1907ApJ....25...95J

ARBITRARY DISTRIBUTION OF LIGHT IN DISPERSION BANDS, AND ITS BEARING ON SPECTROSCOPY AND ASTROPHYSICS[1]

By W. H. JULIUS

In experimental spectroscopy, as well as in the application of its results to astrophysical problems, it is customary to draw conclusions, from the appearance and behavior of spectral lines, as to the temperature, density, and motion of gases in or near the source of light. These conclusions must in many cases be entirely wrong, if the origin of the dark lines is exclusively sought in absorption, and that of the bright ones exclusively in selective emission, without taking into account the fact that the distribution of light in the spectrum is also dependent on the anomalous dispersion of the rays in the absorbing medium.

It is not in exceptional cases only that this influence makes itself felt. Of the vapors of many metals it is already known that they bring about anomalous dispersion with those kinds of light that belong to the neighborhood of several of their absorption lines.[2] In all these cases the appearance of the absorption lines must to a greater or less extent be modified by the above-mentioned influence, since the mass of vapor traversed by the light is never quite homogeneous. Hence it is necessary to investigate the effect of dispersion on spectral lines separately; we must try to distinguish it entirely from the phenomena of pure emission and absorption.

The previously described experiments with a long sodium flame,[3] in which a beam of white light alternately traveled along different paths through that flame, constitute a first attempt in this direction. With these relative displacements of beam and flame the rays of the anomalously dispersed light were much more bent, on account of the

---

[1] The main part of this paper was communicated at the meeting on September 29, 1906, of the Royal Academy of Sciences of Amsterdam.

[2] After Wood, Lummer and Pringsheim, Ebert, especially Puccianti has investigated the anomalous dispersion of various metallic vapors. In *Nuovo Cimento*, (5) **9**, 303, 1905, Puccianti describes over a hundred lines showing the phenomenon.

[3] W. H. Julius, "Dispersion Bands in Absorption Spectra," *Proc. Roy. Acad. Amst.*, **7**, 134–140, 1904; *Astrophysical Journal*, **21**, 271, 1905.

uneven distribution of the sodium vapor, than the other rays of the spectrum; absorption and emission changed relatively little. The result was that the distribution of the light in the neighborhood of D_1 and D_2 could be made very strongly asymmetrical, which could easily be explained in all details as the result of curvature of the rays. The existence of "dispersion bands" was thus proved beyond doubt.

But the pure effect of emission and absorption was not absolutely constant in these experiments, and only conjectures could be made concerning the density of the sodium vapor in the different parts of the flame. Moreover, the whirling ascent of the hot gases caused all rays, also those which suffered no anomalous dispersion, to deviate sensibly from the straight line, so that the phenomena were too complicated and variable to show the effect of dispersion strictly separated from that of emission and absorption. So our object was to obtain a mass of vapor as homogeneous as possible and, besides, an arrangement that would allow us to bring about arbitrarily, in this vapor, local differences of density in such a manner that the average density was not materially altered. The absorbing power might then be regarded as constant. At the same time it would be desirable to investigate the vapor at a relatively low temperature, so that its emission spectrum did not have to be reckoned with.

In a series of fine investigations on the refractive power and the fluorescence of sodium vapor, R. W. Wood[1] caused the vapor to be developed in an electrically heated vacuum tube. It appeared possible, by adjusting the current, to keep the density of the vapor very constant. Availing myself of this experience, I made the following arrangement for the investigation of dispersion bands.

APPARATUS

NN' (see Fig. 1) is a nickel tube of 60 cm length, 5.5 cm diameter, and 0.07 cm thickness. Its middle part, having a length of 30 cm, is placed inside an electrical furnace of Heraeus (pattern *E* 3). Over its extremities covers are placed, the edges of which fit into circular rims soldered to the tube, which consequently shut air-tight when the rims are filled with cement. When the furnace is in action, a steady current of water, passing through the two mantles *M* and *M'*, keeps

---

[1] *Phil. Mag.*, (6) **3**, 128; **6**, 362, 1903.

the ends of the tube cool. Each of the two caps has a rectangular plate-glass window, and also, on both sides of this, openings *a* and *b* (*b′* and *a′*), placed diametrically opposite to each other and provided with short brass tubes, the purpose of which will appear presently.

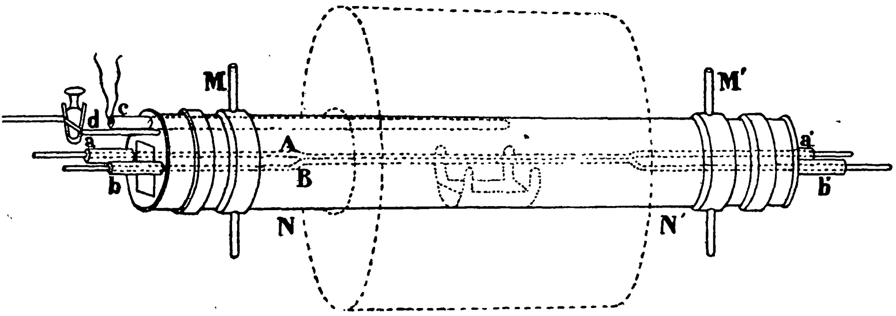


FIG. 1

Moreover, in one of the two caps (see also Fig. 2) two other short tubes *c* and *d* are fastened in openings; through *c* the porcelain tube of a Le Chatelier pyrometer is fitted air-tight, while on *d* a glass cock with mercury lock is cemented, leading to a manometer and a Geryk air-pump. As soon as the sodium (a carefully cleaned piece of about 7 grams) had been pushed to the middle of the tube in a small nickel dish provided with elastic rings, the tube was immediately closed and exhausted.

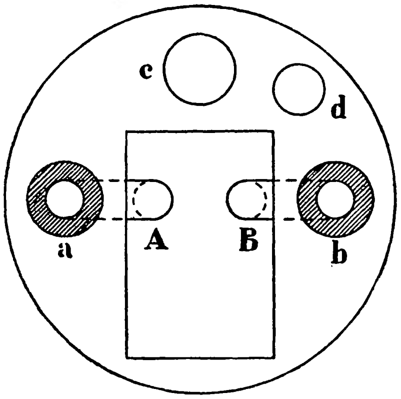


FIG. 2

We shall now describe the arrangement by which arbitrary inequalities in the density distribution were produced inside the mass of vapor. It consists of two nickel tubes *A* and *B* of 0.5 cm diameter, leading from *a* to *a′* and from *b* to *b′*, and so bent that in the heated middle part of the wide tube they run parallel over a length of 30 cm at a distance of only 0.8 cm. In the four openings of the caps, *A* and *B* are fastened air-tight by means of rubber packing. This kind of connection leaves some play, so that by temperature differences between the wide and the narrow tubes these latter need not alter their shape through tension. At the same time the rubber insulates *A* and *B* electrically from *NN′*. The four ends of the narrow tubes which project are kept cool by mantles with running water (these are not represented in the figure).

