 2nd, 3rd, and 4th columns of Table VII. are obtained.

TABLE VII.

1 Area.	2 $\Delta a V_1\eta_1$.	3 $\Delta b V_2\eta_2$.	4 $\Delta c V_3\eta_3$.	5 Σ.	6 Σ sin α	7 Σ′ sin α.	8 25Σ′ sin α.	9 Obs.ΔV.	10 Obs. − Theory.
h								km.	
1	+0·002	−0·078	−0·161	−0·237	−0·061	−0·090	−2·2	+0·4	+2·6
3	−0·004	−0·010	−0·017	−0·031	−0·022	−0·075	−1·9	−3·5	−1·6
5	+0·066	+0·066	+0·082	+0·214	+0·207	+0·142	+3·7	+3·8	+0·1
7	+0·112	+0·100	+0·115	+0·327	+0·316	+0·257	+6·4	+7·5	+1·1
9	+0·123	+0·062	+0·028	+0·213	+0·151	+0·116	+2·9	+2·7	−0·2
11	+0·055	+0·015	−0·015	+0·055	+0·014	+0·011	+0·3	+0·5	+0·2
13	−0·036	+0·008	+0·045	+0·017	−0·044	−0·015	−0·4	−0·2	+0·2
15	−0·151	+0·038	+0·200	+0·087	−0·062	−0·009	−0·2	−1·8	−1·6
17	−0·177	0·000	+0·143	−0·034	+0·033	+0·098	+2·5	+2·8	+0·3
19	−0·072	−0·027	+0·002	−0·097	+0·094	+0·153	+3·8	+5·8	+2·0
21	+0·040	−0·087	−0·211	−0·258	+0·182	+0·217	+5·4	+4·2	−1·2
23	+0·063	−0·078	−0·211	−0·226	+0·058	+0·061	+1·5	+3·5	+2·0

The fifth column contains $\Sigma = \Delta a V_1 \eta_1 + \Delta b V_2 \eta_2 + \Delta c V_3 \eta_3$, and the sixth column shows these sums multiplied into $\sin \alpha$, which, according to the formula, should be equivalent to ΔV expressed on the scale here adopted. It must, however, be borne in mind that terms of the form $(m \cos \alpha + n \sin \alpha)$ would necessarily combine with the mean solar motion which has been eliminated from the observations. An analysis of the figures of column 6 shows that these terms amount to

$$+0{\cdot}062 \sin \alpha + 0{\cdot}013 \cos \alpha.$$

The numerical equivalents of this expression have therefore been subtracted, and the corrected values are given in the 7th column headed $\Sigma' \sin \alpha$. Now these numbers should, barring a constant factor, represent the residuals remaining in the observed radial velocities after subtraction of the mean solar motion, if the stars are grouped symmetrically to the equator. The factor required to make the theoretical and observed values comparable in size is 25. The values of $\Sigma' \sin \alpha$ multiplied into this factor are shown in the 8th column, while the 9th column exhibits the observed ΔV already given in Table I.

The agreement between observation and theory is extremely satisfactory, and must be considered a proof of the correctness of the assertion that the observed inequalities in the radial motions are due to irregularities of distribution of the drifts.

In the unit here chosen the Sun's velocity amounts to 1·02, and is directed towards an apex in R.A. = 270°, Decl. = +33°. Expressed in kilometres, it amounts to 20·8 km. for the generality of stars. The theoretical factor should therefore have been $\frac{20{\cdot}8}{1{\cdot}0} = 21$ approximately. The close agreement with that which fits best the observations, viz. 25, shows that the agreement between theory and observation is also *quantitatively* established.

The very pronounced existence of a double periodic term in the ΔV is naturally due to the prevalence of a single period in the values Σ. Analysing the latter with regard to $\sin \alpha$ and $\cos \alpha$ we find the single periodic terms

$$0{\cdot}189 \sin \alpha - 0{\cdot}179 \cos \alpha.$$

Multiplying into $\sin \alpha$ we obtain

$$0{\cdot}095 + 0{\cdot}131 \cos 2(\alpha - 112°),$$

or expressed in kms.,

$$2{\cdot}4 + 3{\cdot}3 \cos 2(\alpha - 112°).$$

The two maxima in the residuals of the radial motions therefore appear, according to theory, at $\alpha = 7^{h}{\cdot}5$ and $19^{h}{\cdot}5$, which is in accordance with observations (see fig. 1).

