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THE ORBIT OF ALGOL FROM OBSERVATIONS MADE IN 1906 AND 1907.

BY FRANK SCHLESINGER AND R. H. CURTISS.

The material upon which this paper is based was secured by means of the Mellon Spectrograph attached to the Keeler Memorial Reflector used in the Cassegrain form. Accounts of both these instruments will appear later in these pages but a brief description of the spectrograph will be in order here. The dispersing medium is a single prism of O.102 glass having a refracting angle of 63°. It is set for minimum deviation at λ4300. The collimator has an aperture of 27 mm. and a focal length of about 65 cm. During the first few months that the spectrograph was in use the camera lens employed was an achromatic of 40 cm. focus corrected for Hγ light. Some experience with this lens showed the desirability of replacing it with a triplet in which the aberrations should be well corrected over a larger range. This was accordingly done in December, 1906, the new lens having almost exactly the same focal length as the old. The prism-box and in fact practically all the metal parts of the spectrograph are of brass. The instrument is protected by an efficient temperature-case provided with an automatic temperature control. The guiding is done by means of a Huggins slit. The spark spectrum of titanium is used as a comparison, introduced into the slit by means of Wright's convenient device. To focus the stellar image upon the slit the whole spectrograph is moved to and from the mirror; no harm results from doing this during the course of an exposure, as is frequently necessary. The measurable portion of our spectrograms covers 21 mm. and extends from λ3925 to λ4750. The spectrograph was constructed, after specifications by Dr. Curtiss and the writer, at the shops of the John A. Brashear Co., to whom is due the excellence of both its mechanical and optical performance. The observatory owes this instrument, as well as some others, to the generosity of Mr. Andrew Mellon.

Within the region covered by our spectrograms, Algol shows a score or more of lines. Only a few of these are suitable for accurate measurement even on the best plates. The observations were carried on in duplicate by the two observers, care being taken to keep the two sets as independent as possible. The lines

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Table of Observations.

