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ON THE EVOLUTION OF THE SOLAR SYSTEM

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1. *Introduction.*—For about a century it has been quite generally believed that, in an earlier state, our system was a widely extended nebula; and that, contracting under the mutual gravitation of its parts, rings were left off which later developed into the planets. There have been numerous variations from the original theory as formulated by Laplace, but they have all been essentially the same dynamically, for in all of them it was assumed that the original mass was in a state of temporary hydrodynamical equilibrium, maintaining its volume largely by gaseous expansion, and that the planets have developed out of rings left off from the parent mass. In 1900 Professor T. C. Chamberlin and the writer undertook, so far as was possible, to test,[1] by an appeal to the laws of dynamics, the consistency of this ring theory with known phenomena. Contradictions were uniformly found, and in some cases the results were so conclusive as to compel us frankly to abandon it as an untenable hypothesis.

Having given up the ring theory, the problem has been to find, if possible, something more satisfactory. The result has been the

[1] Chamberlin, "An Attempt to Test the Nebular Hypothesis by the Relations of Masses and Momenta," *Journal of Geology*, February-March, 1900. Moulton, "An Attempt to Test the Nebular Hypothesis by an Appeal to the Laws of Dynamics," *Astrophysical Journal*, 11, 103–130, 1900.

formulation of a fairly definite theory, which Professor Chamberlin calls the "Planetesimal Hypothesis," and which he has expounded in his paper, "Fundamental Problems in Geology," in *Year Book* No. 3 of the Carnegie Institution of Washington, pp. 195–258. The present paper is devoted to a brief account of some of the main dynamical features of the theory, and to some comments on the retrograde revolution of *Saturn's* ninth satellite. For the sake of brevity in the exposition, the theory will be given in categorical terms, without implying in the least that it is not yet open to question at every point.

2. *Outline of the theory.*—It is supposed that our system has developed from a spiral nebula, perhaps something like those spiral nebulæ which Keeler showed are many times more numerous than all other kinds together.[1] The spiral nebula is supposed to have originated at a time when another sun passed very near our Sun. The dimensions of the nebula were maintained almost entirely by the orbital motions of the great number of small masses of which it was composed, and only a very little by gaseous expansion. It was never in a state of hydrodynamical equilibrium, and the loss of heat was not necessary for its development into planetary masses. The planets have been formed around primitive nuclei of considerable dimensions by the accretion of the vast amount of scattered material which was spread throughout the system.

Such a spiral nebula as that described, having originated in such a way, will develop into a system having the following properties: The planets will all revolve in the same direction, and approximately (though perhaps not exactly) in the same plane; the sun will rotate in the same direction, and nearly in the same plane, and will have an equatorial acceleration; the more the planets grow by the accretion of scattered matter, the more nearly circular will their orbits become; the planets will rotate in the forward direction, and approximately (though perhaps not exactly) in the planes of their orbits; the more a planet grows by the accretion of scattered matter, the more rapidly will it rotate; the planetary nuclei may be attended originally by many satellite nuclei revolving in any direction, but the scattered material will tend to drive all those satellite nuclei down

[1] *Astrophysical Journal*, 11, 347–348, 1900.

on to the primary nucleus which do not move forward in the general plane of the system; the scattered material develops and preserves circularity in the satellite orbits, if they revolve in the forward direction, but considerable eccentricity, if in the retrograde direction; a satellite may revolve more rapidly than its primary rotates; the system may contain many planetoids whose orbits are interlocked; the small planets will be cool and dense, and the large ones hot and rare; and the greater part of the moment of momentum of the system will belong to the planets. It will now be shown that these statements are true.

3. *A possible origin of spiral nebulæ.*—In view of the relative motions of the stars, it is to be expected that two will sometimes pass near each other, and very much less frequently actually collide. At the time of near approach of two large masses the mutual tidal strains are very great. It is supposed that once a sun, which we shall call S', passed near our Sun, S, and raised on it a huge tide on the side toward S', and an almost equal one on the opposite side. It follows from the well-known theory of tidal forces that the effect on S was equivalent to a diminution of its attraction in the line passing through S', and an increase of its attractions in other directions. Roche has shown that when the bodies are nearer each other than 2.44 times their radii, the self-gravitation of one of them is more than balanced by the differential tidal forces due to the other.