In the transverse motions the residuals in the component

$\mu_\alpha \cos \delta$ for stars grouped symmetrically to the equator are generally expressed by

$$\Delta(\mu_\alpha \cos \delta) = -\varpi[\Delta a V_1 \xi_1 + \Delta b V_2 \xi_2 + \Delta c V_3 \xi_3] \sin \alpha + \varpi[\Delta a V_1 \eta_1 + \Delta b V_2 \eta_2 + \Delta c V_3 \eta_3] \cos \alpha,$$

hence

$$\frac{1}{\varpi}\Delta(\mu_\alpha \cos \delta) = \Sigma \cos \alpha = -0{\cdot}089 + 0{\cdot}131 \cos 2(\alpha - 67^\circ).$$

To convert the numerical terms into seconds of arc we have to consider that the velocity of the parallactic solar motion was found by Boss to be $3''{\cdot}85$ per century. Hence the factor of reduction is $\frac{3{\cdot}85}{1{\cdot}02} = 3{\cdot}8$, and the residuals in $\mu_\alpha \cos \delta$ are theoretically expressed by

$$-0''{\cdot}35 + 0''{\cdot}50 \cos 2(\alpha - 67^\circ).$$

This expression applies, strictly speaking, only to stars situated on the equator; for stars in general the factor $\cos \delta$ has to be added.

Comparing with the observed residuals we find that the amplitude of the second order term agrees well with that derived by Newcomb from the Bradley stars, but is somewhat in excess of that found from the stars of Boss's Catalogue.

There is also a noteworthy difference in phase, the maxima occurring at about 6^h and 18^h, instead of $4^h{\cdot}6$ and $16^h{\cdot}6$ R.A.

It must be borne in mind, however, that in the transverse motions we had to build our theory on an assumption which was not required in dealing with the radial velocities, viz. that the average distances of the stars are the same in all directions. An assumption of this kind is certainly not *a priori* warranted; in fact there is strong evidence militating against it. An inequality in parallax would naturally lead to expressions similar to those resulting from inequality of distribution, and might therefore materially affect the amplitude and phase of the theoretical second order terms. It is interesting to notice that the assumption that the stars at the solar antapex, *i.e.* where they are apparently moving from us, are on the average farther removed than on the opposite side, would be sufficient to bring the observed and computed phases into harmony. The lack of our knowledge of the average distances naturally renders the evidence from transverse motions less conclusive than that derived from the radial motions.

Summarising the results of the present section, the main conclusion is that the existence of the second order terms is incompatible with the assumption of a random distribution of stars. If their presence were confined to the transverse motion, the assumption of an inequality in the average distances might perhaps explain the observed phenomena. But their strongly marked appearance in the radial motions, which could not be interpreted on this assumption, and their very close agreement with the theoretical values derived from the distribution of the drifts, as found from the star counts of Professor Boss's General Catalogue, lends strong support to the conclusion that they are the result of unequal mixture of *several* drifts.

The Maxwellian Law, and the relation between mass and average peculiar motion.—A review of the various attempts recently made to unravel the intricacies of star motion on the basis of Eddington's mathematical analysis must convince us that the applicability of the Maxwellian law to the cosmic motions, at least within the separate drifts, has been fully vindicated by the results. A critical examination of the law on theoretical grounds, which is in course of preparation for press, has also led to the conclusion that its application to the kinetic theory of stars cannot be objected to. In fact, it appears to be applicable, as Jeans has demonstrated, to all forms of conservative mass systems, irrespective of their dimensions and the peculiar mode of mutual interaction between the individual members, either by direct collisions or by attractive forces.

