1910MNRAS..70..527F

The following are the papers which would in the ordinary course have been read at the meeting on May 13:—

On the smallest visible phase of the Moon.

By J. K. Fotheringham, M.A., D.Litt.

It is well known that many calendars, both in ancient and modern times, have reckoned the month from the first appearance of the Moon, and frequent complaints have been made by students of chronology of the absence of any definite information upon the astronomical conditions which govern the visibility of the Moon to the naked eye.* In an article which appeared in the *Journal of Philology* in 1903 † I suggested that "in order to calculate the true date of the phasis we ought to have a table of the requisite depression of the Sun below the horizon at moonset, or of the requisite altitude of the Moon above the horizon at sunset for different angular distances of the Moon from the Sun." But I was not then aware that the observations on which such a table might be constructed were already provided in August Mommsen's *Chronologie* (1883), pp. 69–80. Mommsen here records forty-eight positive observations of first sight of new Moon made by Julius Schmidt at Athens, one such observation (No. 13 below) made by Julius Schmidt at Corinth, and one such (No. 75) made by Mommsen himself at Athens, eighteen negative observations of new Moon (*i.e.* occasions when the Moon might have been expected to be visible, but was not) made by Julius Schmidt at Athens, one such (No. 19) made by Julius Schmidt at Troy, and one such (No. 74) made by Mommsen at Athens, two positive observations of last sight of old Moon (Nos. 36, 47) made by Julius Schmidt at Athens, one such (No. 43) made by Friedrich Schmidt at Athens, two such (Nos. 72, 76) made by Mommsen at Athens, and one negative observation of old Moon (No. 73) made by Mommsen at Athens. The total number of observations is 76. It is probable that all Julius Schmidt's negative observations were made on fine evenings, as they profess to have been made on evenings when the Moon might have been expected to be visible. Mommsen's negative morning observation (No. 73) might, according to his own suggestion, be due to an obscuration of the Moon by Hymettus, but Mommsen himself rejects this suggestion. The observation was made on a walk which extended till the disappearance of the stars. This raises a doubt whether, if the walk had been prolonged a few minutes longer, it might not have had a different effect. Observation No. 74 was made on a cloudy evening, but Mommsen notes that there was a gap in the clouds through which the Moon, if visible, might have been seen. This

* I may instance my paper, "The Date of the Crucifixion," in *The Journal of Philology*, xxix. (1903), pp. 104, 105; Ginzel, *Handbuch der mathematischen und technischen Chronologie*, i. (1906), p. 93; Weissbach, *Zum babylonischen Kalender*, Hilprecht Anniversary Volume (1909), p. 286.

† *Ubi supra.*

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System

1910MNRAS..70..527F

measure of uncertainty attaching to Nos. 73, 74 is the more tantalising, because these observations would otherwise have been among the most valuable of the whole series. For fuller information about observations 1–33, see an article by Schmidt in *Astronomische Nachrichten*, Band 71 (1868), pp. 201 ff.; and for fuller information concerning the whole series, see Mommsen's *Chronologie, ubi supra.*

The whole series of observations was sent by Mommsen to Bruhns, several of whose comments are published by Mommsen. Mommsen expresses himself disappointed that the study of these observations should not have led to the discovery of an exact method of determining the last visible phase of the old Moon and first visible phase of the new. Bruhns's failure to discover such a method was certainly not due to any defect in the observations, and I can only suppose that he did not perfectly understand the purpose of Mommsen's inquiry.

In the following table I give the civil date of each observation, the true altitude of the Moon at sunset for evening observations and at sunrise for morning observations, the true difference in azimuth of Sun and Moon for the same moment, and the result of the observation. The date and result are taken from Mommsen, while I have computed the altitude and azimuths. I have thought it unnecessary to complicate the calculation by introducing the lunar parallax, because the parallax in altitude is practically constant for any given altitude, while the parallax in azimuth is small, and small changes in azimuth produce no perceptible difference in the result.

Reference No.	Date.	Altitude.	Difference in Azimuth.	Result.
		°	°	
1	1859 July 1	12·7	10·0	Visible.
2	Oct. 27	6·1	20·5	Visible.
3	1860 Jan. 23	6·0	3·2	Not visible.
4	Feb. 23	20·2	2·4	Visible.
5	June 20	15·4	12·5	Visible.
6	1861 March 12	12·8	1·8	Visible.
7	Aug. 7	5·2	15·2	Not visible.
8	Aug. 8	11·2	27·1	Visible.
9	Sept. 7	12·9	36·4	Visible.
10	Oct. 5	5·3	19·4	Not visible.
11	Nov. 4	12·7	25·2	Visible.
12	Dec. 3	13·6	16·6	Visible.
13	1862 Jan. 1	12·3	7·1	Visible.
14	March 31	15·9	0·4	Visible.
15	Apr. 29	8·6	0·1	Not visible.
16	July 28	8·6	20·5	Not visible.
17	1864 Jan. 10	18·4	6·5	Visible.
18	March 9	21·1	2·3	Visible.