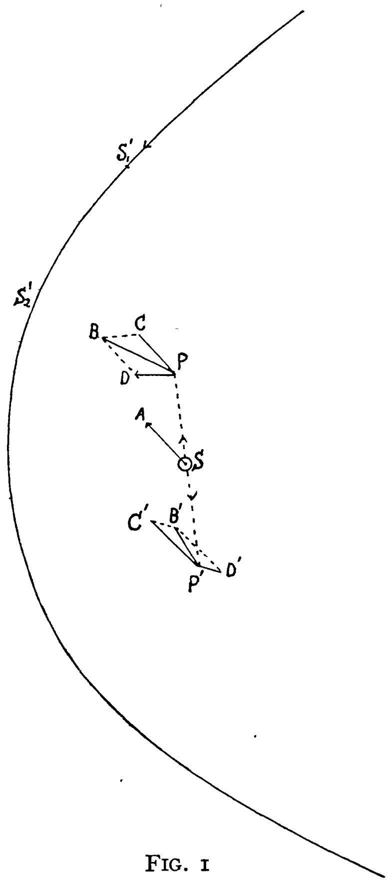

FIG. 1

Our Sun was then agitated by great energies such as now produce the eruptive prominences. The enormous tidal strains attending the near approach of S' increased the eruptive tendencies of S toward and from S', and large quantities of material were ejected with great velocities in both of these directions. If it had not been for the subsequent disturbing effects of S', these ejected masses would have returned to the Sun, but S' drew them from their rectilinear paths and left them describing ellipses around the Sun.

To show how this was done consider Fig. 1. Suppose the masses

P and P' were ejected from S in opposite directions when S' was at S'_1, and consider the character of the disturbing forces when S' has arrived at S'_2. Let $\overline{SA}$ represent the acceleration of S' upon S in direction and amount. In the same units let $\overline{PB}$ and $\overline{P'B'}$ represent the acceleration of S' upon P and P', respectively. Resolve $\overline{PB}$ and $\overline{P'B'}$ each into two components, so that one of them in each case (PC and $P'C'$) shall be equal and parallel to $\overline{SA}$. Since $\overline{SA}$, $\overline{PC}$, and $\overline{P'C'}$ are equal and parallel, they do not disturb the relative positions of S, P, and P'. The disturbing accelerations are therefore $\overline{PD}$ and $\overline{P'D'}$ in direction and amount. It is observed that P and P' are both disturbed so as to start to revolve around S in the direction of the motion of S'.

While the character of the disturbing forces can be shown in this way, the precise results of their continued action can be determined only by computation. The labor of this computation is great, for the disturbing forces are very large, the curve described is complicated, and S' is for several years near enough sensibly to modify the motion of P and P'. The orbits described by the ejected material have depended upon the mass of S', the nearness of its approach to S, the relative position of S' at the time the material was ejected, and the velocity of ejection. At present it seems necessary in order to get a thorough understanding of the dynamics of the subject to treat by numerical processes a large number of special cases. This work is at present under way, and in all the cases so far considered P and P' have been left moving in elliptical orbits.

It remains to show that the nebula will have a spiral form immediately after the departure of S'. Figure 2 shows the positions of the masses ejected at successive intervals. The dotted lines are the actual curves which have been described, and the full lines show the apparent form of the spiral. There will be, of course, a vast quantity of fine material scattered throughout the system. The striking feature of the nebula is that there are two arms of the spiral starting from opposite sides of S. This is precisely the condition revealed by photography in the spiral nebulæ, often most unmistakably. It is to be observed that the motion is not along the lines of the spiral, and that, therefore, as the spiral grows older it will become more and more coiled.

Whether or not computation shall verify this conjecture respecting the origin and nature of spiral nebulæ, it seems probable, both from their appearance and from their spectra,[1] that the theory has led to a correct picture of their physical and dynamical condition. Doubtless those spirals which have been photographed are immensely larger than the one from which our system may have developed, and as a rule have relatively less massive centers.