No. of Plate.	Plate Secured by	Date, G. M. T.	Phase.	Schlesinger.				Curtiss.			
				No. of Lines.	Wt.	Vel.	Resid.	No. of Lines.	Wt.	Vel.	Resid.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
		1906 d h m	days			km.	km.			km.	km.
76	Curtiss	Sept. 23 17 46	1.821	2	3	+ 32.7	+ 2	3	6	+ 38.3	+ 1
81	Schlesinger	24 18 15	2.841	2	4	− 6.9	− 2	2	4	− 6.0	− 4
82	Schlesinger	24 14 20	0.019	2	4	− 21.9	− 13	2	3	− 0.7	+ 5
92	Curtiss	Oct. 7 18 26	1.513	2	2	+ 20.3	+ 12	1	1	+ 20.7	+ 8
93	Curtiss	7 18 58	1.535					3	2	+ 30.4	+ 16
94	Curtiss	7 19 11	1.544	2	1	+ 15.3	+ 4				
99	Curtiss	11 19 08	2.674	2	2	+ 24.9	+ 15	4	4	+ 14.7	0
100	Curtiss	11 19 28	2.688	1	2	+ 7.4	− 1	1	2	+ 3.2	− 10
109	Curtiss	13 19 11	1.809	1	2	+ 41.1	+ 11	2	2	+ 40.0	+ 4
110	Curtiss	13 19 50	1.836					2	2	+ 39.9	+ 2
116	Schlesinger	14 17 36	2.763	1	3	+ 14.8	+ 13	2	5	+ 13.0	+ 7
117	Schlesinger	14 18 04	2.788	1	2	− 1.8	− 1	2	3	+ 5.5	+ 2
118	Schlesinger	14 19 16	2.813	2	4	− 12.3	− 10	1	2	− 4.9	− 6
128	Curtiss	15 19 24	0.951	2				1	1	− 37.3	− 9
129	Curtiss	15 19 50	0.969	2	4	− 28.5	+ 4	2	4	− 32.6	− 5
135	Schlesinger	16 18 08	1.896	1	2	+ 23.9	− 10	1	1	+ 23.3	− 18
136	Schlesinger	16 18 52	1.929	2	3	+ 21.1	− 14	2	4	+ 43.1	0
137	Schlesinger	16 19 13	1.944	2	2	+ 37.4	+ 1	1	1	+ 30.3	− 13
153	Curtiss	21 19 34	1.223	3	5	− 14.5	+ 1	4	7	− 20.2	− 9
154	Curtiss	21 19 41	1.228	1	2	− 20.3	− 5	2	3	− 25.3	− 14
161	Curtiss	23 17 53	0.285	6	12	− 26.8	+ 2	4	11	− 30.8	− 6
162	Curtiss	23 18 18	0.302	5	11	− 35.2	− 6	4	11	− 32.4	− 6
163	Curtiss	23 18 28	0.310	6	11	− 26.4	+ 4	3	9	− 29.9	− 3
169	Curtiss	Nov. 1 17 06	0.650	4	7	− 35.5	+ 5	3	5	− 34.4	+ 1
170	Curtiss	1 17 27	0.665	6	8	− 41.3	− 1	6	11	− 32.3	+ 3
171	Curtiss	1 18 01	0.689	2	3	− 36.8	+ 4	4	6	− 39.7	− 4
180	Schlesinger	2 17 12	1.655	3	2	+ 27.3	+ 8	2	2	+ 31.3	+ 6
181	Schlesinger	2 17 46	1.678	3	3	+ 16.2	− 5	2	3	+ 22.1	− 6
182	Schlesinger	2 17 56	1.685	2	2	+ 17.0	− 5	1	1	+ 28.8	+ 1
183	Schlesinger	2 18 38	1.714	3	2	+ 25.6	+ 2	2	3	+ 38.0	+ 8
184	Schlesinger	2 18 58	1.728	3	2	+ 17.6	− 7	2	5	+ 29.9	− 1
192	Curtiss	5 17 10	1.786	4	8	+ 20.7	− 7	3	7	+ 34.4	0
193	Curtiss	5 17 28	1.797	2	2	+ 39.2	+ 10	2	4	+ 35.4	0
194	Curtiss	5 18 28	1.841	2	4	+ 36.6	+ 5	2	3	+ 47.8	+ 10
202	Schlesinger	6 16 30	2.758	4	7	+ 12.5	+ 10	3	5	+ 12.8	+ 7
205	Schlesinger	6 17 56	2.818	3	7	− 13.8	− 11	3	6	+ 1.9	+ 2
217	Curtiss	7 17 30	0.932	3	6	− 43.3	− 8	2	2	− 28.4	+ 1
218	Curtiss	7 18 02	0.954	4	7	− 36.1	− 2	1	1	− 34.6	− 6
219	Curtiss	7 18 12	0.961	2	5	− 40.2	− 7	2	2	− 33.9	− 6
223	Schlesinger	8 16 58	1.910	2	2	+ 30.8	− 4	2	2	+ 40.9	− 1
224	Schlesinger	8 17 13	1.921	2	1	+ 44.3	+ 9	2	2	+ 47.4	+ 5
225	Schlesinger	8 17 19	1.925	2	4	+ 45.9	+ 10	3	5	+ 51.9	+ 9
226	Schlesinger	8 17 43	1.941					3	2	+ 43.0	0
239	Curtiss	23 17 30	2.597	2	4	+ 12.9	− 3	2	4	+ 14.2	− 7
240	Curtiss	23 17 59	2.617	3	6	+ 7.6	− 7	5	9	+ 10.7	− 9
246	Schlesinger	1907 Jan. 5 13 28	2.421	4	4	+ 29.3	0	6	13	+ 29.8	− 6
247	Baker	5 13 40	2.427	6	10	+ 28.6	− 1	5	8	+ 27.2	− 8
248	Schlesinger	5 14 12	2.449	3	4	+ 29.6	+ 1	5	8	+ 33.6	0
249	Baker	5 14 30	2.461	2	2	+ 32.8	+ 6	4	4	+ 43.7	+ 11
250	Schlesinger	5 14 34	2.464	2	2	+ 30.4	+ 4	3	4	+ 35.2	+ 3

