1901ApJ....13...25M

THE RADIATION OF A BLACK BODY.

By C. E. Mendenhall and F. A. Saunders.

In the present article it is proposed to give, first, a brief review of recent work in connection with the radiation of an absolutely black body, and, second, an account of some experiments of this character carried on in the Physical Laboratory of the Johns Hopkins University. The results of the latter were largely negative; but a statement of methods and difficulties may be of service to others engaged in the same line of work.

In most cases the method of producing the "black body" has been based on Kirchhoff's discussion of the problem of radiation in a uniformly heated enclosure; a hollow body, preferably of good conducting material and having an aperture, being heated as uniformly as possible, the radiation emerging from the aperture has been taken as that of a "black body," and examined by appropriate means. Another method has been suggested and used by Paschen,[1] but it seems to be, on the whole, less satisfactory; in this case a radiating strip is put near the center of a reflecting enclosure having an aperture through which passes the radiation to be examined. In order to consider the subject to the best advantage it will be well to group the materials around the most important of the so-called "laws"—which have been obtained for the most part theoretically, and which have in turn been put to test by recent experiment. For brevity these laws will sometimes be referred to by number, corresponding to those given below, the following symbols being used:

S = total radiant energy at any absolute temperature.
T = this absolute temperature.
$Ed\lambda$ = energy radiated in waves of length $< \lambda + d\lambda$ and $> \lambda$.
λ = any wave-length expressed in thousandths of a millimeter — μ.

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[1] Paschen, *Wied. Ann.*, 60, 1897; Paschen and Wanner, Astrophysical Journal, 9, 40, 1899; 11, 297, 1900.

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λ_m = wave-length of maximum energy at temperature T.
$E_m d\lambda$ = amount of (maximum) energy at temperature T, between the limits, $\lambda_m \pm \frac{d\lambda}{2}$.
A, B, C, c, α, are constants.

The following "laws" will be considered:

(I)	$S = \text{const. } T^4$.	A relation between total radiation and temperature.
(II)	$\lambda T = \text{const.}$	A relation between wave-length and temperature.
(III)	$\lambda_m T = A$.	A relation between wave-length of maximum of energy-curve and the corresponding temperature.
(IV)	$E_m T^{-5} = B$.	A relation between maximum ordinate of energy-curve and the corresponding temperature.
(V)	$E = C\lambda^{-5} e^{-\frac{c}{\lambda T}}$.	Gives distribution of energy in spectrum at any temperature; *i. e.*, is equation of energy-curve.

Equation (I), $S = \text{const. } T$,^{4} expressing the well-known law of Stefan, has been subjected to experimental test recently by Lummer and Pringsheim,[1] and by Paschen.[2] The first named attacked the problem most directly and found, as can be seen from Table I, a fairly satisfactory agreement with theory.

TABLE I.

T.	373°	492°	733°	755°	799°	820°
Obs. S.	156	638	3320	3810	4440	5150
Calc. S.	143	600	3270	3700	4660	5170

T.	877°	1106°	1125°	1403°	1492°	1522°	1561°
Obs. S.	6190	16400	17700	44700	57400	60600	67800
Calc. S.	6180	17200	18500	45000	57600	62400	69100

It is to be noted here that no correction (apparently) was made for the absorption of CO_2 and H_2O vapor in the atmosphere, nor were any precautions taken to diminish this absorption. This work has since been extended to 1700° abs. in one direction and about 100° abs. in the other, and Stefan's law found to be satisfied to within a few per cent. The Stefan relation has also been deduced by Planck,[3] from the basis of the electromagnetic theory of light.

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[1] LUMMER and PRINGSHEIM, *Wied. Ann.*, **63**, 395, 1897.

[2] PASCHEN, *Wied. Ann.*, **58**, 60, 1896, 1897.

[3] M. PLANCK, *Drude's Ann.*, **1**, No. 1, 1900.

The expressions (III), (IV), $\lambda_m T = A$ and $E_m T^{-5} = B$, follow at once from Wien's[1] so-called "Verschiebungsgesetz," $\lambda T =$ const. (II), and this, as originally developed, assumed the truth of Stefan's law. This "Verschiebungsgesetz" of Wien states nothing as to the distribution of energy at any one temperature, but states that this distribution must change with the temperature in such a manner that if there is any definite amount of energy corresponding to a given wave-length λ, at a temperature T, this same amount of energy will at any other temperature T_1 be emitted in waves whose length is determined by the relation $\lambda T = \lambda_1 T_1$. The expressions (III) and (IV) also are necessary consequences of (V), which, in turn, has been theoretically developed by Planck (*loc. cit.*). Thiesen[2] has objected to part of Wien's reasoning, and has deduced the relation $\lambda T =$ const. by another process.

The expressions (III) and (IV) have been very elaborately tested by experiment. Not to mention earlier work, Lummer and Pringsheim[3] obtained the following series of values for A and B:

A	B
2928	2246×10^{17}
2974	2184
2959	2176
2966	2164
2956	2166
2980	2208
2950	2166
2814	2190
2940	2188

Two other series of observations, with different arrangement of apparatus in each case, gave values for A of 2940 and 2930, and equally satisfactory constancy for B.

Paschen[4] finds, as a mean for a number of independent series

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[1] W. WIEN, *Ber. d. Berl. Akad.*, **6**, 1893.

[2] THIESEN, *Verh. d. Deutsch. Phys. Ges.*, **2**, No. 5, 1900.

[3] LUMMER and PRINGSHEIM, *Verh. d. Deutsch. Phys. Ges.*, **1**, 12.