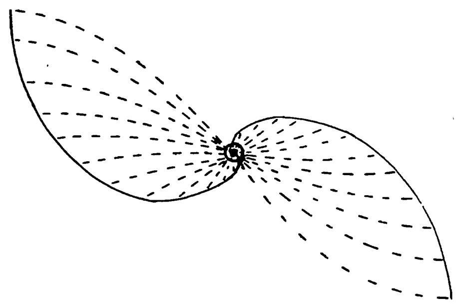

FIG. 2

4. *The revolutions of the planets.*—The matter was originally ejected more or less irregularly with occasional large nuclei which have grown into the planets by the accretion of the finer scattered material. The direction of ejection depended upon the direction of S', the lag of the tides on S, and the original direction and rate of rotation of S. It is very improbable that the plane of the original equator of S coincided with plane of the orbit of S'. Consequently the original ejections were not all exactly in the plane of motion of S'. A little consideration shows that, taking into account the ejections both toward and from S', and both before and after perihelion passage of S', the matter was distributed nearly symmetrically with respect to the plane of the orbit of S'.

It follows from the mode of generation of the elliptic orbits of the ejected material that the planets must all revolve in the same direction, and from the statements just made that their planes will nearly, though not exactly, coincide. It follows from the symmetrical distribution of the ejected material that the more a planet grows by the accretion of the scattered material, the more nearly will the plane of its orbit coincide with that of the orbit traversed by S'. Consequently we should expect but slight divergences in the planes of the orbits of the large planets, which have grown most, and on the average much greater differences in the orbits of such small bodies as *Mercury* and the planetoids. Interpreted according to this

[1] E. g., Scheiner found that the *Andromeda* nebula seems to have a dark line spectrum, *Astronomische Nachrichten*, **148**, 325, 1899.

theory, the high inclination (10°) of the orbit of *Eros*, which lies so close to the orbits of the Earth and *Mars*, is nothing to occasion surprise.

5. *The rotation of the Sun.*—The present rotation of the Sun is the resultant of its original rotation and of the disturbance due to S'. The original direction and rate of rotation of S are quite unknown. Its rotation was affected in two ways by the passage of S'. (*a*) S' raised large tides on the Sun and dragged them around in the direction of its motion. This contributed a certain rotation to the Sun in the direction of motion of S'. (*b*) Some of the ejected particles left the Sun with small velocities, and fell back upon it before their orbits were greatly perturbed by S'. But all of these particles had acquired some moment of momentum in the direction of the motion of S', and, falling obliquely into the Sun, they gave up their moment of momentum to this body. In these two ways a rotation was developed in the Sun agreeing approximately with that of the general motions of the planets.

Both of the influences which have been mentioned were most important in the equatorial zone, and extended to relatively shallow depths. Consequently the Sun was given an equatorial acceleration which still persists. The spots occur where the layers having different rates of rotation flow most rapidly past each other.

6. *The eccentricities of the orbits of the planets.*—The nuclei around which the planets formed were left by S' revolving in ellipses, presumably of considerable eccentricity. The probable amount will be revealed in the course of time by our computations. The orbits of the scattered particles had every possible orientation and crossed the orbits of the nuclei. The nuclei swept up these particles, and in the process had the eccentricities of their orbits changed. The question of interest is whether the eccentricities were increased or decreased. It is observed first that the more nearly two orbits have the same major axis and eccentricity, the more likely are bodies moving in them to collide; for under these circumstances the orbits may intersect at the most acute angle. The case of collision of two bodies moving in such orbits will be treated.

Let a_0 and e_0 be the common major semi-axis and eccentricity before collision, and a and e the corresponding elements of the orbit of the combined mass after collision. Let N represent the mass of the nucleus, and m that of the particle. Let us neglect the slight perturbations that N and m produce in each other's motions, and consider only the effects of a collision at a point where their orbits cross. At the instant before the impact of the two bodies their kinetic energy was

$$\frac{1}{2}(Nv_N^2+mv_m^2) \ ,$$

where v_N and v_m represent their respective velocities. After impact the kinetic energy of their combined mass was

$$\frac{1}{2}(N+m)v^2 \ ,$$

where v is the velocity of the combined mass. Since part of the kinetic energy will have been transformed into heat by the impact, we have