One important point, however, has so far been overlooked, viz. the necessary relation between mass and peculiar motion. We are, of course, familiar with the important conclusion drawn from the Maxwellian law, that in a mixture of two gases, say hydrogen and oxygen, the lighter molecules have greater average speeds than the heavier ones. Their relation to each other is determined by the important law known as the "equipartition of energy," which enunciates that the kinetic energy of the system is evenly divided between the heavier and lighter members. If in a system, in which the Maxwellian law obtains, we were able to examine the motions of the heavier and lighter members separately, we should find that the average kinetic energy of the former is equal to that of the latter, and hence that their average speeds are inversely proportional to the square roots of their masses.

The mathematical demonstration is as follows. According to the Maxwellian law, the number of molecules of mass m whose velocities lie between u, v, w and $u, +du$, $v, +dv$, $w, +dw$, is

$$\mathrm{A} \, . \, e^{-k^2m(u^2+v^2+w^2)}du \, . \, dv \, . \, dw,$$

A and k^2 being constants.

The kinetic energy of a single molecule of mass m is $\frac{1}{2}m(u^2+v^2+w^2)$. Hence the total energy of all the molecules

$$\mathrm{E} = \tfrac{1}{2}\Delta m \iiint_{-\infty}^{+\infty} e^{-k^2m(u^2+v^2+w^2)}(u^2+v^2+w^2)du \, . \, dv \, . \, dw$$

$$= \frac{1}{8}\frac{\mathrm{A}}{k_2}\sqrt{\frac{\pi}{k^2m}}\Big[\iint_{-\infty}^{+\infty} e^{-k^2m(v^2+w^2)}dv \, . \, dw + \iint_{-\infty}^{+\infty} e^{-k^2m(u^2+w^2)}du \, . \, dw + \iint_{-\infty}^{+\infty} e^{-k^2m(u^2+v^2)}du \, . \, dv\Big].$$

The integrals in brackets being identical we may write

$$E = \frac{3}{8}\frac{A}{k^2}\sqrt{\frac{\pi}{k^2 m}}\iint_{-\infty}^{+\infty} e^{-k^2 m(v^2+w^2)} dv \,.\, dw.$$

On the other hand, the total number of molecules is

$$N = A\iiint_{-\infty}^{+\infty} e^{-k^2(u^2+v^2+w^2)} du \,.\, dv \,.\, dw = \tfrac{1}{2}A\sqrt{\frac{\pi}{k^2 m}}\iint_{-\infty}^{+\infty} e^{-k^2 m(v^2+w^2)} du \,.\, dv.$$

Therefore the average kinetic energy

$$\bar{E} = \frac{E}{N} = \frac{3}{4k^2}.$$

In words: the average kinetic energy is independent of the mass. If Ω denotes the average speed we thus find

$$m\Omega^2 = \text{const.},$$

i.e. *the average speed is inversely proportional to the square root of the mass.*

Now the question arises: Are we in a position to sort out, so to speak, the stars according to their masses in order to test the validity of this important law of equipartition of energy? It is maintained that this sorting process is, in some degree at least, accomplished by the spectroscope; *i.e.* that the spectral type of the star offers an indication of its average mass. On *a priori* grounds it is certainly not illogical to associate the rate of development from earlier to more advanced types with the mass of the star. If stars of different masses have started on their career as luminous bodies at the same time, we should, *a priori*, expect the lighter stars to cool down more quickly, and hence to arrive at the more advanced spectroscopic stages sooner than the heavy stars. From this reasoning, then, the conclusion would be reached that the average masses of the early spectral types should be greater than those of the more advanced types. We are now in a position to test this conclusion by means of the principle of equipartition. If our reasoning is correct, the stars of the early types must possess smaller average speeds than those of more advanced types.

Now this is in exact accordance with the results of Kapteyn's investigation, shown in Table VIII.

TABLE VIII.

Type.	Average peculiar speed. km.	No. of Stars.
B to B_9	6·0	64
A to A_5	11·2	18
F to F_8	14·5	17
G to G_5	12·6	26
K to K_5	15·4	55
Ma	19·3	6

Of course, too much importance must not be attached to an *a priori* reasoning, however convincing it may sound; a satisfactory proof of the assertion can only be supplied by the evidence of facts.

