# ON NEW MEMBERS OF THE SYSTEM OF THE STARS $\beta$, $\gamma$, $\delta$, $\epsilon$, $\zeta$, *URSAE MAJORIS*

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It is known that five stars of the constellation *Ursa Major*, $\beta$, $\gamma$, $\delta$, $\epsilon$, $\zeta$, have approximately equal proper motions, and therefore probably are moving in parallel lines in space, with equal velocities. From this point of view, H. Ludendorff has subjected the five stars to a thorough investigation, published in the *Astronomische Nachrichten*, **180**, 265, 1909. He made, in particular, new determinations of the radial velocities of the spectroscopic binaries $\beta$ and $\zeta$ *Ursae Majoris*, thereby making possible a more accurate knowledge of the system. I am informed, in a private letter of the author, that a search was not made for further stars belonging to the group, because Höffler states[2] that he had tested all the stars of the Auwers-Bradley catalogue in a large region, with entirely negative results.

Several stars have, however, incidentally come to my attention which may be suspected of belonging to the system: the first of these are *37 Ursae Majoris* and *a Coronae Borealis*, of which it may be stated as very probable that they belong to the system. Since the angular distance between these two stars is 59°.7, the absolute distance must be of the same order of magnitude as their distance from our sun. It was, therefore, desirable to extend the search to stars still farther removed from these, and to such as had a different value of the proper motion (different parallax). In this search, I first employed Kapteyn's discussion of 2618 Bradley stars (*Publications of the Astronomical Laboratory of Groningen*, No. 9), comparing upon a celestial globe the direction of the proper motion each star would have if it belonged to the system with that observed. In order to base my conclusions on the most accurate proper motions, I first sought out among the stars which were then in the best agreement those which occur in the New Fundamental Catalogue of the Berlin

[1] Translated from the *Nachrichten der K. Gesellschaft der Wissenschaften zu Göttingen*. Mathematisch-physikalische Klasse. 1909.

[2] *Astronomische Nachrichten*, **144**, 369, 1897.

*Astronomisches Jahrbuch.* The eleven stars thus selected make up the first part of Table I. It is to be noted that *Sirius* is found among these stars. The accuracy with which the direction of its large proper motion is known, yields a good geometrical position for the point of radiation. I first reckoned the radiant point as the intersection of the proper motions of *Sirius* and of a fictitious star located at the center of gravity of the five stars named in the title of this paper ($\alpha = 183^\circ.22$, $\delta = +56^\circ.14$, for 1900), the proper motion of which was directed toward Ludendorff's apex ($\alpha = 303^\circ.2$, $\delta = -36^\circ.6$, for 1900). The point was determined, in the second place, on that great circle defined by the proper motion of *Sirius*, which yielded the smallest sum of the squares of the geometrical deviations of the remaining proper motions (Table I, col. 8). In this computation the most doubtful star, *β Eridani*, was omitted. These two determinations, dependent upon the proper motion of *Sirius*, yield values agreeing within $\frac{1}{10}$ of a degree, namely,

$$\alpha = 127^\circ.8,\ \delta = +40^\circ.2, \text{ for } 1900.$$

These co-ordinates are made the basis of the computed values in the table. Ludendorff found $\alpha = 123^\circ.2 \pm 3^\circ.0$, $\delta = +36^\circ.6 \pm 3^\circ.5$ (mean error).

The difference between the observed and computed direction of the proper motion is given in col. 7 of Table I, and in col. 8 the sine of this difference is multiplied by the annual proper motion. These last values, therefore, yield the smallest geometrical variation of the annual proper motions which are competent to bring about an agreement between observation and computation.