$$Nv_N^2+mv_m^2>(N+m)v^2 \ . \qquad (1)$$

But it follows from the theory of elliptic motion in the problem of two bodies that

$$v^2=k^2\left(\frac{2}{r}-\frac{1}{a}\right) \ . \qquad (2)$$

Hence, since r was the same for both N and m the instant before impact, and for the combined mass the instant after impact, we have, substituting in equation (1),

$$Nk^2\left(\frac{2}{r}-\frac{1}{a_0}\right)+mk^2\left(\frac{2}{r}-\frac{1}{a_0}\right)>k^2(N+m)\left(\frac{2}{r}-\frac{1}{a}\right) \ .$$

It follows from this equation that

$$a<a_0 \ . \qquad (3)$$

The moments of momentum of N and m before impact were respectively

$$k^2N\sqrt{a_0(1-e_0^2)} \text{ and } k^2m\sqrt{a_0(1-e_0^2)} \ .$$

After impact the moment of momentum of the combined mass was

$$k^2(N+m)\sqrt{a(1-e^2)} \ .$$

Since the moment of momentum of a system is not changed by collisions, we have

$$k^2N\sqrt{a_0(1-e_0^2)}+k^2m\sqrt{a_0(1-e_0^2)}=k^2(N+m)\sqrt{a(1-e^2)} \ .$$

Making use of the inequality (3), it follows that

$$e < e_0 . \tag{4}$$

That is, in this most important case the eccentricity of the nucleus was always decreased, in whatever manner the collision may have taken place. When the postulated conditions were anywhere nearly fulfilled, an overwhelming majority of collisions operated in the same way, though there were cases where the eccentricity was increased. The conclusion is that the more a planet grew by accretion, the more nearly circular, in general, its orbit became.

Let us compare this with the solar system. The orbits of the terrestrial planets average more than twice as eccentric as those of the great planets. Being nearer the Sun, where friction would have destroyed most irregularities, the ring theory would demand that their orbits should be more circular. The smallest planet, *Mercury*, has an orbit more than twice as eccentric as that of any other. The orbits of the planetoids are, on the average, three times as eccentric as those of the planets, and about one planetoid in four has an orbit more eccentric than that of *Mercury*.

7. *The rotation of the planets.*[1]—There is no reason to assume that the nuclei were originally rotating in any particular direction, and very probably they rotated in various directions. The present rotation of a planet depends upon the original rotation of its nucleus, upon the effects of S' on this nucleus, and upon the effects of the impacts of the scattered material. So far as S' influenced the rotation by the tides it generated, it tended to make it forward. The effects of the impacts will be considered, neglecting for simplicity the eccentricity of the orbit of the nucleus.

Suppose the nucleus N traveled around the Sun so as to fill the space between the curves a and b, and so that its center traveled along the circle c. The orbits of the small masses, m, which entered the path of N will be divided into three classes: (1) those orbits whose perihelia were inside of b and whose aphelia were between b and c; (2) those orbits whose perihelia were inside of c and whose aphelia were outside of c; (3) and those orbits whose perihelia were

[1] This idea was first developed briefly in approximately the present connection by Chamberlin, "A Group of Hypotheses Bearing on Climatic Changes," *Journal of Geology*, **5**, 653, 1897, footnote, pp. 668–669.

PLATE V

SPIRAL NEBULA, *M* 51 *CANUM VENATICORUM*
Photographed with Two-foot Reflector of Yerkes Observatory by G. W. Ritchey, 1902

between c and a and whose aphelia were outside of a. They are represented by the ellipses e_1, e_2, e_3, respectively in Fig. 3.

Case of e_1.—Since the major axis of the orbit of N was greater than that of m, it follows from equation (2) that at the time of their collision N overtook m. It is easily seen from Fig. 3 that this collision tended to give N a forward rotation.