Unfortunately the material from which a knowledge of the masses may be derived is very meagre, but what there is appears to support the present view most emphatically.

According to Campbell (Second Catalogue of Binary Stars, *Lick Obs. Bull.*, No. 181), there are seven spectroscopic binaries of the Orion type and nine of types A–G of which the combined mass of the system $(m_1 + m_2) \sin^3 i$ could be determined. These are stars in the spectra of which the displacements of lines of *both* components could be observed (Group I.).

The material may, however, be considerably increased by including the binaries in which only one component was spectroscopically visible, and which therefore determine the relation of the two masses $\frac{m_1^3}{(m_1 + m_2)^2} \sin^3 i$ (Group II.).

It is a well-known fact that the ratio between the masses of the two components is very nearly the same for all classes of stars. Thus we find for

Spectroscopic binaries	Orion type	$m_1/m_2 =$	0·77
	Other types	,,	0·83
Visual binaries (Boss)		,,	0·81

Hence we may assume that the ratios of the average values of $\frac{m_1^3}{(m_1 + m_2)^2} \sin^3 i$ for different types are practically identical with the ratio of their combined masses, the quantity which we desire to find.

From Group I. we derive

$$\text{Orion type } (m_1 + m_2) \sin^3 i = 16{\cdot}3 \pm 3{\cdot}4.$$

$$\text{Types A–G} \quad . \quad . \quad . \quad 1{\cdot}9 \pm 0{\cdot}3.$$

Ratio of average mass of Orion type to that of other types = 8·6.

Square root of ratio = 2·9.

Group II. has been subdivided into three groups according to the periods of revolution of the binaries. It was found—

for periods $< 5^d$—Ratio of average mass of Orion type to that of other types	= 8·8	No. of stars = 29
,, ,, between 5^d and 30^d .	= 4·6	,, 24
,, ,, $> 30^d$	= 6·2	,, 20
Mean ratio	= 6·5	
Square root of mean ratio .	= 2·5	

So far as the evidence goes, there seems to be good reason to assume that the Orion type stars are on the average more massive than the stars of more advanced type.

Now, according to the Maxwellian law, the ratio of the peculiar speeds of these types should equal the inverse square root of the ratio of their masses, *i.e.*—

Average speed of Orion star: average speed of later types

$$= 1 : 2·5 = 0·40.$$

From the preceding Table VIII. we find

$$6·0 : 14·3 = 0·42.$$

The relation between spectral type and mass is also borne out by the results obtained by Mr. Russell (*Astron. Journ.*, vol. xxvi., Nos. 18–19) from a discussion of visual binaries. From six stars of spectral type A–G he finds the average mass 2·4 in good agreement with the value obtained from spectroscopic binaries of the same type. But three stars of type $K_5 - Ma$ yield a considerably smaller value, viz. 0·3. The evidence, considered by itself, would perhaps appear of little value owing to the fewness of stars, but, taken in conjunction with the preceding data, assumes some significance.

Another important fact pointing in the same direction, and fully borne out by the researches of Kapteyn and Russell, is the relation between type and intrinsic brightness. Knowing the apparent magnitude m and parallax π, the absolute magnitude M—*i.e.* magnitude at distance for which $\pi = 0''·1$—is found from the well-known equation

$$M = m + 5(1 + \log \pi).$$

The following table shows the result of Kapteyn's investigation of the brighter stars of various types for which he determined the average parallaxes by means of his ingenious statistical method:—

	Observed App. Mag.	Observed Aver. Parallax.	Computed Absolute Mag.
Orion type	5·0	0″·0066	−0·9
Type A	5·0	0·0098	0·0
Type F–K	5·0	0·0224	+1·8

Mr. Russell's result refers to fainter stars of large proper motions for which direct determinations of parallax have been made. He finds—

Type.	Observed App. Mag.	Observed Parallax.	Computed Absolute Mag.
F_8	7·0	0″·044	5·2
G, G_2	7·8	0·029	5·1
G_5	8·6	0·064	7·6
K	7·4	0·119	7·8
K_5	8·2	0·254	10·2
M	8·3	0·221	10·0

The result is in both cases undoubtedly that stars of early types are intrinsically brighter than stars of advanced types. And combining this result with the preceding conclusion that the early type stars contain the heavier masses, we may also say that *intrinsic brightness and mass are in direct relationship.*

Are there any facts which bear out this remarkable relation?

