The factor of S is zero, and the factor of R reduces to

$$-\frac{1}{k\sqrt{p}}\cdot 2r\sqrt{1-e^2}.$$

Hence

$$dM = -\sqrt{1-e^2}\cdot d\omega - \frac{1}{k\sqrt{p}}\cdot 2r\cdot\sqrt{1-e^2}\cdot R$$

This value requires two corrections; one for the motion of the orbit plane, and one for the variation of the mean motion. When we pass from one value of M to another we must use the actual, or disturbed value of μ; and hence must add to $dM \int\frac{d\mu}{dt}\cdot dt$. We have, therefore, a double integral in the value of M.

The component W is the disturbing force that alters the node and inclination. This force makes the orbit plane revolve about the radius-vector (Hansen, *Auseinandersetzung*, p. 67), through the small angle $d\xi$ in the time dt. Consider at any moment the spherical triangle, equinox, node, and planet, or EAP. The side EA is Ω; the side AP is u; and the side EP is constant. The angle at E is constant; the angle at A is $180° - i$; and the variation of the angle at P is

$$d\xi = \frac{W\cdot dt}{c\sin\psi} = \frac{r\cdot W\cdot dt}{k\sqrt{p}}$$

The differential formulas of a spherical triangle give

$$\begin{aligned} di &= \cos u\cdot d\xi \\ du &= -\cos i\cdot d\Omega \\ \sin i\cdot d\Omega &= \sin u\cdot d\xi \end{aligned}$$

Hence we have

$$\frac{di}{dt} = \frac{W}{k\sqrt{p}}\cdot r\cos u$$

$$\frac{d\Omega}{dt} = \frac{W}{k\sqrt{p}}\cdot\frac{r\sin u}{\sin i}$$

The variation du, arising from the rotation of the orbit plane, must be applied to arcs measured on the orbit; such as ω, the distance from the node to the perihelion, the longitude of the perihelion, $\pi = \Omega + \omega$, and the longitude in the orbit, $L = \Omega + u$.

Collecting results we have

$$\begin{aligned}
\frac{da}{dt} &= \frac{2a^2}{k\sqrt{p}}\cdot\left(e\sin v\cdot R + \frac{p}{r}\cdot S\right)\\
\frac{d\mu}{dt} &= -\frac{3a\mu}{k\sqrt{p}}\cdot\left(e\sin v\cdot R + \frac{p}{r}\cdot S\right)\\
\frac{de}{dt} &= \frac{1}{k\sqrt{p}}\cdot\{p\sin v\cdot R + p(\cos v + \cos E)\cdot S\}\\
e\cdot\frac{d\omega}{dt} &= \frac{1}{k\sqrt{p}}\cdot\{-p\cos v\cdot R + (p+r)\sin v\cdot S\} - e\cos i\cdot\frac{d\Omega}{dt}\\
\frac{di}{dt} &= \frac{r\cos u}{k\sqrt{p}}\cdot W\\
\frac{d\Omega}{dt} &= \frac{r\sin u}{k\sqrt{p}}\cdot\frac{1}{\sin i}\cdot W\\
\frac{dM}{dt} &= -\sqrt{1-e^2}\left\{\frac{d\omega}{dt} + \cos i\frac{d\Omega}{dt}\right\} - \frac{2r\sqrt{1-e^2}}{k\sqrt{p}}\cdot R + \int\frac{d\mu}{dt}\cdot dt
\end{aligned}$$

This method of resolving the disturbing force into the components R, S, and W, is old-fashioned, as well as that of T, N, and W, but the former can be easily computed, while the latter cannot. The above formulas give an idea of the action of the disturbing force on the orbit of the planet. The results which Airy has pointed out in his book on Elementary Celestial Mechanics can be deduced immediately from these formulas. The formula for $e\cdot d\omega$ explains the retrograde motion of the line of apsides of the orbit of *Hyperion* through the action of *Titan*.

1906 *March* 29.

NOTES ON SOME LONG-PERIOD VARIABLE STARS,

By A. STANLEY WILLIAMS.

The following notes are in continuation of those already published in the *Astronomical Journal*. The introductory remarks in No. 559, p. 62, will apply generally to the present series. The observations were all made with a 6½-in. reflector, a power of 73 being usually employed. The maxima and minima have all been derived from single curves.

RW Andromedae (189.1904).

R.A. = $0^h\ 41^m\ 56^s$. Decl. = $+32°\ 8'.4$ (1900).

Hartwig suggested a period of 435 days for this variable, with a maximum on 1904 Dec. 10 (*A.N.*, No. 4009). According to observations obtained here on 13 nights, between 1904 Dec. 7 and 1905 Feb. 2, the maximum occurred in 1904 on Dec. 23 (about $8^M.0$). From 1905 Aug. 5 to Nov. 23 the variable remained below 12^M, though on several nights a faint star, about 12^M, was seen at or near its place. On Dec. 27 of last year it was still invisible in a 6½-in. reflector (fainter than $11\frac{1}{2}^M$); but on 1906 Jan. 22, it was nearly equal in brightness to DM. +31°114 ($9^M.0$), and further observations on 5 nights, between this date and Feb. 25, indicate a well-defined maximum for 1906 Feb. 5 (about $8^M.6$). The interval between the two maxima is 409 days, nearly the same as that suggested by Hartwig, so that the resulting elements of variation will be,

Maximum = 1904 Dec. 23 (J.D. 2416838) + 409^d E.