[4] PASCHEN, ASTROPHYSICAL JOURNAL, **10**, 40, 1899; **11**, 288, 1900; Paschen and Wanner, *ibid.*, **9**, 300, 1899.

of observations, extending from about 150° C. to 1300° C. and over a range of wave-lengths from 0.5 μ to 9.2 μ, the value of A to be 2907; while the maximum variation of B is 4 per cent.

In the later work of Lummer and Pringsheim the apparatus was so arranged as largely to exclude CO_2 and H_2O vapor from the atmosphere between the radiator and bolometer strip, so that the gaps due to the selective absorption of these two substances were very greatly reduced in extent and depth. On the other hand, in Paschen's later work, the bolometer strip was placed at the center of a reflecting hemisphere, in order to approach more closely to the condition of a perfect absorber. According to Kurlbaum[1] a thinly-coated bolometer strip, which appears, however, black to the eye, may absorb but 40 per cent. of the incident total radiation, so that the difference between the results of Lummer and Pringsheim and of Paschen may be partly due to the imperfect absorption of Lummer and Pringsheim's bolometer strip.

So far, then, as the expressions (III) and (IV) are concerned, which tell us nothing as to the distribution of energy in the spectrum, the predictions of theory seem to be verified to within the outstanding experimental errors, and these are being steadily reduced. Equation (V), $E = C\lambda^{-5} e^{-\frac{c}{\lambda T}}$ which concerns this distribution of energy, remains to be considered. As regards its theoretical foundation, Wien's original method is not rigorous, as has been pointed out by Lummer and Pringsheim.[2]

The most careful attempts at experimental verification have been made by Paschen[3] and by Lummer and Pringsheim.[2] Full accounts of the work of the former have been published in this JOURNAL He finds c (see Table II for meaning of these constants) to be 14531, with a possible error of 80 from a series of experiments, while C is more variable. No systematic variation

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[1] KURLBAUM, *Wied. Ann.*, 67, 1899.

[2] LUMMER and PRINGSHEIM, *Verh. Deutsch. Phys. Ges.*, 1, 1, 1900.

[3] PASCHEN, ASTROPHYSICAL JOURNAL, 10, 40, 1899; 11, 288, 1900.

of either C or c is evident; he further tests (V) by plotting it logarithmically.

The theoretical and observed curves agree to within the errors of experiment, except in the region of very short wave-lengths, where the observed energy is greater than the theoretical; and precisely here, as Paschen points out, it is very difficult to avoid stray light.

On the other hand, Lummer and Pringsheim find that while the computed and observed curves (graphs of V) agree quite well in the region of wave-lengths near the maximum, particularly at lower temperatures, this agreement is not so good for the long wave-lengths, and that this disagreement increases as the temperature increases. This can be stated in another way. If the expression $E = C\lambda^{-5}\, e^{-\frac{c}{\lambda T}}$ is transformed by the introduction of logarithms we have $\log E = \log C - \frac{c}{\lambda T} \log e - 5 \log \lambda$, and if $\log E$ is plotted against $\frac{1}{T}$ we have the so-called "isochromatic" curve, evidently a straight line. This may be compared with the curve plotted for corresponding values of E and T, observed at a fixed point in the spectrum (λ constant). Evidently the slope of this line is proportional to c, while from the constant term can be obtained the value of C. According to Lummer and Pringsheim's observations the observed isochromatic is convex toward the $\frac{1}{T}$ axis, and the values for C and c obtained from these isochromatic curves increase systematically as the temperature rises. These investigations of Lummer and Pringsheim are still in progress. Thiesen has, however, found, from a recalculation of the results of Lummer and Pringsheim, that a modification of V, by changing the coefficient of λ from -5 to -4.5, would completely satisfy their observations. The law thus modified would be satisfied by the observations of Beckmann,[1] and is further strengthened by some recent work of Lummer and Pringsheim.[2]

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[1] BECKMANN, *Inaug. Diss.*, Tübingen, 1898.

[2] Referred to by THIESEN, *Verh. d. Deutsch. Phys. Ges.*, **2**, 5, 1900.

As regards the variation of these constants, Rubens[1] has discussed the results of Beckmann, who used a hollow "black body" as source of radiation, and produced a more or less perfect isolation of certain wave-lengths by repeated reflection from fluor spar, and thus determined an approximate "isochromatic" curve in a spectral region not heretofore studied in this connection, viz., for a mean wave-length of about 28 μ. In order that the Wien formula (V) should represent Beckmann's work (as recalculated by Rubens) it is necessary that c should have the value 26000. On account of the method used, this result ought not, perhaps, to be considered conclusive. However, Rubens points out that the change in the ordinate of the energy curve at $\lambda = 25\ \mu$ produced by a change in c from 26000 to 14500 would be (at 2000° C.) $\frac{2}{10000}$ of the maximum ordinate; so that it would be difficult to detect such a change in c by study of the energy curves.

The present knowledge respecting these various laws of radiation can perhaps best be summed up in the following table:

TABLE II.

(I) $S = \int E d\lambda = \text{const. } T^4$.	First given by Stefan.[2] Thermodynamically deduced by Boltzmann[3] with certain assumptions. Experimentally tested by Paschen[4] and more especially by Lummer and Pringsheim.[5] Deduced by Planck from electro-magnetic theory, involving electro-magnetic definition of entropy and temperature.

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[1] RUBENS, *Wied. Ann.*, **69**, 1899.

[2] STEFAN, *Sitzber. d. k. Gesellsch. zu Wien*, **79**, 1879.