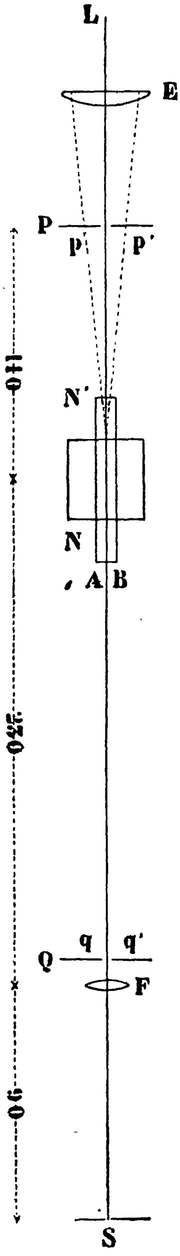


Fig. 3

If an electric current is now passed through *A* or *B*, the temperature of this tube rises a little above that of its surroundings; if an air-current is passed through it, the temperature falls a little below that of its surroundings. The intensities of the currents, and consequently the differences of temperature, can in either case be easily regulated and kept constant for a long time.

Fig. 3 gives a sketch of the whole arrangement. The light of the positive carbon *L* is concentrated by the lens *E* on a screen *Q* having a slit-shaped aperture of adjustable breadth. The lens *F* forms in the plane of the slit *S* of the spectrograph a sharp image of the diaphragm *P*. The optical axis of the two lenses passes through the middle of the tube containing the sodium vapor, exactly between the two small tubes *A* and *B*.

If now the opening in the diaphragm *P* has the shape of a vertical narrow slit, and if its image falls exactly on the slit of the spectrograph, then the continuous spectrum of the arc-light appears with great brightness. If the tube *NN′* is not heated, D_1 and D_2 are seen as extremely fine dark lines, attributed to absorption by the sodium, which is always present in the neighborhood of the carbons. In order that this phenomenon might always be present in the field of view of the spectrograph as a comparison spectrum, also when the tube is heated, a small totally reflecting prism was placed before part of the slit *S*, to which part of the principal beam of light was led by a simple combination of lenses and mirrors without passing the electric furnace. Thus the unmodified spectrum of the source is also seen on each photograph that was taken.

The spectral arrangement used consists of a plane diffraction grating 10 cm in diameter (ruled surface 8 by 5 cm) with 14,436 lines to the inch, and two silvered mirrors of Zeiss; the collimator mirror has a focal length of 150 cm, the other, of 250 cm. Most of the work was done in the second spectrum.

When heating the sodium for the first time a pretty large quantity of gas (according to Wood, hydrogen) escaped from it, which of course was pumped off. After the apparatus had been operated a couple of times, the tension within the tube remained for weeks less than 1 mm of mercury; also during the heating, which, in the experiments described in this paper, never went beyond 450°. The inner wall of NN', and also the small tubes A and B, are after a short time covered with a layer of condensed sodium, which favors the homogeneous development of the vapor in subsequent heatings. It is remarkable that scarcely any sodium condenses on the parts of the tube that project from the furnace, so that the windows also remain perfectly clear. The density of saturated sodium vapor at temperatures between 368° and 420° has been experimentally determined by F. B. Jewett.[1] He gives the following table:

Temperature	Density	Temperature	Density
368°	0.00000009	395°	0.00000270
373	0.00000020	400	0.00000350
376	0.00000035	406	0.00000480
380	0.00000043	408	0.00000543
385	0.00000103	412	0.00000590
387	0.00000135	418	0.00000714
390	0.00000160	420	0.00000750

These densities are of the same order of magnitude as those of mercury vapor between 70° and 120°. At 387° the density of saturated sodium vapor is about one-thousandth of that of the atmospheric air at 0° and 76 cm.

OBSERVATIONS

If we now regulate the intensity of the current in the furnace in such a manner that the thermo-couple indicates a steady temperature

---

[1] "A New Method of Determining the Vapor-Density of Metallic Vapors, and an Experimental Application to the Cases of Sodium and Mercury," *Phil. Mag.*, (6) **4**, 546, 1902.