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System

1910MNRAS..70..527F

Reference No.	Date.	Altitude.	Difference in Azimuth.	Result.
19	1864 May 6	7°9	4°6	Not visible.
20	June 6	18·5	20·0	Visible.
21	Aug. 4	8·7	22·1	Not visible.
22	Sept. 3	11·4	25·8	Visible.
23	Nov. 1	13·7	18·7	Visible.
24	1865 Jan. 28	17·8	3·4	Visible.
25	March 28	20·0	5·4	Visible.
26	Apr. 26	13·6	7·2	Visible.
27	June 24	9·2	16·0	Not visible.
28	July 24	9·7	21·3	Visible.
29	Oct. 21	12·8	17·3	Visible.
30	1866 Jan. 17	10·7	0·8	Not visible.
31	Apr. 16	18·0	9·2	Visible.
32	1867 Feb. 5	10·5	1·8	Not visible.
33	Nov. 27	13·7	9·6	Visible.
34	1868 June 22	19·7	22·8	Visible.
35	1869 May 12	9·1	9·4	Not visible.
36	1870 July 25 M.	32·8	23·3	Visible.
37	1871 Feb. 20	11·4	8·9	Visible.
38	Apr. 20	8·4	7·0	Not visible.
39	May 20	11·2	8·0	Not visible.
40	June 18	5·8	4·6	Not visible.
41	June 19	14·2	10·9	Visible.
42	Aug. 17	12·9	12·1	Visible.
43	Sept. 14 M.	8·8	2·8	Visible.
44	1872 June 7	15·3	9·0	Visible.
45	July 6	9·8	5·6	Not visible.
46	Sept. 4	13·1	14·3	Visible.
47	Sept. 30 M.	28·7	2·1	Visible.
48	Oct. 3	8·9	9·0	Not visible.
49	Oct. 4	13·6	20·3	Visible.
50	Dec. 31	12·2	15·3	Visible.
51	1873 Apr. 27	9·1	4·2	Not visible.
52	May 27	15·2	6·5	Visible.
53	Dec. 20	4·9	10·4	Not visible.
54	1874 Apr. 17	15·3	4·2	Visible.
55	1875 July 4	17·4	12·2	Visible.
56	1876 Feb. 26	16·1	5·5	Visible.
57	June 22	11·9	4·6	Not visible.
58	July 22	14·6	15·9	Visible.
59	1877 March 16	18·0	5·3	Visible.

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System

1910MNRAS..70..527F

Reference No.	Date.		Altitude.	Difference in Azimuth.	Result.
60	1877 June	12	14°·7	6°·4	Visible.
61	Nov.	7	11·1	27·2	Visible.
62	Dec.	6	10·5	18·1	Visible.
63	1878 Jan.	5	17·8	14·9	Visible.
64	June	2	18·6	8·7	Visible.
65	July	1	12·2	10·4	Visible.
66	July	31	12·2	21·7	Visible.
67	Oct.	27	7·9	23·0	Visible.
68	Nov.	26	15·8	26·2	Visible.
69	1879 May	23	24·9	12·3	Visible.
70	June	22	23·5	24·9	Visible.
71	July	22	20·0	37·1	Visible.
72	Dec.	11 M.	21·6	25·9	Visible.
73	Dec.	12 M.	11·2	13·3	Not visible.
74	Dec.	14	10·8	12·1	Not visible.
75	Dec.	15	21·1	21·6	Visible.
76	1880 Jan.	10 M.	13·4	19·4	Visible.

These observations give a very clear dividing line between the conditions of positive and negative observations.

The following table satisfies all the observations except Nos. 2 and 43.

True Difference in Azimuth at Sunset (or Sunrise).	Minimum True Altitude at Sunset (or Sunrise) for Moon to be visible same evening (or morning).
0°	12°·0
5	11·9
10	11·4
15	11·0
20	10·0
23	7·7

The two discordant observations are both positive, where a negative observation might have been expected. One (No. 43) is a morning observation, and may be evidence that the same rule does not apply to morning and evening observations. As Mommsen only records six morning observations, the evidence for morning conditions is much weaker than for evening conditions. That there should be only one discordant evening observation among so many is remarkable, and seems to show that, given a clear sky, *the problem is almost purely astronomical, and not atmospheric.* As the solution given above takes into account nothing except the relative positions of the Sun, Moon, and horizon, it is independent of differences in latitude, and *ought therefore to be applicable to any place,* subject to a slight modification for permanent differences in the clearness of the air. The Moon should,

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System

1910MNRAS..70..527F

unless obscured by clouds, appear on the first evening when her altitude at sunset has reached the height required by the difference in azimuth.