In connection with the angular distances of the stars from the assumed radiant point (Table I, col. 9), the radial velocities found for the four stars: *α Canis Majoris*, *β* and *ζ Ursae Majoris*, and *α Coronae*, yield, by least squares, a velocity of 18.4 km per sec. relative to the sun for the system. The radial velocity computed from this for each star is found in col. 10. Finally, the absolute parallaxes in col. 12 were computed with the assistance of the amounts of the annual proper motions. The absolute point of convergence of the system lies near $\alpha = 285^\circ$, $\delta = -2^\circ$, or, in galactic co-ordinates, $l = 0^\circ$, $b = -5^\circ$, and the absolute velocity amounts to 28.8 km per sec.

TABLE I

| | R. A. 1900 | Decl. 1900 | Annual Proper Motion $\mu$ | Direction of Proper Motion | | O.−C. | $\mu$ sin (O.−C.) | Distance from Star to Convergence Point RS | Radial Velocity | | Computed Parallax $\pi$ | Angle Pole Radiant-Star R | Rectangular Co-ordinates (Unit=Cosec 1″ Astron. Units) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | O. | C. | | | | C. km | O. km. | | | $x$ | $y$ | $z$ |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| | ° | ° | ″ | ° | ° | ° | ″ | ° | | | ″ | ° | | | |
| *β Eridani* | 75.73 | − 5.22 | 0.1188 | 228.1 | 221.3 | +6.8 | +.0141 | 65.9 | − 7.5 | | 0.034 | 239.4 | −13.8 | −23.3 | 12.1 |
| *β Aurigae* | 88.05 | +44.94 | .0455 | 260.1 | 265.2 | −5.1 | −.0040 | 29.4 | −16.0 | (−21) | .030 | 292.6 | 6.2 | −15.0 | 28.8 |
| *α Canis maj* | 100.19 | −16.58 | 1.3235 | 203.6 | 203.6 | .0 | .0000 | 62.3 | − 8.5 | − 7.4 | .387 | 210.1 | − 2.0 | − 1.1 | 1.2 |
| *37 Ursae maj* | 157.18 | +57.60 | .0756 | 61.8 | 60.0 | +1.7 | +.0023 | 25.6 | −16.6 | | .045 | 37.4 | 7.6 | 5.8 | 19.9 |
| *β Ursae maj* | 163.95 | +56.92 | .0871 | 72.3 | 70.0 | +2.3 | +.0035 | 28.7 | −16.1 | −16.8 | .047 | 42.2 | 7.6 | 6.9 | 18.7 |
| *δ Leonis* | 167.20 | +21.07 | .2017 | 132.5 | 128.8 | +3.6 | +.0032 | 38.5 | −14.4 | | .084 | 107.9 | − 2.3 | 7.1 | 9.3 |
| *γ Ursae maj* | 177.14 | +54.25 | .0945 | 88.5 | 87.5 | +1.1 | +.0018 | 35.5 | −15.0 | | .042 | 49.8 | 8.9 | 10.5 | 19.3 |
| *δ Ursae maj* | 182.62 | +57.59 | .1095 | 88.6 | 87.6 | +1.0 | +.0018 | 38.7 | −14.4 | | .045 | 44.5 | 9.8 | 9.7 | 17.2 |
| *ε Ursae maj* | 192.41 | +56.50 | .1140 | 95.6 | 96.9 | −1.3 | −.0026 | 44.0 | −13.2 | (−13) | .042 | 45.8 | 11.4 | 11.7 | 16.9 |
| *ζ Ursae maj* | 199.98 | +55.45 | .1250 | 101.8 | 103.4 | −1.7 | −.0036 | 48.4 | −12.2 | −12.6 | .043 | 46.2 | 11.9 | 12.5 | 15.3 |
| *α Coronae* | 232.61 | +27.05 | .1580 | 128.5 | 132.0 | −3.4 | −.0094 | 83.1 | − 2.2 | + .4 | .041 | 60.1 | 12.0 | 20.9 | 2.9 |
| *Groombr. 1930* | 191.08 | +60.87 | .075 | 88.5 | 91.4 | −2.9 | −.0037 | 43.0 | −13.4 | | .028 | 39.1 | 18.6 | 15.1 | 25.7 |
| *78 Ursae maj* | 194.11 | +56.91 | .115 | 99.5 | 98.0 | +1.5 | +.0031 | 44.9 | −13.0 | | .042 | 44.6 | 12.0 | 11.8 | 16.8 |