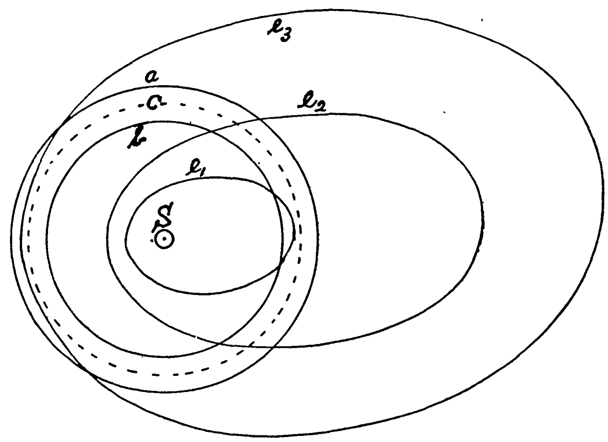

FIG. 3

Case of e_2.—In this case N and m were moving with nearly equal velocities, and it follows that the blow on N was largely radial. Therefore the effect of any single impact was small, and the combined effects of many, which tended to give rotations in various directions, cannot have been important.

Case of e_3.—In this case m overtook N outside of its center and tended to give it a forward rotation.

Thus, in the two cases where the impacts were most effective in changing the rotation, N was given a forward rotation. Therefore we should expect to find the planets rotating forward approximately in the planes of their orbits. The inclinations of the planes of their equators to the planes of their orbits might differ, because of differences in the original rotations of the nuclei, from planet to planet as in the case of *Saturn* and *Jupiter*. If there were any marked exception to the rule, we should expect to find it in the outermost planets where the particles moving in the orbits of the third type would be few. Moreover, the larger a nucleus was, the greater were the relative velocities of impact, the greater was the momentum contributed for a given mass and velocity, and therefore the larger was the amount of the moment of momentum contributed to N. For this reason, as well as for the fact that the larger nuclei probably acquired relatively larger amounts of scattered material than the smaller nuclei, we should expect to find the larger planets rotating more rapidly than the smaller.

8. *The satellites.*—When the planetary nuclei left the Sun, they were attended by many smaller secondary nuclei. When the velocity of the secondary nucleus was very small relatively to its primary nucleus, it was attracted into the larger mass and lost its separate existence. When the velocity of the secondary nucleus was large compared to its primary nucleus, it passed away from its gravitative control and began a career as an independent body. In all other cases the secondary nuclei revolved around the primary nuclei, and there is no reason for supposing they had originally any general direction of revolution.

For the purposes of the discussion, these secondary nuclei will be divided into three classes depending upon the positions of the planes of their orbits and their directions of revolution. The first class will consist of those whose orbits were highly inclined to the planes of motion of their respective primary nuclei; the second class, of those which revolved nearly in the planes of motion of the primary nuclei, but in the forward direction; and the third, of those whose revolution was nearly in the planes of motion of the primary nuclei and retrograde.

Secondary nuclei of the first class.—The problem for consideration is the effect of the impact of the scattered material with the secondary nuclei.

There is not room to go into details here, but it is easy to see that whenever the motion of the secondary nucleus was perpendicular to the plane of the general system, the scattered material acted upon it like a resisting medium. The result was that the size of the orbit was diminished. When its mass had become doubled by the accretion of the scattered material, its orbit had shrunk, by this cause alone, to one-fourth of its former dimension. In addition to this, the increase in mass of the planetary nucleus still further reduced its orbit. Consequently the secondary nuclei of the first class were in general precipitated on their respective primary nuclei. Only those have survived which were originally very remote, but not beyond the gravitative control of the primary, and which developed under the exceptional conditions prevailing near the borders of the system.

Secondary nuclei of the second class.—It can be shown, by a discussion similar to that given in section 7, that in this case the collisions tended to increase the dimensions of the orbits of secondary nuclei, and that, therefore, some of these bodies have maintained their separate existence. The increase of velocity was greatest when the secondary nuclei were remotest from their respective planetary nuclei. The principles of celestial mechanics show that this has made their orbits continually more nearly circular.

Secondary nuclei of the third class.—It can be shown, by a discussion similar to that given in section 7, that in this case the scattered material acted as a resisting medium. Moreover, up to a certain point the eccentricities were increased. Consequently most of those secondary nuclei which moved in the retrograde direction were precipitated upon the primary nuclei around which they revolved.