RU Andromedae (11.1903).

R.A. = $1^{h}\ 32^{m}\ 47^{s}$, Decl. = +38° 9′.5 (1900).

Observations made on 21 nights, between 1905 Sept. 27 and 1906 Jan. 13, show that a well-defined maximum occurred on 1905 Dec. 9 ($9^{M}.5$). The following maxima and minima have now been observed:

E	Date	J.D.	Mag.
	MAXIMA.		
5	1903 Dec. 11	2416460	9.9
8	1905 Dec. 9	7189	9.5
	MINIMA.		
4	1902 Dec. 18	2416102	12.7
5	1903 Aug. 25	6352	12.2

From which the following elements have been derived:

Maximum = 1900 Aug. 2 (J.D. 2415234) + $244^{d}.7$ E.

the interval $M-m$ being 108 days.

RV Andromedae.

R.A. = $2^{h}\ 4^{m}\ 34^{s}$, Decl. = +48° 27′.6 (1900).

According to 13 observations, between 1904 Nov. 12 and 1905 Mar. 19, a well-defined minimum ($10^{M}.4$) occurred on 1905 Jan. 29, two days later than the expected date. Six observations in the early part of 1905, by Miss MARY WHITNEY, at Vassar College Observatory, have been published in the *A.N.*, No. 4050. Making allowance for a systematic difference of scale,* these observations are in very good agreement with those made here, excepting that the first estimate makes the star about one-fifth magnitude brighter than mine.

In 1905 the star rose rapidly from $10^{M}.1$ on Aug. 5 to a sharply-defined maximum on Sept. 14 ($8^{M}.0$), and then declined, at first nearly as quickly, to a minimum ($10^{M}.3$) on Nov. 27. The variable then rose to $9^{M}.9$ on Dec. 19, but then remained nearly stationary until 1906 Jan. 22, when a slow rise set in again. This minimum occurred nearly two months earlier than the expected date, and the maximum preceding it nearly a month earlier. There are, perhaps, traces of a secondary minimum about 1906 Jan. 14. Observations were made on 35 nights, between 1905 Aug. 5 and 1906 Feb. 20, so that the light-changes during the intervening period are well ascertained. Owing to the irregularities referred to above, it is doubtful at present what modification of the published elements is indicated. It would seem as though the recent prolonged minimum, or standstill, will have the effect of making the next maximum occur nearly on the computed date.

1205. *Y Persei.*

Observations made on 9 nights, between 1904 Dec. 13 and 1905 Mar. 19, indicate a minimum for 1905 Feb. 11 ($9^{M}.8$), but the results are insufficient to give a good determination of the date. Observations on 22 nights, between 1905 Aug. 5 and 1906 Jan. 13, give a good determination of another minimum for 1905 Oct. 29 ($9^{M}.8$). The above dates of minimum are 11 and 12 days respectively later than those given by the elements of GRAFF.*

RU Persei (187.1904).

R.A. = $3^{h}\ 23^{m}\ 57^{s}$, Decl. = +39° 18′.9 (1900).

Observations on 36 nights, between 1905 Aug. 5 and 1906 Feb. 25, show that the star was at minimum brightness ($10^{M}.4$) on 1905 Sept. 17; and that a well-defined maximum ($9^{M}.4$) occurred on 1905 Nov. 26. At the end of February of this year the star was apparently just about at minimum again.

RY Lyrae.

R.A. = $18^{h}\ 41^{m}\ 15^{s}$, Decl. = +34° 34′.3 (1900).

Observations were made on 9 nights, between 1905 May 5 and Oct. 19. On the former date the variable was $9^{M}.2$, and it diminished steadily in brightness to $12^{M}.4$ on July 27. On Oct. 19 it was invisible in a 6½-in. reflector, and must have been fainter than 12^{M}. Maximum brightness was probably attained about 1905 May 2. As four maxima have now been observed, fresh elements of variation have been derived as under.

Maximum = 1900 Nov. 13 (J.D. 2415337) + 327^{d} E.

The observed maxima are:

E	Date	J.D. 241+	Mag.	O—C
0	1900 Oct. 23	5316	. .	-21^{d}
3	1903 July 25	6321	9.5	+ 3
4	1904 June 10	6642	10.6	− 3
5	1905 May 2±	6968±	9.2	− 4

The first maximum is a photographic one, but it appears to be quite well determined, and it can hardly be more than a few days in error. The maximum of 1905 cannot well be later than May 2, whilst it might be somewhat earlier. It appears probable, therefore, that the period is not of uniform length. According to M. and G. WOLF the variable is not visible, and fainter than 13^{M}, on a Heidelberg plate, dated 1896 Nov. 4 (see *A.N.*, No. 4046). The computed date of maximum in that year is May 22, so that the star should have been faint at the date of the photograph, which thus confirms the general correctness of the elements.