[3] BOLTZMANN, *Wied. Ann.*, **22**, 1884.

[4] PASCHEN, *Wied. Ann.*, **58**, 1896; **60**, 1897.

[5] LUMMER and PRINGSHEIM, *Wied. Ann.*, **63**, 1897.

(II) $\lambda T = \text{const.}$ from which follow: (III) $\lambda_m T = A$. (IV) $E_m T^{-5} = B$.	Developed theoretically by Wien,[1] assuming Stefan's law (I). Tested experimentally by Paschen,[2] Lummer and Pringsheim,[3] Paschen and Wanner,[4] and found to hold with increasing accuracy as experimental methods are improved. Outstanding difference between Paschen and Lummer and Pringsheim of about 1 per cent. in value of *A*; *B* not so good. (II) Theoretically developed by Thiesen,[5] who questions the rigor of Wien's original method, and by Planck[6] from an electro-magnetic basis.
(V) $E = C\lambda^{-5} e^{-\frac{c}{\lambda T}}$. (III) and (IV) followed by differentiation from (V). (VI) $E = C\lambda^{-\alpha} e^{-\frac{c}{\lambda T}}$.	Theoretically deduced by Wien[7] —but with rather arbitrary assumptions and not altogether rigorous reasoning.[8] An expression of the same form (VI) was given by Paschen as best representing the energy curves of various radiating surfaces. Tested by Lummer and Pringsheim,[3] who found systematic variations in *C* and *c* with temperature; also by Paschen and Wanner[4] and Paschen,[2] who finds no systematic variation of the constants and a quite satisfactory agreement of the observed and computed curves. Also deduced theoretically by Planck. (III) and (IV) are necessary, but not sufficient conditions for the truth of (V).

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[1] WIEN, *Ber. d. Berl. Akad.*, **6**, 1893; *Wied. Ann.*, **52**, 1894.

[2] PASCHEN (*loc. cit.*) and ASTROPHYSICAL JOURNAL, June 1899, May 1900.

[3] LUMMER and PRINGSHEIM, *Verh. d. Deutsch. Phys. Ges.*, **1**, 1 and 12, 1899.

[4] PASCHEN and WANNER, *Ber. d. Berl. Akad.*, January 1899.

[5] THIESEN, *Verh. d. Deutsch. Phys. Ges.*, **2**, 5, 1900.

[6] PLANCK, *Drude's Annalen*, **1**, 1900; **4**, 1900.

[7] WIEN, *Wied. Ann.*, **58**, 662, 1896.

[8] LUMMER and PRINGSHEIM, *Verh. d. Deutsch. Phys. Ges.*, **1**, 1, 1899.

[9] PASCHEN, *Wied. Ann.*, **60**, 1897.

Here

$S =$ total energy of radiation at any absolute temperature.

$T =$ total energy of radiation at this absolute temperature.

$Ed\lambda =$ energy radiated in waves of length $< \lambda + d\lambda$ and $> \lambda$.

$\lambda =$ any wave-length in μ, (0.001 mm.)

$\lambda_m =$ wave-length of maximum energy at temperature T.

$E_m d\lambda =$ amount of maximum energy at temperature T, between the limits

$$\lambda_m \pm \frac{d\lambda}{2} .$$

A, B, C, c, α, are constants.

The following work on the radiation of an "absolutely black body" was begun by Mr. C. E. Mendenhall in conjunction with Dr. H. F. Reid, the part relating to temperatures above 500° C. being carried out by him, while that relating to temperatures below 500° C. was carried out by Mr. F. A. Saunders. The work was done in the Physical Laboratory of the Johns Hopkins University, Baltimore.

We shall first consider the work at temperatures above 500° C., then point out the differences in procedure adopted for the low temperature curves, and give the corresponding results.

The method of realizing the "black body," based on Kirchhoff's theoretical investigation of the radiation inside a uniformly heated inclosure, had been suggested by Dr. Reid in this JOURNAL,[1] and independently by Wien and Lummer.[2]

Our black body was either a cast-iron or copper cylinder, about 8 cm in diameter and 12 to 18 cm high, with a slit in the side through which passed the radiation to be examined; a furnace, or, for the lower temperatures, appropriate baths, served for the heating.

The spectrometer was practically a reproduction of Langley's early one, used at Allegheny.

The available rock salt consisted of a 60° prism, having faces about 5 cm × 7 cm, and a lens, about 10 cm diameter and 40 cm focus, with Brashear surfaces — the material for which had been kindly loaned by Professor Langley.

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[1] H. F. REID, ASTROPHYSICAL JOURNAL, **2**, 160, 1895.

[2] W. WIEN and O. LUMMER, *Wied. Ann.*, **56**, 1895.

In order to increase the sensitiveness of the bolometer, two diagonal arms of the Wheatstone bridge quadrilateral were exposed to radiation. Theoretically this should be better than a single strip of the same width in the ratio $\sqrt{2}$ to 1. Considerations of freedom from "drift" with general changes in temperature made it desirable that the balancing arms should be as nearly like the exposed strips as possible, and similarly situated. All four arms were accordingly made of similar strips of annealed platinum foil, and mounted in the same cell. Balancing was accomplished by moving the galvanometer terminals independently along two copper wires; one, being comparatively fine (No. 24 about), gave rough adjustment, while the other (No. 12), gave fine adjustment.