TABLE OF OBSERVATIONS.—*Continued.*

No. of Plate.	Plate Secured by	Date, G. M. T.	Phase.	Schlesinger. No. of Lines.	Schlesinger. Wt.	Schlesinger. Vel.	Schlesinger. Resid.	Curtiss. No. of Lines.	Curtiss. Wt.	Curtiss. Vel.	Curtiss. Resid.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
		1907 d h m	days			km.	km.			km.	km.
251	Baker	Jan. 5 14 53	2.477	3	3	+33.8	+8	3	4	+28.3	−3
252	Schlesinger	5 14 57	2.480	3	2	+30.0	+5	3	4	+30.2	−1
255	Schlesinger	9 13 16	0.675	5	7	−34.9	+5	2	2	−32.4	+3
256	Baker	9 13 42	0.689	3	5	−44.6	−4	6	11	−33.1	+2
261	Schlesinger	21 13 06	1.200					3	4	−14.9	−1
263	Curtiss	22 13 48	2.228	3	2	+50.1	+12	6	9	+48.5	+3
264	Curtiss	22 14 14	2.246	5	7	+39.1	+2	8	12	+45.6	+1
265	Curtiss	22 14 36	2.261	3	5	+29.2	−8	4	8	+42.6	−2
266	Curtiss	22 15 08	2.283	3	3	+46.8	+10	5	9	+54.3	+11
267	Curtiss	22 16 16	2.332	3	7	+39.0	+5	4	9	+45.4	+4
270	Baker	23 13 35	0.351	2	4	−40.7	−8	4	8	−32.3	−3
271	Schlesinger	23 14 14	0.378	6	10	−34.5	−1	6	14	−27.2	+3
272	Baker	23 14 48	0.402	5	4	−33.2	+2	5	9	−30.1	+1
273	Schlesinger	23 15 07	0.415	6	11	−30.1	+5	6	9	−25.5	+6
278	Curtiss	26 12 50	0.454	4	5	−40.3	−3	4	6	−24.4	+9
279	Curtiss	26 13 43	0.491	5	6	−34.0	+4	4	4	−28.9	+5
280	Curtiss	26 14 18	0.515	5	5	−34.9	+4	3	7	−28.3	+6
281	Curtiss	26 14 36	0.527	2	3	−34.8	+4	2	5	−35.5	−1
286	Schlesinger	27 13 08	1.466	2	2	+4.4	0	4	3	+14.7	+6
287	Schlesinger	27 13 42	1.490	4	2	+14.2	+8	8	11	+9.0	−1
288	Schlesinger	27 14 36	1.527	5	7	+17.8	+8	5	7	+13.1	0
291	Baker	28 12 58	2.459					1	1	+35.3	+2
292	Curtiss	28 13 23	2.476					3	2	+33.3	+2
293	Baker	28 14 04	2.503	1	4	+24.9	+1	5	6	+40.1	+10
297	Curtiss	30 12 46	1.583	3	3	+5.7	−8	7	8	+23.1	+4
298	Curtiss	30 13 26	1.610	4	6	+12.8	−4	6	11	+26.3	+5
299	Curtiss	30 14 08	1.640	8	14	+19.4	+1	9	18	+25.8	+1
300	Curtiss	30 15 14	1.686	8	13	+27.8	+6	6	11	+26.0	−2
303	Schlesinger	Feb. 2 14 12	1.776	3	5	+22.7	−5	4	7	+31.4	−3
305	Curtiss	5 14 33	1.923	2	2	+32.2	−3	2	3	+37.4	−5
306	Baker	5 15 18	1.954	4	6	+25.0	−12	5	8	+35.5	−9
315	Curtiss	11 15 55	2.245					1	2	+39.0	−6
316	Curtiss	11 15 32	2.270	2	3	+42.7	+6	2	4	+56.7	+13
322	Curtiss	27 14 41	0.990	8	16	−38.9	−7	7	13	−29.4	−3
328	Schlesinger	Mar. 6 12 42	2.172	2	3	+48.7	+10	2	2	+55.9	+9
329	Schlesinger	6 13 14	2.194	4	5	+39.6	+1	3	6	+48.8	+3
338	Schlesinger	16 13 04	0.719	3	4	−40.5	0	2	2	−31.8	+3
339	Schlesinger	16 13 39	0.743	6	12	−48.4	−8	8	12	−38.9	−4
347	Baker	20 12 38	1.832	5	12	+21.3	−10	4	8	+33.3	−5
348	Schlesinger	20 13 19	1.861	8	15	+30.9	−2	9	21	+37.1	−2
356	Curtiss	23 13 59	2.021	3	6	+42.6	+4	6	8	+39.5	−6
363	Curtiss	25 12 56	1.110	6	12	−18.4	+6	7	12	−13.4	+6
364	Curtiss	25 13 50	1.147	2	2	−26.6	−5	3	3	−20.5	−3