In a system having both direct and retrograde satellites it is to be expected, on the basis of this theory, that those moving in a retrograde direction will be very remote and have considerable eccentricities. This is the case with the ninth satellite of *Saturn*, whose orbit is about twice as eccentric as that of any other satellite.[1] The peculiar positions of the orbits of the satellites of *Uranus* seem to indicate that in this case the original secondary nuclei were moving in a special way. While the conditions prevailing in the satellite system of *Uranus* would not have been predicted on the basis of this theory, yet they do not definitely contradict it as they do the ring theory. The large planetary nuclei had gravitative control over secondary nuclei with large relative velocities, and for this reason the chances for their having several satellites are more favorable than in the case of small planets.

There is no reason why a satellite may not revolve more rapidly than its primary rotates, especially as its period continually diminishes as the mass of the planet increases.

9. *The planetoids.*—The planetoids have formed from material whose orbits did not cross a region swept by a large nucleus. Their ability to grow has been small because of their small dimensions and feeble gravitating power. The variations of the eccentricities

[1] According to the latest data, received since this statement was in print, the orbit of the seventh satellite of *Jupiter* seems to be retrograde and very eccentric.

and inclinations of their orbits are a measure of the heterogeneity prevailing after the passage of S' and before the equalizing effects of collisions had been realized. *Eros* is the one of considerable dimensions between the orbit of the Earth and *Mars*, whose orbit was so highly inclined that it did not lose its separate existence by collision with one of these larger bodies.

10. *The physical condition of the planets.*—The original nuclei which have grown into the terrestrial planets were of such small mass that they did not possess sufficient gravitative power to control true atmospheres. Consequently they cooled rapidly and became solid. The scattered material likewise lost its heat rapidly. Hence these planets have grown up from solid material, and have been solid throughout nearly their whole history since the visit of S', and their atmospheres have been acquired in a later stage of their evolution from the occluded gases which escaped as they contracted. The present internal heat and the past igneous action, of which there is abundant evidence in the Earth, are due partly to the residue of original heat which was not lost by radiation, and much more to the heat generated by the contraction of the planets from the density of the material when it fell on them to their present density. The writer has shown the quantitative effectiveness of such moderate contractions in bodies having the dimensions of the planets.[1]

The original nuclei which have grown into the great planets were large enough to retain true atmospheres. Consequently their original heat has been much more largely retained. For the same reason it is probable that they contain a larger proportion of the light volatile substances than the smaller planets do. The larger nuclei attracted the scattered material upon themselves with considerable velocities, and the heat generated by the impacts was very great. In the case of the small planets this surface heat was rapidly radiated away, but the atmospheres of the great planets retained it largely, and for this reason their original fluid state has been immensely prolonged.

11. *The moment of momentum of the system.*—It follows from the origin of the system that the remote planets should possess most of the moment of momentum of the system. This is precisely what is

[1] Chamberlin, "Hypotheses Bearing on Climatic Changes," *Journal of Geology*, **5**, 674, 1897, and Moulton, *Introduction to Celestial Mechanics*, p. 61, problem 5.

found. As Professor Chamberlin has pointed out, *Jupiter* contains about one-tenth of 1 per cent. of the mass within the orbit of *Saturn*, and more than 95 per cent. of the moment of momentum. This is an inevitable consequence of the spiral theory, but, on the contrary, the whole question of moment of momentum is a rock on which the ring theory breaks.[1]

12. *The ninth satellite of Saturn.*—The retrograde revolution of the ninth satellite of *Saturn*, taken in connection with the forward revolution of the other satellites and the direction of rotation of the planet, contradicts what has been considered up to the present time to be an inevitable consequence of the ring theory.[2] Nevertheless, the discoverer, Professor W. H. Pickering, has attempted to explain how it may have originated without contradicting the ring theory.[3] This section is devoted to an examination of these explanations.