The spectrometer, balancing bridge, etc., were inclosed in a double-walled box — for the purpose not only of protecting the bolometer and its appendages from temperature changes, but also of protecting the rock salt from moisture. No attempt was made to exclude or remove CO_2, nor water vapor, except in so far as was needed for the protection of the rock salt. The battery consisted of a number of Edison-Lalande cells, connected in multiple, and, on the advice of Mr. C. G. Abbot, of the Smithsonian Institution, carefully protected, as were the main leads, also, from temperature changes. The galvanometer was of the Thomson 4-coil pattern, of low resistance (about 4 ohms), and with the needle used in the first part of the work gave 1 mm deflection at 1 m with about 2.5×10^{-10} amp., with a complete period of 10 sec., though only rarely was this maximum sensibility needed.

The bolometer showed a very satisfactory freedom from "drift" and from disturbance in general. The galvanometer, however, located, as it was, in the midst of a city, was, with the best needle-system which we were able to produce, and with a quadruple iron magnetic shield, quite unusable in the day time, so that all observations had to be carried on at night. For temperatures above 500° the black body was heated in a furnace of fire-clay and the temperatures were determined by the use of several platinum, platinum-iridium thermo-couples, according to the potentiometer method, much as outlined by Barus. These were calibrated by the use of a number of standard melting and boiling temperatures, viz., water, napthalin, mercury, potassium chloride and gold. The higher temperature determinations were perhaps in error by 5° or 10°. Temperatures were measured at four points, two at the top of the cylinder (black body), and two at the bottom. With the furnace method of heating the black body, as we used it, differences of temperature of from 10° to 20° were usually found between some of these four points.

PART I.

With the above apparatus observations of the distribution of energy in the black body spectrum were taken at many temperatures between 500° C. and 1100° C.; and a few sets of observations of energy at various (fixed) points in the spectrum while the temperature varied — giving data for the so-called isochromatic curves. When the observations were used with the corresponding observation of minimum deviation to plot energy curves, the characteristic absorption bands of CO_2 and H_2O vapor were very marked. These curves were then put in the normal form by changing from minimum deviation to wave-length, using the dispersion curve of rock salt found by Rubens,[1] and by Rubens and Trowbridge.[2] The corresponding change in the ordinates of these curves, viz., multiplication by $\frac{d\lambda}{d\delta}$, was made — also corrections for impurity of spectrum, according to Runge,[3] and for variation in sensibility of apparatus.

It was at first attempted to allow for the absorption bands of H_2O and CO_2 in the usual way by "bridging over" the gaps with a free-hand curve. Upon comparing these curves, however, it was concluded that the amount of absorption had, over part

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[1] RUBENS, *Wied. Ann.*, 54, 436, 1895.

[2] RUBENS and TROWBRIDGE, *Am. Jour. Sci.*, January 1898.

[3] RUNGE (Paschen) *Wied. Ann.*, 60, 1897, and Schlömilch's *Zeit. fur Math. u. Phys.*, 43, 1897.

of the curve, been greatly underestimated. This made the entire middle portion of the curves uncertain; especially it made the wave-length of maximum energy very difficult to determine, and hence made it impossible to test accurately equation (III). In fact, by properly bridging over the absorption gaps, the curves can be made to satisfy (III) as exactly as may be desired. Paschen[1] has stated that the expression

$$\lambda_m = \frac{(\log \lambda_2 - \log \lambda_1)\, \lambda_2\, \lambda_1}{(\lambda_2 - \lambda_1) \log \epsilon},$$

where (λ_1, λ_2) are any two wave-lengths on opposite sides of the maximum corresponding to equal energies of radiation, serves to give consistent values for λ_m, and it has accordingly been used in connection with these curves. The wave-lengths (λ_1, λ_2) were taken at points where the absorption was as small as possible, and for each of seven curves several values of λ_m were calculated; these values agreed usually to about 0.1 μ. The resulting values of $\lambda_m\, T$ are as follows:

T, $C°$	λ_m	$\lambda_m\, T$
570	3.34	2815
704	2.72	2657
771	2.53	2641
837	3.36	2619
896	2.20	2571
944	2.20	2611
1030	2.00	2586

With the exception of the first one, the numbers in the last column are as nearly constant as could be expected considering the possible errors of measurement—but the mean value differs by nearly 300 from Paschen's mean value 2907, or Lummer and Pringsheim's 2930. This could be accounted for by imperfect "blackness" of our radiator, but this seems a rather improbable explanation considering the size and form of our enclosure. It is perhaps more probable that the heavy absorption on the descending side of our curves has led to an apparent shifting of all the λ_m toward the short wave-lengths.

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[1] PASCHEN, *Wied. Ann.*, 50, 409.

As far as (I) is concerned our method is at best a poor one—analyzing the radiation only to integrate the energy-curve afterwards; with the absorption as large a part of the total energy as the curves would indicate, an attempt to confirm (I) becomes still less fruitful.

As for (IV) E_m is rendered uncertain by the bands above referred to—but not to the same extent as λ_m; for the entire change in λ_m through the temperature range used in the high temperature work is but about 1.2 μ; and an examination of the curves shows that the uncertainties are a large part of this.

The causes of this extremely strong absorption undoubtedly lay in the use of a furnace to heat the black body, which became filled with the products of combustion, notably CO_2 and H_2O. That no more elaborate means to prevent this were taken was due to the conclusion, drawn from a comparison of some of the final curves roughly plotted, with some curves previously taken with slightly different arrangements—that the amount of furnace gas in the black body was not sufficient to produce extraordinary absorption. This conclusion was evidently in error.

