THE ECHELON SPECTROSCOPE.¹

By A. A. MICHELSON.

The resolving power of a diffraction grating is proportional to the product of the total number of lines by the order of the spectrum observed. But little effort seems to have been made to make a decided step in the direction of increasing the *order* of the spectrum observed, and this is doubtless because for a grating acting by *opacity* the brightness of the spectrum diminishes very rapidly as the order increases. This difficulty has been successfully overcome by ruling the lines in such a way as to concentrate the greater proportion of light in one spectrum, but so far as I am aware such attempts have been limited to the first, second or third spectrum and the results even here are somewhat fortuitous.

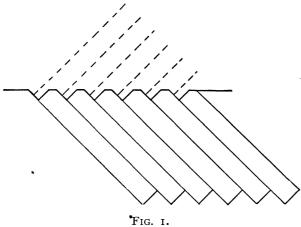
It seems nevertheless quite possible to construct gratings which shall throw a quite large proportion of the light in very high orders of spectra—say the hundredth—in which case the grating space must be of the order of a hundred waves or say twenty to the millimeter, instead of a thousand. The lines would have to be drawn with no more accuracy than before, and the grating could be completed in a very short time and temperature changes would have a much smaller effect than at present.

It may be that there are more serious practical difficulties in the way of such a ruling as is represented in Fig. I than would be anticipated. Especially may this be true if the greater part of the light is to be returned in the direction from which it came; for that the grooves must be correspondingly deep, and the grating space would vary with the depth.² Fig. I at once suggests a possible method of effecting the same result, by

¹The first part of this paper was published as a preliminary notice in the Am. Jour. Sci. for March.

²This question will receive a practical test as soon as a ruling machine now under construction is completed.

building up the steps by equal thicknesses of optical glass. Here the difficulty, even supposing the optical work to be



practically perfect, would be the joining of the separate plates in such a way as to have always the same distance between them.

By using the same arrangement for transmission instead of reflection this difficulty is avoided - and there remains absolutely nothing but the difficulty of making a considerable number of plane-parallel plates of the same thickness—to an order of accuracy only one-fourth that required in the former arrangement, or even one-tenth of this if the other medium be water or oil instead of air.

Probably the surprising thing is the smallness of the number of plates required to give results which are comparable with those of the best gratings. This can be shown as follows: Let abd (Fig. 2) be one step in the series of plates and let ab = s and bd = t. If m is the order of the spectrum observed, $m\lambda = \mu \cdot b d - a c$ or

$$m\lambda = \mu t - t \cos \theta + s \sin \theta$$

$$\frac{d\theta}{d\lambda} = \frac{m - t \frac{d\mu}{d\lambda}}{t \sin \theta + s \cos \theta}$$

$$\frac{d\theta_{x}}{dm} = \frac{\lambda}{t \sin \theta + s \cos \theta}$$

and if $\delta\theta$ is the displacement corresponding to $\delta\lambda$ and $\delta\theta_{\rm r}$ that corresponding to $\delta m=1$, assuming Cauchy's formula

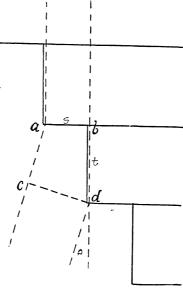


FIG. 2.

 $\mu=a+b/\lambda^2$, and taking the approximate value of $m=(\mu-1)$ t/λ , we have

$$\delta \theta / \delta \theta_{r} = [(\mu - 1) + 2(\mu - a)] \frac{t}{\lambda} \cdot \frac{\delta \lambda}{\lambda}$$

For flint glass the coefficient of t/λ is approximately equal to unity; so that if $\frac{\delta\lambda}{\lambda}$ =.001, as in the case of the two sodium lines, and $t=5^{\rm mm}$ =10000 λ , then $d\theta$ =10 $d\theta_{\rm r}$, that is, the sodium lines would be separated by ten times the distance between the spectra.

The resolving power of such a combination is mn, exactly as in the case of gratings; so that with but *twenty* elements 5^{mm} thick and hence m = 5000 the resolving power would be 100000, which is about that of the best gratings.

The experiment was actually tried with but seven elements, placed between a collimator and an observing telescope and the collimator slit illuminated with light from a sodium flame.

The images were so distinct that the broadening of the lines could be very easily detected, and the Zeeman effect was readily observed when the sodium flame was placed in a magnetic field.

It is important to note that the resolving power is independent of the number of plates but depends only on the total thickness, and the only advantage in a large number of elements is the greater separation of the spectra. The overlapping of spectra is doubtless a disadvantage, which however could be overcome by a preliminary analysis; and for the examination of single lines and especially in the investigation of effects of broadening, shifting, or doubling of lines, the method seems especially well adapted.

An echelon spectroscope of twenty elements, the essential parts of which were constructed in the Ryerson Physical Laboratory, is represented in Fig. 3.