As to the probability that the separate stars belong to the system, let us first consider *Sirius*. Since we know for this star the amount and direction of its proper motion, as well as its radial velocity and parallax, the three components of its velocity relative to the sun may be computed. All three agree, within the precision of the observations, with the velocity found for the five stars in *Ursa*. Let it be remarked that the assumed radiant point ($\alpha = 127°.8$, $\delta = +40°.2$) lies 97° distant from the apex of the solar motion ($\alpha = 266°$, $\delta = +33°$), so that the members of the group here under consideration may be separated with comparative ease from the principal streams of the other stars. Only about one among a thousand arbitrarily selected stars would yield as good agreement as *Sirius* does. On the assumption, which is accordingly very plausible, that *Sirius* and the five stars of *Ursa Major* have equal absolute velocities, we may say that the radial velocities of these stars may be checked by measurement of the parallax of *Sirius*.

In the case of several of the other stars, we unfortunately still lack a knowledge of the radial velocities. Ludendorff has in prospect a remeasurement of this quantity for the center of gravity of $\beta$ *Aurigae*. For the radial velocity of the center of gravity of $\epsilon$ *Ursae Majoris*, Ludendorff gives, with reserve, the provisional value of $-13$ km per sec.

The parallax of $\delta$ *Leonis* may be expected to be measurable.

Ludendorff has already called attention to a peculiar progression in the deviation between observed and computed directions of the proper motions of the five stars in *Ursa Major*. This progression (Table I, cols. 7 and 8) becomes more evident in the extension of the system which I have here undertaken, but it nevertheless always remains within the permissible errors of observations. I have found no plausible physical explanation of this progression; perhaps, indeed, stellar systems of the sort here discussed might serve in the determination of systematic errors of observation.

I have looked up a number of other stars, whose proper motion is directed approximately toward the above point of convergence. Determinations of the radial velocities and of the parallax are needed, however, before anything further can be said about them. Nevertheless the membership in the system of *78 Ursae Majoris* may be

stated to be very probable, and that of *Groombridge 1930* as probable. These two stars are added to the eleven principal stars in Table I. *78 Ursae Majoris* is a visual double star (Burnham, *General Catalogue*, No. 6348). The position angle has increased about 20° in fourteen years, with a distance of 1″.4. This orbital motion is in good accord with the computed parallax of 0″.042. *78 Ursae Majoris*, by the way, was already mentioned by Höffler. *κ Bootis* (Σ 1821), is, further, somewhat suspicious, but the different determinations of its proper motion differ by a pretty large amount.

TABLE II

| 1 | Stellar Magnitude H. R. $m$ 2 | Magnitude Reduced to $\pi=1''$ $m+5\log\pi$ 3 | Spectrum | |
|---|---|---|---|---|
| | | | Maury 4 | H. R. 5 |
| *β Aurigae* | 2.07 | −5.53 | VIII a | Ap |
| *ε Ursae maj.* | 1.68 | −5.18 | VIII P | Ap |
| *α Coronae* | 2.31 | −4.62 | VIII ab | A |
| *β Eridani* | 2.92 | −4.44 | IX b | A 2 |
| *ζ Ursae maj.* A | 2.40 | −4.42 | VIII a | Ap |
| *γ Ursae maj.* | 2.54 | −4.34 | VIII b | A |
| *β Ursae maj.* | 2.44 | −4.20 | VIII a | A |
| *α Canis maj.* | −1.58 | −3.64 | VII a | A |
| *δ Ursae maj.* | 3.44 | −3.28 | IX b | A 2 |
| *ζ Ursae maj.* B | 3.96 | −2.86 | | A 2 |
| *δ Leonis* | 2.58 | −2.80 | IX b | A 2 |
| *g Ursae maj.* | 4.02 | −2.80 | IX b | A 5 |
| *78 Ursae maj.* | 4.89 | −1.99 | | F |
| *Groombr. 1930* | 5.87 | −1.86 | | F |
| *37 Ursae maj.* | 5.16 | −1.56 | | F |

It is striking that so many of the stars here in question are double: this is certainly true of nine of the fifteen stars in Table II.