The following table gives the values of (B) for seven curves:

°C	T	$E_m\,T^{-5}\times$ const.
1020	1293	59
914	1187	65
896	1169	61
837	1110	50
771	1044	55
704	977	51
570	843	54

As to Stefan's law (I), S can be approximately determined from the area of the various curves as finally corrected. From these we obtain the following table of values of $\frac{S}{[T^4-T_1^4]}$, where T_1 is the absolute temperature of the shutter used to exclude radiation.

T	$\frac{S}{[T^4-T_1^4]} \times$ const.
1293	493
1187	528
1169	457
1110	436
1044	470
977	458
843	433

Here also there is very unsatisfactory constancy. The five lower temperature curves agree fairly well among themselves, but we think it probable that the absorption has not been completely allowed for in these curves. The error in the 1187° curve seems to be rather larger than could be accounted for by an error in the absorption correction alone; unless, as suggested above, the other curves have been undercorrected. If this is the case, then the coincidence of an error of 8° in the estimation of temperature (for the 1187° curve) with an easily allowable overestimation of the absorption correction would account for the discrepancy.

PART II.

On account of the extremely small amount of radiation with which one has to deal in measuring the energy in the spectrum of a radiating body at comparatively low temperatures, it was absolutely necessary to have a more sensitive needle system in the galvanometer for this part of the work, and accordingly a series of experiments was undertaken to determine what form of system would be most efficient. The vertical needle system of Weiss having proved inadequate when the highest sensibility was required, the ordinary form of system was used and a number of modifications in it were tried, with results which it may be of interest to give. It seemed obvious from the work of Paschen and others that the lighter the system was, other things being equal, the more sensitive it was, and also that the greater the proportion of the weights of the magnets to the total weight, the greater the sensitiveness. Starting from these facts, fourteen different systems were made and tested. The magnets used throughout were made from watch and clock-spring material and were tempered, magnetized and boiled before being mounted. A great many magnets were made at the beginning and from these the best were selected by observing their activity when laid upon a glass plate and tapped in a vertical magnetic field; only the best were used. The sensitiveness of each system was found by mounting it on a fine quartz fiber in the galvanometer and observing the deflection produced by a measured current. The figures given for this result are the currents in amperes which produced a deflection of 1 mm on a scale 1 meter distant when the complete period of the system was ten seconds.

Each system was built on a very fine glass rod and furnished with a minute copper wire loop at its upper end, by means of which it could be hung on a corresponding hook on the lower end of the fiber. The mirrors used were fragments of the finest microscope cover-glasses obtainable, silvered, and cut into pieces roughly circular in shape with an area of about 1.5 sq. mm. Their weights varied from 0.4 to 0.7 mg.

A study of the results obtained brought out the following conclusions, which, of course, apply exactly only to a galvanometer whose "free space" is circular and of the same diameter as ours (3 mm):

1. It is unwise to make the magnets shorter than 1 mm.
2. It is unwise to make the system as deep (measured along the stem) as the diameter of the free space.
3. It is somewhat disadvantageous to make the magnets themselves as long as the free space is wide; such systems are also very troublesome to use.
4. There is a slight disadvantage in making the magnets of material thinner than 0.05 mm.

One system was finally chosen as the best of all, and the systems used in the subsequent work were made after the same pattern. Each group in the system consisted of three magnets, two of which were 1.6 mm long while the third was 2.3 mm long; the width of each magnet was 0.2 mm and its thickness 0.05 mm. Each group was spaced along the stem so as to cover about 1.5 mm. The total weight was 2.5 mg; of which 1.3 mg was of steel, 0.6 mg of mirror and 0.6 mg of glass and shellac.

An effort was next made to bring a number of these systems to a highly astatic condition. This, of course, requires that the magnets shall lie in the same plane, or in parallel planes, and that the upper and lower sets shall be equal in magnetic strength but opposite. As this occupied an unexpectedly long time, it may perhaps be well to give an account of the difficulties encountered in this apparently simple operation. In the first systems the magnets were all fastened on the same side of the glass rod and the mirror was on the opposite one. The magnets of the systems being fastened in place while lying on a piece of plate glass they were nearly all in the same plane, and the fine adjustment was made by loosening one of the magnets a little by heating and turning it through a small angle. For a considerable time no consistent results were obtained, owing to the proximity of a magnetized steam-pipe which had an unexpectedly great effect on the uniformity of the Earth's magnetic field in the place in which the systems were kept. Having moved the systems to a place where the field was uniform, we once more adjusted them by turning one magnet of one set so that the plane of the equivalent magnet was parallel to that of the other set. This process, since it involved heating, usually resulted in a slight weakening of the magnet turned, which was corrected by bringing near it a powerful permanent magnet. Any system the planes of whose magnets are not parallel should, when the magnetic strength of the two sets are made equal, stand with its magnets pointing east and west, and its period should be longer the nearer its magnet sets are to being in the same plane. This was found to be the case in about one system only out of ten. The others as they approached an astatic condition reached a condition when they would oscillate about *two* positions of equilibrium, usually in the two directions northwest to southeast and northeast to southwest. When this was the case, there was no means of finding, by its positions of equilibrium or by its period of oscillation, which of the two magnet sets was the stronger, nor in what direction one of them must be turned in order to bring the sets more nearly into the same plane. It was therefore impossible to proceed with the astaticizing at all.

This anomalous condition was in no way due to torsion in the supporting fiber, for a complete revolution of this produced no more than five degrees change in the natural position of the system, and six whole turns were necessary to force the system to revolve. The fibers were from 15 to 25 cm in length and not more than 0.0025 mm in diameter, and usually somewhat less. They were made by the blowing process invented by Boys,[1] in which the oxy-hydrogen flame melts and at the same time blows out a minute fragment of quartz into a long fiber, a process which proved extremely efficient and simple after a little practice had been obtained with it.