The rapidity with which the Minimum Altitude diminishes when the difference in azimuth exceeds 20° is worthy of attention. The observations under discussion include five (Nos. 2, 16, 21, 28, 67) where a difference of more than 20° in azimuth is combined with an altitude of less than 10°. In three of these five instances the Moon was visible. On the other hand, No. 43 is the only instance of the Moon being visible where the difference in azimuth is less than 20° and the altitude is less than 10°. A rough approximation to the conditions of visibility of the Moon found above is given by the formula

$$\text{Minimum Altitude} = 12^\circ{\cdot}0 - 0^\circ{\cdot}008\ Z^2,$$

where Z = the number of degrees in difference of azimuth; but the summary table given above is more accurate.

The only previous attempt to give an exact rule for determining the smallest visible phase of the Moon, so far as I know, is that of the Jewish philosopher Maimonides in his treatise on the Sanctification of the New Moon, best studied in von Littrow's article in *Sitzungsberichte der Wiener Akademie*, *Math.-Naturw. Classe*, lxvi. (1872), pp. 459–480. Maimonides makes the result depend on two variables, the true elongation of the Moon and the apparent "angle of vision," both computed for a time which is on the average twenty minutes after sunset. If "angle of vision" means difference of Sun and Moon in zenith-distance, his rule has very nearly the same effect as mine, but gives slightly lower minimum altitudes, from which it may follow that observations are slightly easier at Jerusalem than at Athens. I do not, however, sufficiently understand his arithmetical method to be able to state with certainty whether this is what he intends by "angle of vision."

It is to be hoped that this solution will prove valuable to chronologists and to astronomers who may be consulted by chronologists. Dates dependent on the first appearance of the Moon are found in ancient Egypt, Palestine, and Babylonia, and in the Mohammedan world down to the present day. The earliest Greek calendars also began the month with the appearance of the Moon, but we have few, if any, definite dates belonging to these calendars. All these countries have a very clear atmosphere, though they may differ slightly one from another. Hitherto it has been the practice to assume that the Moon becomes visible on the first evening when she is more than 30 hours old at sunset. Some important dates in Egyptian and early Christian history have been made to depend on this, and I shall be glad if this paper makes it possible to fix them on a surer foundation.

I have to thank Professor Turner for some labour-saving suggestions in the preparation of this paper.

24 *The Avenue, Muswell Hill,*
London N.: 1910 *May* 9.

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System

1910MNRAS..70..527F

On the Mass-Determination of Parallaxes.* By Karl Pearson, F.R.S., and Julia Bell, M.A. (Plates 16, 17.)

(1) In the course of some recent work we were very desirous of obtaining the value of the parallax constant, V_D, the coefficient of variation of stellar distance. The constant can be found indirectly in a number of ways, but its direct determination depends on the knowledge of a very large number of parallaxes. In endeavouring to determine it directly we have examined the memoir of Professor Kapteyn and Dr. de Sitter, with its details of the parallaxes of 3650 stars.† We found for this material that the values of V_D and V_π ‡ came out very differently from those obtained from other series of parallaxes, from values found indirectly, and from the theoretical values which result from various hypotheses as to the distribution of the stars. They are in fact of a wholly different order. This led us to table various points as to the frequency distributions of the Kapteyn parallaxes, which it is possible may be of some general interest.

It will be remembered that Kapteyn tables two parallaxes,—Π_0, the final reduction value without magnitude corrections, and Π, the final value including corrections for magnitude errors. We have dealt with both these quantities.

(2) Before discussing the frequency distributions, we may note that the same stars occur in some 406 cases on two plates. Table I. is a correlation table for the parallax of the same star as determined from different plates. Taking A and B to represent the two measurements we have, all measurements being in seconds of arc, §

$$m_A = -\cdot 00207, \qquad \sigma_A = \cdot 05595,$$
$$m_B = -\cdot 00071, \qquad \sigma_B = \cdot 05105,$$
$$r_{AB} = \cdot 301 \pm \cdot 0304.$$

The table, and these constants for it, show that the determination of a parallax from one plate gives very little idea of what its value will be found from a second plate. We have on the average—

$$\frac{\text{variability of second measure of parallax}}{\text{variability of all parallaxes}} = \sqrt{1 - r_{AB}^2} = \cdot 95.$$

To determine how far this was due to the measurer or the plate, we correlated the cases, 28 in number, in which duplicate parallaxes

* The word "mass" is used here in the technical statistical sense of the *Theorie der Massenerscheinungen* of Professor W. Lexis. Kapteyn himself uses "wholesale."

† "The Parallaxes of 3650 Stars of Different Galactic Latitudes, derived from Photographic Plates prepared by Professor Anders Donner," *Publications of the Astronomical Laboratory at Groningen*, No. 20, Groningen, 1908.

‡ See *Royal Soc. Proc.*, June 1910, for the application and significance of these constants of parallax.

§ All duplicate measurements on the same plate excluded.

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System