selected for measurement by Schlesinger with the adopted wave-lengths and normal screw-readings are as follows:

Ca, λ	3933.825,	82.7284*R*	Si, λ	4131.040,	96.5876*R*
He,	4026.370,	89.6284	H,	4340.634,	108.3744
H,	4102.000,	94.7378	He,	4471.676,	114.5867
Si,	4128.204,	96.4096	Mg,	4481.397,	115.0177

These lines were measured and reduced according to the method described on pages 8 and following of this volume. At the time of measurement a weight was assigned to each line and the sum of these weights for each plate appears in column 6 of the table, the mean velocities in column 7 being taken in accordance with the separate line weights. These velocities were then examined to see whether an improvement could not be effected by slight changes in the wave-lengths. It was found that such changes would all be under 0.04Å and it was decided not to use them. Had these empirical corrections been applied a better accordance would have resulted among the plates, but the effect on the velocity curve would have been very small.

Dr. Curtiss' procedure is described by him as follows: "From seven plates of Rigel reduced to a standard dispersion I prepared, by taking simple means, a standard unknown velocity table. With this table I reduced all the Algol plates, thus obtaining velocities relative to my arbitrary unknown standard. I then readjusted the lines in my standard table by means of the residuals for each line obtained from the measures of thirty-two of the Algol plates themselves, and with this new homogeneous adjusted table of unknown velocity, I readjusted all the Algol plates. The resulting velocities were corrected for the earth's motions and used to determine the oscillation curve of Algol. From this I obtained the preliminary elements of the orbit including the relative velocity of the center of mass of the system referred to my standard table of unknown velocity. In order then to determine the absolute velocity, I reduced my final adjusted table of unknown velocity by the usual wave-length method." The wave-lengths upon which Dr. Curtiss' velocities rest are therefore as given below:

Ca, 3933.701	Fe, 4233.375
He, 4026.406	H, 4340.784
H, 4101.869	He, 4471.804
Si, 4128.254	Mg, 4481.374
Si, 4130.930	

The two sets of observations were now plotted and it at once became evident that there is a large systematic difference between the 1906 plates and those of 1907. By two successive approximations it was found from Schlesinger's measures that a correction of about − 10.5 km. would bring the early plates into best accord with the later; and from Curtiss' measures similarly treated a correction of about — 9.5 resulted. As this quantity is uncertain by one or two kilometers, − 10 km. was applied to both series and this correction is included in the velocities printed in the tables, for the plates up to and including number 240. It is not possible to state definitely at the present time what is the cause of this difference. As the camera lens was altered in the interval it is natural to lay the change to this circumstance. This explanation can be tested when spectrograms of other stars taken with the old lens have been measured. It is not impossible that the difference is due to a real change in the star such as was suspected by Belopolsky,* but the interval between our sets would seem too short to warrant much confidence in this explanation. Nevertheless, it is noteworthy that the absolute velocity of the system as determined by our later plates is in accord with Belopolsky's first and third series, while our earlier plates agree with his second and fourth series:

Vogel,	1889–90,	− 3 km.	Schlesinger,	1906,	+ 9 km.
Belopolsky,	1897,	− 2	Curtiss,	1906,	+ 13
"	1902–3,	+ 12	Schlesinger,	1907,	− 2
"	1903–4,	− 3	Curtiss,	1907,	+ 3
"	1904–5,	+ 12			

As our 1906 plates are well distributed in phase and as they supply only about one-third of the total material here discussed, the effect upon the orbit of an uncertainty of as much as 3 km. in this correction will not be large.†

The observations were now treated according to the method of Lehmann-Filhés and preliminary elements derived.‡ These elements are in fair agreement except for the value for ϖ, Doctor Curtiss having derived $-3°$, and the writer $+44°$. Large uncertainties in this element and in the time of periastron passage must be expected when the eccentricity is small. As a point comes up later in the discussion that makes it desirable to avoid any arbitrary step, it was decided to deduce the definitive elements by means of least-square solutions. For this purpose the observations were grouped into thirteen normal places, as shown on page 30, and an ephemeris was computed on the following elements, which are round means between the two sets of preliminary elements:

$$K = 40.9 \text{ km.}, \quad e = 0.05 \quad T = 2.25 \text{ days}, \quad \varpi = +21°,$$
$$\gamma(\text{Schl.}) = -1.91 \text{ km.}, \quad \gamma(\text{Curt.}) = +4.09 \text{ km.}$$

* Mitteilungen der Sternwarte zu Pulkowa, I, 101.

† This statement is based upon the following test: A set of elements had been deduced by Doctor Curtiss from his observations in which this correction was assumed to be -7 km.:

$$K = 40.9 \text{ km.}, \ e = 0.06, \qquad \varpi = -3° \qquad \gamma = +3.8 \text{ km.}$$

Comparing these with the definitive elements from the same observations, in which -10 km. was adopted as the correction, we see that the agreement is close except in the case of ϖ. This element was, however, not included in the least-square solution.

‡ Up to this point each observer had reduced his own observations and it was our intention to follow this plan to the end. Owing however, to Doctor Curtiss' resignation in September, 1907, and his assumption of other duties at the University of Michigan, he was not enabled to carry the work beyond the determination of preliminary elements. The responsibility for the discussion which follows rests with the writer alone.

No.	Limits of Phase.		Wt.	Schlesinger.			Curtiss.		
				Mean Phase.	Mean Velocity.	Resid.	Mean Phase.	Mean Velocity.	Resid.
	days	days		days	km.	km.	days	km.	km.
1	0.28 to	0.31	1	0.299	− 29.4	+ 0.17	0.299	− 31.1	− 5.09
2	0.35	0.42	1	0.392	− 33.5	+ 0.85	0.386	− 28.5	+ 1.88
3	0.45	0.53	1	0.493	− 36.0	+ 2.20	0.497	− 29.0	+ 4.79
4	0.65	0.75	3	0.693	− 41.3	− 0.88	0.693	− 35.2	+ 0.18
5	0.93	0.99	1	0.968	− 38.2	− 5.61	0.976	− 30.8	− 3.26
6	1.11	1.23	2	1.152	− 18.4	+ 2.83	1.160	− 17.6	− 1.09
7	1.46	1.61	3	1.547	+ 13.5	+ 2.87	1.545	+ 18.4	+ 2.84
8	1.64	1.79	3	1.699	+ 22.3	− 0.41	1.699	+ 28.6	+ 0.24
9	1.79	1.86	3	1.840	+ 29.6	− 2.09	1.841	+ 37.4	− 0.87
10	1.89	2.03	3	1.950	+ 33.8	− 2.78	1.950	+ 40.6	− 3.23
11	2.17	2.33	3	2.256	+ 40.2	+ 3.00	2.257	+ 48.0	+ 3.42
12	2.42	2.50	3	2.452	+ 29.3	+ 1.54	2.453	+ 32.7	− 0.98
13	2.60	2.89	2	2.753	+ 0.6	− 3.48	2.735	+ 7.2	− 0.40