Pickering's first suggestion is that *Phoebe* may have been originally a comet which has been captured by *Saturn.* It can scarcely be possible that the eccentricity of *Saturn's* orbit, or that the perturbations of *Saturn* by *Phoebe*, would assist in the supposed capture. Therefore we shall neglect in this discussion the eccentricity of the orbit of *Saturn* and the mass of *Phoebe.* This reduces the problem to that treated in the writer's *Introduction to Celestial Mechanics*, Chapter VII. It is shown there that if the constant of the Jacobian integral is sufficiently large, the surfaces of zero relative velocity are closed around the finite bodies, and consequently that a satellite or inferior planet in this case must remain permanently a satellite or inferior planet.[4] By the methods explained there it is found that if the constant is greater than 3.0180, the surface around *Saturn* is closed. The constant that belongs to the ninth satellite is found to

[1] Chamberlin, "An Attempt to Test the Nebular Hypothesis by the Relations of Masses and Momenta," *Journal of Geology*, **8**, 58–73, 1900. Moulton, "An Attempt to Test the Nebular Hypothesis by an Appeal to the Laws of Dynamics," *Astrophysical Journal*, **11**, 128, 1900.

[2] The observations at present seem to indicate that the sixth satellite of *Jupiter* revolves in the forward direction, and the seventh retrogrades at just about the same distance.

[3] *Annals of the Harvard College Observatory*, **53**, No. III, pp. 60–61.

[4] Hill used this method in connection with his special differential equations to prove the existence of a superior limit to the Moon's radius vector. *American Journal of Mathematics*, Vol. I.

have the value 3.0626. Consequently it can never escape from *Saturn's* control, and conversely it has never come under *Saturn's* control from a remote region. The radius of *Saturn's* largest closed surface of zero relative velocity is only about 40,000,000 miles, and it follows that *Jupiter* cannot have assisted in making the capture.

The other suggestion, which evidently was considered much more probable, is that *Saturn* once extended out to the orbit of *Phoebe*, and rotated in the retrograde direction with the angular velocity of the orbital motion of this satellite. The Sun is supposed to have raised large tides in this widely extended mass. Now, tides are generated on the side toward and the side opposite the tide-raising body, and it follows that the viscosity of the mass would in time reduce the rotation so that one side would be constantly toward the Sun. That is, tidal friction would in time give it a forward rotation with the angular velocity of the planet's motion around the Sun. As the planetary mass contracted, it would rotate more rapidly in the forward direction, and the interior nine satellites and the rings are supposed to have been left behind in this stage.

Let us examine the question quantitatively. Since the moment of momentum is a signed quantity, we may suppose that it is negative in the case of a retrograde rotation, and positive in the case of a direct rotation. Then, according to the explanation suggested by Pickering, the Saturnian nebula originally had a negative moment of momentum. The tides raised by the Sun destroyed this moment of momentum, and contributed positive moment of momentum until the rotation and the revolution of the mass were made in the same period. But when the rotation in the forward direction became faster than the revolution as the mass contracted, then the tides acted in the opposite direction and decreased the moment of momentum. Consequently, *the Saturnian system had its maximum moment of momentum with respect to the center of Saturn when its rotation was forward, with a period exactly equaling the period of Saturn's revolution around the Sun.* When the mass had shrunk down to the size of the orbit of *Japetus*, it must have rotated at the rate *Japetus* now moves, and we must find, if the theory is true, that its moment of momentum had been decreased by the retarding effects of the tides raised by the Sun.

Let us first compute an upper limit to the maximum moment of momentum. We do not know the dimensions of the mass, except that it was somewhere between the orbits of *Phoebe* and *Japetus*, when it had its maximum. But the greater its radius was, the greater was its maximum moment of momentum. We shall certainly get something *too large*, if we suppose that this condition was realized when it extended entirely out to the orbit of *Phoebe*. We do not know the law of density; but the mass was certainly densest at its center. If we assume that it was homogeneous, we shall certainly get *too large* a result. Since the mass rotated on its axis but once in 29.5 years, it cannot have been sensibly flattened at its poles. Hence we shall certainly have an upper limit to the maximum moment of momentum, if we compute it under the assumptions that the mass extended out to the orbit of *Phoebe* at this time, and that it was homogeneous.

Consider the case when the mass had shrunk to the dimensions of the orbit of *Japetus*. It rotated in the period of revolution of this satellite, and may have been considerably flattened at the poles. If we assume that it was not flattened, we shall get a result which is *certainly too small*. If we assume that it is homogeneous, we shall get a result which is, so far as this factor is concerned, too large. We shall undoubtedly be nearer the truth if we assume that it obeyed the Laplacian law of density, and that its surface density was very small compared with the mean density.