Each fiber was hung in a glass tube so that its lower end projected beyond the tube into a small wooden box whose front was closed by a glass plate fastened by a bit of wax. It was found that the act of detaching this glass plate charged it quite highly with electricity, and it was hoped that the action between this charge and the charge induced on the magnets might account for the existence of two positions of equilibrium. That it did not, however, was amply shown by the use of a brass plate instead of the glass one, which was carefully discharged whenever handled, by passing it through a flame. The systems behaved just as before.

A lack of symmetry in the system itself was next suggested as a possible explanation, though by no means, at first sight at least, a very rational one. Accordingly, several systems were constructed and astaticized in which the following precautions were taken: the weights of the two sets of magnets were made equal to within 0.1 mg; the weight of the magnets fastened on one side of the glass stem was made equal to the weight of the magnets and mirror on the other side of the stem, to the same degree of approximation (it was not possible to do this more accurately, as the total weight of the parts was not often more than 1 mg and the weighings were not certain to less than

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[1] V. C. BOYS, *London Electrician*, December 11, 1896.

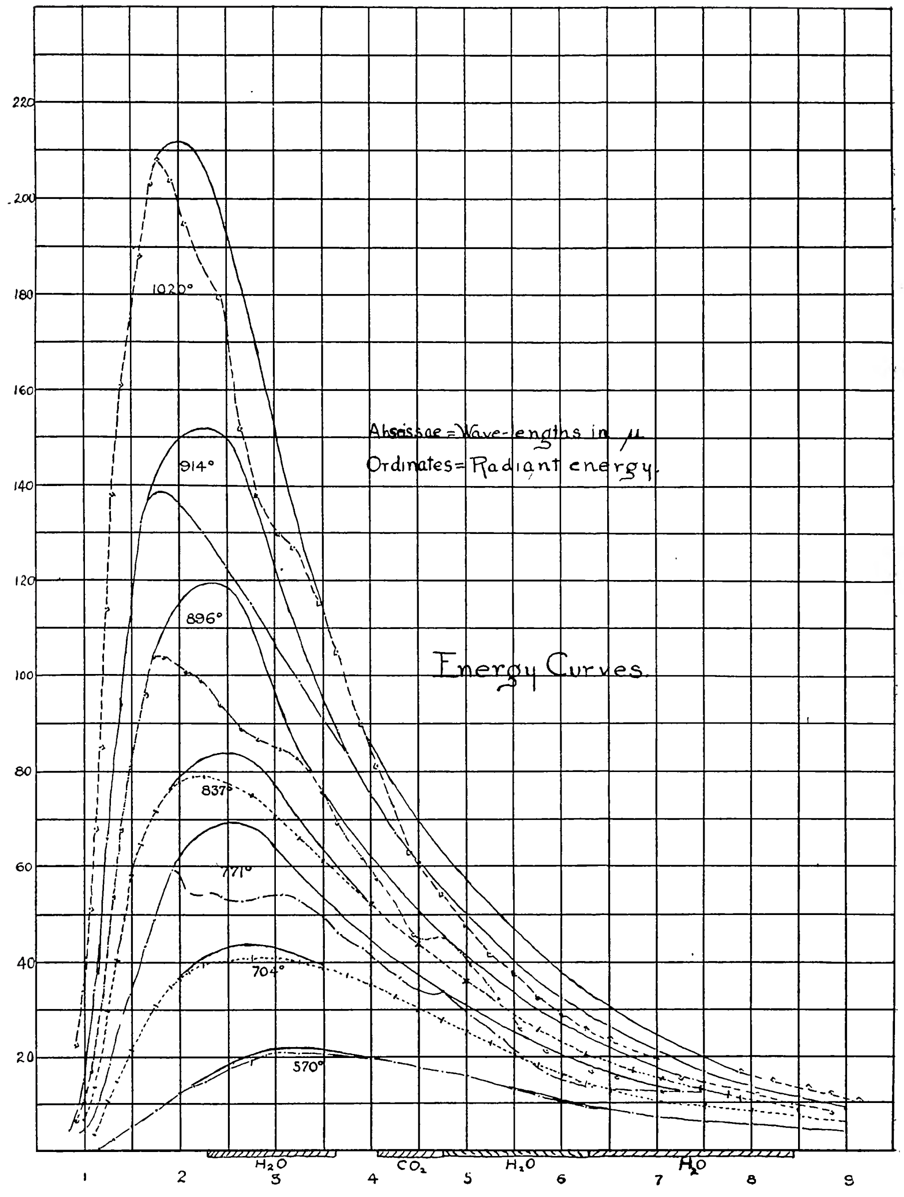

FIG. 1.

Broken lines, uncorrected or partially corrected curves.
Full lines, corrected curves, drawn to fit relation $\lambda_m T = \text{const.}$
Absorption bands indicated at bottom.

0.1 mg or to 10 per cent.); the greatest care was taken to have the glass stem fastened at the middle points of the magnets, and the mirror was so adjusted as to make the oscillations of the whole system aperiodic when it was supported horizontally so as to be free to turn about the stem as axis; and, finally, the loop of wire at the upper end of the system was dispensed with and the quartz fiber was fastened directly to the stem. The systems that were produced after all these precautions were taken were in no way better than those made before, and as no further changes could be thought of which would prevent this anomalous behavior all the systems that exhibited it were discarded.