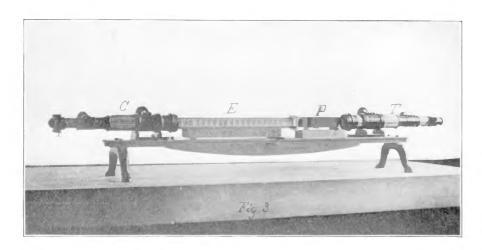


Fig. 3.

 \mathcal{C} is the collimator and T the observing telescope; E the echelon consisting of twenty plates, each 18^{mm} thick, and diminishing in width from 22^{mm} to 2^{mm} , so that the width of the elementary pencils is 1^{mm} , and the successive retardations are of the order of twenty thousand waves.

From what precedes it follows that practically all the light may be concentrated in one spectrum, so that the only losses are those due to the reflections and absorptions, and these are much less than in either gratings or prisms of equal resolving power. The intensity is given by the formula

$$I = \begin{bmatrix} \sin \pi \frac{s}{\lambda} \theta \\ \frac{s}{\lambda} \theta \end{bmatrix}$$

or by the curve Fig. 4.

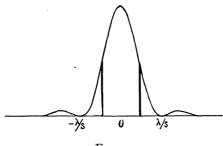


FIG. 4.

All the light is practically included between the deviations $-\frac{\lambda}{s}$ and $\frac{\lambda}{s}$. But the distance between successive spectra is $\frac{\lambda}{s}$, so that in general there will be *two* spectra visible. By slightly inclining the echelon however, the directions of the spectra may be varied so that one falls at O, where the intensity is a maximum, while the adjoining ones disappear.

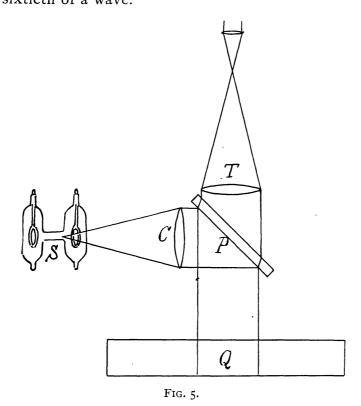
The theoretical resolving power of the twenty element echelon is 15000×20 or 300000; and the results which are given below show that this limit is nearly reached in practice.

The overlapping of the spectra is overcome by a direct vision prism of moderate dispersion, but the distance between the spectra is so small in comparison with the dispersion of the echelon that the spectrum of the source under examination must consist of rather fine lines if overlapping is to be avoided. Thus, in the present case the range of wave-length corresponding to two successive spectra is $\frac{\delta\lambda}{\lambda} = \frac{2\lambda}{t} = \frac{1}{15000}$; that is,

the lines to be examined must have a total width of less than one-fifteenth of the distance between the sodium lines.

This caused some difficulty in the examination of certain phenomena, as for instance, the Zeeman effect, and it will doubtless be an improvement to have three times the number of elements of one-third the thickness—for then with equal resolving power, the range would be three times as great.

The increased number of the plates will of course increase the losses by reflections, and the degree of accuracy in working the plates must also be correspondingly increased. Thus for twenty plates the retardations must differ by less than onetwentieth of a wave, while with sixty the difference must be less than a sixtieth of a wave.



The method of testing the plate (from which the elements are afterwards cut) is shown in Fig. 5.

S is an end-on vacuum tube containing mercury vapor

which is illuminated by the spark of an induction coil. The green radiation is so much brighter than the others that it is not necessary to use a spectroscope. The light, after being brought to an approximately parallel beam by the collimator C, is reflected from the lightly silvered plate P to the plate Q to be tested. From both surfaces of Q it is returned normally, and part passing through P, is examined by a low-power telescope T.

When the two surfaces of \mathcal{Q} are nearly parallel, circular interference fringes begin to appear, which become more clearly defined as the accuracy of parallelism increases. At this stage the plate \mathcal{Q} is moved so that all parts of the surface are examined in turn, and any residual errors of parallelism are at once detected by a contraction or expansion of the circles. The surface is locally corrected till these disappear. Under ordinary circumstances it is quite easy to distinguish a difference in phase of one-twentieth, which would correspond to a difference of retardation of the transmitted light of one-eightieth of a wave, so that there would be no serious difficulty in constructing an echelon of eighty or even a hundred elements.

In order to test the practical efficiency of the instrument, a somewhat extended investigation of the Zeeman effect was undertaken.

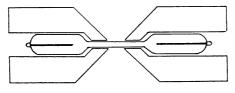


Fig. 6.

As has been already indicated in a previous paper,^r in order to observe the complete analysis of the radiations in a magnetic field, it is necessary that these should be as nearly homogeneous as possible, and this is the case only when the radiations take place under low pressure, so that the substance must be placed in a vacuum tube from which the air is exhausted to 5–10 mm residual pressure, and illuminated by the electric dis-

¹ Phil. Mag., September 1892.

charge. If the discharge takes place in a direction perpendicular to the field there is a lateral displacement of the discharge current, which interferes seriously with the result when the field is strong. In order to avoid this difficulty the pole pieces were arranged as in Fig. 6.