1907ApJ....25...95J

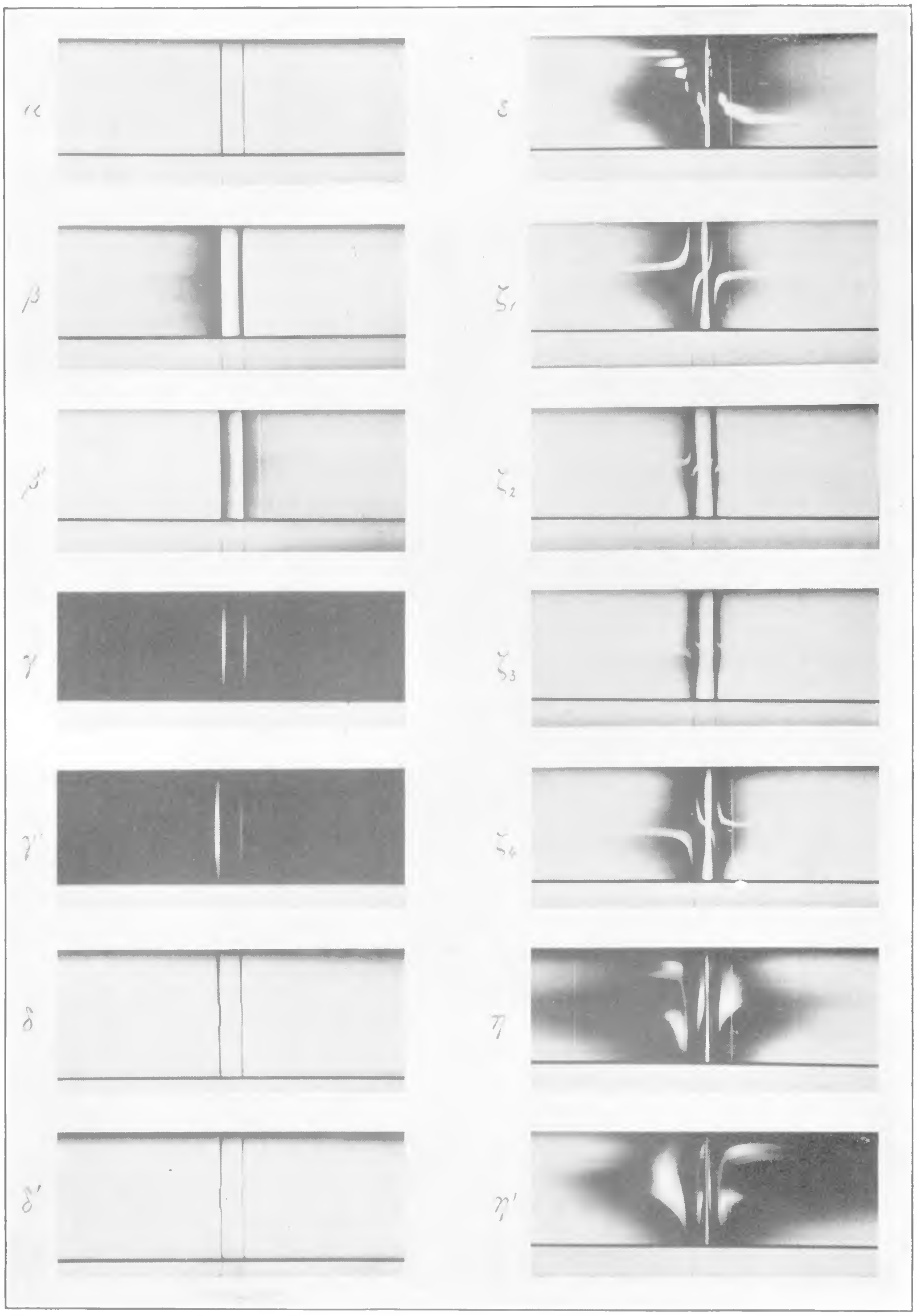
(in many of our experiments 390°), then the density of the vapor is not everywhere the same within the tube, for the temperature falls from the middle toward the ends; but since the surfaces of equal temperature are practically perpendicular to the beam of light, all rays pass nearly rectilinearly through the vapor. Accordingly the spectrum is only little changed; the two D lines have become somewhat stronger, which we shall, for the present, ascribe to absorption by the sodium vapor in the tube.

We now blow a feeble current of air through the tube *A*, which thus is slightly cooled, so that sodium condenses on it, the vapor-density in its neighborhood diminishing. We soon see the sodium lines broaden considerably. This cannot be the consequence of increased absorption, since the average vapor-density has decreased a little. The reason is that rays of light with very great refractive indices are now bent toward *q′* (Fig. 3), and rays with very small indices toward *q;* hence in the image of the slit *P* which is formed on *S*, rays belonging to regions on both sides of the D lines no longer occur, while yet this image remains perfectly sharp, since the course of all other rays of the spectrum has not been perceptibly altered. If now at the same time the tube *B* is heated by a current of, say, 20 amperes, by which the density-gradient in the space between the tubes is increased, the breadth of the lines becomes distinctly greater still. The heat generated in the tube by the current is about 1 calorie per second; it is, however, for the greater part conducted away to the cooled ends of the tube, so that the rise of temperature can only be small.

By switching a current key and a cock, *A* and *B* can be made to suddenly exchange parts, so that *A* is heated, *B* cooled. The dark bands then shrink, pass into sharp D lines and then expand again, until, after a few minutes, they have recovered their former breadth.

The lines in the transition stage are fine and sharp, however, only if the temperature of the furnace is very constant. If it rises or sinks, the minimal breadth appears to be not so small. In this case, however, there certainly exist currents in the mass of vapor which cause the distribution of density to be less regular. When, therefore, *A* and *B* being at equal temperatures, we still sometimes see the sodium lines slightly broadened, it stands to reason to attribute this also to refraction in such accidental irregularities.

PLATE V



That spectral lines possess some breadth is commonly ascribed either to motion of the light-emitting and absorbing molecules in the line of sight, or to changes in the vibrational period of the electrons by the collisions of the molecules. We now have a third cause—anomalous dispersion in the absorbing medium. The whole series of phenomena observed in our sodium tube corroborates the opinion that this latter cause must in many cases be regarded as by far the most important. It will appear that this conclusion holds not only for dark, but also for bright spectral lines.

If the slit in the diaphragm P is made much broader toward p', this has no influence on the spectrum as long as A and B are at the surrounding temperature. The D lines appear narrow, as in α, Plate V. If A is now cooled below this temperature, and B is raised above it, the dark D lines broaden only in the direction of the shorter wave-lengths, while at the side of the longer wave-lengths the intensity of the light is even increased. Indeed, the deficiency of these longer waves, which has been observed in the case of the *narrow* slit in P, is now overcompensated by the anomalously bent rays coming from the broad radiating field p' and finding their way through the slit Q to S. The resulting aspect is shown in β on the plate.

The spectrum β passes into β' when the temperature difference between A and B is made to change its sign, or also when the original temperature difference is maintained, but the slit in P is made much wider toward p instead of toward p'; for with both alterations the rays of longer and those of shorter wave-lengths than D_1 and D_2 only exchange parts.

A small shifting of the diaphragm P in the direction toward p' (starting from the conditions fulfilled when taking β) brings the image of the screen p upon the slit S, and thus prevents all the light not undergoing anomalous dispersion from reaching S. This causes the spectrum γ to appear, which makes the impression of an emission spectrum of sodium with slightly shifted lines, although it is evidently only due to rays from the field p' which have undergone anomalous dispersion in the vapor.

In a similar way the pseudo-emission spectrum γ' is obtained by shifting the diaphragm a little, starting from the conditions that gave β'.

1907ApJ....25...95J

The cases β and β' may be combined by using a diaphragm P with an opening of the shape of Fig. 4. When the slit S occupies the position of the dotted line in the image of this opening, then, if A is cooled, we shall have the conditions of β in the upper and lower parts of the spectrum, and the conditions of β' in the middle part. The resulting combination of the spectra β and β' may be easily imagined, and has not, therefore, been reproduced. But it is of some interest to notice the appearance of the same combination when the density-gradient is made much smaller than it was when taking β and β'; for now we get δ and, after reversing the gradient, δ'. The line-shifting here produced has, of course, nothing to do with Doppler's principle. The two photographs further prove that even these narrow lines are almost totally due to anomalous dispersion instead of to absorption; indeed, the real, straight absorption line must be common to the three sections, and we see that there is scarcely any room left for it.