With the assumptions made, the formula for the upper limit of the maximum moment of momentum is

$$M_M=\frac{2}{5}mR_P^2\omega_S ,$$

where m is the mass of the Saturnian system inside of the orbit of *Phoebe*, R_P the mean radius of the orbit of *Phoebe*, and ω_S the mean angular velocity of *Saturn* around the Sun. The moment of momentum at the time the mass extended to the orbit of *Japetus* was less than, or perhaps about equal to,

$$M_J=\frac{2}{3}\left[1-\frac{6}{\pi^2}\right]mR_J^2\omega_J=0.2614\, mR_J^2\omega_J ,$$

where R_J represents the radius of the orbit of *Japetus* and ω_J is its angular rate of revolution.

Hence we find for the ratio of the upper limit of the maximum moment of momentum to that at the time the mass extended to the orbit of *Japetus*,

$$\frac{M_M}{M_J}=\frac{0.4R_P^2\omega_S}{0.2614R_J^2\omega_J}=\frac{1}{0.6535}\left(\frac{R_P}{R_J}\right)^2\left(\frac{T_J}{T_S}\right),$$

where T_J and T_S are the periods of *Japetus* around *Saturn* and *Saturn* around the Sun respectively. It is known that

$$R_J=2,225,000 \text{ miles},$$
$$T_J=79.3 \text{ days},$$
$$T_S=29.46\times 365.25 \text{ days},$$

and from Pickering's elements of the orbit of *Phoebe* that

$$R_P=7,996,000 \text{ miles}.$$

Consequently

$$\frac{M_M}{M_J}=0.1456 .$$

That is, we have found that the upper limit to the maximum moment of momentum is only one-seventh of something certainly less than the maximum. The only explanation of this contradiction is that the supposed development of the Saturnian system upon which the computation was based is erroneous. It follows that this retrograde revolution of *Phoebe* is actually, as well as apparently, squarely contradictory to the ring theory.

13. *Conclusions.*—While only abstracts of a portion of the discussions have been made in this paper,[1] enough has been said to show that the spiral theory is even now a good working hypothesis. It explains all the phenomena upon which the ring theory rested, and many others which are contradictory to the ring theory. Nothing has yet been found which seems seriously to question its validity.

The spiral theory raises a whole series of new and very difficult questions in celestial mechanics. These are the immediate effects of the tidal forces which are developed by the near approach of two suns, the perturbations of the orbits of matter which has been ejected by one of them under a variety of conditions, and the secular evolution of the orbits of this ejected material. A large amount of labor

[1] More details are given in the second volume of the *Geology* by Chamberlin and Salisbury, and in the writer's *Introduction to Astronomy*, both of which are to appear soon.

will be required to carry the discussion of these questions to a successful conclusion.

The spiral theory is fertile in suggesting new considerations for interpreting the immense variety of special phenomena of the system. It is not too much to expect that it may suggest new questions for observational investigation. It affords geologists new conceptions of the early history of the Earth. But perhaps its most interesting contribution is to our general philosophy of nature. Heretofore we have regarded the cosmical processes as forever aggregating matter into larger and still larger bodies, and dissipating energy more and more uniformly. Now we recognize important tendencies for the dispersion of matter.[1] This idea has introduced an element of possible cyclical character in the evolution of the heavenly bodies, though the question of the source of the requisite energy is serious.[2] There is hope that the difficulties of this question may soon be relieved, for recent discoveries respecting the internal energies of atoms suggest the possibility that the Helmholtzian contraction theory explains the origin of only a part of the energy given up by the stars.

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[1] This idea was first sharply worked out in Chamberlin's paper "On a Possible Function of Disruptive Approach in the Formation of Meteorites, Comets, and Nebulae," *Astrophysical Journal*, **14**, 17–40, 1901. The phenomena of radioactivity point in the same direction, though in quite a different way.

[2] This trouble is equally serious under the ring theory. See Chamberlin's "On Lord Kelvin's Address on the Age of the Earth as an Abode Fitted for Life," *Smithsonian Report* for 1899, p. 240.