SU Lyrae (59.1905).

R.A. = $18^{h}\ 50^{m}\ 7^{s}$, Decl. = +36° 23′.1 (1900).

This star, the variability of which was discovered at Heidelberg by M. and G. WOLF, was about 11^{M} in the early part of May, 1905, and from this time it decreased in brightness at a nearly uniform rate until July 30, after

* Miss WHITNEY gives $9^{M}.4$ for the magnitude of DM. +48°614. I had assumed it to be $8^{M}.7$, after the DM.

* *Mitteilungen der Hamburger Sternwarte*, Nr. 8, p. 24.

which it could no longer be seen with a $6\frac{1}{2}$-in. reflector. The last negative observation was made on Oct. 4.

6816. *Z Lyrae.*

A well-defined maximum ($9^M.9$) occurred 1905 June 17. Nineteen observations between May 5 and Sept. 3.

6783. *RX Lyrae.*

A maximum ($10^M.9$) occurred 1905 May 19, but this is not a good determination, the curve being somewhat flat, and the observations not well distributed. Twelve observations between the limiting dates May 5 and June 24.

6895. *RU Lyrae.*

Nineteen observations, between 1905 June 22 and Oct. 27, give a good determination of the time of maximum, namely, 1905 Sept. 2 ($10^M.2$). The maximum was a well-defined one.

7019. *T Cygni.*

A well-defined maximum occurred 1905 Sept. 19 ($8^M.6$). Twenty observations between June 22 and Nov. 16.

20 *Hove Park Villas, Hove,* 1906 *March* 9.

7571*a*. *TW Cygni.*

A well-defined maximum occurred 1905 May 26 ($9^M.0$). Fourteen observations between May 5 and Aug. 5.

RV Pegasi (159.1904).

R.A. = $22^h\ 21^m\ 2^s$, Decl. = $+29°\ 57'.9$ (1900).

This star was invisible ($<12\frac{1}{2}^M$) in a $6\frac{1}{2}$-in. reflector on 1905 June 21, but on July 30 it was observed about 11^M, and a pretty definite maximum ($10\frac{1}{2}^M$) was reached on Aug. 5. From this date there was a slow decline to below 12^M by Nov. 23, with a distinct secondary maximum about Aug. 27. The date of the previous maximum is somewhat uncertain, though, having regard to the photographic observations of Götz at Heidelberg, it probably occurred somewhere about 1904 Aug. 5, so that the period is evidently nearly equal in length to a year. It is noteworthy that the recent maximum was about two whole magnitudes fainter than that of 1904, since in the latter year the star must have attained to a visual magnitude of $8\frac{1}{2}$ or brighter. The decline after the maximum of 1904 was also very much more rapid than that after the recent maximum. The comparatively rapid decline in 1904 is shown by the observations of Mr. Wickham at the Radcliffe Observatory, Oxford, published in the *Monthly Notices,* Vol. 65, p. 161.

MICROMETRICAL OBSERVATIONS OF THE FIFTH SATELLITE OF *JUPITER*,

By E. E. BARNARD.

The present season has been very bad for observations of the Fifth Satellite of *Jupiter.*

Though the planet has been splendidly placed for observation, only a few measures of the satellite have been obtained, and those with very great difficulty, with the exception of the latter set of measures on 1905 Dec. 5, at which time the satellite was very easy to measure. Unfortunately the measures were interrupted at midnight on that date, and the complete elongation could not be observed.

I have devoted most of the time to measures from the polar limbs of the planet, because such measures have been very few in the previous observations, and are much needed to determine the inclination of the satellite's orbit. The results are not so satisfactory as I could wish. In bad seeing the polar limbs are not so well seen as the equatorial, because of the duskiness of the polar regions of *Jupiter*, and hence the settings are not so accurate.

Following are all the observations I have been able to secure. The times are $6^h\ 0^m$ slow of Greenwich Mean Time.

Observations of the Satellite.

1905	Central Stand. Time		Appt. lat.	Comp.	
	h m s	From N. limb ″	″		
Oct. 28	10 3 54	23.48	−0.78	5	Satel. preceding. Seen for only a few minutes
		From S. limb			
Oct. 28	10 10 53	22.45	−0.24	1	
		From S. limb			
Nov. 25	12 55 56	25.05	+1.84	4	Satel. following. Seeing very poor.
		From N. limb			
Nov. 25	13 8 21	22.18	+1.03	4	

1905	Central Stand. Time		Appt. lat.	Comp.	
	h m s	From S. limb ″	″		
Nov. 25	13 12 31	24.28	+1.07	4	Very difficult.
		From N. limb			
Nov. 25	13 15 43	22.65	+0.56	4	
		From S. limb			
Nov. 25	13 19 1	24.18	+0.97	4	
		From f. limb	From center		
Nov. 25	13 24 18	34.21	58.96	4	
	13 26 52	34.28	59.04	3	