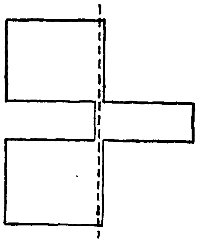


Fig. 4

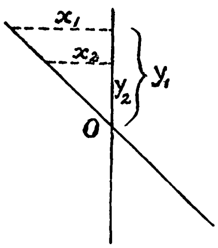


FIG. 5

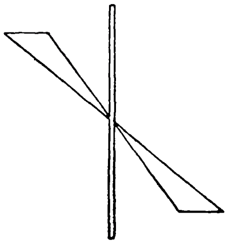


FIG. 6

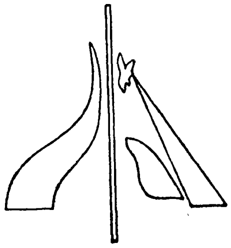


FIG. 7

(On several of the photographs a few narrow bright lines appear;

they are emission lines of the arc, belong to the extreme violet of the third spectrum, and bear no relation to the phenomena with which we are concerned. The line a little to the right of D, for instance, is probably the calcium line λ 3933.83, for $3933.83 \times \frac{3}{2} = 5900.74$.)

Let us now return to the diaphragm *P* with a narrow slit placed in the optical axis. (A piece of glass coated with tin-foil, in which a slit was cut out, was generally used.) The spectrum then shows broad bands when there is a sufficient density-gradient between *A* and *B*. If an opening is cut in the tin-foil beside the slit, a group of rays of definite refractivities (and consequently also of definite wave-lengths) is given an opportunity to reach *S* through *Q*, and a bright spot is formed in the dark band, the shape of which depends on the shape of the opening in the tin foil, but is by no means identical with it. Thus, for instance, the spectrum ϵ shows the effect of a series of rectangular openings in the screens p and p'.

The law connecting the form of bright areas in the dispersion bands with the shape of openings in the screen is not very simple, because it depends on the configuration of the surfaces of equal density in the space between the tubes *A* and *B*. Some idea of the connection may be got if we simplify the problem by supposing those surfaces to be parallel planes, perpendicular to the plane containing the axes of *A* and *B*. The latter plane may cut the slit *P* in the point *O* and the slit *S* in O'. We shall take *O* as the origin of rectangular co-ordinates x (horizontal) and y (vertical), by which the points in the plane of the screens p and p' may be determined. Points in the image on *S* may be designated by x' and y' with respect to O'.

Now let us suppose a pin-hole xy to be made in p'. Light coming from this point will be focused by the lens *F* at a point $x'y'$ beside the slit *S*, provided it has not deviated in the sodium vapor. It does not get into the spectrograph. But the rays undergoing anomalous dispersion will spread out nearly horizontally; the lens *F* unites them in a continuous series of points having about the same y', but various values of x'. Only those for which $x' = 0$ enter into the spectrograph. If in the spectrum the middle of one of the sodium lines be called O'', the co-ordinates of the bright spot, produced in the dark dispersion band by the beam that entered, will be y'' (proportional to y') and z, the abscissa z depending on the wave-length λ of that beam.

The connection between this wave-length and the abscissa x of the hole is given by the dispersion-curve of the sodium vapor. Indeed, we can easily prove that x is proportional to $n-1$, the factor only depending on linear dimensions of the arrangement, and on the density-gradient of the vapor.[1] So, for a given x, n may be computed; the corresponding λ is taken from the dispersion curve, and in the spectrum we have $z=\lambda-\lambda_D$. The ordinate y'' is derived from y by merely introducing focal distances. We shall thus have expressed the co-ordinates of the bright spot in terms of the co-ordinates of the pin-hole.

The following instance may serve to elucidate the connection between corresponding figures in the plane P and in the spectrum, without calculation.

Instead of the pin-hole we make a second straight slit in the diaphragm, cutting the first one obliquely in O (Fig. 5). Now all positive and negative values of x, and therefore of $n-1$, are represented each of them belonging to a separate value of y which is proportional to it:

$$y_1 : y_2 = x_1 : x_2 = (n_1-1) : (n_2-1) \, .$$

As the ordinates y'' in the spectrum are proportional to y, we have also

$$y_1'' : y_2'' = (n_1-1) : (n_2-1) \, .$$

At the same time

$$z_1 : z_2 = (\lambda_1-\lambda_D) : (\lambda_2-\lambda_D) \, .$$

The bright curve in the spectrum, therefore, is the dispersion-curve itself, with the point $n=1$, $\lambda=\lambda_D$ taken for the origin of co-ordinates.

The spectrum ζ_1 of Plate V realizes this case. It has been obtained by using a diaphragm with an opening of the shape of Fig. 6. The width of the oblique slit was enlarged toward the ends in order to increase the luminosity of the ascending and descending branches of

---

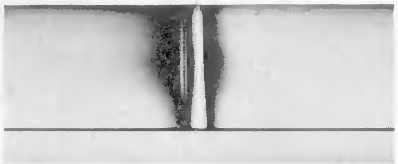
[1] From the equations (1) and (2) on pages 106 and 107 follows immediately

$$x = dlR\frac{d\Delta}{ds} = dl\frac{d\Delta}{ds} \cdot \frac{1}{\Delta}(n-1) \, .$$