If we examine the relation between apparent and absolute magnitude of the generality of stars, irrespective of type, by means of their average parallaxes, we arrive at the data collected in the following table, which are based on Kapteyn's investigations.

Observed App. Mag.	Observed Aver. Parallax.	Computed Absolute Mag.
2·7	0″·0383	0·6
4·1	0·0205	0·7
5·1	0·0147	0·9
6·0	0·0129	1·5
6·9	0·0089	1·7
8·6	0·0063	2·6
10·5	0·0048	3·9

It appears, then, that the apparently fainter stars are also on the average intrinsically fainter, *i.e.* their lustre is reduced, not only on account of greater distance, but also on account of inherent feebler luminosity.

Now, if our previous conclusions are correct, the fainter stars must, on the average, be less massive than the bright stars; and hence, according to the Maxwellian relation between mass and peculiar motion, *the fainter stars must show greater peculiar speeds than the bright stars.*

This is exactly what Campbell found to be the case in his discussion of the radial velocities of the northern stars in *Ap. J.*, vol. xiii., No. 1.

On the whole, then, there is a tolerably strong chain of empirical evidence supporting the view that the principle of equipartition of energy is applicable to the system of bodies constituting the visible universe.

The importance of this principle, from the evolutionary point of view, cannot be too much emphasised. It enables us to weigh the stars—not individually, of course, but on the average—through the medium of their *motus peculiares.* We are in a position to separate the stars in order of weight, and to examine the influence of mass on the character of the spectrum and on the surface luminosity, and there can be little doubt that fresh information thus gained will ultimately further our progress towards the ultimate goal of astrophysical research, viz. to comprehend the cause and true meaning of the evolution of the universe.

Throughout the paper I have endeavoured to let the facts speak for themselves and to abstain from theoretical speculations. But even the facts can be accepted only at their face value. Though they may be trustworthy in themselves, the mere fact that they are necessarily incomplete may bias the general conclusions derived from them. All we can do is to proceed step by step and as carefully as possible, but to be always prepared that our conclusions may have to be revised on the evidence of additional empirical data.

On the Progress of the New Tables of the Moon's Motion.

By Ernest W. Brown, F.R.S.

1. In the *Monthly Notices* for 1909 December a general plan for the formation of the new tables was outlined. The forms to be given to the single- and double-entry tables were described and the numerical details of the tables for the principal solar terms in longitude were shown. At that time the single-entry tables of the longitude were in process of formation. In the interval, the single- and double-entry tables for the solar terms in both longitude and parallax have been practically finished, and the numerical transformation of the latitude into the form adopted for tabulation has also been completed.* The plans for the tabulation of all three coordinates, previously somewhat indefinite, have been worked out in detail: in some cases changes have been made, but the material differences are not great. The stage which has been reached will be described below.

Four more tables have been added to the 22 single-entry tables in longitude, previously described, and two of the tables have been combined into one, resulting in a net increase to 25 single-entry tables. Tables for dealing with the numerous remainder terms with periods of $\frac{1}{3}$ of a month or less have been devised; as the forms of these are somewhat different from those previously used, they are described in detail. In the previous paper it was suggested that the sum of these terms might be tabulated at half-daily intervals for 100 years; this labour is avoided and the period suggested is increased to 200 years. Although a mechanical device by which the former plan could alone be economically carried out is no longer necessary, it is still a desideratum, though not a necessity, for obtaining the sums at 10-day intervals, and is under consideration.

The tabulation of the parallax is altered to the extent of adding two more single-entry tables.

The most troublesome problem was the tabulation of the latitude. It will be seen that in the form finally adopted the

* See a paper entitled "The Transformation of the Moon's Latitude" in the present number of the *Monthly Notices*.