The direction of the proper motion of *α Geminorum* might arouse the suspicion that it also belonged to the system. Its spectrum, VIII *a*, according to Miss Maury, is very similar to that of the other bright members, and *Castor* has, in common with *Mizar*, the spectroscopic duplicity of the two components. Nevertheless, the radial velocities of the centers of gravity of the systems of $a_1$ and $a_2$ *Geminorum*, respectively −1.0 and +6.2 km per sec., sufficiently show that these stars do not belong to the system. The computed parallax also turns out too large.

The absolute brightness, reduced to a parallax of 1″, was com-

puted with the aid of the computed parallaxes and the stellar magnitudes of the *Revised Harvard Photometry* (*Harvard Annals*, **50**), and the stars are arranged accordingly in col. 3 of Table II. The combined light was taken for the spectroscopic binaries. A comparison of the spectra, given in cols. 4 and 5, suggests an increasing development of the spectrum, with a decline in the absolute brightness. In this respect, the system here treated behaves in a manner analogous to the physically connected *Pleiades;* while for the *Hyades* it is known that this is true only for a part of the stars, and that marked deviations occur—all of which hitherto pointed in the direction that these stars are more red than would be expected from their absolute brightness, a phenomenon that is also known to us in case of other stars like *α Boötis*, *α Tauri*, *α Orionis*, *α Scorpii*. For 16 *Hyades* (with spectra between A and F, average A 4 F) the brightness, reduced to a parallax of 1″, averages −2.86 magnitudes; this, together with the data given in Table II, indicates that the *Hyades*, and the stars of the system under discussion, have on the average equal absolute brightness for spectra between A and F.

The question is of particular interest how many stars of low absolute brightness may belong to the group. Among the 307 stars having an annual proper motion of more than 0″.5, which Kobold treated in his book *Bau des Fixsternsystems*, there occurs no strikingly large number for which the proper motion is directed with sufficient accuracy toward the above point of convergence. For some of these stars, determinations of parallax are available, but in all cases, with the exception of *Sirius*, a much smaller value was found than corresponded with the system. The parallax, as may readily be seen, cannot fall below $\frac{1}{4}$ of the magnitude of the annual proper motion, and this is, for very few stars with large proper motion, the case. It therefore remains highly probable that the distances between one member of the system and its neighbors are to be designated as decidedly "stellar." In case this also applies to other systems, it becomes clear why Kobold found among the above 307 stars only uncertain traces of the systems; even if it is assumed that the majority of the stars group themselves in such systems. To find this out, it would be necessary to go down to decidedly smaller proper motions, but for this the accuracy hitherto has been rarely adequate.