1907ApJ....25...95J

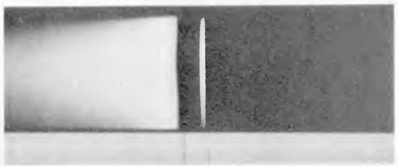
PLATE VI

θ



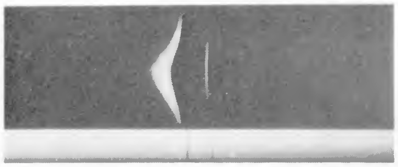
ι

κ



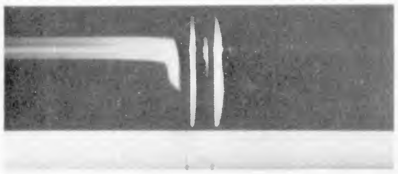
λ

μ



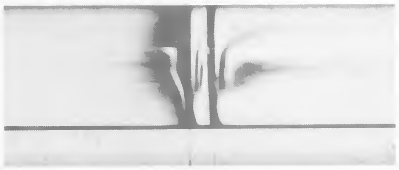
ν

ν′

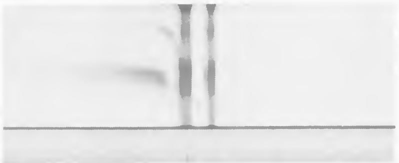


ξ

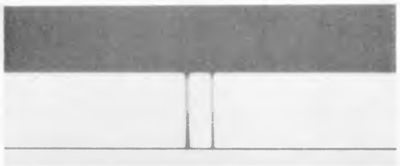
ξ′



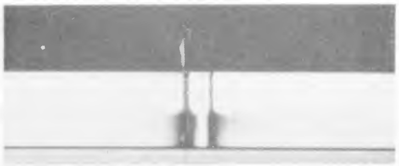
ο



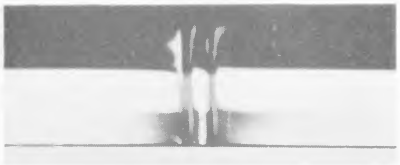
π



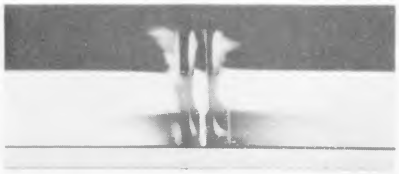
ρ



σ



τ



the curve. When the electric current and the air current through the tubes are diminished, the figure shrinks to ζ_2; when they are stopped, we return to a; reversing the gradient makes the spectrum proceed through ζ_3 to ζ_4.

Having thus experimentally found the relation between the two figures for a simple case, it is not difficult to design for any desired distribution of light the shape of the required opening in the diaphragm. The flower η, for instance, requires the diaphragm represented in Fig. 7; by reversing the gradient the image η passes into η'.

Thus we possess the means for arbitrarily producing all stages of enhancement, wingedness, reversal, shifting, duplication, ramification of bright or dark spectral lines, and it seems possible faithfully to reproduce all phenomena observed in this respect in the spectra of sun-spots, faculae, flocculi, or prominences. On Plate VI a number of arbitrary distributions of light have been collected. They were all produced in sodium vapor of 390° on the average. In θ on the dark dispersion band D_2 a bright double line is seen, reminding us of the spectrum of the calcium flocculi described by Hale. In the same negative D_1 also shows a fine double line which, I fear, will be invisible in the reproduction. The spectrum ι is not unlike that of a prominence taken with the tangential slit; κ reminds us of certain star spectra; etc. The photographs π, ρ, σ imitate the development of a prominence and a sun-spot spectrum: π represents the spectrum of the quiet solar limb with radially placed slit; in ρ a prominence appears and a spot with phenomena of reversal; σ shows all of this in a stronger degree. If now the density-gradient is made to change sign, the image first shrinks again to π, after which it expands to τ, in a certain sense the inversion of σ.

The striking spectacle of these phenomena, the gradual changes of which admit of perfect control, is only poorly reproduced by the photographs.

Plate VII shows on a slightly larger scale some photographs taken in the third spectrum with sodium vapor of only 320°. The density of the saturated vapor at this temperature is unknown. If the temperature-density-curve found by Jewett is extended beyond the observations so as to be in harmony with the shape of the better-known curve of mercury, we may infer that at 320° the density will probably be inferior to 0.00000003. The density-gradient produced by cooling or heating our tubes must have been of the order of magnitude 0.00000001 in these experiments. The diaphragm used was the same as that which served with δ and δ'. When taking υ and ϕ, the slit S occupied the position of the dotted line in the image of the opening, Fig. 4; with χ the slit was a little to the left; with ψ and ω a little to the right. We see from these photographs that the real absorption lines of the sodium vapor must have been excessively narrow; indeed, it is dubious whether they can be distinguished at all and the distribution of the light seems to be wholly governed by anomalous dispersion.

THE RELATION BETWEEN THE CURVATURE OF THE RAYS AND THE DENSITY-GRADIENT

The question arises whether it is *probable* that circumstances such as were realized in our experiments are also met with in nature, or in ordinary spectroscopical investigations undertaken with entirely different purposes.

We remark, in the first place, that curiously shaped diaphragm openings are not absolutely essential for the production of phenomena as those described above. If, for instance, our source of light had a constant, say circular, shape; if, on the other hand, the direction and magnitude of the density-gradient in our tube had not been so regular, but very different in various places of the field reproduced by the lens F, then the D lines would also have shown all sorts of excrescences, now determined by the configuration of the density distribution.

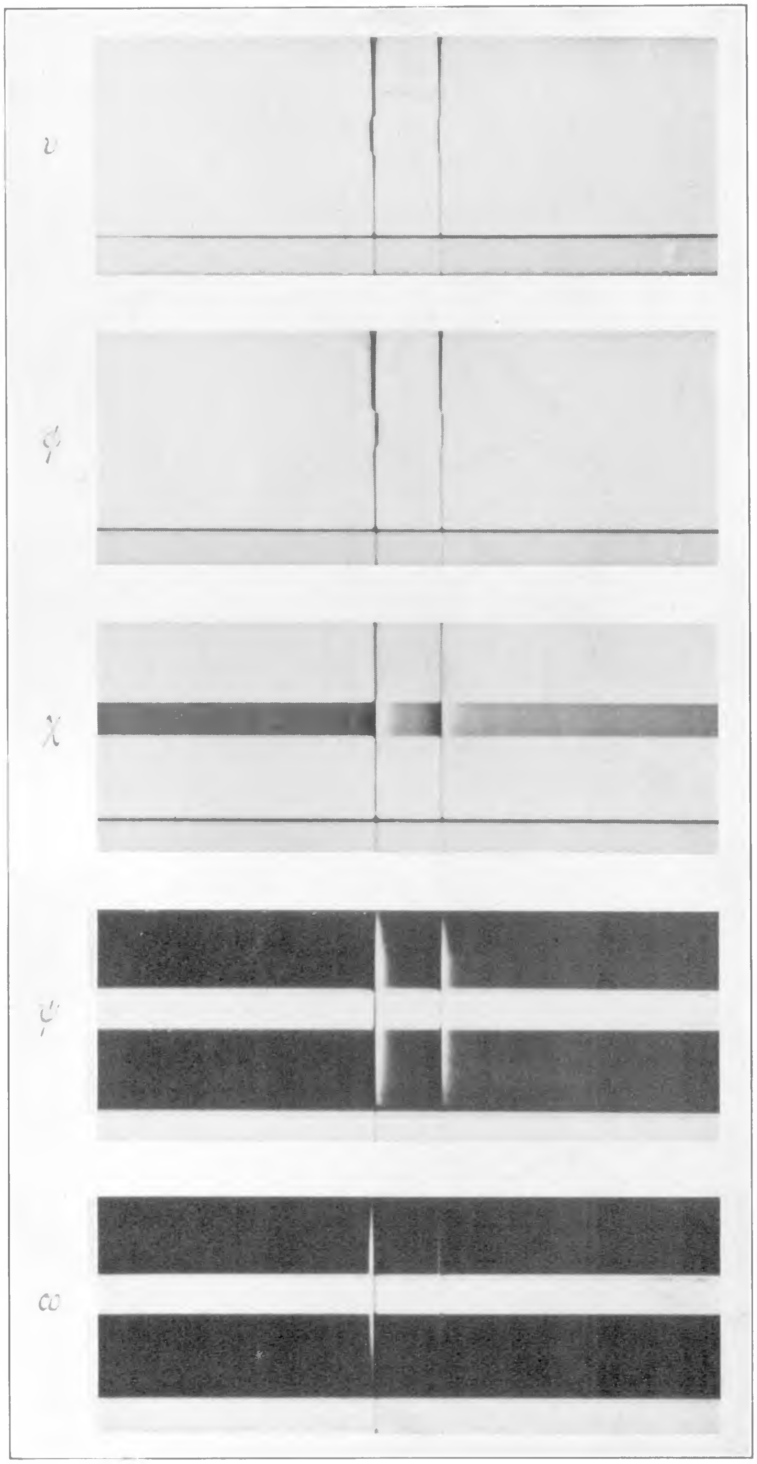
In the second place, we will try to form some idea of the quantitative relations.