It would be quite possible to account for this difficulty by remembering that an absolutely astatic system, *i. e.*, one whose magnets were all in the same plane and whose sets were exactly equal and opposite in strength when they were in an east and west plane, would be thrown out of astaticism when the plane of the magnets became north and south by the magnetism induced in the magnets by the Earth's field, and would hence oscillate about two positions of equilibrium. It seemed very unlikely that these systems were so nearly perfect that this explanation could apply to them, particularly as the induced magnetism must be very feeble in hardened watch-spring steel. No other explanation has, however, been thought of.

The best systems finally chosen, three or four in all, were then brought to a fairly astatic condition, as shown by their period of oscillation (complete period six or seven seconds), and they were then examined from day to day, being as far as possible undisturbed in the intervals, to see how their condition altered with the time. This alteration proved to be very great, and was apparently as great in the case of systems composed of boiled magnets as it was with some constructed of "raw" material. All of them at first lost their astaticism rapidly, but, after readjusting them once or twice a day, at the end of two weeks we obtained one which held a period of about five seconds for the ensuing two weeks without appreciable alteration. This system was therefore mounted in the galvanometer and was used in the subsequent work. It had a sensitiveness of about 1×10^{-10} amp. At the close of the investigation, however, after three months use, it was found to have a period of about three seconds, indicating a considerable fall in astaticism.

Seven curves in all were obtained, all of them duplicated in important regions, namely, at the temperatures 100°, 175°, 243°, 313°, 399°, 503°, and 578° C. Of these curves the four taken at the lowest temperatures are drawn in Fig. 2, and reveal at once the presence of the absorption bands due to the presence of carbon dioxide and of water vapor in the air, which are to be found in all the curves taken. The full lines with which the observed curves in part coincide indicate the curves filled in according to a method explained below. It was hoped that five or six vessels containing concentrated sulphuric acid, which were put inside the spectrometer box, would keep the air inside reasonably dry. They did keep the rock-salt surfaces from being fogged, but did not prevent the water-vapor bands from being prominent in every curve. With the spectrum as impure as it was, these bands in some places overlap and affect the curves continuously for a considerable distance. A few points in the spectrum seemed, however, to be free from absorption, and the observations at these points only were used.

Paschen[1] has found that if Wien's law be true, and if the radiation curves be plotted, not as is usually done, with wave-length and intensity as coördinates, but with the logarithms of these quantities instead, then the curves have the property of congruency. By this is meant that any one curve is an exact copy of any other, but shifted, unaltered in shape, to another part of the diagram. Now, since the maximum energy occurs at different wave-lengths as the temperature is changed, it is evident that any region of absorption will fall upon a different part of each curve, and hence if the curves are congruent as above explained, the points cut out of any curve by absorption can be supplied from another curve by merely laying one above the other. In this way, by the use of curves taken at different temperatures,

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[1] F. PASCHEN, *Wied. Ann.*, 60, 1897.

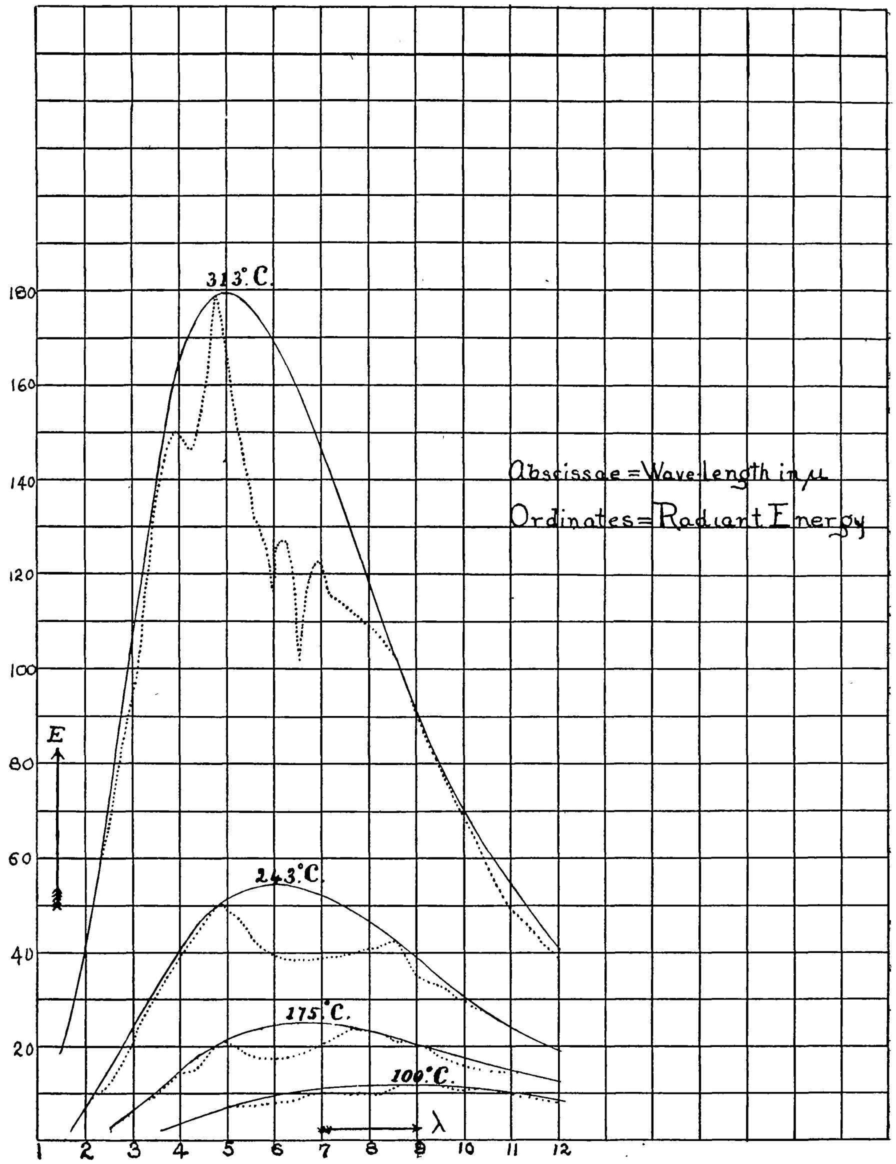

FIG. 2.