Putting $\delta T' = \dfrac{K.\mu}{(1-e^2)^{\frac{3}{2}}}, \quad \delta e' = \delta e \dfrac{K}{1-e^2}, \quad \delta\varpi' = K.\delta\varpi$

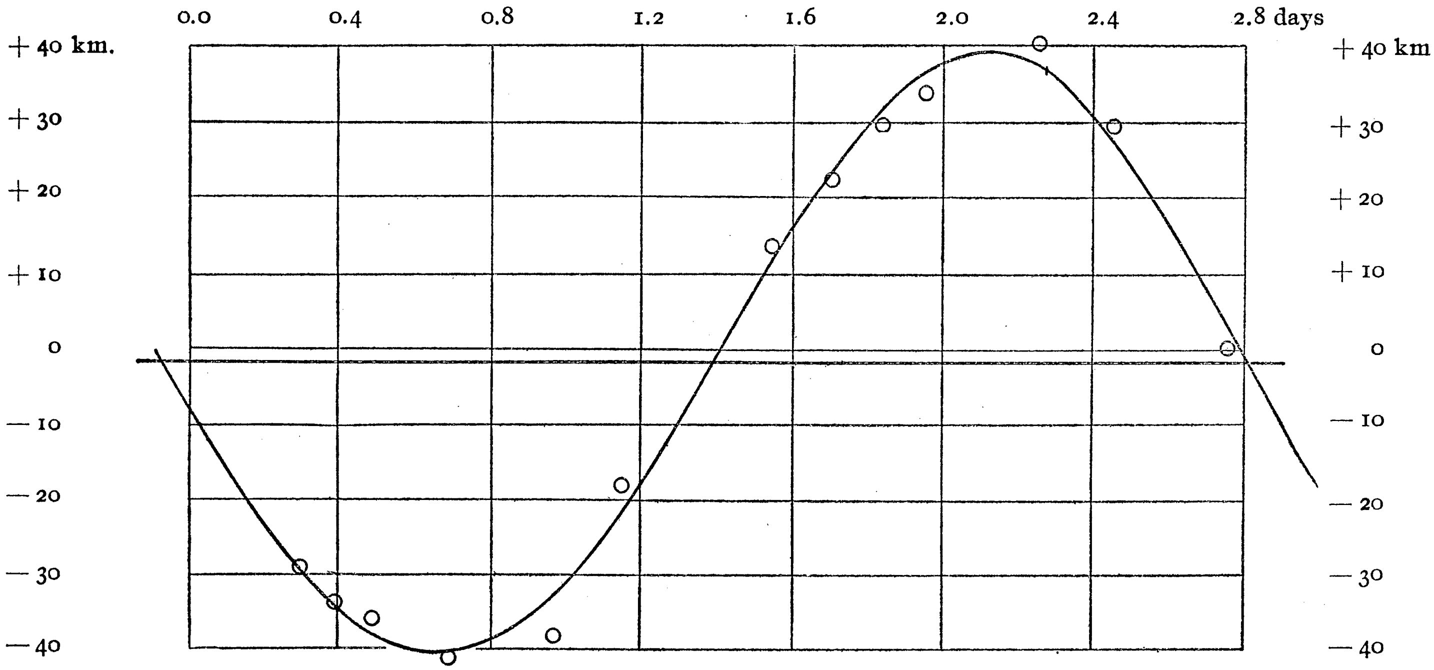
Putting $\delta T' = T\delta$ we obtain these normal equations:

$\delta\gamma$	δK	$\delta T'$	$\delta e'$	$\delta\varpi'$		Schlesinger.	Curtiss.
29.000	+ 6.709	− 3.738	+ 2.179	+ 3.533	=	− 9.00	− 21.20
	+ 16.784	− 1.717	− 1.584	+ 1.590	=	− 17.31	− 0.87
		+ 13.215	+ 3.226	− 12.915	=	+ 21.62	+ 19.49
			+ 12.572	− 3.175	=	− 2.25	+ 7.21
				+ 12.651	=	——	——

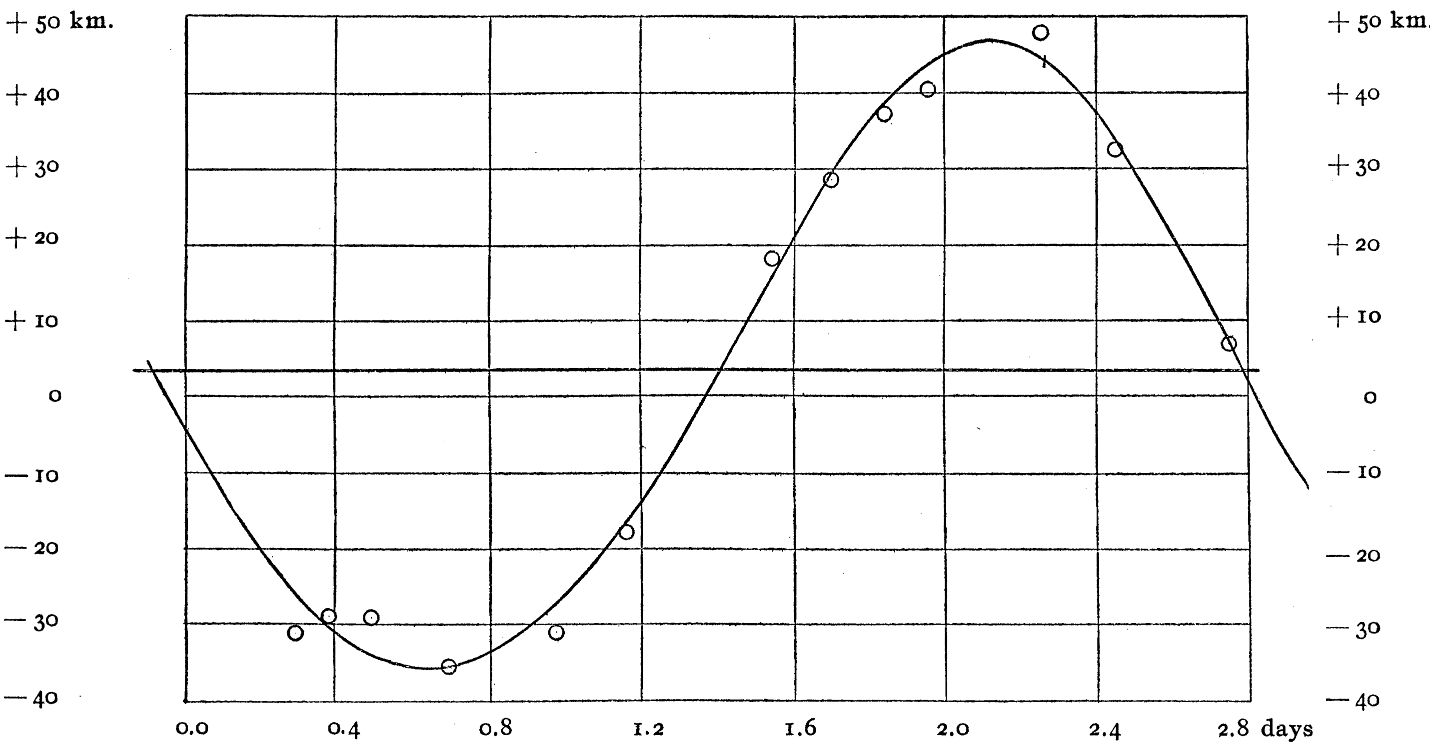
If two columns in a set of normal equations differ by only a constant factor it is impossible to determine separately the corresponding unknowns. This is practically the case here between $\delta T'$ and $\delta\varpi'$. I have therefore assumed the latter to be zero. Solving the remaining equations we obtain these definitive elements and their probable errors:

	Schlesinger.	Curtiss.
e	0.031 ± 0.022	0.060 ± 0.021
T (after light minimum).	2.270 ± 0.0096 days	2.264 ± 0.0094 days.
ϖ	+ 21° ± 24°	+ 21° ± 24°
K	39.9 ± 0.77 km.	41.3 ± 0.75 km.
A	41.0 km.	43.6 km.
B	38.8 km.	39.0 km.
γ	− 1.70 ± 0.60 km.	+ 3.40 ± 0.58 km.
$a \sin i$	1,570,000 km.	1,630,000 km.

For the probable error of ϖ I have assumed half the difference between the two graphical determinations. It will of course be understood that a correspondingly large uncertainty attaches to T, the probable error indicated above only applying when ϖ is assumed. The residuals that result from these solutions are shown in the table of normal places and the separate residuals for each plate are given in the table of observations. The latter were scaled from plots corresponding to those here reproduced.



Velocity Curve from Schlesinger's Observations.



Velocity Curve from Curtiss' Observations.

If we form the weighted mean of the residuals for all the plates taken with the old camera lens we find — 1.3 km. from Doctor Curtiss' observations and — 0.8 km. from the writer's. These indicate that it would have been better to have adopted a somewhat smaller difference between the new and old lens plates. But as already pointed out the effect upon the orbit of a change in this correction is not large.

The two determinations of the velocity of the system (γ) differ by 5.1 km. Part of this is due to differences of assumed wave-lengths combined with inequalities in the mean weights assigned to the various lines. But doubtless the greater part of the difference is due to personalities in the measurements such as are easily possible in the case of the ill-defined lines that this spectrum presents. If we deduce the velocity of the system separately from each line employed, adopting solar and laboratory wave-lengths, we find that Doctor Curtiss' observations have a range of 16 km., and those of the writer only 6 km. On the other hand, basing his velocities on the places of these same lines as they appear in Rigel, Doctor Curtiss also obtains a range of 6 km. The two observers adopted somewhat different procedures for measuring, for not only did they use different parts of the micrometer screw but one of them (Doctor Curtiss) reversed the plate in the usual way in order to obtain the "red to left" measures while the other reversed the measuring machine instead.

On the plausible hypothesis that the light changes in an Algol-type variable are caused by an eclipse, the orbit of one of these stars should fulfill a certain condition. When the light is a minimum the angle between the line joining the two stars and that directed toward us, should also be a minimum; or in other words, we should have $u = 90°$ when $t = 0$. Applying this to our computed elements we find the surprising result that from Doctor Curtiss' orbit $u = 103°.5$, and from the writer's, $u = 99°.2$. As the difference in phase implied in even the smaller of these corresponds to more than 6 km. in some parts of the orbit it does not seem at all likely that this result can be due to errors of observation. A different value of ϖ would somewhat modify matters, but no admissible assumption can be made in this respect that would bring the discrepancy within the region of probable inaccuracy. Furthermore an independent confirmation is furnished by the elements published by Belopolsky in the *Mitteilungen der Pulkowa Sternwarte*, Band 1, page 103. Applying the same test to these we find that $u = 96°.2$ at light minimum, corresponding to a discrepancy of 4.5 km. in some parts of the orbit. These results may therefore be said to indicate that the light minimum lags from one and a half to two hours behind the time demanded by the velocity determinations.

It was at first our intention to supplement these observations by additional ones during the present season. Instead, however, an entirely new set has been secured from which it should be possible to deduce an independent set of elements and thus permit of a conclusive test of the reality of the discrepancy just discussed. The writer hopes to publish these observations during the first half of the current year.

24 March, 1908.