The radius of curvature ρ of the path of the most deviated rays occurring in our photographs may be easily estimated from the distance d of the diaphragm to the middle of the furnace, the distance x of one of the most distant diaphragm openings to the optical axis, and the length l of the space in which the incurvation of the rays is brought about. For

$$\rho : l = d : x \, . \tag{1}$$

1907ApJ....25...95J

PLATE VII



Putting $x=1$ cm, $d=110$ cm, $l=27$ cm, this gives $\rho=3000$ cm. The average density Δ of the sodium vapor was in this case about one one-thousandth of that of the atmospheric air.

Let us see how ρ changes with the density-gradient. We always have

$$\rho=\frac{n}{n'},$$

if n represents the local index of refraction of the medium for the ray under consideration and $n'=\frac{dn}{ds}$ the change of this index per centimeter in the direction of the center of curvature. We have approximately for a given kind of light

$$\frac{n-1}{\Delta}=\text{constant}=R,$$

$$n=R\Delta+1,$$

$$n'=\frac{dn}{ds}=R\frac{d\Delta}{ds}.$$

From this follows

$$\rho=\frac{R\Delta+1}{R\frac{d\Delta}{ds}};$$

but since for rarefied gases n differs little from unity, even for the anomalously dispersed rays which we consider, $R\Delta$ may be neglected with regard to 1 and we may write

$$\rho=\frac{1}{R\frac{d\Delta}{ds}}. \tag{2}$$

For every kind of light ρ is consequently inversely proportional to the density-gradient of the vapor in the direction perpendicular to that of propagation.

An estimate of the magnitude of the density-gradient existing, in our experiments, between A and B may be obtained in two ways. It may be inferred either from the difference of temperature produced, or from formula (2). The temperature difference between A and B would have been pretty easy to determine thermo-electrically; up to the present, however, I have had no opportunity to make the necessary arrangement. Besides, the relation between the density distribution in the space traversed by the rays, and the temperatures of A and B, cannot be so very simple, since we have to deal, not with two parallel planes, but with tubes, from which, moreover, many drops of liquid sodium hang.

The second method at once gives an average value of $\frac{d\Delta}{ds}$ for the space traversed by the rays. It requires a knowledge of $R=\frac{n-1}{\Delta}$ for a kind of ray for which in our experiments also ρ has been determined.

Now, Wood[1] gives a table for the values of n for rays from the immediate vicinity of the D lines. These data, however, refer to saturated sodium vapor of 644°; but we may deduce from them the values of n for vapor of 390° by means of the table which he gives in his paper on page 317.

For, when we heat from 389° to 508°, the refractive power of the vapor (measured by the number of passing interference fringes of helium light $\lambda=5875$) becomes $\frac{98}{9}=11$ times greater, and at further heating from 506° to 644° again $\frac{50}{4}=12.5$ times greater (now found by interference measurement with light from the mercury line $\lambda=5461$); hence from 390° to 644° the refractive power increases in ratio of 1 to $11\times 12.5=137$.

Since now for rays situated at 0.4 Ångström unit from the D lines[2] we have $n-1=\pm 0.36$ (as the average of three values taken from Wood's table on page 319), we ought to have with sodium vapor at 390° for the same kind of rays

$$n-1=\frac{0.36}{137}=0.0026\ .$$

The density Δ at 390° is, according to Jewett, 0.0000016, whence

$$R=\frac{n-1}{\Delta}=\frac{0.0026}{0.0000016}=1600\ .$$

---

[1] *Phil. Mag.*, (6) **8**, 319, 1904.

[2] The spectrum ζ_1 in our plate shows that the extremities of the peaks correspond pretty well to light of this wave-length; for they approach the D lines to a distance which certainly is no more than one-fifteenth of the distance of the D lines which amounts to 6 Ångström units. For these rays the opening of the diaphragm was 1 cm distant from the optical axis.

Then from formula (2) follows

$$\frac{d\Delta}{ds}=\frac{1}{R\rho}=\frac{1}{1600\times 3000}=0.0000002\ .$$

DISPERSION BANDS IN THE SPECTRA OF TERRESTRIAL SOURCES

It is very probable that, when metals evaporate in the electric arc, values of the density-gradient are found in the neighborhood of the carbons that are more than a thousand times greater than the feeble density-gradient in our tube with rarefied sodium vapor.[1]

The radius of curvature will, therefore, in these cases be over a thousand times smaller than 30 meters, and so may be no more than a few centimeters or even less. A short path through the vapor mass is then already sufficient to alter the direction of certain rays very perceptibly.

If now an image of the carbon points is produced on the slit of a spectroscope, then this is a *pure* image only as far as it is formed by rays that have been little refracted in the arc, but the rays which undergo anomalous dispersion do not contribute to it. Light of this latter kind, coming from the crater, may be lacking in the image of the crater, and, on the other hand, penetrate the slit between the images of the carbon points. Thus, in ordinary spectroscopic observations, broadening, not only of absorption lines, but also of emission lines, must often to a considerable extent be attributed to anomalous dispersion.