In the case of non-volatile substances the self-induction spark, between terminals of the substance to be examined, was used. The arrangement is shown in Fig. 7.

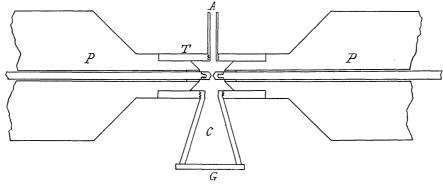


FIG. 7.

The pole-pieces are bored axially to receive the rods rr, to the ends of which the terminals are screwed. The brass tube T is fitted air-tight by melted beeswax to the pole-pieces, and communicates by the tube A with a mercury pump, and a glass plate G closes the conical tube C. One of the rods r is fitted with a screw so that the corresponding terminal may be adjusted midway between the pole-pieces. The other rod is given a rapid oscillatory motion by a small electric motor, the joint being closed by a rubber tube.

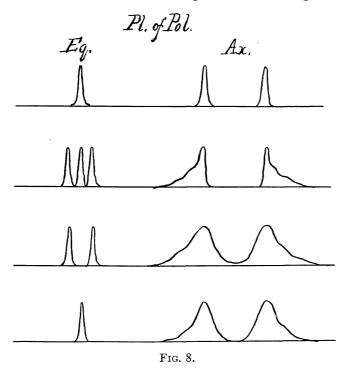
Under these circumstances a spark of considerable intensity is produced at every break of contact between the terminals with a storage battery of ten cells. The beeswax joints permit

¹ By making the rods and one of the terminals of iron, the field may be increased several times.

²This arrangement was only partially successful. The spark in the magnetic field was frequently too faint to observe, though occasionally the reverse was true.

an exhaustion to within a few millimeters of mercury, which may be maintained as long as necessary.

The observations completely confirm the experiments made by the method of visibility curves. In particular the distribution for the cadmium lines is as represented in Fig. 8.



In addition to the lines previously classified the following are added:

Gold, yellow line, class II.

Gold, green line, class I.

Silver, yellow line, class I.

Silver, green line, class I.

Copper, yellow line, class IV.

Copper, green lines, class I.

This JOURNAL, February 1898. The "outer lines" of the Zeeman triplet show indications of complex structure, but were actually resolved in the case of the green mercury line only; the difficulty arising from the overlapping of adjacent spectra when the field was sufficiently powerful to separate the elements of the group.

²Ibid.

Magnesium, green line (5183), class III.
Magnesium, green line (5172), class II.
Magnesium, green line (5167), class I.
Manganese, green (5340), class IV.
Argon, red line, class I.
Tin, red line (6450), class II.
Tin, yellow line (5798), class I.
Tin, yellow line (5587), class I.
Tin, yellow line (5564), class I.
Iron, yellow lines, class I.
Iron, blue lines, class I.

The central line of the iron triplets are all broadened, especially the blue lines.

A very remarkable effect is observed in the case of the yellow copper line. The line without the field is a close double, the distance being one-one hundred and fiftieth of the distance between the D lines, or 0.04 A. U. As the field increases the lines merge together without broadening, and with a strong field there is but a single very narrow line.

The behavior of the yellow-green line of manganese is even more striking. The line is a quadruple line, just resolvable. In a weak magnetic field the light accumulates at the center of the group, the lines becoming indistinct and merging together. In a strong field the quadruple band is reduced to a single fine line at the center of the group.

These two cases are the only ones of this character thus far observed, though doubtless a systematic search will reveal many others. They may be considered as a new type of line to be added to the three classes previously described.

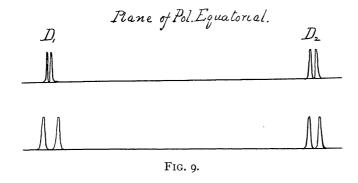
Another interesting case is that of sodium. Without the field each component is a close double, the distribution being that represented in Fig. 9a.

With a strong magnetic field and plane of polarization equatorial, the distribution is represented in Fig. 9b.

In α the elements of D_2 are about two-thirds as far apart as D_1 , while in b the reverse is the case.

¹ This had been pointed out before. See Phil. Mag., September 1892.

Finally, the distance between the components in a is a function of the density of the sodium vapor, increasing from zero for low densities to about one-sixtieth of D_1-D_2 for the greatest



density obtained without broadening, and finally obliterating the lines.

For the investigation of very close lines the present instrument has not quite sufficient resolving power. For instance, it has been shown that the green mercury line is a complex system whose principal components are less than 0.01 A. U. apart, while the theoretical limit of resolution of the echelon used is about twice as great.

A new echelon is now under construction which will have a resolving power about five times as great as the present instrument, and this will amply suffice for the analysis of the closes groups that are likely to occur.

¹ Phil. Mag., September 1892.