TABLE III

O = *observed*; C = *calculated*

| 1 | R. A. 1900 2 | Decl. 1900 3 | Annual Proper Motion 4 | Direction of Proper Motion O. 5 | Direction of Proper Motion C. 6 | Distance from Star to Point of Convergence 7 | Radial Velocity C. km. 8 | Magnitude 9 | $m+5 \log \pi$ 10 | Spectrum 11 | Parallax C. $\pi$ 12 | Parallax O. 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ° | ° | ″ | ° | ° | ° | | | | | ″ | ″ |
| *41 H. Androm.* | 23.92 | +42.11 | 0.82 | 99.4 | 99.4 | 69.3 | +15.6 | 5.10 | −0.01 | F | 0.095 | 0.116 S. |
| *δ Trianguli* | 32.74 | +33.77 | 1.18 | 101.6 | 101.7 | 60.9 | +21.4 | 5.07 | 0.89 | G | .146 | .108 F., .123 S. |
| *θ Ursae maj* | 141.54 | +52.13 | 1.10 | 240.2 | 239.9 | 60.3 | +21.8 | 3.26 | −1.06 | F 8 | .137 | .098 F., .054 K., .098 P. |
| *11 Leonis min* | 142.42 | +36.26 | 0.76 | 250.0 | 250.0 | 54.1 | +25.8 | 5.48 | 0.61 | K | .106 | −.016 C., .075 J. |
| AOe 10603 | 151.31 | +49.96 | 1.44 | 249.8 | 249.9 | 65.0 | +18.6 | 6.76 | 2.94 | K | .172 | .333 B., .185 K., .178 P., .138 F. |
| Lal 27742 | 227.06 | +19.65 | 0.68 | 296.3 | 296.5 | 128.1 | −27.1 | 6.41 | 1.26 | G | .093 | .054 S. E. |
| P. XXIII, 267 | 359.91 | +34.10 | 0.76 | 82.8 | 82.8 | 92.0 | − 1.5 | 6.23 | 0.81 | F | .082 | .004 C. |

The computed rectangular co-ordinates in space of the separate stars are given in cols. 14, 15, and 16 of Table I. The distance of a star with a parallax of 1″ was taken as the unit. The sun lies at the origin, and the Z-axis is directed toward the radiant point; the X–Z plane contains the earth's axis. If we designate the triangle Pole-Radiant-Star with *PRS*, we have, $\pi$ being the parallax,

$$x=\frac{\sin RS \cos R}{\pi}, \quad y=\frac{\sin RS \sin R}{\pi}, \quad z=\frac{\cos RS}{\pi}.$$

The quantities *RS*, *R*, and $\pi$ are found in cols. 9, 12, and 13 of Table I.

## ADDENDUM REGARDING THE *HYADES*

According to Lewis Boss,[1] the *Hyades* have a convergent point at $\alpha=92^\circ.15$, $\delta=6^\circ.93$, for 1900. I assumed in the first values the values $\alpha=92^\circ.3$, $\delta=+7^\circ.1$, for 1900, and a velocity relative to the sun of 44.0 km per sec. Among the above-mentioned 307 stars investigated by Kobold, I have sought out those which gave the best agreement between observed and computed direction of proper motion, and I have enumerated seven of them in Table III. As will be seen from a comparison of cols. 5 and 6, the proper motions of these seven stars are so accurately directed toward the convergent point of the *Hyades*, that they can scarcely all of them be strangers to this group. Determinations of the radial velocity are available only for *θ Ursae Majoris*. Küstner found +15 km, which value differs by 7 km from the computed value of +22 km. The star has a physical companion of magnitude $13\frac{1}{2}$ at a distance of 5″ (Burnham, *General Catalogue*, No. 5123), so that the radial velocity is possibly influenced a little by the orbital motion.

In the observed parallax given in col. 13, the following abbreviations are used:

B=Ball; C=Chase; E=Elkin; F=Flint; J=Jost; K=Kapteyn; P=Peter; S=Smith.

The parallax of the comparison stars, 0″.008, was added to the original figures.

For the double star *55 Tauri*, which physically belongs to the *Hyades*, a parallax of 0″.025 is computed from the orbital motion,

[1] *Astronomical Journal*, **26**, 31, 1908.

on the assumption that the sum of the two masses is equal to that of the sun. This value agrees exactly with that computed by Lewis Boss, so that we here again arrive at masses similar to that of the sun. The two double stars, *8o Tauri* and *Hu* 1080 (Burnham, *General Catalogue*, No. 12,992), similarly belong to the group, and give promise, within a reasonable time, of offering the possibility of an orbital motion.

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