When we bear this in mind, many until now mysterious phenomena will find a ready explanation. So, for instance, the fact that Liveing and Dewar[2] saw the sodium lines strongly broadened each time when vapor was vividly developed after bringing in fresh material, but saw them become narrower again when the mass came to rest, although the density of the vapor did not diminish. If by pumping nitrogen into the evaporated space the pressure was gradu-

---

[1] If, for instance, we put the vapor-density of the metal in the crater, where it boils, at 0.001, the density of the vapor outside the arc at a distance of 1 cm from the crater at 0.00001, then we have already an average gradient 5000 times as large as that used in our experiments.

[2] "On the Reversal of the Lines of Metallic Vapors," *Proc. R. S.*, **27**, 132–136; **28**, 367–372, 1878–1879.

ally increased, the lines remained sharp; but if the pressure was suddenly released, they were broadened. All this becomes clear as soon as one has recognized in the lines dispersion bands, which must be broad when the density of the absorbing vapor is irregular, but narrow, even with dense vapor, if only the vapor is evenly spread through the space.

Another instance. According to the investigations of Kayser and Runge, the lines belonging to the second secondary series in the spectra of magnesium, calcium, cadmium, zinc, mercury, are always hazy toward the red and are sharply bordered toward the violet; whereas lines belonging to the first secondary series or to other series are often distinctly more widened toward the violet. With regard to the spectrum of magnesium they say:

> Auffallend ist bei mehreren Linien, die wir nach Roth verbreitert gefunden haben, dass sie im ROWLAND'schen Atlas ganz scharf sind, und dann stets etwas kleinere Wellenlänge haben. So haben wir 4703.33, ROWLAND 4703.17; wir 5528.75, ROWLAND 5528.62. Unschärfe nach Roth verleitet ja leicht der Linie grössere Wellenlänge zuzuschreiben; so gross kann aber der Fehler nicht sein, denn die ROWLAND'sche Ablesung liegt ganz ausserhalb des Randes unserer Linie. Wir wissen daher nicht, woher diese Differenz rührt.[1]

Kayser has later[2] given an explanation of this fact, based on a combination of reversal with asymmetrical widening; but a more probable solution is, in my opinion, obtained when we regard the widened serial lines partly as dispersion bands.

If we assume that, when we proceed from the positive carbon point, which emits the brightest light, to the middle of the arc, the number of the particles associated with the second secondary series decreases, then rays coming from the crater, whose wave-length is slightly greater than that of the said serial lines, will be curved so as to turn their concave side to the carbon point. Their origin is erroneously supposed to be in the prolongation of their final direction, so they *seem* to come from the arc, and we believe we see light emitted by the vapor, in which light different wave-lengths occur, all greater than the exact wave-length of the serial lines. The observed displaced lines of the second secondary series are consequently comparable to apparent emission lines of the spectrum γ of Plate V.

---

[1] Kayser und Runge, *Über die Spektren der Elemente*, **4**, 13.

[2] Kayser, *Handbuch der Spektroskopie*, **2**, 366.

In this explanation things have been represented as if the light of these serial lines had to be *exclusively* attributed to anomalous dispersion. Probably, however, in the majority of cases, emission proper will indeed perceptibly contribute to the formation of the line; the sharp edge must then appear in the exact place belonging to the particular wave-length.

How can we now explain that lines of other series are diffuse at the opposite side? Also this may perhaps be explained as the result of anomalous dispersion, if we assume that of the emission centers of these other series the density *increases* when we move away from the positive carbon point. In this case, namely, the rays originating in the crater, which are concave toward the carbon point and consequently seem to come from the arc, possess shorter wave-lengths than the serial lines; i. e., the serial lines appear widened toward the violet. This supposition is not unlikely. For the positive and negative atomic ions which, according to Stark's theory, are formed in the arc by the impact of negative electronic ions, move in opposite directions under the influence of the electric field; hence the density-gradients will have opposite signs for the two kinds. Series whose lines are diffuse toward the red, and series whose lines flow out toward the violet, would, according to this conception, belong to, or be produced by, ions of opposite signs—a conclusion which at all events deserves nearer investigation.

The examples given may suffice to show that it is necessary systematically to investigate to what extent the already known spectral phenomena may be the result of anomalous dispersion. A number of cases in which the hitherto neglected principle of ray-curving has undoubtedly been at the root of the matter are found in Kayser's handbook, **2**, 292–298, 304, 306, 348–351, 359–361, 366.

DISPERSION BANDS IN THE SPECTRA OF CELESTIAL BODIES

Since almost any peculiarity in the appearance of spectral lines may be explained by anomalous dispersion, if only we are at liberty to assume the required density distributions, we must ask, when applying this principle to astrophysical phenomena: Can the values of the density-gradient for the different absorbing gases in celestial bodies really be such that the rays are sufficiently curved to exert such a distinct influence on the distribution of light in the spectrum?

In former communications[1] I showed that the sun, for instance, may be conceived as a gaseous body, the constituents of which are intimately mixed, since all luminous phenomena giving the impression as if the substances occurring in the sun were separated, may be brought about in such a gaseous mixture by anomalous dispersion. We will now try to prove, not only that this *may* be the case, but that it *must* be so on account of the most likely distribution of density.

Let us put the density of our atmosphere at the surface of the earth at 0.001293. At a height of 1050 cm it is smaller by $\frac{1}{760}$ of this amount, so that the vertical density gradient is

$$\frac{0.001293}{1050\times 760}=16\times 10^{-10}.$$

The horizontal gradients occurring in the vicinity of depressions are much smaller; even during storms they are only about one one-thousandth of the said value.[2] Over small distances the density-gradient in the atmosphere may of course occasionally be larger, through local heating or other causes.

Similar considerations applied to the sun, *mutatis mutandis*, cannot, however, lead to a reliable estimate of the density-gradients there occurring. A principal reason why this is for the present impossible is found in our inadequate knowledge of the magnitude of the influence, exerted by *radiation-pressure* on the distribution of matter in the sun. If there were no radiation-pressure, we might presuppose, as is always done, that at the level of the photosphere gravitation is twenty-eight times as great as on the earth; but it is counteracted by radiation-pressure to a degree, dependent on the size of the particles; for some particles it may even be entirely abolished. The radial density-gradient must, therefore, in any case be much smaller than one might be inclined to calculate on the basis of gravitational action only.