the entire curve can be constructed from a few points. This has been the method used. Our curves when plotted logarithmically are roughly congruent, though not accurately so, as there seems to be a slight change in form as the temperature changes. The curve is roughly of an inverted U shape, and our curves show a slight tendency towards a widening of the U as the temperature increases. The mean curve was, however, taken, as the deviations did not seem too large, and by means of this the missing points in the observed curves were supplied.

Four tests were applied to this set of curves. Law I was first tested, with the results shown by the following tables:

T	λ_m	$\lambda_m T$
373°	8.30	3090
448	6.50	2910
516	5.90	3040
586	4.90	2870
672	4.37	2940
776	3.81	2950
851	3.24	2760
	Average	2940

These figures are somewhat irregular, but the determination of the maximum of the energy curve is an extremely uncertain thing when the curves are as much cut up by absorption bands as these were. The average value of this constant, 2940, is somewhat higher than that found by Paschen, and there is a slight tendency shown in the series of values towards an increase in the value of the constant with decrease of temperature. The marked difference between this value and that found in the earlier part of this work may perhaps point to the same result. The expression (IV) was next tested. This set of observations fails entirely to conform with this law. The maximum energy varies nearly as the sixth power of the absolute temperature.

The following relation should also hold:

$$\frac{E}{E_m} = \left\{ \frac{\lambda_m}{\lambda} \cdot e^{\frac{\lambda - \lambda_m}{\lambda}} \right\}^{a},$$

where E is the intensity at wave-length λ, and E_m is the maximum intensity. This relation was found experimentally by Paschen, and according to Wien's law the constant a should be 5. This constant was calculated from a great number of points on all the curves, and the average value obtained was slightly less than 5, but very near it.

Finally, Wien's expression (V) for the energy at any wave-length, gives, on integration with respect to λ from *O* to λ,

$$\int_0^\lambda C\lambda^{-5}\,\epsilon^{-\frac{c}{\lambda T}}\cdot d\lambda = C\left(\frac{T}{c}\right)^4\cdot\lambda^{-3}\cdot\epsilon^{-\frac{c}{\lambda T}}\cdot$$

$$\left\{\left(\frac{c}{T}\right)^3+3\left(\frac{c}{T}\right)^2\lambda+6\left(\frac{c}{T}\right)\lambda^2+6\lambda^3\right\}.$$

This enables us to compare the areas of the curves up to a certain wave-length with those required by the formula. If we knew the entire curve we could simply integrate it and then its area, representing as it does the total radiation, should follow Stefan's law, *i. e.*, should be proportional to the fourth power of the absolute temperature. It is not possible, however, to do this on account of the large part of the curve lying in the region of the longer wave-lengths, which is influenced by the absorption of the prism, etc. This circuitous method was tried on our curves, and since Wien's law is based on Stefan's, should lead to the same result. This was not found to be the case here, our areas being nearly proportional to the seventh power of the absolute temperature; but this method of obtaining the total radiation is too indirect to lead to accurate results.

The conclusions to be drawn from our results are, as before stated, rather negative in character. It is evident that some of the deductions from Wien's law are satisfied, while others are not; but no results of sufficient accuracy can be obtained without taking excessive precautions in regard to the "blackness" of both radiator and bolometer strip, and in excluding from the air about the apparatus all traces of carbon dioxide and water vapor.

It is interesting to note that if law III holds, the maximum of the radiation curve of a body at the temperature of the boiling of liquid air under atmospheric pressure should lie at about 30 μ, and would therefore be beyond the reach of any rock-salt dispersion apparatus. An effort was made to test this by cooling the black body with liquid air, but at the last it proved impossible to obtain enough for our purposes. With the small quantity that we had, however, the body was cooled to about —90° C., and at this point caused the greatest deflection at about $10\,\mu$, which is roughly where the maximum should lie, according to law III. The deflection was also of the order to be expected, though the working conditions at the time were not good enough to allow any accurate measurements to be taken.

The writers wish to express their sincere indebtedness to Dr. H. F. Reid for his continued interest and valuable advice, and to Professors Rowland and Ames, not only for their kind supervision, but for the generosity with which they placed all the facilities of the laboratory at our disposal.

NOTE.—Mr. H. C. Dickinson of Williams College has suggested that the two positions of equilibrium of the delicate galvanometer needles, above referred to, may be due to the disturbance of the co-planarity of the two systems of magnets, by the couples between these systems and the magnetic field, opposed by the torsional rigidity of the connecting glass rod. This assumes that the two groups of magnets are initially very closely in the same plane—and of very nearly equal moments. Some experiments have been performed which indicated that a change of several minutes might be expected in the angular relation of the two groups, and this seems sufficient to account for the observed phenomena. C. E. M.

November 1900.