---

[1] *Proc. Roy. Academy Amsterdam*, **2**, 575; **4**, 195; **5**, 162, 589, and 662; **6**, 270; **8**, 134, 140, and 323. *Astrophysical Journal*, **12**, 185–200; **15**, 28–37; **18**, 50–64; **21**, 271–291. *Physikalische Zeitschrift*, **4**, 85–90; 132–136; **6**, 239–248. A sketch of a solar theory, in which refraction and dispersion have been considered, is to be found in the *Revue générale de sciences*, **15**, 480–495, 1904.

[2] Arrhenius, *Lehrbuch der kosmischen Physik*, p. 676.

Fortunately we possess another means for determining the radial density-gradient in the photosphere, at any rate as far as the order of magnitude is concerned. According to Schmidt's theory, the photosphere is nothing but a critical sphere the radius of which is equal to the radius of curvature of luminous rays whose path is horizontal at a point of its surface. This radius of curvature is consequently $\rho = 7 \times 10^{10}$ cm, a value which we may introduce into the expression for the density-gradient:

$$\frac{d\Delta}{ds} = \frac{1}{R\rho} .$$

The refractive equivalent R for rays that undergo no anomalous dispersion varies with different substances, to be sure; but in an approximate calculation we may put $R = 0.5$. Then at the height of the critical sphere we shall have

$$\frac{d\Delta}{ds} = \frac{1}{0.5 \times 7 \times 10^{10}} = 0.29 \times 10^{-10} ,$$

(this is 50 times less than the density-gradient in our atmosphere). All arguments supporting Schmidt's explanation of the sun's limb are at the same time in favor of this estimate of the radial density-gradient in the gaseous mixture. It should be observed, on the other hand, that, when things are considered from other points of view than from Schmidt's theory, this density-gradient appears by no means improbably large. Yet gradients of this order of magnitude will produce ray-curving in a degree amply sufficient for giving rise to very conspicuous dispersion phenomena, as we shall see presently. If, therefore, arguments are found for assuming larger density-gradients, our explanations will thereby only be corroborated.

Let us now consider rays that do undergo anomalous dispersion. In order that light, the wave-length of which differs but very little from that of one of the sodium lines, may seem to come from points situated some seconds of arc outside the sun's limb, the radius of curvature of such anomalously bent rays need only be slightly smaller than 3×10^{10} cm. Let us put, for instance,

$$\rho' = 6 \times 10^{10} \text{ cm} .$$

If we further assume that of the kind of light under consideration the wave-length is 0.4 Ångström units greater than that of D_1, then for this kind of light $R'=1600$, as may be derived from the observations of Wood and of Jewett;[1] we thus find for the density-gradient of the sodium vapor,

$$\frac{d\Delta'}{ds}=\frac{1}{R'\rho'}=\frac{1}{1600\times 6\times 10^{10}}=0.0001\times 10^{-10},$$

a quantity 2900 times smaller than the density-gradient of the gaseous mixture.

Hence if only one three-thousandth part of the gaseous mixture consists of sodium vapor, then, on account of the assumed radial density-gradient of the mixture, the critical sphere (or the photosphere) will already seem to be surrounded by a "chromosphere" of light, this light having a striking resemblance with sodium light. This kind of light has, so to say, its own critical sphere which is larger than the critical sphere of the light not anomalously refracted. If the percentage of sodium were larger, the "sodium chromosphere" would appear higher.

It is customary to draw conclusions from the size of the chromospheric and flash crescents, observed during a total eclipse with the prismatic camera, as to the *height* to which various vapors occur in the solar atmosphere. According to us, this is an unjustified conclusion. On the other hand, it will be possible to derive from these observations data concerning *the ratio in which these substances are present in the gaseous mixture*, provided that the dispersion-curves of the metallic vapors, at known densities, shall first have been investigated in the laboratory.

Until now we have dealt only with the normal radial density-gradient. By convection and vortex motion, however, irregularities in the density distribution arise, with gradients of various direction and magnitude. And since on the sun the resultant of gravitation and radiation-pressure is relatively small, there the irregular density-gradients may reach values that approach the radial gradient sooner than on the earth, or may be occasionally larger.

The incurvation of the rays in these irregularities must produce capriciously shaped sodium prominences, the size of which depends, among other causes, on the percentage of sodium vapor in the gaseous mixture.

So the large hydrogen and calcium prominences prove that rela-

---

[1] See page 108.

tively much hydrogen and calcium vapor is present in the outer parts of the sun; but perhaps even an amount of a few per cent. would already suffice to account for the phenomena.[1]

If we justly supposed that non-radially directed density-gradients are of frequent occurrence in the sun, and there disturb the general radial gradient much more than on the earth, then not only rays from the marginal region, but also rays from the other parts of the solar disk, must sensibly deviate from the straight line. Chiefly concerned are, of course, the rays that undergo anomalous dispersion. *Every absorption line of the solar spectrum must consequently be enveloped in a dispersion band.*

To be sure, absorption lines of elements which in the gaseous mixture occur only in a highly rarefied condition, present themselves as almost sharp lines, since for these substances all density-gradients are much smaller than for the chief constituents, and so the curvature of the rays from the vicinity of these lines becomes imperceptible. Also some lines of strongly represented elements may appear sharp, since not all lines of the same element, with given density, cause anomalous dispersion in the same degree. Perhaps there are even absorption lines which under no condition give rise to this phenomenon; though this would be rather improbable from the point of view of the theory of light.

Be this as it may, the limitations mentioned do not invalidate our principal conclusion: that the general interpretation of the solar spectrum has to be modified. We are obliged to see in Fraunhofer's lines not only absorption lines, as Kirchhoff does, but chiefly dispersion bands (or dispersion lines). And that refraction has a preponderant influence also on the distribution of light in stellar spectra cannot be doubted either.

We must become familiar with the idea that in the neighborhood of the celestial bodies the rays of light are in general curved, and that consequently the whole interstellar space is filled with *nonhomogeneous radiation fields*[2] of different structure for the various kinds of light.

---

[1] This result would be in accordance with a hypothesis of Schmidt (*Physikalische Zeitschrift*, 4, 232 and 341), according to which the chief constituent of the solar atmosphere would be a very light gas, until now unknown.

[2] "Das ungleichmässige Strahlungsfeld und die Dispersionsbanden," *Physikalische Zeitschrift*, 6, 239–248, 1905.